Background

Dot enumeration is the basic mathematics competency in young children and a significant indicator of later mathematics achievement.

Aim

The present study focused on (a) how children's dot enumeration ability changed as they progressed from late kindergarten years (K3) to the second year of primary school in Hong Kong (P2); and (b) the extent to which such changes are associated with students' mathematics outcomes assessed at the fourth grade, including standardized mathematics achievement, whole number magnitude understanding and rational number concept.

Sample (s)

Two hundred and eleven Hong Kong kindergarteners were recruited.

Methods

The participants' dot enumeration was assessed from K3 to P2. Their mathematics outcomes were assessed at P4, including standardized mathematics achievement, whole number magnitude understanding and rational number concept.

Results

The changes in their dot enumeration speed reflected a linear growth pattern. Further, individual growth rates of dot enumeration predicted whole number magnitude understanding and rational number concept two years later.

Conclusions

The results indicate the importance of focusing on children's growth in a specific mathematics skill in addition to their status at one single time point. Practical implications are discussed in this article.

Key words: dot enumeration; growth rate; rational number concept; latent growth curve modeling

The Growth Rates of Dot Enumeration Ability Predict Mathematics Achievements: A 5-Year Longitudinal Study

Two number processing skills have been proposed as core mathematics competencies, dot enumeration and magnitude comparison (e.g., Reeve, Reynolds, Humberstone, & Butterworth, 2012). While the dot enumeration refers to the representation of the numerosity of an array of dots using number symbols, magnitude comparison is more about the comparison of relative magnitude of two numerosities, be it symbolic (e.g., Arabic numeral) or nonsymbolic (e.g., array of dots; Reeve et al., 2012). As numerosity has been proposed to be mentally represented with approximate magnitude codes (Dehaene, 1997), an enumeration process is needed to precisely access the representation of numerosities (Schleifer & Landerl, 2011). Dot enumeration has received much less attention than magnitude comparison although it has been reported to be a significant indicator of mathematical competency in both typically-developing children and mathematical learning difficulties in atypical development (e.g., Gray & Reeve, 2014; Gray & Reeve, 2016). Very little is known about the developmental trajectories of dot enumeration in young children (Reeve et al., 2012). Further, it is unknown if the growth rate of dot enumeration can be treated as an important predictor of mathematics achievement beyond the dot enumeration status assessed at a single time point.

The present study aimed, first, to quantify the changes in dot enumeration and, second, to use the changes as an indicator to predict mathematics achievement. It sought to contribute to understanding of dot enumeration and its power to predict longitudinal mathematical development.

Dot Enumeration

To succeed in dot enumeration, children need to have precise mental representation of small quantities, knowledge of numbers (i.e., counting list) and understanding of the mapping principle between the two. Typically, when the quantity is below four, the enumeration is assumed to be a subitizing process. Subitizing describes a process of identifying the amount of small quantities rapidly without consciously accessing to individual objects (Kaufman, Lord, Reese, & Volkmann, 1949). When the quantity goes beyond four, the enumeration is assumed to be a counting process where each single object in an array is accessed so that the total number of objects can be identified (Schleifer & Landerl, 2011). Some skilled enumerator may also make use of the addition/decomposition strategy in the numeration process (e.g., decomposing 7 dots into 4 dots + 3 dots; Camos, 2003). During development, such enumeration process enables us to establish the first symbolic number representation. The standard dot enumeration task also involves mapping knowledge, that is, the knowledge that there is a connection between quantities and numbers.

The Development of Dot Enumeration

Dot enumeration skills improve along development. Starkey and Cooper (1995) found that the subitizing range increased with age from 1-3 among infants, 1-4 among 3-to 5-year-olds, and up to 1-5 in adulthood. In Hong Kong, counting is formally introduced since 3 years old in school. Kindergarteners, by their third year in the kindergartens (i.e., 6 years old), are able to count to around 50 (Liu, Lin, & Zhang, 2016). Although accuracy in subitizing and counting in the range of 1-9 (which are typically assessed using the standard dot enumeration task) are considered to develop to a certain level of maturity by the end of kindergarten years (around 5-6 years old) (Liu, Lin, & Zhang, 2016), the efficiency of dot enumeration is hypothesized to improve progressively with age due to children’s growth in number knowledge in school.

