Probabilistic Assessment of Transport Network Vulnerability with Equilibrium Flows

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Abstract

This paper develops a probabilistic approach for assessing transport network vulnerability. A novel performance measure is proposed to evaluate the expected impact when multiple transport network components fail simultaneously at various degrees. The proposed measure captures both the likelihood and consequence of a combination of transport network component failures. The most critical combination of transport network component failures is obtained by solving a bi-level optimization problem. The upper-level problem is to solve for the combination of transport network components together with their corresponding disruption levels, which induces the maximum reduction in the performance measure. The lower-level problem is to capture the response of travelers to network changes due to network component failures and is formulated as a traffic assignment problem. The clonal selection algorithm (CSA), a biologically inspired approach, is adopted to tackle the proposed bi-level optimization problem. Numerical results indicate that neglecting partial capacity degradation and its probability of occurrence could misestimate the worst scenario, and different vulnerability assessment approaches could identify similar critical components but our approach can discover some components that are not found by other existing approaches. Moreover, it is shown that the CSA outperforms the well-known genetic algorithm (GA) in terms of solution quality in a large network.

Keywords: Transport Network Vulnerability; Bi-level Optimization; Clonal Selection Algorithm; Genetic Algorithm

1 Introduction

The prosperity of a society is closely related to the performance of the supporting transportation infrastructure, which is a long-term investment with significant impacts on sustainability (Fiksel, 2006). Nevertheless, transportation infrastructure is vulnerable to various kinds of disruptions (Chow et al., 2015), which induce direct economic loss and impede sustainable urban development. Disrupted transportation infrastructure requires a significant cost for repairs or reconstruction to rehabilitate it to its designed capacity to ensure an adequate level of service. In Europe, the transportation network was severely disrupted by extreme winter weather in January 2013. The UK Department for Transport (DfT, 2011) highlights that the severe winter weather experienced by the United Kingdom caused extensive disruptions to transport networks and brought travel misery to millions of people. The total cost of delayed journeys of both businesses and individuals was estimated to be around £280 million per day in England alone. We also cannot forget the devastating losses due to a major earthquake in the Sichuan region, China in 2008, and the losses brought upon by two major earthquakes in 2011 in Japan and New Zealand. In particular, the 9.0-magnitude tsunami that struck off the Tohoku region, Japan in March 2011 caused 15,800 deaths, with 6,100 injured, 2,600 missings, and a $170 billion monetary loss (PwC, 2013). In addition to these extreme disasters, floods, landslides, and adverse weather conditions deteriorate free-flow speed and road capacity (Lam et al., 2013; Tsapakis et al., 2013; Pregnolato et al., 2017). It is expected that, in the era of climate change, the vulnerability of infrastructures tends to increase (Nagurney et al., 2010). Thus, vulnerability analysis that can assist the decision-maker in identifying vulnerable infrastructures and preparing for unpredictable disruptions and natural disasters needs more attention.

A vital issue in the vulnerability analysis of a transportation network is to identify the most critical network component(s) (Wang et al., 2016). A critical component is a link or a node, the failure of which would bring the most severe deterioration to the system (Chen et al., 2012). Once critical components are identified, the robustness of the overall network can then be enhanced by reinforcing these elements subject to budgetary constraints. A number of studies have contributed to the quantification of the criticality of transport network components in terms of transport network performance. For a thorough literature review, interested readers are welcome to consult the studies of Nagurney and Qiang (2009), Sullivan et al. (2009), Wang et al. (2014), Zhao et al. (2014), Mattsson and Jenelius (2015), and Gu et al. (2020). In this study, the representative works of the state-of-the-art approaches are listed in chronological order in Table 1 (a), which focuses on four aspects, namely, type of measure, degree of closure, number of disrupted components, and the traffic assignment models/principles used once the network is disrupted[1].

One of the first studies that explicitly mention vulnerability is by Nicholson and Du (1997). They suggested that vulnerability analysis is governed by the equilibrium of demand and supply and network vulnerability is evaluated by the system surplus. For the demand side, a given demand function was adopted, while for the supply side, the user-equilibrium (UE) traffic assignment problem was solved. Murray-Tuite and Mahmassani (2004) proposed a disruption index to measure the importance of a link to a network. The disruption index is obtained by aggregating the vulnerability indices that measure the importance of a specific link to the connectivity of an origin-destination (OD) pair considering the availability of alternative paths, excess capacity, and travel time. They formulated a bilevel optimization model where the upper-level problem is to determine one or multiple links that maximize the disruption index and the lower-level problem is a system optimal (SO) (instead of UE) traffic assignment

[1] If the vulnerability calculation involves a benchmark scenario (i.e., undisrupted network), sometimes two assignment scenarios can be involved, one for undisrupted and one for disrupted.

problem, assuming that a traffic management agency provides route guidance and travelers comply with the guidance. Both Jenelius et al. (2006) and Scott (2006), in line with Nicholson and Du (1997), embraced a UE assignment problem in their vulnerability analyses. Jenelius et al. (2006) used both weighted origin-destination travel cost and unsatisfied demand to define performance indices and evaluate the importance of a link. Meanwhile, Scott (2006) established a Network Robustness Index (NRI) that uses the increase in the total system travel time/cost of the removal of a link to measure its criticality. Since then, the NRI has been widely adopted. Sullivan et al. (2010) applied such an index for evaluating the impacts of partial capacity reductions, meaning that capacity drops to given levels, such as 99%, 95%, etc. Knoop et al. (2012) and Zhou and Wang (2018) applied the index to investigate the network-wide effect of short-term capacity variations. Rupi et al. (2014) extended the measure by using the weighted sum of the total cost and the average daily traffic.

