The association between visual attention and arithmetic competence: The mediating role of enumeration

Highlights

- The relation between visual attention and arithmetic competence was investigated
- Goal-directed, but not stimulus-driven, visual attention significantly predicted arithmetic
- The relation was mediated through enumeration speed

Abstract

The current study aimed at clarifying the nature of relation between visual attention and arithmetic competence. A group of 301 Chinese second graders were assessed. Children's visual attention was measured using two versions of visual search task, with an efficient visual search task (the similarity between the target and the distractors is low) tapping automatic, stimulus-driven visual attention and an inefficient visual search task (the similarity between the target and the distractors is high) tapping effortful, goal-directed visual attention. Children's arithmetic competence, enumeration skills (assessed in about half of the participants), as well as other domain-general cognitive abilities were also assessed. The results suggested that only inefficient visual search significantly predicted arithmetic competence, and such relation was mediated through their enumeration skills. The findings highlight the role of fundamental cognitive capacities in mathematics learning and provide insights to potential interventions for improving children's arithmetic competence.

Key words: visual attention, arithmetic, enumeration, goal-directed processing

Introduction

Visual attention, a mechanism for selectively processing the visual information exposed to our visual receptor (Carrasco, 2011), has been found to be important for mathematical development in a number of studies recently (Anobile, Stievano, & Burr, 2013; Commodari & DiBlasi, 2014; Steele, Karmiloff-Smith, Cornish, & Scerif, 2012). Adults with developmental dyscalculia, a learning disability indicated by a substandard mathematical achievement, have also been shown to have deficits in their attentional networks (Askenazi & Henik, 2010). Although these studies demonstrated the link between visual attention and mathematical competence, various issues about the nature of this link remained unclear. First, the processing of focusing visual attention comes in stages (Treisman & Gelade, 1980). While the first stage of visual attention process is automatic and parallel across the visual field, the second stage requires effortful, serial processing of stimuli. It was unclear how the two visual attention processes relate to children's mathematical competence. Second, very few, if any, studies have examined the potential mechanisms involved in the relation between visual attention and children's mathematical competence. The current study was therefore set to examine (1) which of the attentional processes drives the relation between visual attention and arithmetic competence (one core aspect of mathematical competence), and (2) whether such relation could be explained by ones enumeration skills.

Sub-processes of visual attention

In a seminal paper, Treisman and Gelade (1980) proposed the feature-integration theory of attention. The theory suggested that the process of focusing attention comes in two stages. In the first stage, the major focus is to register the visual features presented in the visual field, and this registration process is automatic and parallel across the visual field. After the features are registered, we can then focus our attention to individual objects. At this stage, separable features are combined so that the objects in the visual field can be identified, and the focused attention serves as the “glue” that integrates different features.

The examination of different stages of visual attention processing was usually done through the use of visual search task (Wolfe, 1998; Wolfe & Horowitz, 2004). As indicated by Treisman and Gelade (1980) and Wolfe (1998), there is a continuum range of visual search from efficient ones to inefficient ones. In efficient visual search, the contrast between target and distractors is salient, so that it is very easy to find the target (e.g., search for letter “I” among a number of “O”s). The target will “pop out” in efficient visual search just like the automatic detection of visual features, and the search performance will not be influenced significantly by the increment of distractors (Wolfe, 1998; Wolfe & Horowitz, 2004). Therefore, efficient visual search was thought to reflect first stage of the attentional process: an automatic, stimulus-driven parallel processing.

In inefficient visual search, on the other hand, the saliency of the target is low, and thus it becomes difficult to find the target (e.g., search for a red vertical line among a number of green vertical lines and red horizontal lines, or search for letter “C” among a number of “G”s). Attention has to be focused on each individual stimulus in order to distinguish the target from the distractors. The searching performance of inefficient search will therefore be significantly influenced by the number of distractors (e.g., the searching latency will become longer when searching for a “C” among more “G”s; Wolfe, 1998; Wolfe & Horowitz, 2004). Inefficient visual search, thus, was thought to involve the second stage of the attentional processing: an effortful, goal-directed serial processing (Wolfe, 1998).

The link between visual attention and arithmetic competence

A number of recent studies converged to support the link between visual attention and mathematical competence. For instance, among a group of children aged 8 to 11, Anobile and colleagues (2013) showed that visual attention was significantly correlated with the “magnitude factor” of children’s mathematical achievement (covering number comparison and complex calculation skills). On the other hand, Steele and colleagues (2012) found that visual sustained-selective attention of pre-schoolers longitudinally predicted their numeracy skills one year later. These studies illustrated the link between visual attention and children’s mathematical competence (but see Tinelli et al., 2015, for a null finding on this relation).

Despite the fact that the relations between visual attention and mathematics received empirical support, we seem to have little knowledge concerning this visual attention-math link. Is visual attention related to mathematics achievement through a similar mechanism as it is related to reading (e.g., reducing distraction and distinguishing orthographically similar words; Bednarek et al., 2004; Reynolds & Chelazzi, 2004)? Although solving mathematical problems also require children to focus their attention to the particular number sentences and distinguish similar numbers (e.g., 107 versus 170), research in numerical cognition seems to suggest more deep-rooted connections between visual attention and arithmetic competence, a core aspect of mathematical competence among a garden variety of other competence (e.g., geometry).

Specific mechanisms between visual attention and arithmetic competence

The search for the specific mechanism between visual attention and arithmetic competence may start from the basic numerical skills that are proposed to be underlying our numerical cognition. Among the wide range of basic numerical skills that have been investigated (e.g., approximate number system; Halberda, Mazzocco, & Feigenson, 2008; cardinality understanding, Geary, Chu, Rouder, Hoard, & Nugent, 2018; enumeration skills, Reigosa-Crespo et al., 2012; number line estimation, M. Schneider et al., 2018; and place-value understanding, Chan, Au, & Tang, 2014), enumeration skills appear to be particularly relevant to the process of focusing visual attention.