There are two major shapes of developmental trajectory (Elman, Bates, Johnson, Karmiloff-Smith, Parisi, & Plunkett, 1998; Rojas & Iglesias, 2013). Linear growth indicates that dot enumeration competency increases continuously and linearly during a period. Nonlinear growth indicates that children's dot enumeration may increase quickly at some times and slowly at others. Regarding nonlinear developmental patterns, we focused on two types, quadratic patterns and latent basis growth. The former indicates that, with time, changing rates of knowledge/skills become faster/slower. The second indicates that, with time, children's growth in dot enumeration is not stable, rather, it is alternately fast and slow. The present study tested which of the three growth patterns applies to the development of dot enumeration ability in young Hong Kong children from late kindergarten to early primary school.

Importance of Growth Rates in Predicting Mathematics Learning

The novelty of the present study lies in the examination of the developmental progression of dot enumeration, which touches on a different concept which is unique in longitudinal studies. In studies focusing on predicting children's mathematical abilities, the typical approach is to find precursors of some abilities a certain length of time later by assessing children's performances of the precursors at one single time point. Doing this may possibly neglect how the abilities change over the study period.

Previous studies mainly have two gaps in investigating the importance of changes in dot enumeration ability in mathematics outcomes. First, there is no empirical studies, to the best of our knowledge, that has investigated the predictive power of growth rates of dot enumeration in mathematics outcomes beyond the initial level of dot enumeration. Theoretically, when one focuses on the developmental trajectories of children's knowledge/skills, there are two concepts, initial level and growth rate. These two concepts are related but distinct in many ways. This is because performance at one time point reflects the total amount of a child's specific knowledge/skills for that moment, which is a description of static status. The growth of children's abilities is related not only to quantitative changes over development, but also to qualitative changes in conceptual understanding. In our study, initial level of dot enumeration was assessed at the kindergarten stage, which is assumed to reflect Hong Kong children's basic knowledge in counting, e.g., knowledge about counting list (Fuson et al., 1982; Liu, Lin, & Zhang, 2016). Over the development during the next two years, whole number knowledge is introduced intensively in school instruction. Whole number knowledge highly emphasizes magnitude understanding of all whole numbers (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Siegler & Opfer, 2003; Siegler & Booth, 2004). During the first two years of primary school, children may undergo a process in which they are instructed to develop their magnitude understanding of whole numbers and then, either spontaneously or instructed, employ this magnitude understanding to reciprocally advance their previously learned dot enumeration processing. Thus, the growth of dot enumeration during this period is very likely to be affected by integration of old and new interpretation of numbers. Our study thus investigated the importance of this dynamic development of dot enumeration in predicting three mathematics outcomes while the initial level of dot enumeration is controlled.

Second, very few studies have investigated whether dot enumeration is a strong predictor of mathematics achievement, especially rational number concept, while whole number knowledge is controlled. Counting is considered to somewhat impede young children's rational number learning to a large extent because the two are different in many ways (see Ni & Zhou, 2005 for a review). For example, counting involves concrete objects and numbers but rational number concept is related to processing continuous quantities. Whole number knowledge, in contrast to counting, is often used by the researchers in the field to refer to children's knowledge of the whole number system, the number-space relation over the number line, the arithmetic rules over whole numbers (e.g., Laski & Siegler, 2007; Siegler & Opfer, 2003). Previous studies have identified the strong predictive power of whole number knowledge in fraction learning (Bailey, Siegler, & Geary, 2014; Hansen et al., 2015; Jordan et al., 2013; Vukovic et al., 2014). However, it is not known in empirical studies whether dot enumeration would predict rational number concept beyond whole number knowledge. In the present study, we investigated both the initial level and growth rate of dot enumeration in predicting rational number concept. Whole number knowledge was assessed with children's whole number line estimation at second grade (Bailey, Siegler, & Geary, 2014; Hansen et al., 2015; Jordan et al., 2013; Vukovic et al., 2014).

Reeve et al. (2012) examined children's dot enumeration competency in late kindergarten and primary school children. Their findings revealed that children's performance in dot enumeration could be divided into three subgroups and the subgroups were quite consistent over development. Their focus was whether individuals' performance category was consistent over development. Our focus was on the growth shape of dot enumeration across three years and asked if it was linear or not. Further, their study used children's subgroup on sixth grades to predict further computation competency. As stated above, our predictor, growth rates of dot enumeration, is different from these studies because it demonstrates dynamic changes in knowledge rather than static status which is point estimation. Moreover, our study examined three mathematics outcomes (including rational number concept, a high level of number knowledge) and controlled three domain-general cognitive variables and most proximal whole number magnitude understanding when the strong effect of dot enumeration was investigated. Together with the approach and results by Reeve et al. (2012), our study would make contribution to understanding of the fundamental role of dot enumeration in the development of mathematics learning.