Nevertheless, the above time/cost-based indices suffer a shortcoming that they are not methodologically sound to address the problem of disconnected networks (e.g., isolating links). Qiang and Nagurney (2008) overcame this technical disadvantage and developed the Unified Network Performance Measure (UNPM), which can be interpreted as a demand-weighted generalization of the measure for evaluating the topological network efficiency introduced by Latora and Marchiori (2001) (named as the L-M measure herein). Their unified measure can be applied to assess the importance of either links or nodes or both and is applicable to both fixed and elastic demand network equilibrium problems. A number of applications to real-world problems using this measure can be found in the studies of Nagurney and Qiang (2009) and Nagurney and Qiang (2012). Similar to Qiang and Nagurney (2008), Chen et al. (2012) were also motivated from the L-M measure and devised a network efficiency index analyzing the vulnerability of networks with demand uncertainty, wherein UE was extended to reliability-based user equilibrium (RUE) to capture the effect of uncertainty on travelers' route choice behavior. Dehghani et al. (2014) extended the network efficiency index by considering the probability of each failure scenario, but the equilibrated travel behavior has not been taken into account. Jansuwan and Chen (2015) applied the network efficiency index to examine the effect of travelers' perception in determining the importance of network components. It is worth noting that both the UNPM and network efficiency index reduce to the L-M measure under certain conditions.

Other than the prevailing NRI and UNPM measures, more measures have been introduced. Taylor et al. (2006) measured the network degradation based on accessibility and adopted three standard accessibility measures, including generalized travel cost, the Hansen integral accessibility index, and the ARIA index. On the other hand, Chen et al. (2007) proposed a utility-based index for measuring network connectivity and Bell et al. (2017) identified the potential flow bottlenecks of transport networks by finding cuts with the least capacity without knowing demand information.

Table 1 Studies of road transport network vulnerabilities

(a) Chronological summary of the vulnerability studies

	References	**Measures**	**Degree of closure***	**No. of disrupted components**	**Traffic assignment models/principles used**
1	Nicholson and Du (1997)	System surplus/net utility	Partial	Single	User equilibrium (UE)
2	Murray-Tuite and Mahmassani (2004)	Disruption index	Full	Multiple	System optimal (SO)
3	Jenelius et al. (2006)	Weighted origin-destination travel cost; unsatisfied demand	Full	Multiple	UE
4	Scott et al. (2006)	Network Robustness Index (NRI)	Full	Single	UE
5	Taylor et al. (2006)	Accessibility index	Full	Single	Shortest path
6	Chen et al. (2007)	Utility-based index	Full	Multiple	UE

7	Nagurney and Qiang (2007a, b, c),	Unified Network Performance Measure (UNPM)	Full	Single	UE
8	Qiang and Nagurney (2008)	UNPM	Full	Single	UE
9	Sullivan et al. (2010)	NRI	Partial	Single	UE
10	Nagurney and Qiang (2012)	UNPM	Both	Single	UE; SO
11	Chen et al. (2012)	Network efficiency index	Full	Single	Reliability-based UE
12	Knoop et al. (2012)	NRI	Full	Single	Dynamic UE/Non-equilibrium**
13	Rupi et al. (2014)	Weighted sum of NRI and average daily traffic	Full	Single	UE
14	Dehghani et al. (2014)	Network efficiency index	Full	Multiple	Shortest path
15	Jansuwan and Chen (2015)	Network efficiency index	Full	Single	SUE
16	Wang et al. (2016)	Total travel cost	Full	Multiple	UE
17	Bagloee et al. (2017)	Total travel cost	Full	Multiple	UE; SO
18	Bell et al. (2017)	The second smallest eigenvalue of the graph Laplacian	Full	Multiple	No traffic assignment model
19	Xu et al. (2018)	The remaining network throughput after disruptions	Full	Multiple	Shortest path
20	Zhou and Wang (2018)	NRI	Partial	Single	Non-equilibrium

* Full closure refers to either the removal of the network component or setting the component's capacity to be extremely low. Partial closure refers to that a component's capacity is characterized by different reduced levels

**A dynamic equilibrium traffic assignment model is used for non-incident situations. For a disrupted network, a dynamic non-equilibrium traffic simulator is used.

(b) Classification

Categories		References
Topology-based	Accessibility index	5
	Capacity-based index	18, 19
System-based	NRI	4, 9, 12, 13, 20
	Network efficiency index	11, 14, 15
	Total travel cost	3, 16, 17
	UNPM	7, 8, 10
	Utility-based index	1, 6
	Disruption index	2

Table 1 (b) further classifies the vulnerability analysis into two categories, namely, topology-based and system-based, according to the reviews of Mattsson and Jenelius (2015) and Gu et al. (2020). The topology-based analysis relies upon the graph theory such as shortest path (e.g., Taylor et al., 2006; Xu et al., 2018) and maximum flow (e.g., Bell et al., 2017) theories to calculate the index, while the system-based analysis usually requires a traffic assignment model (e.g., UE, SUE or DUE model, see the last column in Table 1 (a)) to capture the travelers' response to the network disruption to calculate the index. Thus, the system-based analysis is more suitable to be applied in a congested network as the impact of travel flow on transportation supply is considered (Gu et al., 2020).

As indicated by Table 1, most studies only looked at a single component failure in order to obtain their criticality ranking. As to the studies that considered multiple component failures, they did not consider the possibility of partial closure. Therefore, this study aims at addressing this research gap by developing an approach for identifying the most critical combination of network component disruptions, where various degradations of the network component's capacity are considered. To identify the critical components, most of the existing studies use the brute-force simulation-based approach (e.g., Taylor et al., 2006; Scott et al., 2006; Taylor, 2007; Chen et al., 2007; Qiang and Nagurney, 2008; Dehghani et al., 2014, etc.) or the optimization approach (e.g., Wang et al., 2016; Bagloee et al., 2017; Xu et al., 2018). In the brute-force approach, each link or node or a combination of links and nodes is iteratively removed, and the corresponding consequence is estimated based on the measure adopted. However, the number of possible combinations can be substantial, so that it becomes time-consuming to enumerate all possible combinations and to assess their outcomes for a real network. Meanwhile, the brute force method of removing links or nodes implicitly assumes that a network component completely fails after the degradation. However, when a disaster occurs, the affected network components may still be operational, but at a degraded capacity level. Thus, the removal method can be best suited for analyzing extreme destruction events; otherwise, it can overestimate the damage of some components. For the optimization approach, an integer programming model is formulated to determine whether a network component is critical or not. Existing optimization studies focus on either devising a formulation that can be solved by an off-the-shelf solver (e.g., Xu et al., 2018), or developing exact solution methods, such as Bender's decomposition (e.g., Bagloee et al., 2017) and linearization (e.g., Wang et al., 2016). Nevertheless, the preceding optimization models overlook the stochastic occurrence of disruption and partial closure, which will induce additional complexity that restrains the application of their methodologies to a large network.