Enumeration refers to the task of specifying the total number of items in an array. This process contrasts with estimation because exactness is emphasized. As the enumeration process involves the access of the representation of numerical magnitude, the efficiency of such process has been suggested to be one of the indices of our core number abilities (Reeve, Reynolds, Humberstone, & Butterworth, 2012). The findings that individual difference in enumeration efficiency is relatively stable across time (Reeve et al., 2012), and that such individual difference predicts children's arithmetic competence throughout the whole elementary school period (Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Moore & Ashcraft, 2015; Reigosa-Crespo et al., 2012) further support such a claim (but see Anobile, Arrighi, & Burr, 2019, for a null finding on the relation between subitizing and arithmetic). On the other hand, children with persistent low achievement in mathematics have been shown to have deficient enumeration skills prior to school entry (Wong & Chan, 2019). These findings illustrated the strong relation between children's enumeration skills and their arithmetic competence.

There are two processes underlying the enumeration process. When the number of objects to be enumerated falls within a limit of 3 to 4, accuracy is close to ceiling, and the reaction time increases very slowly with increasing number of objects to be enumerated (40 to 120 ms per item). This process is known as subitizing (Kaufman, Lord, Reese, & Volkmann, 1949). However, when the number of objects go beyond 4, accuracy starts to fall, and the reaction time needed to enumerate each additional item increases considerably (250 to 350 ms per item). This process is known as counting (Trick & Pylyshyn, 1993). The presence of two distinctive processes in enumeration is usually illustrated through the discontinuity in slope when enumeration latency is plotted against the number of objects to be enumerated: a relatively flat slope in the range between 1 to 4, and a steeper slope in the range beyond 4 (Trick & Pylyshyn, 1993).

Trick and Pylyshyn (1993) have attempted to explain the existence of two enumeration processes through our attention system. Echoing with Treisman and Gelade (1980), they suggested that our visual processing of numerosity involves two stages. In the first stage, the spatially parallel preattentive stage, features within a visual scene are registered all at once. Yet, it is not until the second stage, or the attentive stage, where attentional focus is put onto individual objects and analyses are done at the item level. The discontinuity in enumeration slope is suggested to be due to the limited number of registers in the preattentive mechanisms: when the number of objects to be enumerated is fewer than the number of registers in the preattentive mechanism, quick, spatially parallel processing of numerosity is allowed and thus result in very high enumeration accuracy and a very small increase in reaction time required for enumerating an additional object. However, when the number of objects to be enumerated is greater than the number of available registers, each item has to be processed individually in the attentive stage. Attention has to be focused on each individual object to be enumerated, and hence result in a much greater decrease in accuracy and a much higher enumeration slope. Trick and Pylyshyn (1993) further showed with their data that subitizing occurs only when the objects to be enumerated allow preattentive processing (e.g., non-overlapping rectangles instead of concentric rectangles). When such preattentive processing is not allowed (e.g., search for Os from Qs), people fall back to counting even within the subitizing range. On the other hand, Simon and Vaishnavi (1996) illustrated that counting, but not subitizing, is dependent on eye movement. When eye movement is prevented either by the use of afterimage or by restricting the presentation time of the objects, participants' enumeration accuracy was severely affected within the post-subitizing range, although their accuracy within the subitizing range remained high. This further suggested that different mechanisms were underlying the subitizing versus the counting processes: the former relies on preattentive, parallel processing where eye movement is not necessary, while the latter relies on at attentive, serial processing that involve eye movements.

While earlier studies seem to suggest that subitizing is preattentive (Simon & Vaishnavi, 1996; Trick & Pylyshyn, 1993), more recent studies have revealed a more complicated picture concerning whether subitizing requires attention. Using a dual task paradigm, Vetter, Butterworth, and Bahrami (2008) found that adults' performance in enumeration was affected when their attention was captured by a primary visual target detection task. In particular, enumeration performance within the subitizing range seem to be more severely affected. The findings have been replicated by Burr, Turi and Anobile (2010), who found a primary task of either letter recall or spatial target detection resulted in less accurate enumeration of numerosity, especially within the subitizing range. Piazza, Fumarola, Chinello, and Melcher

(2011), on the other hand, observed a reduction of adults' subitizing limit when their visual attention capacity was occupied. These findings suggest that the subitizing process may also require the same effortful, serial processing of individual objects as the counting process.

The present study

Based on the aforementioned findings, while the link between visual attention and arithmetic competence appears to be empirically supported (Anobile et al., 2013; Steele et al., 2012), the nature of this link deserves further investigation. First, given the existence of two different attentional processes (i.e., automatic, stimulus-driven parallel processing versus effortful, goal-directed serial processing; Wolfe, 1998), it is unclear which attentional process actually drives the relation between visual attention and arithmetic competence. Second, although it has been shown that that visual attention is involved in the enumeration process (Simon & Vaishnavi, 1996; Vetter et al., 2008), and the enumeration skills serve as an important precursor of our arithmetic competence (Lyons & Ansari, 2015; Reigosa-Crespo et al., 2012), the issue of whether enumeration skills serve as the mechanism underlying the link between visual attention and arithmetic competence remains unexplored.

The current study was therefore conducted to address the two aforementioned hypotheses concerning the link between visual attention and arithmetic competence. Two cohorts of second graders were recruited. They were assessed on the visual attention skills, arithmetic competence, as well as other domain-general cognitive factors and reading skills (which served as controls). In particular, visual attention was assessed using two different visual search tasks, with efficient visual search measuring automatic, stimulus-driven parallel processing and inefficient visual search measuring effortful, goal-directed serial processing on top of the effortless, stimulus-driven parallel processing. A dot enumeration task was further introduced to the second cohort of participants to test whether children's enumeration skills mediate the relation between visual attention and arithmetic competence. Based on the existing literature (Simon & Vaishnavi, 1996; Vetter et al., 2008), effortful, goal-directed serial processing is expected to be driving the relation between visual attention and arithmetic competence, and the relation is expected to be mediated by enumeration skills.

