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Constraining the state space in any physical theory with the principle of information symmetry

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Symmetry shares an entwined history with the structure of physical theory. We propose a consequence of symmetry towards the axiomatic derivation of Hilbert space quantum theory. We introduce the notion of information symmetry (IS) and show that it constraints the state-space structure in any physical theory. To this end, we study the minimal error binary state discrimination problem in the framework of generalized probabilistic theories. A theory is said to satisfy IS if the probability of incorrectly identifying each of two randomly prepared states is the same for both the states. It is found that this simple principle rules out several classes of theories while being perfectly compatible with quantum theory.

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Introduction. Obtaining a physical perspective of the abstract mathematical description of quantum theory is a longstanding aspiration in quantum foundations. A variety of different approaches, some as old as the theory itself, have attempted to addressed this question, providing deeper understanding about the Hilbert space formulation of the theory [1–11]. The advent of quantum information theory introduces a new direction to this endeavor. It identifies physically motivated principles excluding a class of multipartite *nonlocal* correlations that are strong enough to be incompatible with quantum theory, though weak enough to satisfy relativistic causality or the no-signalling (NS) principle [12–21], thus providing a device-independent outlook about the correlations al lowed in the physical world [22,23]. Another approach is to identify rudimentary rule(s) that directly derive the state space structure or some crucial features of quantum theory [24–41].

Despite a number of nontrivial achievements, a complete physical or first-principles motivation of Hilbert space quantum mechanics is still elusive. In the present work, we consider a different approach to address this issue, by investigating the state space structure of physical theories from the perspective of *symmetry, as a principle*. Symmetry has played a long and widespread role in formulating theories of the physical world. Rather than being the byproduct of dynamical laws, symmetry principles have been appreciated as primary features of nature, that in turn, determine the fundamental physical laws [42,43]. For instance, while formulating the special theory of relativity, Einstein recognized relativistic invariance as a principle, which stipulates the form of transformation rules to be Lorentzian. Later, a similar approach guided him to develop his seminal theory of gravity where the principle of equivalence—a principle of local symmetry—determines the dynamics of space-time. In the context of the present work too, we take symmetry as the guiding feature, though the symmetry we explore here has different consequences. Rather than guiding directly the dynamics, it imposes constraints on the ways of information gain in the act of measurement, and consequently puts restrictions on the structure of state space.

In order to study the implications of the proposed symmetry, we consider a very generic mathematical framework that allows the largest possible class of convex operational theories, also called generalized probability theories (GPTs). The state space of such a theory is a convex set in $\mathbb{R}^n$ with extreme points [44] denoting pure states or states of maximal knowledge. This framework embraces the notion of indistinguishable states—members of a set of states that can not be identified perfectly given a single copy of the system prepared in one of these states. For a completely random ensemble of two such states, the most general strategy for minimum-error discrimination comprises of a two-outcome measurement—the two different outcomes correspond to two different preparations. While extracting information through such a binary measurement, error can occur in two ways: (i) outcome-1 that should correspond to state-1 may click even when the system is prepared in state-2, and (ii) outcome-2 may click when the system is prepared in state-1. Our proposed *information symmetry* (IS) assumes that for any randomly prepared binary ensemble of pure states, optimal information about the preparation is obtained symmetrically from both the states. In other words, the two possible sources of error contribute equally in minimal error state discrimination. Throughout the paper we consider that the pair of states are prepared with uniform probability distribution.

Through the analysis presented herein, we find that this seemingly naive symmetry condition is not satisfied by a large class of GPTs. In particular, we show that regular polygonal state spaces [45] with more than four pure states are incompatible with IS. Polygonal state spaces with four pure states, known by the name *squit*, also become incompatible with IS when it is applied to the binary ensembles of mixed states. This newly identified symmetry property turns out to be pivotal in determining the state space structure of physical theories as we find that both classical and quantum theory are perfectly compatible with IS. We begin our analysis with a brief discussion on the mathematical framework of GPTs.