To address the above issues, this paper develops a probabilistic approach for identifying the most critical combination of network component disruptions and analyzing network vulnerability under equilibrium traffic flows. Instead of measuring the changes due to the removal of one component, the study postulates a more realistic assumption that multiple network components could fail simultaneously, and both the degradation degrees and the occurrence of the failures are stochastic. The network vulnerability is measured by the maximum expected impact, which extends the unified network performance measure proposed by Qiang and Nagurney (2008) by incorporating the likelihood of the occurrence of the failures. To obtain this value and to determine its corresponding vulnerable infrastructure, a bi-level optimization model is developed. The upper-level problem is to identify the most critical combinations of network components, while the lower-level problem is the user equilibrium traffic assignment problem that captures travelers' responses to disruptions. This traffic assignment model is considered appropriate for the case of prolonged roadblocks due to natural disasters, because of the availability of the current advanced traveler information system and the scope for traffic to adjust and move towards a new equilibrium state (Chen et al., 2007).

It is well-known that bi-level programs are inherently non-convex and challenging to solve (Meng et al., 2001; Meng and Yang, 2002; Ban et al., 2006). Concerning the solution method, metaheuristics are gaining popularity in handling bi-level transportation optimization problems, owing to their insensitivity to the mathematical property of the problems. A number of metaheuristics or their hybrids have been extensively applied for tackling bi-level transportation optimization problems, including the Genetic Algorithm (GA) (e.g., Unnikrishnan and Lin, 2012), Ant Colony Optimization (e.g., Vitins and Axhausen, 2009), Chemical Reaction Optimization (e.g., Szeto et al., 2014), and Artificial Bee Colony (e.g., Jiang et al., 2013; Szeto and Jiang, 2012, 2014). This study adopts an evolutionary algorithm named the Clonal Selection Algorithm (CSA) to solve the proposed bi-level optimization problem. The CSA is inspired by Burnet's clonal selection theory, which exploits the diversity and learning properties of the acquired immune system of vertebrates (Brownlee, 2007). It has been reported that the algorithm is capable of solving several benchmark problems in machine learning and optimization (Castro and Zuben, 2000, 2002) and the algorithm performs better than other heuristics such as GA in some cases (Ulutas and Kulturel-Konak, 2011). In the field of transportation research, to the best of our knowledge, only Miandoabchi et al. (2012a, 2012b, 2013) applied the algorithm to solve their studied bi-level transportation network design problems, which motivates us to investigate its capability for solving the proposed bi-level optimization problem.

To sum up, the main contributions of this paper include the following:

1. Proposing a performance measure to assess network vulnerability, in which the consequence and likelihood of multiple network component failures are considered simultaneously. Compared with existing measures, the innovation of the proposed measure is that it considers both the concurrent failures of a set of network components and the probability of such a scenario as well as the various degrees of the components' disruptions. This is more realistic for non-extreme disruptions compared with the consideration in existing studies that a set of network components completely fails after the degradation;
2. Developing a bi-level optimization model to identify the most critical combination of network component disruptions based on the proposed measure, while travelers' response to the network failure is captured via the lower-level traffic assignment model;
3. Proposing the Clonal Selection Algorithm to solve the bi-level optimization model and demonstrating the performance of this algorithm. To our best knowledge, the performance of such an algorithm has not been examined in the network vulnerability literature.

This remainder of this paper is organized as follows: Section 2 presents the model formulation. Section 3 introduces the CSA solution approach. Numerical examples are presented and discussed in Section 4. Finally, Section 5 provides concluding remarks and future extensions.

2 Formulation

We consider a general network $G(V, A)$, where V and A, respectively, denote the set of nodes and the set of links. Based on this notation, we present the lower-level traffic assignment model followed by the upper-level formulation. Further notations are explained when used.

2.1 Lower-Level Problem

In this study, the transportation network under consideration is subject to a range of uncertain failure scenarios and is used by a predefined set of OD demands under the user equilibrium condition (Wardrop, 1952), which is formulated as follows:

$$x_p^w \begin{cases} > 0 \Rightarrow t_p^w = \pi^w \\ = 0 \Rightarrow t_p^w \geq \pi^w \end{cases}, \forall p \in \mathcal{P}^w, w \in \mathcal{W}, \tag{1}$$

where x_p^w is the flow of traffic assigned to path p between OD pair w, $\mathcal{P}^w$ is the set of routes connecting OD pair w, $\mathcal{W}$ is the set of OD pairs, t_p^w is the travel time on route p between OD pair w, and π^w is the minimum travel time between OD pair w. The equilibrium flows are subject to the following flow conservation and non-negativity constraints:

$$\sum_{p \in \mathcal{P}^w} x_p^w = d^w, \ \forall w \in \mathcal{W} \text{ and} \tag{2}$$

$$x_p^w \geq 0, \forall w \in \mathcal{W}, p \in \mathcal{P}^w, \tag{3}$$

where d^w is the travel demand associated with OD pair w. Beckmann et al. (1956) formulated the UE traffic assignment problem as the following minimization problem.

$$\min_{\mathbf{v}} \sum_{a \in A} \int_0^{v_a} c_a(v, C_a) dv \tag{4}$$

subject to Eqs. (2) and (3), and

$$v_a = \sum_{w \in \mathcal{W}} \sum_{p \in \mathcal{P}^w} x_p^w \delta_p^a, \forall a \in A, \tag{5}$$

where v_a represents the traffic flow on link a and $\mathbf{v} = (v_a)_{a \in A}$; C_a is the capacity of link a, which is subject to the disruptions determined by the upper-level problem. Given v_a and C_a, $c_a(v_a, C_a)$ computes the travel time on link a. Following Lo and Tung (2003), we adopt the commonly used Bureau of Public Roads (BPR) link performance function in this study, which is

$$c_a(v_a, C_a) = t_{a,0}\left(1 + 0.15\left(\frac{v_a}{C_a}\right)^4\right), \forall a \in A, \tag{6}$$

where $t_{a,0}$ denotes the free-flow travel time of link *a*. δ_p^a is a binary indicator defined as

$$\delta_p^a = \begin{cases} 1 & \text{if link } a \text{ is on route } p, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