Method

Participants

Two cohort of second graders were recruited for the current study. The first cohort consists of 137 second graders (88 boys and 49 girls), with a mean age of 8.20 years (S.D. = .45 year). The second cohort consists of 164 second graders (99 boys and 65 girls), with a mean age of 8.05 years (S.D. = .37 year). The participants were recruited from 17 different primary schools in Hong Kong. All the participants were Cantonese-speaking Chinese. Thirteen of the participants were found to have IQ falling below 80, and they were excluded from the final sample as they might have difficulties understanding the task instructions. This resulted in a final sample of 288. Written parental consent was obtained from all the participants. The current project was approved by the Human Research Ethics Committee of the University to which the first author was affiliated.

Measures

A total of nine measures were administered. Except for the two arithmetic outcome measures (i.e., arithmetic fact retrieval and arithmetic computation, which were conducted in groups in the classroom settings), all the measures were conducted individually.

Visual attention

The visual search task asked the participants to circle target items intermixed with many distractor items as quickly and accurately as possible (adapted from Liu, Chen, & Chung, 2015; Liu, Chen, & Wang, 2016). For example, they had to circle the target *E*s in an array of *F*s. Each trial presented a 20-item × 3-line matrix printed on a separate piece of paper, in which targets and distractors were arranged randomly. There were two types of items: alphabetic letters and symbols (e.g., ↗ in an array of ↙). They were presented in the Microsoft Jheng Hei font with a font size of 11. The number of targets varied from 11 to 13 across trials to prevent the participants from guessing how many targets they should find. The task included 24 trials, 12 for each item type.

To measure inefficient and efficient visual searches, we adopted the task items with high and low visual similarity. Visual similarity was rated by 30 undergraduate students on a scale ranging from 1 (very different) to 7 (very similar). As Table 1 displays, the target-distractor pairs in the inefficient search trials were more similar than those in the efficient search trials. To exclude the possibility that search performance was impacted by visual complexity of items rather than similarity, we measured visual complexity by dividing the squared perimeter of the stimulus by its ink area (Pelli, Burns, Farell, & Moore-Page, 2006). Table 1 shows that the visual complexity scores of items in the inefficient and efficient trials were comparable, suggesting that the items in the two types of trials were similarly complex and thus difference in search performances was not caused by this factor. All items used in the visual search task is shown in Table 2. Statistical analyses showed that the letter search and the symbol search did not differ from each other in visual complexity, F (2, 9) = 1.79, p = .22, or similarity, F (2, 9) = .24, p = .79.

Each trial first presented two lines of the real items to familiarize them with the target and distractor items. A sample trial is shown in Figure 1. We recorded completion time in each trial using a stopwatch. The number of errors were calculated by adding the number of targets that were missed and the number of distractors that were circled. The Cronbach's αs of the accuracies of inefficient and efficient visual search tasks were .55 and .40 respectively. The corresponding αs of the reaction times were .87 and .94 respectively.

Table 1

Complexity and similarity of visual search stimuli.

Stimuli type	Complexity condition	Similarity condition	Complexity value	Similarity value
Letter	High	High	19.03	5.08
	High	Low	16.88	1.77
	Low	High	11.17	5.23
	Low	Low	12.78	2.12
Symbol	High	High	21.43	3.58
	High	Low	17.73	1.27
	Low	High	8.67	4.08
	Low	Low	11.01	2.08

Table 2

All items used in the visual search task.

Item type	High-similarity pairs				Low-similarity pairs			
Symbol	◎-⊙	∠-↙	▽-◇	↗-↙	△-□	↘-◇	○-☆	↙-匚
Letter	B-D	O-Q	E-F	C-G	F-K	g-w	t-x	G-E

Note: The right item in each pair is the target, while the left is the distractor.

Practice: Circle the “S” from the “V”

V V S V V V S V V V V S V V V V V V V V
V V V V V V V S V V V V V V V S V V V V

V V S V V V V V V S V V V V S V V S V V
V V V S V V V V V V V S V V V V S V V V
V V V V V S V V V S V V V S V V V S V V

Figure 1 – Sample item in the Visual search task

Arithmetic outcomes

Two arithmetic tasks were administered to the participants to assess their arithmetic competence. In the arithmetic fact retrieval task, the participants were given two sets of arithmetic items. One of the sets involved 45 single-digit additions, while the other set consisted of 45 subtraction items with single-digit subtrahend and difference. For each set of items, the participants were instructed to complete as many items as possible within one minute. The arithmetic computation task was adapted from Wong, Ho, and Tang (2014, 2016). In this task, a total of 28 arithmetic items with varying difficulties (from single-digit addition to multi-digit subtraction to multiplication and division) were presented to the participants, and they were asked to complete as many items as possible. They were given 15 minutes to complete these problems. Rough work sheets were provided to the participants. Performance was measured by accuracy for both tasks. The Cronbach's αs were .97 and .92 for arithmetic fact retrieval and arithmetic computation respectively.

Intelligence

The short form (sets A to C) of the Raven's standard progressive matrices (Raven, 1976) was used to assess participants' nonverbal intelligence. In each of the items, the participants saw a pattern with a missing piece, and they had to identify the correct piece, among six to eight options, that would fit the pattern. Performance was measured in terms of accuracy. The Cronbach's α of this task was .86.