Framework. The structure of any operational theory consists of three basic notions—state or preparation, observable or measurement, and transformation [24,26,30,31]. While observables correspond to the possible choices of measurement on the system, its initial preparation is represented by a state, and the time evolution of the state is governed by some transformation rule. In the *prepare and measure* scenario, the state and observable together yields the statistical prediction of an outcome event.

Preparation or state ω of a system specifies outcome probabilities for all measurements that can be performed on it. A complete specification of the state is achieved by listing the outcome probabilities for measurements belonging to a "fiducial set' [24,26]. The set Ω of all possible states is a compact and convex set embedded in the positive convex cone V_+ (see Ref. [44] for precise definition) of some real vector space V. Convexity of Ω assures that any statistical mixture of states is a valid state. The extremal points of the set Ω are called pure states. For example, state of a quantum system associated with Hilbert space $\mathcal{H}$ is described by positive semidefinite operator with unit trace, *i.e.*, a density operator $\rho \in \mathcal{D}(\mathcal{H})$, where $\mathcal{D}(\mathcal{H})$ denotes the set of density operators acting on $\mathcal{H}$. For the simplest two level quantum syste m (also called a qubit) $\mathcal{D}(\mathbb{C}^2)$ is isomorphic to a unit sphere in $\mathbb{R}^3$ centered at the origin, where points on the surface correspond to pure states.

An effect e is a linear functional on Ω that maps each state onto a probability $p(e|\omega)$ representing successful filter of the effect e on the state ω. Unit effect u is defined as, $p(u|\omega) = 1,\ \forall\, \omega \in \Omega$. The set of all linear functionals forms a convex set embedded in the cone V_+^* dual to the state cone V_+. The set of effects is occasionally denoted as $\Omega^* \subset V_+^*$. A d-outcome measurement M is specified by a collection of d effects, *i.e.*, $M \equiv \{e_j \mid \sum_j e_j = u\}$. For every effect e one can always construct a dichotomic measurement $M := \{e, \bar{e}\}$ such that $p(e|\omega) + p(\bar{e}|\omega) = 1,\ \forall\, \omega \in \Omega$; $\bar{e}$ is called the complementary effect of e. Likewise the states, effects can also be characterized as pure and mixed ones. Framework of GPTs may assume, a priori, that not all mathematically well-defined states are allowed physical states and not all mathematically well-defined observables are allowed physical operations. For example, the set of physically allowed effects $\mathcal{E}$ may be a strict subset of Ω^*. A theory is called 'dual' if it allows all elements of Ω^* as valid effects [46]. In this generic framework of probabilistic theory, one can define the notion of distinguishable states.

Definition 1. Members of a set of n states $\{\omega_i\}_{i=1}^n \subset \Omega$ are called distinguishable if they can be perfectly identified in a single shot measurement, i.e., if there exists an n-outcome measurement $M = \{e_j \mid \sum_{j=1}^n e_j = u\}$ such that $p(e_j|w_i) = \delta_{ij}$.

Not every set of states can be perfectly discriminated. However, a set of such indistinguishable states can be distinguished probabilistically allowing one to define the following state discrimination task. Suppose one of the states chosen randomly from the pair $\{\omega_1, \omega_2\} \subset \Omega$ is given. The aim is to optimally guess the correct state while one copy of the system is provided. Without loss of generality one can perform a two outcome measurement $M = \{e_1, e_2 \mid e_1 + e_2 = u\}$ and guess the state as ω_i while the effect e_i clicks. The error in guessing can occur in two ways—effect e_1 clicks when the given state is actually ω_2 which happen with probability $p_{12} := p(e_1|\omega_2)$, and with $p_{21} := p(e_2|\omega_1)$ probability effect e_2 clicks when the given state is actually ω_1. As the states are chosen with uniform probability, the total error is therefore $p_E = \frac{1}{2}(p_{12} + p_{21})$, and hence, the probability of successful guessing is $p_I = 1 - p_E$. The measurement that minimizes the error $p_E^{\min} := \min_M p_E$ is known as the Helstrom measurement, initially studied for quantum ensembles in 1970's [47–49] and more recently, also studied in the GPT framework [50–53]. While e_1 and e_2 used in the above discrimination task are mixed effects in general, however in the Helstrom measurement one of them is a pure effect.