It is worth noting that the objective function is formulated in terms of link flow, while the constraints are formulated in terms of route flow. Hence, the definitional constraint (5) is necessary to interrelate the link and the route flows. Beckmann et al. (1956) showed that solving this mathematical programming formulation is equivalent to solving the user equilibrium traffic assignment problem. The equivalency can be established by verifying that the Karush-Kuhn-Tucker (KKT) necessary conditions for a minimum point of the problem (see Sheffi 1985, pp. 63 – 66) are exactly the conditions of user equilibrium. The sufficient conditions for the uniqueness of user equilibrium are as follows: the feasible region is convex and the objective function is strictly convex in the vicinity of the optimal link flow vector $\mathbf{v}^*$ (and convex elsewhere). The convexity of the feasible region is guaranteed by the linearity of the constraints, while the convexity of the objective function is assured as long as the link cost function is strictly convex. One should note that the above analysis is only confined to link flow, and indeed the equilibrium solution is not unique with respect to route flow (Sheffi, 1985). Many efficient solution algorithms are later developed and used to solve Beckmann et al.'s (1956) mathematical programming formulation and its extensions effectively. Examples of the algorithms can be found in the studies of Bar-Gera (2002) and Perederieieva et al. (2015). Nevertheless, it is worth noting that the above formulation only applies to a single modal traffic equilibrium where the link cost functions are separable (Nagurney, 1984).

2.2 Upper-Level Problem

Given the equilibrium flows, the objective of this study is to identify the most critical combination of network component failures that induces the most severe disruption to a network. The severity of the disruption to a network is measured by a novel impact value developed based on the Unified Network Performance Measure (UNPM, Qiang and Nagurney 2008). To start with, we briefly recall the UNPM, which is defined by

$$\varepsilon\left(\mathbf{v}^*\right)=\frac{1}{|\mathcal{W}|}\sum_{w\in\mathcal{W}}\frac{d^w}{\pi^w\left(\mathbf{v}^*\right)}, \tag{8}$$

in which $|\mathcal{W}|$ is the cardinality of the set of OD pairs $\mathcal{W}$ (i.e., the number of OD pairs in the network) and π^w is the equilibrium travel time between OD pair *w*. The above equation states that the UNPM value depends on the equilibrated link flow vector $\mathbf{v}^*$ obtained by solving the lower-level traffic assignment problem.

To elaborate our innovation, which consists of adopting a probabilistic approach to incorporating the likelihood together with the consequence of disruptions, we first define that the components whose capacity could degrade are vulnerable. In a transport network, both nodes and links can be degraded. Nevertheless, as stated by Nicholson and Du (1997), any node can be treated as a collection of nodes with connecting arcs and, thus, node degradations can be treated as link degradations. Therefore, without loss of generality, the vulnerable components in a transport network refer to the links of the network in this study. For vulnerable component *a*, let integer l be its capacity reduction level between $[0,\omega_a]$, where ω_a is the total number of non-zero capacity reduction levels. The degradation of all vulnerable components can be described using a joint distribution where the marginal distribution of each component can be depicted by the following two equations:

$$\Psi_a=\left\{\psi_a^l \mid 0\le\psi_a^l\le 1, l=0,1,\ldots,\omega_a\right\}, \forall a\in A \text{ and} \tag{9}$$

$$\Phi_a=\left\{\phi_a^l \mid 0\le\phi_a^l\le 1, \sum_{l'=0}^{\omega_a}\phi_a^{l'}=1, l=0,1,\ldots,\omega_a\right\}, \forall a\in A. \tag{10}$$

In Eq. (9), ψ_a^l denotes the proportion of capacity degradation with respect to the original capacity of component a when the reduction level is l. To keep the notation simple, we define $\psi_a^0 = 0$ and $\psi_a^{\omega_a} = 1$ for the normal and total failure scenarios of component a, respectively. Eq. (10) expresses the requirement of the probability mass function where ϕ_a^l denotes the probability of capacity degradation of component a at level l.

The decision variable vector of the upper-level optimization problem is $\{ y_a, \forall a \in A \}$ where y_a is the capacity reduction level of component a. After a disruption, the probability that y_a is equal to a certain realized capacity reduction level is given by

$$\Pr\left(y_a = l_a\right) = \phi_a^{l_a}, \tag{11}$$

where l_a denotes the realized capacity reduction level of component a.

When a disruption occurs, one or more components' capacities can drop below their design capacities simultaneously and we define this as a failure scenario. One failure scenario corresponds to a set of realized network components' capacity reduction levels in this scenario. Hence, a failure scenario can be mathematically expressed as

$$\mathbf{y} = \left\{ y_1 = l_1, y_2 = l_2, \ldots, y_{|A|} = l_{|A|} \right\}, \forall \mathbf{y} \in S, \tag{12}$$

where S denotes all the possible failure scenarios. Hence, the probability of the occurrence of such a failure scenario $R(\mathbf{y})$ can be calculated by the chain rule of probability:

$$\begin{aligned} R(\mathbf{y}) &= R\left(y_1 = l_1, y_2 = l_2, \ldots, y_{|A|} = l_{|A|}\right) \\ &= \Pr\left(y_{|A|} = l_{|A|} \middle| y_{|A|-1} = l_{|A|-1}, \ldots, y_1 = l_1\right) \cdot \Pr\left(y_{|A|-1} = l_{|A|-1} \middle| y_{|A|-2} = l_{|A|-2}, \ldots, y_1 = l_1\right) \cdot \ldots \cdot \\ &\quad \Pr\left(y_2 = l_2 \middle| y_1 = l_1\right) \cdot \Pr\left(y_1 = l_1\right), \forall \mathbf{y} \in S. \end{aligned} \tag{13}$$

To calculate such a joint probability, the conditional probabilities involved can be estimated from empirical data, for example, via the conventional risk-modeling framework (Koks et al., 2019). The determination of such input data is an important issue, but it is beyond the scope of the present study.