Working memory

Two working memory tasks were administered to test the participants' working memory capacity. In the backward syllable span task that assessed participants' verbal working memory, the participants were orally presented with a sequence of Cantonese syllables in a pace of one syllable per second. After listening, they had to repeat the syllables in a reversed manner. In the backward Corci span task (Corsi, 1973) that assessed participants' visuospatial working memory, the participants were presented with a Corsi block (a black board with nine small boxes on it). In each item, the participants saw a video in which an experimenter tapped the boxes in a particular sequence, and the task of the participants was to repeat the sequence in a reversed manner. For both tasks, two practice trials were provided to familiarize the participants with the tasks. Starting with the span size of two, the span size was increased by one unit every three trials, resulting in a total of 21 trials (span size of eight) for each task. The task would be terminated if the participants failed all three items within the same span size. Performance was measured in terms of accuracy. The Cronbach's αs were .55 and .77 for backward syllable and backward Corsi respectively.

Word reading

The word reading subtest of the Hong Kong Test of Specific Learning Difficulties in Reading and Writing for Primary School Students – Third Edition [HKT-P(III); Ho et al., 2015] was adopted to assess the word reading skills of the participants. Participants were shown a list of 120 two-character Chinese words, and they were asked to read the words aloud. Each correctly read word yielded one mark. The task would be terminated when the participant got 0 marks for 30 consecutive words. Performance was measured in terms of accuracy. The task was highly reliable, with Cronbach's α being .98.

Dot enumeration

The dot-enumeration task was adopted by Andersson and Östergren (2012) to assess children's efficiency in enumeration, and it was conducted only to the second cohort of participants. It was programmed with E-prime (Version 2.0; W. Schneider, Eschman, & Zuccolotto, 2012). In each trial, participants were shown an array of dots with numerosity ranging from 1 to 9. Their task was to say aloud the number of dots presented, and the experimenter would press the spacebar to record the reaction time and write down their answer on the record booklet[1]. The dots were randomly scattered around the screen. In half of the trials, the total area of the dots was controlled, while the total perimeter was controlled in the other half of the trials. There were five trials for each numerosity, resulting in a total of 45 trials. Participants' performance was indicated by accuracy and reaction time. The Cronbach's α was .60 for accuracy and .92 for reaction time.

Procedures

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[1] The effect of experimenter on participants' reaction time was not statistically significant, F(11,113) = .72, *p* = .57; suggesting that the manual responding had minimal effect on participants' reaction time.

Informed consents were obtained from the parents of the participants before the data collection began. The participants were tested on their arithmetic and reading performance in their own primary schools, and they were tested on their cognitive performance at home. Their parents were allowed to stay besides the participants during the assessment but were advised not to communicate with the participants during the assessment. The arithmetic and reading tests took about an hour, and the cognitive assessment took around 2.5 hours. Participants received supermarket coupons and stationaries as compensation for their time and effort. All the assessments were conducted by trained psychology undergraduates.

Analyses

Two sets of analyses were conducted, with each set of analyses addressing one research question. To address the first research question (i.e., whether it is the effortful, goal-direct serial processing or the automatic, stimulus-driven parallel processing that drives the relation between visual attention and arithmetic competence), the interrelations among these variables were examined based on the data from both cohorts. Multiple linear regression was then conducted to examine the relative contributions of efficient versus inefficient visual search performance to arithmetic competence. To address the second research question (i.e., whether dot enumeration skills mediate the relation between visual attention and arithmetic competence), the correlations between various dot enumeration performance indices and other variables were examined based on the data from the second cohort, and a mediation analysis using the PROCESS macro (Hayes, 2013) in the Statistical Package for the Social Sciences (SPSS) was conducted to see if the indirect effect from visual attention to arithmetic through dot enumeration performance was significant.

Prior to the aforementioned analyses, two issues had to be addressed. The first one concerns about the performance indicator of speeded measures (i.e., visual search tasks, dot enumeration task). Bruyer and Brysbaert (2011) suggested that error rate and reaction time can be combined as the inverse efficiency score only when there were positive correlations between these two indices. However, these positive correlations were not observed in the current study (all *r*s < .06). Therefore, separate analyses were conducted for accuracy versus reaction time. The second issue concerns about the best indicator of dot enumeration performance. Although the enumeration process is usually divided into the subitizing and counting sub-processes, it may be difficult to tease apart the two sub-processes as children may groupitize (i.e., grouping stimuli into subsets of subitizable quantities; Starkey & McCandliss, 2014). In order to identify the best indicator of dot enumeration performance, the subitizing ranges of the participants were calculated using the paradigm introduced by Leibovich-Raveh, Lewis, Al-Rubaiey Kadhim, and Ansari (2018). Other indices, such as the accuracy and reaction time for subitizing, counting, and the overall enumeration process, were then calculated. The indices with the highest correlations with visual attention and arithmetic outcomes were considered the best indicator of dot enumeration performance and was included in the mediation model as the mediator.

Results

Descriptive statistics

The data was first screened for univariate outliers. The data points which were 3 S.D. beyond the corresponding means were identified (n = 29, coming from 26 different participants[2]) and deleted. After that, we screened for multivariate outliers using Mahalanobis distance in SPSS. Two multivariate outliers were detected and deleted. A total sample of 260 participants was left for the following analyses (138 from the second cohort). Except for the accuracy measures for overall enumeration and counting (with kurtosis being 3.55 and 3.27 respectively, mainly due to ceiling performance), all other variables were normally distributed, with the skewness and kurtosis values ranging between -1.78 and 1.86.