Remark 1. For any pair of indistinguishable states in a GPT the measurement that optimally discriminates the states consists of a pure effect and its complementary effect.

In a GPT a pure state corresponds to the state of maximal knowledge. While in binary state-discrimination problem, a pair of such states are given randomly with uniform probability distribution, it seems that both states should contribute identically in the error probability of optimal guessing. This leads us to the following definition.

Definition 2. A GPT is said to satisfy information symmetry (IS) if $p_{12} = p_{21}$ in $p_E^{\min}$ for every pair of pure states allowed in that GPT. In orther words, for any pair of pure states, maximum information about the ensemble is obtained only if both states contribute symmetrically to this quantity.

Classical theory trivially satisfies IS as all the pure states are perfectly distinguishable. The classical state space with d number of perfectly distinguishable states is a $(d-1)$-simplex. In quantum mechanics, there however exists indistinguishable pure states. For a pair of such pure states, $\psi \equiv |\psi\rangle\langle\psi|,\ \phi \equiv |\phi\rangle\langle\phi| \in \mathcal{D}(\mathcal{H})$ the minimum error state discrimination (MESD) is obtained through Helstrom measurement [47–49]. While ψ and ϕ are prepared randomly with equal probability, the measurement $M \equiv \{E_\psi,\ E_\phi \mid E_\psi,\ E_\phi \in \mathcal{L}^+(\mathcal{H})\ s.t.\ E_\psi + E_\phi = \mathbb{I}\}$ achieving MESD is the one consisting of projectors onto the basis that "straddles" ψ and ϕ in Hilbert space, and we have $p_E^{\min} = \frac{1}{2}(1 - \sqrt{1 - |\langle\psi|\phi\rangle|^2})$ [44]; $\mathcal{L}^+(\mathcal{H})$ i s the set of positive operators on $\mathcal{H}$. Although IS holds true in classical and quantum theory, we now show that the class of GPTs with regular polygonal state spaces are not compatible with it.

Regular polygonal state spaces. An associated toy theory for bipartite systems was first proposed to demonstrate the possibility of no-signaling theories which can have nonlocal behavior similar to quantum mechanics [54]. This entails the need to exclude such theories by providing new *physical* principles. In fact, several successful attempts have been made to exclude stronger than quantum nonlocal correlations [12–21]. Here we take a different approach. We aim to exclude a large class of such theories by invoking principle(s) that consider only the elementary system, i.e., single partite system.

For an elementary system the state space Ω_n is a regular polygon with n vertices [45,55–60]. For a fixed n, Ω_n is the convex hull of n pure states $\{\omega_i\}_{i=0}^{n-1}$ with $\omega_i := (r_n \cos(\frac{2\pi i}{n}), r_n \sin(\frac{2\pi i}{n}), 1)^T \in \mathbb{R}^3$; where T denotes