As a special case, when the capacity degradation on each component is independent of the others, the probability of occurrence of a failure scenario $R(\mathbf{y})$ reduces to the product of all the marginal probabilities:

$$R(\mathbf{y}) = \prod_{a \in A} \Pr\left(y_a = l_a\right), \forall \mathbf{y} \in S. \tag{14}$$

Given a failure scenario defined by $\mathbf{y}$, the capacity reductions of all vulnerable links can be retrieved from the set defined by Eq. (9) and, accordingly, the capacity for each link is then obtained by

$$C_a\left(y_a\right) = C_{a,0}\left(1 - \psi_a^{y_a}\right), \forall a \in A. \tag{15}$$

The reduced capacity induces the changes in the travelers' route choice so as the resultant flow pattern. Accordingly, $\mathbf{v}^*$ can be expressed as a function of capacity reduction levels, $\mathbf{v}^*(\mathbf{y})$, and the UNPM defined in Eq. (8) can be written as $\varepsilon\left(\mathbf{v}^*(\mathbf{y})\right)$. Define $\mathbf{y}^0 = \left\{y_a^0 = 0, \forall a \in A\right\}$ as the base scenario where all network components perform at their design capacities and $\varepsilon\left(\mathbf{v}^*\left(\mathbf{y}^0\right)\right)$ as the corresponding UNPM. Following Qiang and Nagurney (2008), the proposed impact value of a failure scenario $\mathbf{y}$ is given by the following severity indicator:

$$I\left(\mathbf{y} \middle| \mathbf{y}^0\right) = \frac{\varepsilon\left(\mathbf{v}^*\left(\mathbf{y}^0\right)\right) - \varepsilon\left(\mathbf{v}^*(\mathbf{y})\right)}{\varepsilon\left(\mathbf{v}^*(\mathbf{y})\right)}, \forall \mathbf{y} \in S. \tag{16}$$

The larger the value of $I\left(\mathbf{y} \middle| \mathbf{y}^0\right)$, the more disruptive to the network the failure scenario is. If $I\left(\mathbf{y} \middle| \mathbf{y}^0\right) = 0$, it implies that the failure scenario causes no impact on the network, while $I\left(\mathbf{y} \middle| \mathbf{y}^0\right) = 1$ means that the network is completely damaged and can no longer serve any traffic.

Given $I\left(\mathbf{y}\middle|\mathbf{y}^0\right)$ and $R\left(\mathbf{y}\right)$, we can then derive the expected impact value associated with failure scenario $\mathbf{y}$ as

$$e\left(\mathbf{y}\right)=I\left(\mathbf{y}\middle|\mathbf{y}^0\right)\cdot R\left(\mathbf{y}\right),\forall\mathbf{y}\in S. \tag{17}$$

Once all the scenarios are known and their impact values are computed, the 'worst' disrupted scenario of the network is determined by

$$\mathbf{y}^*=\arg\max_{\mathbf{y}\in S} e\left(\mathbf{y}\right). \tag{18}$$

2.3 Bi-level Formulation

Based on the preceding sub-problem formulations, we can develop the following bi-level program to solve for $\mathbf{y}^*$.

Upper-level problem:

$$\max_{\mathbf{y}} e\left(\mathbf{y}\right) \tag{19}$$

subject to

Eqs. (8), (13), (15), (16), and (17).

Lower-level problem:

$$\min_{\mathbf{v}}\sum_{a\in A}\int_0^{v_a} c_a\left(v,C_a\left(y_a\right)\right)dv \tag{20}$$

subject to

Eqs. (2), (3), and (5).

However, in the above formulation, the variable y_a appears in the superscript in Eq. (15). To amend this, we introduce an auxiliary binary variable z_a^l to the upper-level problem and replace Eq. (15) with the following constraints:

$$y_a-l_a\le M\left(1-z_a^l\right),\forall a\in A,y_a\in\mathbf{y}\in S,l_a=0,1,...,\omega_a, \tag{21}$$

$$y_a-l_a\ge M\left(z_a^l-1\right),\forall a\in A,y_a\in\mathbf{y}\in S,l_a=0,1,...,\omega_a, \tag{22}$$

$$z_a^l\in\left\{0,1\right\},\forall a\in A,l_a=0,1,...,\omega_a, \tag{23}$$

$$\sum_{l=0}^{\omega_a} z_a^l = 1, \forall a \in A \text{, and} \tag{24}$$

$$C_a = C_{a,0}\left(1 - \sum_{l=0}^{\omega_a} z_a^l \psi_a^l\right), \forall a \in A. \tag{25}$$

Conditions (21)-(23) ensure that if $y_a = l_a$, then $z_a^l = 1$; otherwise $z_a^l = 0$. Eq. (25) ensures that exactly one of the reduction level is chosen. Eq. (25) reformulates Eq. (15) using the binary variable z_a^l.

The revised upper-level formulation is given below.

Revised upper-level problem:

$$\max_{\mathbf{z},\mathbf{y}} e(\mathbf{y}) \tag{26}$$

subject to

Eqs. (8), (13), (16), (17), (21)-(25),

where $\mathbf{z} = \left\{ z_a^l, \forall a \in A, l = 0,1,\ldots,\omega_a \right\}$.

3 Solution Method

One exact method to solve the bi-level formulation is to enumerate all the possible failure scenarios, which, however, is computationally prohibitive for realistic network applications. Therefore, this study adopts a meta-heuristic approach, i.e., the Clonal Selection Algorithm (CSA) to solve the bi-level optimization problem.

3.1 Overview of the Algorithm

The procedure of the CSA used in this study is as follows:

Step 1. Initialize the population.

Step 1.1. Generate n_{pop} solutions, where n_{pop} is the predefined population size.

Step 1.2. Compute the fitness value of each solution, which is $e(\mathbf{y})$ defined in Eq. (17).