Table 3 presented the means, standard deviations, and the reliability (Cronbach's α) of the variables measured. Except for backward syllable span (α = .55) and the accuracies of efficient and inefficient visual search accuracies (αs = .40 and .55 respectively) and dot enumeration (α = .60), all the other variables yielded good reliabilities (α ≥ .70). The

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[2] Number of univariate outliers in the following measures: 4 from backward syllable span, 1 from backward Corsi span, 4 from inefficient visual search accuracy, 1 from inefficient visual search RT, 6 from efficient visual search accuracy, 2 from efficient visual search RT, 2 from arithmetic fluency, 6 from dot enumeration accuracy, 2 from dot enumeration RT, 1 from dot enumeration subtiizing range; 3 participants were univariate outliers in 2 variables

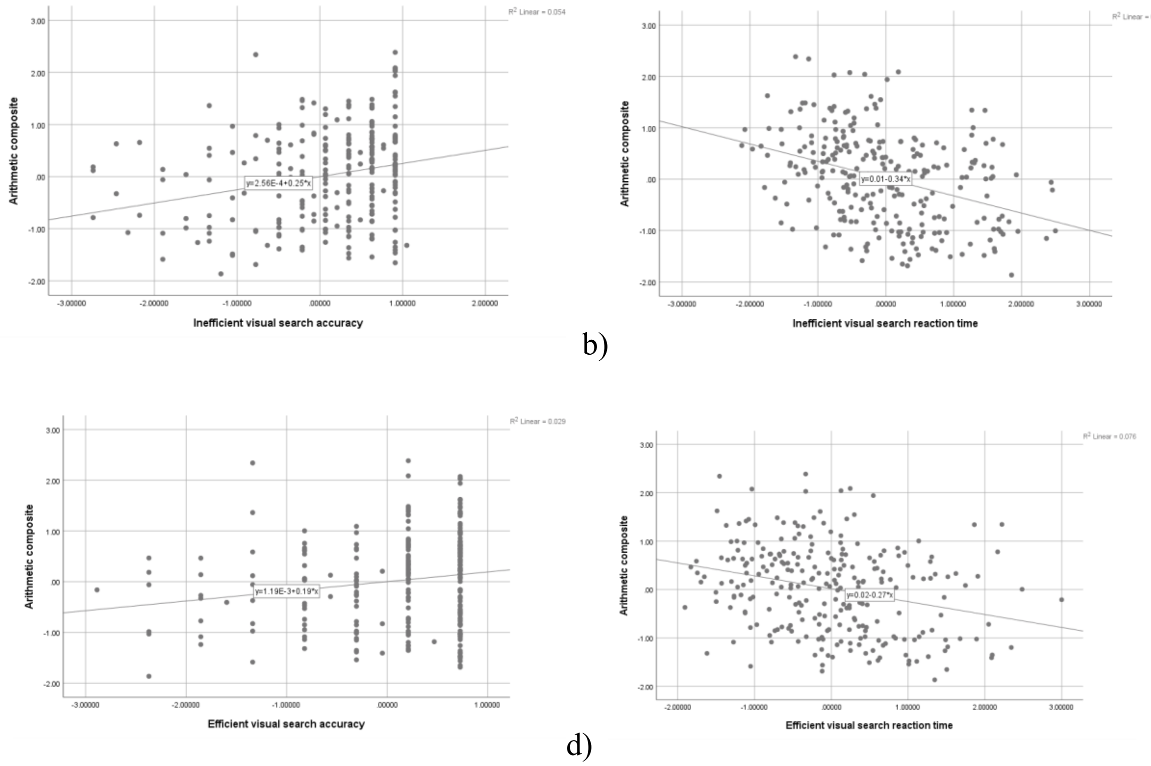
correlations among the variables were also shown in Table 3. Both the accuracy ($r = .47$, $p < .001$) and the reaction time ($r = .80$, $p < .001$) of the two visual search tasks were strongly related with each other, and all visual search indices were significantly related to the two arithmetic outcome measures ($|r|$s ranged from .13 to .24 for accuracy, ps<.05; $|r|$s ranged from .26 to .35 for reaction time, ps<.001). All other variables (except for age) were significantly related to the arithmetic outcomes ($|r|$s ranged from .27 to .57, ps < .001). Given the high correlations between the two arithmetic measures ($r = .75$, $p < .001$), an arithmetic composite score was computed by averaging the standardized score of the two arithmetic measures. The scatter plots of the between various visual search indices and the arithmetic composite were shown in Figures 2a to 2d.

Table 3

Descriptive statistics and correlations among variables

Variable	Mean	S.D.	α	Correlations										
				Age	Raven	BSS	BCS	WR	IVS (acc.)	IVS (RT)	EVS (acc.)	EVS (RT)	AF	AC
Age	8.10	.40	--	--										
Raven	23.97	5.24	.86	.00	--									
Backward syllable span	4.99	1.71	.55	.00	.33***	--								
Backward Corsi span	7.20	2.88	.77	.10	.44***	.25***	--							
Word reading	33.21	26.31	.98	.05	.36***.	.30***	.30***	--						
Inefficient visual search (acc.)	.94	.01	.55	-.04	42***	.25***	.21**	.23***	--					
Inefficiency visual search (RT)	19.58	3.74	.87	-.11	-.12	-.19**	-.29***	-.13*	.15*	--				
Efficient visual search (acc.)	.98	.03	.40	-.12	.35***	.22***	.10	.27***	.47***	.13*	--			
Efficient visual search (RT)	12.95	2.82	.94	-.17**	-.06	-.15*	-.22***	-.05	.23***	.80***	.23***	--		
Arithmetic fact retrieval	28.19	14.10	.97	.12	.49***	.27***	.49***	.57***	.24***	-.33***	.19**	-.26***	--	
Arithmetic computation	13.38	6.26	.92	.07	.48***	.34***	.48***	.57***	.19**	-.35***	.13*	-.26***	.75***	--

* $p < .05$, ** $p < .01$, *** $p < .001$



a) b)

c) d)

Figures 2a and 2d – Scatter plots on the relation between arithmetic composite and (a) inefficient visual search accuracy, (b) inefficient visual search reaction time, (c) efficient visual search accuracy, (d) efficient visual search reaction time.