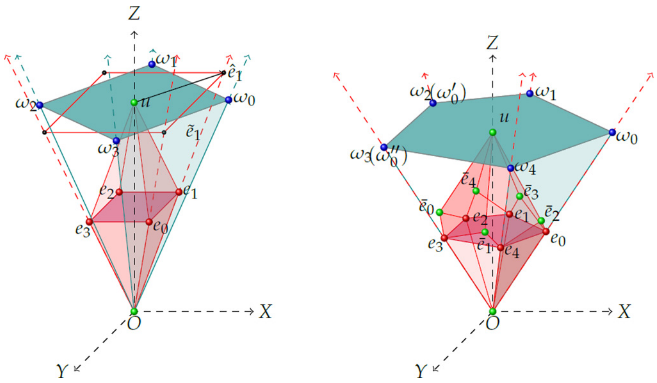

FIG. 1. State and effect spaces of *squit* (left) and *pentagon* (right) models. Blue dots are the extremal states and red dots denote the ray extremal effects. Green dots denote extremal effects that are not ray extremal. In the squit model the effect e_1 is scaled up to $\tilde{e}_1$ so that its tip (black dot) lies on the normalized states space (green surface) and it can be represented as $\tilde{e}_1 = u + \hat{e}_1$. In the pentagon model, the state ω_0 and the states $\omega_0^{(\eta)} := \eta\omega_0' + (1-\eta)\omega_0'' \equiv \eta\omega_2 + (1-\eta)\omega_3$ are perfectly distinguishable by the dichotomic measurement $M \equiv\equiv \{e_0, \bar{e}_0\}$, for all $\eta \in [0, 1]$.

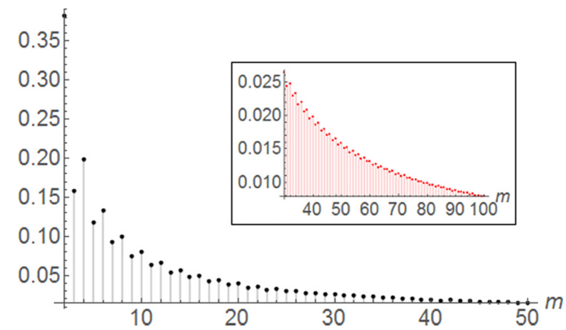

FIG. 2. Absolute difference between p and $\bar{p}$ is plotted against $m \geqslant 2, m \in \mathbb{Z}$, where $\Omega_{(2m+1)}$ is the corresponding odd-gon state space. Inset depicts magnified plot for higher values of m.

transpose and $r_n := \sqrt{\sec(\pi/n)}$. The unit effect is given by $u := (0, 0, 1)^T$. The set $\mathcal{E}$ of all possible measurement effects consists of convex hull of zero effect, unit effect, and the extremal effects $\{e_i, \bar{e}_i\}_{i=0}^{n-1}$, where $e_i := \frac{1}{2}(r_n \cos(\frac{(2i-1)\pi}{n}), r_n \sin(\frac{(2i-1)\pi}{n}), 1)^T$ for even n and $e_i := \frac{1}{1+{r_n}^2}(r_n \cos(\frac{2\pi i}{n}), r_n \sin(\frac{2\pi i}{n}), 1)^T$ for odd n.

The pure effects $\{e_i\}_{i=0}^{n-1}$ correspond to exposed rays and consequently the extreme rays of V_+^* [44,61]. For odd-gonal cases, due to self-duality of state cone V_+ and its effect cone V_+^* [62] every pure effect e_i has one to one ray-correspondence to the pure state ω_i. Consequently, for every pure state ω_i there exist exactly two other pure states ω_i' and ω_i'' such that ω_i and $\bar{\omega}_i^{(\eta)} := \eta\omega_i' + (1-\eta)\omega_i''$ are always perfectly distinguishable for all $\eta \in [0, 1]$ (see Fig. 1). The discriminating measurement consists of the effects $\{e_i, \bar{e}_i\}$ such that $p(e_i|\omega_i) = 1$ and $p(e_i|\bar{\omega}_i^{(\eta)}) = 0$. The effects $\{\bar{e}_i\}_{i=0}^{n-1}$ are extremal elements of $\mathcal{E}$ but they are not ray extremal, i.e., they do not lie on an extremal ray of the cone V_+^* [63]. For an even-gon, the scenario is quite different as the self duality between V_+ and V_+^* is absent. Here, all the e_i's and their complementary effects $\bar{e}_i$'s correspond to extreme rays of V_+^*.