Step 2: Repeat the following steps until a pre-defined termination criterion, e.g., a fixed number of iterations or a fixed number of the lower-level problems solved, is satisfied.

Step 2.1. Select $\lfloor \beta_{clone} n_{pop} \rfloor$ solutions from the population based on the fitness value, where β_{clone} is a parameter to determine the number of solutions to be cloned (replicated) and $\lfloor \cdot \rfloor$ is the operator that truncates a real number to its closest integer.

Step 2.2. Replicate (clone) each selected solution to form the clone population.

Step 2.3. Hypermutate each solution in the clone population.

Step 2.4. Generate $\lfloor \beta_{receptor} n_{pop} \rfloor$ candidate solutions, where $\beta_{receptor}$ is the proportion of the population that undergoes receptor editing, compute their fitness values, and add them to the clone population.

Step 2.5. If the predefined termination criterion is satisfied, then stop and output the best solution; otherwise, proceed to Step 2.6.

Step 2.6. Select the best n_{pop} candidate solutions from the existing and clone populations to form the sorted population for the next iteration.

Step 2.7. Return to Step 2.1.

In Step 1.2, the fitness value of each candidate solution is obtained by calculating the upper-level objective function, $e(\mathbf{y})$, which involves solving the lower-level traffic assignment problem. In Step 2.2, the total number of clones is fixed. The solution to be cloned is determined by a roulette wheel selection method based on its fitness value. Accordingly, solutions with higher fitness values are cloned more frequently. Step 2.3 is described in Section 3.3.

3.2 Solution Representation and Generation

To apply the CSA to solve the bi-level optimization problem efficiently, the solution representation should be designed to cater to the solution structure of the problem. We encode the capacity reduction levels for reducing the length of the solution representation. Figure 1 illustrates the solution representation for one failure scenario. Each bit corresponds to one network component and consists of its realized capacity reduction level l_a in this scenario. The number of bits is the number of network components that could fail (i.e., vulnerable components). The initial solution is generated by randomly selecting a potential capacity reduction level for each component.

l_1	l_2	$\cdots\cdots$	$l_{\lvert A\rvert-1}$	$l_{\lvert A\rvert}$

Fig. 1 Solution representation

3.3 Hypermutation

To carry out the hypermutation process in Step 2.3, two basic operators, denoted as Operator I and Operator II, are designed. They are described as follows.

Operator I: Reassign the $l_a, a = 1,2,...,|A|$ value of each bit in the candidate solution.

Operator II: Randomly perform one of the following procedures for $\lfloor \theta \cdot n_{pop} \rfloor$ times, where θ is an input parameter between 0 and 1.

a) Add a new disrupted component (i.e., randomly select an undisrupted component and change the corresponding l_a from 0 to a random integer value between $[1, \omega_a]$); if all the vulnerable components are disrupted, no change will be made to the candidate solution.

b) restore a currently disrupted component to its original capacity (i.e., randomly select a disrupted component and set the corresponding l_a to be 0).

c) swap the values of a randomly selected bit representing an undisrupted network component (i.e., $l_a = 0$) and a randomly selected bit representing a disrupted network component (i.e., $l_b \neq 0$).

By varying the probability of using these two operators, the hypermutation process can put different priorities associated with conducting the neighbor search and generating new solutions. One of the following is performed randomly:

Mutation 1 Randomly increase or decrease the level of capacity reduction of each component by one. If $l_a = 0$ $\left(l_a = \omega_a\right)$, then only increase (decrease) the capacity reduction level.

Mutation 2: 10% probability to perform Operator I and 90% probability to perform Operator II with $\theta = 0.2$.

Mutation 3: 20% probability to perform Operator I and 80% probability to perform Operator II with $\theta = 0.4$.

Mutation 4: 30% probability to perform Operator I and 70% probability to perform Operator II with $\theta = 0.6$.

Mutation 5: 40% probability to perform Operator I and 60% probability to perform Operator II with $\theta = 0.8$.

Mutation 6: 50% probability to perform Operator I and 50% probability to perform Operator II with $\theta = 1.0$.

4 Numerical Examples

4.1 Comparison between the proposed and deterministic approaches

To demonstrate the property of the proposed probabilistic approach to determining the critical combination of network component failures, we conducted experiments to compare the results obtained from the probabilistic approach with two deterministic approaches. The first approach was developed by Qiang and Nagurney (2008) while the second one was by Wang et al. (2016), which uses total travel cost as the vulnerability measure. The former is a special case of our proposed measure with the same probability of occurrence to all scenarios, while the latter considers multiple component failures without considering partial closures as well as the probability of failures.

For illustration purposes, we adopted a four-node network shown in Figure 2. In this example network, there are two OD pairs: OD pair 1-3 and OD pair 1-4. Their demands were set as $d_{13} = 10$ and $d_{14} = 20$, respectively. The link data is given in Table 2(a). It is assumed that all links possess the same set of capacity reduction proportions and the probability distribution as shown in Table 2(b). Moreover, to simplify the calculation, it is assumed that the failure probability of each link is independent of others. The classic BPR function was adopted to calculate the link travel time. To obtain the optimal solution to the small network using the proposed probabilistic approach, a brute force method was adopted and all possible scenarios that preserve the connectivity of all OD pairs were examined. To apply the method by Wang et al. (2016), it is required specifying the number of vulnerable components. In our experiment, the number increased from one to three, and all the combinations of components that preserve the connectivity of each OD pair were examined. The results obtained from the two deterministic methods and the proposed probabilistic approach are presented in Table 3.