Regression analyses

Two hierarchical regression analyses were conducted to examine whether the two visual search indices (efficient vs. inefficient visual search) significantly predicted the arithmetic outcomes. All the control variables (i.e., age, intelligence, working memory, word reading) were put in the first block, while the visual search indices were placed in the second block. The arithmetic composite was the dependent variable. Separate regression analyses were done for accuracy versus reaction time of the visual search tasks. All variables were standardized prior to the regression analyses. The two visual search accuracy measures did not account for any unique variance of arithmetic composite after controlling for the control variables, $\Delta R^2 = .04$, $F(2,252) = 1.08$, $p = .34$; and neither of them significantly predicted arithmetic composite ($|\beta| < .1$, *p*s > .2). However, the two visual search reaction time measures did account for a significant amount of variance of arithmetic composite, $\Delta R^2 = .04$, $F(2,252) = 11.67$, $p < .001$, after considering the control variables in the model (see Table 4 for details). The examination of the coefficients, however, suggested that only inefficient visual search reaction time ($\beta = -.15$, $p = .018$), but not efficient visual search reaction time ($\beta = .05$, $p = .45$), significantly predicted the arithmetic composite. Other significant predictors of arithmetic composite included intelligence ($\beta = .22$, $p < .001$), visuospatial working memory ($\beta = .21$, $p < .001$) and word reading ($\beta = .39$, $p < .001$). Multicollinearity was not an issue here as all the predictors had variance inflation factor (VIF) of 3 or smaller. The results suggested that only inefficiency visual search, but not efficient visual search, was significantly related to children’s arithmetic performance.

Table 4

Regression predicting arithmetic composite

Step	Predictors	Based on accuracy					Based on reaction time				
		R^2	ΔR^2	β	t	VIF	R^2	ΔR^2	β	t	VIF
1	Age	.55	.55***	.04	1.10	1.03	.55	.55***	.03	.82	1.04
	Intelligence			.23	4.67***	1.59			.22	4.93***	1.40
	Verbal WM			.06	1.36	1.20			.03	.60	1.20
	Visuospatial WM			.25	5.53***	1.32			.21	4.63***	1.38
	Word reading			.40	9.29***	1.26			.39	9.45***	1.24
2	Inefficient visual search	.55	.00	-.01	-.22	1.44	.59	.04***	-.15	-2.38*	2.88
	Efficient visual search			-.07	-1.27	1.40			-.05	-.76	2.82

* $p < .05$, ** $p < .01$, *** $p < .001$

Mediation analysis

Given the significant relation observed between effortful, goal-directed serial processing and children's arithmetic competence, the dot enumeration task was further introduced to examine whether children's enumeration skills mediated the aforementioned relation. However, prior to the mediation analysis, the correlations between different dot enumeration indices (i.e., subitizing range, accuracy and reaction time for subitizing, counting, and the overall enumeration process) and the major variables (i.e., inefficient visual search reaction time, arithmetic composite) were compared (see Table 5). Although all the enumeration indices correlated significantly with the arithmetic outcomes ($|r|$s ranged from .22 to .62, ps $< .01$), only subitizing range ($r = -.19$, $p = .03$) and the reaction time measures ($|r|$s ranged from .25 to .29, ps $< .01$) significantly correlated with the reaction time of inefficient visual search. Among these indices, the correlation between overall enumeration reaction time and other variables were slightly but consistently stronger than those of the other enumeration indices (i.e., subitizing range, subitizing and counting reaction time). The overall enumeration reaction time was therefore taken as the indicator of children's enumeration skills.

Table 3

Descriptive statistics of the enumeration variables and correlations between enumeration variables and other variables

Variable	Mean	S.D.	α	Correlations										
				Age	Raven	BSS	BCS	WR	IVS (acc.)	IVS (RT)	EVS (acc.)	EVS (RT)	AF	AC
Subitizing range	3.16	.56	NA	.18*	.26**	.04	.23**	.18*	.12	-.19*	-.04	-.15	.33***	.22**
Subitizing acc.[†]	1.00	0.00	NA[††]	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
Subitizing RT	1.33	.22	NA[††]	-.26**	-.13	-.22*	-.20*	-.25**	-.18*	.27**	-.11	.17*	-.41***	-.36***
Counting acc.	.96	.04	NA[††]	.01	.14	.15	.20*	.16	.10	-.15	.06	-.09	.30***	.28**
Counting RT	3.64	.71	NA[††]	-.15	-.17*	-.28**	-.23**	-.35***	-.23**	.25**	-.19*	.14	-.53***	-.49***
Enumeration acc.	.98	.03	.60	.02	.16	.17*	.22**	.17*	.11	-.16	.05	-.11	.31***	.30***
Enumeration RT	2.79	.50	.92	-.23**	-.24**	-.30***	-.29***	-.42***	-.27**	.29**	-.17*	.18*	-.62***	-.54***

* $p < .05$, ** $p < .01$, *** $p < .001$

[†] After deleting the outliers, no variability was observed for subitizing accuracy.

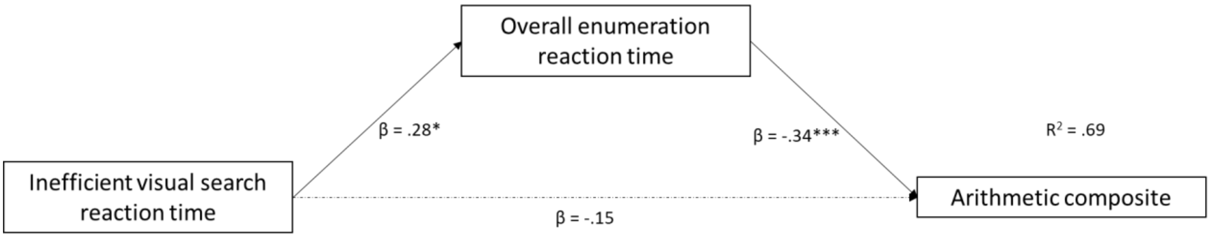
[††] Reliability was not calculated because different participants had different subitizing range, so the accuracy and RT were based on different number of items.