Every ray-extremal effect e generates an extreme ray λe for the cone V_+^*, where $\lambda \geqslant 0$. With proper choice of λ any such e can be scaled up to a new $\tilde{e} \equiv \lambda e$, such that the tip of this scaled effect vector $\tilde{e}$ lies on the normalized state plane. Let us consider a particular direct sum decomposition of the space $\mathbb{R}^3$, i.e., $\mathbb{R}^3 = \mathbb{R}u \oplus \hat{V}$, where $\hat{V}$ is a two-dimensional subspace of $\mathbb{R}^3$ parallel to the X-Y plane. This allows a particular representation of $\tilde{e}$ in the following way $\tilde{e} = u + \hat{e}$, where $\hat{e} \in \hat{V}$. Similarly, every $\omega \in \Omega$ has a representation $\omega = u + \hat{w}$, with $\hat{w} \in \hat{V}$ (see Fig. 1). In this representation, the outcome probability of the effect e on the state ω reads $p(e|w) = \lambda p(\tilde{e}|\omega) = \lambda(u + \hat{e}).(u + \hat{\omega}) = \lambda(1 + \hat{e}.\hat{\omega})$ [64], where dot represents euclidean inner-product in $\mathbb{R}^n$. Set of the vectors $\hat{\omega}$ corresponding to the states $\omega \in \Omega$ forms a convex-compact set $\hat{W}_s \subset \hat{V}$. For the n-gonal case, norm of these vectors satisfy the bound $||\hat{\omega}||_2 \leqslant r_n$ with exactly n vectors saturating the bound. Similarly the vectors $\hat{e}$ forms another convex-compact set $\hat{W}_e \subset \hat{V}$ and $||\hat{e}||_2 \leqslant r_n$ with exactly n vectors saturating the bound. Self-duality for the odd-gonal cases imply $\hat{W}_s = \hat{W}_e$ which is not the case for even n.

Theorem 1. GPTs with state space Ω_n with $n > 3$ (for odd n) and with $n > 4$ (for even n) are not compatible with IS.

Proof. Let us first consider an n-gonal state space with odd n and $n > 3$. Without loss of generality, consider the two neighboring states $\omega_0, \omega_1 \in \Omega_n$. According to Remark 1 the measurement that optimally discriminate these states consists of one of the effects corresponding to the vectors $\hat{e}_{n-k} \in \hat{W}_e$ such that $||\hat{e}_{n-k}||_2 = r_n$, with $k \in \{0, \cdots, n-1\}$ and its complementary effects. With such a measurement the error reads as

$$p_E = \frac{1}{2}\left[1 + \frac{1}{1+r_n^2}\hat{e}_{n-k}.(\hat{\omega}_0 - \hat{\omega}_1)\right]. \qquad (1)$$

Let us denote the angle between $\hat{e}_{n-k}$ and $(\hat{\omega}_0 - \hat{\omega}_1)$ as θ_k. It is evident from (1) that for minimal error k should be chosen in a way that $|\theta_k - \pi| \to 0$. However, the self-duality of odd-gonal theory demands that $\theta_k = \frac{\pi}{2} + (2k+1)\frac{\pi}{n}$. Then, a straightforward calculation shows that minimal error discrimination is achieved for $k = [\frac{n}{4}]$. For this optimal measurement, probability of clicking e_{n-k} when the input state is ω_0 is given by $p = \frac{1}{1+r_n^2}[1 + r_n^2 \cos\{\frac{2\pi}{n}(k+1)\}]$ and probability of clicking $\bar{e}_{n-k}$ on ω_1 is given by $\bar{p} = \frac{r_n^2}{1+r_n^2}[1 - \cos(\frac{2\pi}{n}k)]$. An elementary trigonometric argument ensures that the probabilities p and $\bar{p}$ are not same for any Ω_n, with odd n and $n \geqslant 3$. The absolute difference of these two probabilities, however, decreases with increasing n (see Fig. 2).

The proof for the even-gonal case is similar to the odd-gonal case. We provide the detailed proof in Ref. [44]. ■

We have shown that all polygonal state spaces Ω_n, with $n \geqslant 5$ are incompatible with IS. Now the question arises as to what happens for $n = 4$ which corresponds to the marginal state space of the most general two-input-two output bipartite NS correlations. This state apace is also known by the name *squit* whose center corresponds to the marginal state of the famous Popescu-Rohrlich correlation [54]. Here we have the following observation about squit state space.

Observation 1. Any pair of pure states in squit can be discriminated perfectly.