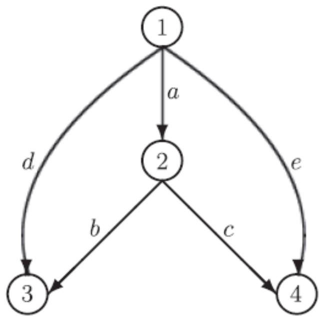

Fig. 2 Four-node network

Table 2 Data for the network in Figure 2

(a) Link data

	Link *a*	Link *b*	Link *c*	Link *d*	Link *e*
Free flow travel time	10	10	10	10	10
Capacity	100	20	60	10	20

(b) Link disruption probability matrix

	Probability ϕ_n^l				
Capacity reduction proportions (ψ_n^l)	Link *a*	Link *b*	Link *c*	Link *d*	Link *e*
0.00	0.35	0.35	0.35	0.35	0.35
0.30	0.30	0.30	0.30	0.30	0.30
0.60	0.30	0.30	0.30	0.30	0.30
1.00	0.05	0.05	0.05	0.05	0.05

Comparing Table 3(a) with Table 3(c), it is observed that the result of the proposed probabilistic approach differs from that of Qiang and Nagurney's (2008) in the following aspects: 1) More than one component disruption is involved in the most critical scenario identified; 2) The components with zero impact values in Table 3(a) could also contribute to the deterioration of the network performance in the most critical failure scenario, for example, link *a*. This is because once links *d* and *e* are disrupted, travelers going to their destinations will divert to alternative paths (*a*-*b*) and (*a*-*c*) to reach destination nodes 3 and 4, respectively. Accordingly, if link *a*'s capacity deteriorates simultaneously with links *d*'s and *e*'s, travelers' travel time increases; 3) Neglecting partial capacity degradation and its probability of occurrence could misestimate the worst scenario, in which not all the disrupted links are completely broken down. This is because the worst scenario, defined by the expected impact value, captures both the consequence and the probability of the occurrence of failures. A scenario that some links are completely broken down could induce a high consequence, but the probability of occurrence of such a scenario is low. For example, in Table 3 (c), the capacities of links *d* and *e* are only reduced by 0.6. If instead both links were completely broken down, the resultant scenario induces a higher impact value, but the associated occurrence probability is much lower. Comparing Table 3(b) with Table 3(c), it is noticed that the total travel cost of the critical scenario is substantially larger than that in Table 3(b). This is due to the feature that the proposed approach captures partial failures of the network component, allowing more levels of capacity reduction without destroying the connectivity between OD pairs.

Table 3 Comparison with the deterministic approaches

(a) Qiang and Nagurney's (2008) approach

Ranking	Link	Impact value*
1	*d*	0.0101
2	*e*	0.0046
3	*a/b/c*	0.0000

(b) Wang et al.'s (2016) approach

Ranking	Link combination	Total cost
1	(*d*, *e*)	611.15
2	(*c*, *d*)	606.26
3	(*b*, *e*)	602.79

(c) The result of the most critical scenario obtained from the proposed probabilistic approach

Link	*a*	*b*	*c*	*d*	*e*
Capacity reduction proportion	1.00	0.00	0.00	0.60	0.60
Impact value I	0.9501				
Scenario probability R	0.0006				
Expected impact e	0.0005				
Total cost	12018.75				

*The impact value is equivalent to the importance of a network component in the study of Qiang and Nagurney (2008)

Interestingly, by comparing the three methods, it is found that the top two critical components identified by Qiang and Nagurney (2008) form the most critical failure scenario found by Wang et al. (2016). Their capacity degradations are also significant (i.e., by 60%) in the most critical failure scenario found in this study. This implies that different approaches may identify similar critical network components involved. Nevertheless, our approach can discover some components that are not found by other existing approaches. Moreover, as shown in Table 4, the path flow distributions and travel times obtained by various approaches are completely different, implying that the impacts are also different.

Table 4 Comparison of link flow and travel time obtained by different approaches

		Qiang and Nagurney (2008)		Wang et al. (2016)		Proposed Probabilistic Approach	
		Removing link *d*		Removing (*d*, *e*)		Most critical scenario	
OD	Path	Flow	Travel Time	Flow	Travel Time	Flow	Travel Time
1-3	*a-b*	10.00	20.626	10.00	20.706	0.00	400.63

	d	0.00	-	0.00	-	10.00	400.62
1-4	*a-c*	0.00	20.001	20.00	20.204	0.00	400.62
	e	20.00	20.000	0.00	-	20.00	400.62

4.2 Algorithmic Performance

To test the performance of the proposed algorithm, the CSA was coded in C++ and compiled using Microsoft Visual Studio Community 2017. All the tests were run on a desktop with an Intel® Xeon (TM) E5-2697M CPU @2.3 GHz and 12.0 GB RAM. The parameters for the CSA were set as follows: $n_{pop} = 20$, $\beta_{clone} = 0.8$, $\beta_{receptor} = 0.2$, and the CSA was set to stop after evaluating 10000 UE solutions. The experiments were carried out using Sioux-Falls and Anaheim networks. Their link and demand data were obtained from Transportation Networks for Research Core Team (2017). To solve the lower-level traffic assignment problem, the bi-conjugate Frank–Wolfe method (CFW, Mitradjieva and Lindberg, 2013) was adopted, since it has been concluded by Perederieieva (2015) that, compared with other state-of-the-art methods, CFW can achieve a good trade-off between memory consumption and solution accuracy for large instances.

4.2.1 Comparison of the performance between CSA and the brute-force method

In this test, we randomly selected 10 links from the Sioux-fall network as vulnerable links and each link has the same failure probability distribution with 3 levels of capacity reduction proportion (i.e., reduction proportion equals 0.2 with a probability of 20%, equals 0.4 with a probability of 20%, and remains unchanged with a probability of 60%). The brute force method was applied to enumerate all the possible solutions in order to obtain the benchmark result for this network. In comparison, the CSA algorithm was run 20 times with different initial seeds for generating random numbers. Moreover, we varied the tolerance in determining the termination condition of the traffic assignment problem. The value was reduced from 1.0e-2 to

1.0e-6 and the results are presented in Table 5.