BSS = backward syllable span, BCS = backward Corsi span, WR = word reading, IVS = inefficient visual search, EVS = efficient visual search, AF = arithmetic fact retrieval, AC = arithmetic computation

A mediation analysis was then conducted to examine whether children's enumeration skills mediated the relation between visual attention and arithmetic competence. As it was shown that only inefficient visual search reaction time, but not other visual search indices, contributed to children's arithmetic skills, the inefficient visual search reaction time served as the independent variable in the mediation model, while the efficient visual search reaction time, together with other potential confounding variables (i.e., age, intelligence, working memory and reading) were put into the models as control variables. Children's overall enumeration reaction time served as the mediator, while the arithmetic composite served as the dependent variable in the mediation model. The mediation analysis was conducted using the PROCESS macro (Hayes, 2013) in SPSS. The bootstrapping procedure with 10,000 bootstrap samples and bias-corrected confidence intervals was applied. Such procedure involved selecting 10,000 bootstrap samples with replacement, and the point estimates for the indirect effects were calculated within each of the bootstrap sample. Based on the sampling distributions of these estimates, the 95% confidence intervals were then calculated. The indirect effects were considered statistically significant if the 95% confidence interval does not include 0.

In the mediation model (see Figure 3 for details), the inefficient visual search reaction time significantly predicted overall enumeration reaction time ($\beta = .28$, $p = .02$, 95% CI = .05 to .52), while the efficient visual search reaction time did not ($\beta = -.16$, $p = .21$, 95% CI =

-.40 to .09). The overall enumeration reaction time in turn predicted the arithmetic composite (β = -.34, $p < .001$, 95% CI = -.45 to -.22). The direct effect of inefficient visual search reaction time on the arithmetic composite became marginally significant when the mediator was included in the model (β = -.15, $p = .05$, 95% CI = -.31 to .00). Other significant predictors of arithmetic composite included intelligence (β = .30, $p < .001$, 95% CI = .19 to .41), visuospatial working memory (β = .14, $p < .01$, 95% CI = .04 to .25) and word reading skills (β = .28, $p < .001$, 95% CI = .17 to .39). A total of 68.64% of the variance in arithmetic composite was explained by the model. The investigation into the indirect paths suggested that the indirect path through overall enumeration reaction time was significant (β = -.09, 95% CI = -.17 to -.03). These results suggested that enumeration speed significantly mediated the relation between effortful, goal-directed serial attentional processing and children’s arithmetic competence.



* *p* < .05, ** *p* < .01, *** *p* < .001

Figure 3 – The mediation pathway from inefficient visual search reaction time to arithmetic composite through overall enumeration reaction time. For simplicity, control variables were not shown in the figure.

Discussion

Based on the established relation between visual attention and mathematical competence (Anobile et al., 2013; Steele et al., 2012), the current study aimed at clarifying such relation in two important ways: (1) to examine whether it was the automatic, stimulus-driven parallel processing or the effortful, goal-directed serial processing that drove the relation between visual attention and arithmetic competence, and (2) to explore the potential mechanism explaining the relation between visual attention and arithmetic competence. The current findings suggested that it was the effortful, goal-directed visual attention, but not automatic, stimulus-driven visual attention, that predicted children's arithmetic competence. Furthermore, such relation was accounted for by children's enumeration skills. All these relations were driven by reaction time instead of accuracy, probably due to the close to ceiling performance in the accuracy measures. Theoretical and practical implications were discussed in the following.

The role of effortful versus automatic visual attention in arithmetic competence

The relation between visual attention and arithmetic competence had been established in two recent studies (Anobile et al., 2013; Steele et al., 2012). While Anobile et al. (2013) had established a concurrent relation between visual attention and the magnitude factor of mathematics achievement, Steele et al. (2012) had demonstrated that the sustained-selective attention factor (in which visual search was a major component) longitudinally predicted numeracy skills one year later. Although converged to support the link between visual attention and arithmetic, these studies did not clarify whether it was automatic, stimulus-driven parallel processing or effortful, goal-directed serial processing that drove the relation. The current study clarified this issue through including two visual search tasks concurrently so that their relative contribution to arithmetic competence could be examined. The finding suggested that only effortful, goal-directed serial processing was accounted for unique variance in children's arithmetic competence, thus confirming the role of goal-directed serial processing, but not automatic, stimulus-driven parallel processing, in children's arithmetic competence. Given the role of the former attentional process in basic numerical competence such as subitizing (Vetter et al., 2008) and counting (Simon & Vaishnavi, 1996), the current finding is not surprising. Future researchers may focus on the effortful, goal-directed serial attention process when they further explore the visual attention-arithmetic link.