It is possible to generalize IS that applies to the ensemble of mixed states. A GPT is said to be compatible with generalized information symmetry (GIS) if every pair of states each having identical *minimal* type subjective ignorance can be optimally discriminated with symmetric error measurement. While a pure state is the state of maximal knowledge, *i.e.*, contains no subjective ignorance, a state ω is said to have *minimal* type subjective ignorance if it allows a convex decomposition in terms of two distinguishable pure states, *i.e.*, $\omega = p\omega_i + (1-p)\omega_j$ for some perfectly distinguishable pair of pure states ω_i and ω_j. Two such states $\omega = p\omega_l + (1-p)\omega_j$ and $\omega' = q\omega_k + (1-q)\omega_l$ are said to have identical subjective ignorance when $p = q$. It turns out that *squit* state space does not satisfy GIS while quantum theory is perfectly compatible with GIS [44].

Discussions. The newly identified symmetric primitive, namely, the information symmetry, has important implications in the axiomatic derivation of Hilbert space quantum mechanics as it puts nontrivial restrictions on the state space structure of generalized probabilistic models. While the state space of quantum theory is perfectly compatible with IS, we find that the polygonal state spaces do not satisfy this elementary symmetry condition, or its generalized version.

In this context, it is worth mentioning a couple of other features of the structure of GPTs, which though interesting, are not powerful enough to exclude various categories of models while allowing for quantum and classical mechanics in the manner of IS. First, the notion of logical bit-symmetry [33] imparts self-duality on the state space leading to the exclusion of even-gonal state spaces only, but not the odd-gonal ones [45]. Secondly, polygonal state spaces lack well defined purification for all states [65]. However, the state space of the classical bit also lacks this particular property, whereas it satisfies IS. On the other extreme, the "toy bit" model of Spekkens [66] does not satisfy IS though it may allow well defined purification [44].

To summarize, IS imparts a remarkable restriction on the state space structure, excluding all regular polygonal state spaces as well as the Spekkens model, thus representing a more stringent structural constraint compared to self-duality. Moreover, unlike bit-symmetry, IS assumes no constraint on the dynamics of the theory. Before concluding, note that though it can be shown that the state space of the bipartite NS box with a Bell measurement is equivalent to a Bloch ball [39], the formulation of IS is more general and does not involve any structure from composite systems. Finally, it may be interesting to explore implications of IS on other state space structures as well as generalizations of IS for ensembles prepared with bias.

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[63] In quantum mechanics, this happens when dimension d of the associated Hilbert space $\mathcal{H}_d$ is larger than two. For every pure state $\psi \equiv |\psi\rangle\langle\psi|$ the effect $(\mathbb{I} - \psi)$ is extremal in the set of proper effects but not a ray extremal effect; $|\psi\rangle \in \mathcal{H}_d$, and $\mathbb{I}$ is the identity operator on $\mathcal{H}_d$.
[64] This expression has close similarity with the corresponding qubit expression. If a two outcome positive-operator-valued measurement $M \equiv \{E := \frac{\mu_1}{2}(\mathbb{I} + \mu_2 \vec{a}.\vec{\sigma}), \mathbb{I} - E | 0 \leqslant \mu_1 \leqslant 2, 0 \leqslant \mu_2 \leqslant \min\{\mu_1, 2 - \mu_1\}\}$ is performed on a qubit state $\rho = \frac{1}{2}(\mathbb{I} + \vec{n}.\vec{\sigma}) \in \mathcal{D}(\mathbb{C}^2)$, probability of clicking the effect E is given by $p(E|\rho) = \mathrm{Tr}(\rho E) = \frac{1}{2}(\mu_1 + \mu_2 \vec{a}.\vec{n})$; here $\vec{a}.\vec{\sigma} \equiv a_x\sigma_x + a_y\sigma_y + a_z\sigma_z$; $\vec{a}, \vec{n} \in \mathbb{R}^3$ with $|\vec{n}| \leqslant 1$ and σ's are the Pauli operators.
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