Table 5 Comparison of the performance between CSA and the brute-force method

	Capacity reduction proportion of the vulnerable links										Fitness (1.0e-5)		Computation time (Seconds)	
Tolerance	1	7	9	17	32	44	55	58	71	75	Brute force	CSA*	Brute force	CSA**
1.0e-2	0	0	0	0	0	0.4	0	0	0	0	1.78	1.78	717.4	121.1
1.0e-3	0	0	0	0	0	0	0	0	0.4	0	2.56	2.56	1544.8	255.1
1.0e-4	0	0	0	0	0	0	0	0	0.4	0	2.23	2.23	3299.7	546.2
1.0e-5	0	0	0	0	0	0	0	0	0.4	0	2.13	2.13	8331.3	1372.2
1.0e-6	0	0	0	0	0	0	0	0	0.4	0	2.12	2.12	33316.5	5262.2

*All 20 runs of CSA successfully obtain the same fitness value

** Average time of 20 runs

Table 5 shows that except when the tolerance is 1.0e-2, both experiments identify the same most critical scenario, in which the capacity of link 71 is reduced by 40%. This implies that when the tolerance is less than a threshold value, it does not affect the determination of the most critical scenario. However, the tolerance still has a minor effect on fitness values (i.e., objective function values) because the tolerance affects the convergence of the bi-conjugate Frank–Wolfe method for the lower-level problem and hence the flow values used to determine the corresponding objective function value. For the experiments, the CSA successfully attains the same fitness values as those from the brute force method at a considerably lower computation time. The results demonstrate that the proposed CSA algorithm could find the optimal solution to the problem considered with a significantly reduced computational burden.

4.2.2 Comparison of the performance between CSA and GA

The performance of the CSA was then compared with that of GA using the Anaheim network. We randomly generated 100 vulnerable links, and, for simplicity, all the links are assumed to be subject to 4 levels of capacity reductions. For a fair comparison, the GA used the same solution representation and initialization procedure as in the proposed CSA. In GA, two operators, i.e., mutation and crossover operators are designed to search for new solutions. For the mutation operator, the hypermutation mechanism developed for the CSA was adopted. For the crossover operator, a one-point crossover operator was used. The crossover rate and mutation rate were set to be 0.8 and 0.2, respectively, and the population size was set to be the same as that of the CSA. In comparison, both two algorithms were run 20 times and stopped when a predefined number of lower-level problems, which is 20000, was solved. Such a termination criterion is adopted for a fair comparison between the two algorithms.

The results presented in Table 6 state that on average, CSA obtained better solutions than GA, but took more computation time than GA did. Theoretically, to compare the complexity of the two algorithms, we can decompose the computation tasks into two parts. One is evaluating the newly generated solution by solving the lower-level traffic assignment problem, which induces the main computation burden. In every iteration of GA, two parents produce two offspring; hence, the total number of newly generated solutions equals the population size of the GA. In CSA, the number is determined by the size of cloned solutions, which can either be static or increase over iterations (Ulutas and Kulturel-Konak, 2011). Hence, depending on the cloning method, the number of solutions generated in CSA varies. The other parts of the computation task include several procedures in the algorithm, including (hyper) mutation (used by both GA and CSA), one-point crossover (used by GA), and sorting algorithm (used by both GA and CSA). For the (hyper) mutation designed in Section 3.3, the complexity of Operator 1 is linear with respect to the number of vulnerable components in the network, while that of Operator 2 is linear with respect to the size of the population. For the one-point crossover adopted in this study, its complexity is also linear with respect to the number of vulnerable components. For the sorting algorithm, its complexity depends on the data structure and algorithm used (Cormen et al., 2009). In our implementation, we have tried to ensure the fairness of the comparison via using the common function and code as much as possible.

Table 6 Comparison of results between CSA and GA

	CSA	GA
Average of Fitness (10^{-47})	4.22	1.04
Average CPU Time (second)	3362	2773
Best Fitness (10^{-46})	7.12	1.74

5 Concluding Remarks

This paper develops a probabilistic approach for analyzing network vulnerability. Instead of determining the criticality of a single network component, the paper proposes to evaluate network failure scenarios considering both the consequences and the probabilities of simultaneous failures of multiple network vulnerable components. A bi-level optimization model is formulated to obtain the most critical failure scenario. The Clonal Selection Algorithm is adopted to solve the bi-level optimization model. The numerical studies were conducted to compare the proposed probabilistic approach with the two deterministic approaches. The results show that neglecting partial capacity degradation and its probability of occurrence could underestimate the impact of the worst scenario and different vulnerability assessment approaches may identify similar critical components, but different equilibrium traffic flow. In addition, our approach can discover some components that are not found by other existing deterministic approaches. Moreover, the results of computational experiments demonstrate the effect of convergence tolerance used in solving the user equilibrium assignment problem in determining the critical components and show that the proposed CSA algorithm outperforms GA in finding better solutions in a large instance on average.

The study opens various future research directions for network vulnerability and sustainability analysis: 1) The lower-level problem is formulated as a static traffic assignment model. It could be interesting to adopt a dynamic traffic assignment model (e.g., Jiang et al., 2016; Wang et al., 2018) in future studies to investigate how the network vulnerability measures (e.g., total travel time) change with a higher temporal and spatial resolution (see, for example, Nagurney and Qiang, 2008); 2) The lower-level problem only considers a single transport mode, wherein the link travel time functions are assumed to be separable. When multimodal traffic equilibrium is considered, the assumption is required to be relaxed. Therefore, one of the future directions should be developing a generalized lower-level formulation, such as utilizing a variational inequality formulation, to handle asymmetric multimodal traffic network equilibrium problems; 3) The lower-level problem assumes deterministic travel time and ignores the stochasticity in travel time as well as travelers' response to travel time uncertainty. In future studies, these issues can be addressed by utilizing a reliability-based assignment framework (e.g., Szeto et al., 2011a,b; Jiang and Szeto, 2016), leading to reliability-based network design models (e.g., Chootinan et al., 2005; Chen et al., 2010, 2011; Yim et al., 2011); 4) In this study, we only compared the performance of CSA with GA. There are other recent metaheuristics with good success. Therefore, one possible future research direction is to compare the performance of CSA with other metaheuristics; 5) This study only considers the economic dimension of sustainability. However, other dimensions could be incorporated into the proposed framework to form a multiobjective bi-level optimization model (e.g., Chen et al., 2010; Szeto et al., 2015; Jiang and Szeto, 2015; Xu et al., 2016). This extension is left for future studies.

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