The mechanism underlying the relation between visual attention and arithmetic

After confirming the component of visual attention that drove the relation between visual attention and arithmetic competence, the current study went one step further by exploring the potential mechanism underlying the relation. As both the subitizing (Vetter et al., 2008) and the counting (Simon & Vaishnavi, 1996) processes were found to involve effortful, goal-directed serial processing, and that enumeration skills as a whole was found to be significantly correlated with children’s arithmetic competence (Lyons et al., 2014; Reigosa-Crespo et al., 2012), children’s enumeration skills were considered as a potential mediator in the current study. While it would be ideal to further disentangle the two enumeration sub-processes, this separation is empirically difficult given that some children might group the stimuli into subitizable units (Starkey & McCandliss, 2014). Based on the possibility of groupitization, as well as the slightly but consistently stronger correlations between overall enumeration speed (compared to subitzing range, and subitizing and counting speeds) and inefficient visual search speed and arithmetic competence, the overall enumeration speed was included in the mediation analysis as the mediator. The mediation turned out to be significant, suggesting that the relation between effortful, goal-directed serial processing and arithmetic competence could be at least partially explained by enumeration speed. In other words, children with more efficient effortful, goal-directed visual attention are quicker in enumerating quantities, and this higher enumeration speed in turn relates to higher arithmetic competence. Future studies may employ different dot enumeration paradigms (e.g., unstructured versus grouped conditions as in Starkey and McCandliss, 2014) to further explore which enumeration sub-processes (i.e., subitizing, counting, or groupitzing) actually drives such a relation. It should also be noted that the direct effect from inefficient visual search speed to arithmetic competence remained to be marginally significant after accounting for the effect of enumeration speed, suggesting that other mechanisms might be in play.

Future studies may explore the cognitive capacities that explain the remaining variance of the relation between effortful, goal-directed visual attention and arithmetic competence.

Theoretical and practical significance

The current study has enriched the literature by elaborating the nature of the relation between visual attention and arithmetic competence (Anobile et al., 2013; Steele et al., 2012). Only effortful, goal-directed visual attention was shown to be related to one's arithmetic competence, and such relation was mediated through enumeration speed. The current behavioural findings echo with the existing neurological findings, which suggest that the intraparietal sulcus are involved in both goal-directed visual attention (Corbetta & Shulman, 2002) and symbolic number processing (Bugden, Price, McLean, & Ansari, 2012). The current findings, together with other similar findings in the field (Vukovic et al., 2014), highlight the importance of fundamental cognitive ability in learning arithmetic through illustrating how they may facilitate the basic numerical skills. Children who have difficulties in focusing their visual attention may be less efficient when they need to enumerate a set of objects, and such inefficient enumeration may further result in a slower development in their arithmetic competence. Such findings have important implications on the population with dyscalculia as well. Research on dyscalculia, or mathematics learning disability (MLD), suggested that at least some of children with dyscalculia/MLD suffer from a deficit in enumeration (Chan & Wong, 2019; Olsson, Östergren, & Träff, 2016). As enumeration was shown to be related to our effortful, goal-directed visual attention system, it is worth investigating whether a deficit in such visual attention system underlies the difficulties faced by children with this subtype of MLD.

Practically, the current findings may provide insights concerning how to improve children's arithmetic competence. Various previous studies suggested that our visual attention can be improved through action video game playing (Ashkenazi & Henik, 2012; Green & Bavelier, 2003, 2007). Green and Bavelier (2003), for instance, have shown that non-video-game-players who had been trained on an action video game for 10 days demonstrated significant improvement in various aspects of attention, including a reduction in attentional blink and an enhancement in the allocation of spatial attention over the visual field. More importantly, the enumeration performance among this group of participants also improved. Such improvement was not observed among the control group participants who were trained on a non-action video game. Ashkenazi and Henik (2012) and Libertus and colleagues (2017) went one step further to test whether such action video game playing improved the arithmetic competence of adults. The results revealed a rather complicated picture. Ashkenazi and Henik (2012) showed that the attentional performance of participants did improve after video game playing, such effect shows limited generalizability to their mathematical competence. On the other hand, Libertus and colleagues (2017) did find a significant improvement in the computational skills of participants who had played action video games, but their attentional capacities and basic numerical skills did not show similar improvement. The small sample size in these studies (n = 9 and 12 in each group) might have limited the power for detecting significant differences, and future studies may further investigate the issue with a larger sample size.

Limitations

While the current findings demonstrated that children’s effortful, goal-directed serial visual attention is related to their arithmetic competence, readers should be reminded that the current study was a correlational study and thus the issue of causality cannot be addressed. Readers should also be aware that the current study focused only on visual attention, which did not allow us to tell whether the relation is modality general (applies to all modalities of attention tasks) or modality specific (applies only to the visual modality). Future studies may include attention tasks of different modalities to investigate the specificity of this relation. Furthermore, the current study focused on second graders only. The role of visual attention on arithmetic competence may vary across the development, whether such relation, as well as the mechanism behind such relation, holds for other age groups remains to be explored by future studies. Finally, the current study focused only on arithmetic competence among a large variety of mathematical competence. In order to obtain a more comprehensive understanding on this topic, other aspects of mathematical competence should be investigated. Word problems, for example, is another important aspect of mathematical competence, and the process of solving word problems is expected to be related to visual attention because in addition to the computation process, solving word problems require students to read (Fuchs, Fuchs, Compton, Hamlett, & Wang, 2015; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008), and reading has been shown to require visual attention (Franceschini, Gori, Ruffino, Pedrolli, & Facoetti, 2012; Liu et al., 2015; Plaza & Cohen, 2007). Furthermore, in the word-problem solving process, students need to identify key words from the word problems and sometimes even suppress the irrelevant information that are present (Krawec, 2014). Such process may also require visual attention. Future studies should therefore look into the relation between visual attention and word-problem solving.

Conclusion

The current study has clarified the nature of relation between visual attention and children's arithmetic competence in two important ways: first, it was effortful, goal-directed serial visual attention that drove the relation between visual attention and arithmetic competence, and second, the relation was mediated through children's enumeration speed Children who have poor goal-directed visual attention may enumerate objects at a slower rate, which is further linked to their low arithmetic competence. The findings have highlighted the importance of fundamental, domain-general cognitive capacities in mathematics learning and provided insights to the potential intervention strategies for improving children's arithmetic competence.

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