The Present Study

Our hypothesis is that the growth rates of dot enumeration would predict mathematics outcomes, including standardized mathematics achievement, whole number magnitude understanding and more importantly, rational number concept. During the study, children's domain-general abilities (phonological processing, visual-spatial skills and working memory) and the most proximal whole number knowledge were included as control variables.

The present study followed a group of children for five years, from the third year of kindergarten (K3) to the fourth year of primary school (P4). The children's dot enumeration ability was tracked for the first three years and the growth rates were calculated. It is important to choose the appropriate criteria to measure developmental growth to avoid any ceiling effect in studying longitudinal development of children's ability (Petscher, Quinn, & Wagner, 2016). If the focus is on counting accuracy in the range of 1-10 in primary school students, research may notice that the growth is faster at the beginning but relatively flat later. Thus, the present study focused on the speed of dot enumeration to avoid any developmental ceiling effect. Reeve et al. (2012) have revealed that speed of dot enumeration reflects individual differences specific to the representation and processing of numbers rather than general processing speed.

Method

Participants and Procedure

Two hundred and eleven Hong Kong kindergarteners (111 boys and 100 girls) were recruited (mean age = 73 months; *SD* = 4 months), and 124 of them (66 boys and 58 girls; mean age = 121 months, *SD* = 4 months) retained in the final sample at Time 5 (fourth grade).

Although these children came from different socioeconomic backgrounds, all spoke Cantonese as their native language; had normal or corrected-to-normal vision; and, according to school personnel, had no known developmental disabilities. The corresponding author had obtained approval from Ethics Committee for the research. Written consent to participate was obtained from children's parents/guardians.

The participants were in K3 when they were first assessed (T1). Their dot enumeration skills were assessed four times, in the second term of the third year of kindergarten (i.e., K3 [T1], at which time they were also assessed for their phonological processing, visual-spatial skills and working memory capacity), in the first semester of the first grade (i.e., P1S1 [T2]), the second semester of the first grade (i.e., P1S2 [T3]), and the second semester of the second grade (i.e., P2S2 [T4]), respectively. At T4, we assessed the children’s whole number line estimation to control for their existing whole number knowledge. Two years later, in the second semester of fourth grade (i.e., P4S2 [T5]), we assessed the outcomes, namely the standardized mathematics achievement, whole number magnitude understanding and rational number concept. All tasks were conducted individually by trained experimenters, who were undergraduates with a major in either psychology or education, or the second author.

Measures

Dot enumeration. Butterworth (2003)’s dot-number matching task has been widely used to assess dot enumeration (e.g., Bartelet, Vaessen, Blomert, & Ansari, 2014; Iuculano, Tang, Hall, & Butterworth, 2008; Reigosa-Crespo et al., 2012; Rodic et al., 2015). Four practice trials were presented to familiarize children with the task. For the test trials, dots ranging from 1-9 were presented on half of the screen. Meanwhile, on the other half, a number in the form of an Arabic digit was shown. The participants were asked to judge if the Arabic number matched the quantity of dots. The participants were asked to press F when they matched and J when they did not. There were 36 trials in total. The total numbers of correct responses and reaction times were recorded.

Standardized mathematics achievement. It was assessed with the Learning Achievement Measurement Kit (LAMK). The LAMK is a locally normed, standardized mathematics achievement test developed by the Hong Kong Education Bureau. It is adopted by the Hong Kong Education Bureau to understand primary school students' math competencies, identify students with delays in mathematics for possible intervention, and possibly influence distribution of education resources (Education Bureau, 2012). The task has been used frequently in Hong Kong to assess young children's mathematics achievement (e.g., Chan, 2014; Wong et al., 2018).

The task included 41 items which assessed children's whole number knowledge, fractions, arithmetic competency using fractions and whole numbers, knowledge of shapes and time. The questions were presented in the forms of word problem and pictures.

Whole number magnitude understanding. A computerized number line estimation task was administered to each child (Siegler & Opfer, 2003; Siegler & Booth, 2004). On each trial, a number line was presented with 0 at one end and 100 at the other end. At the centre and above of the number line, there was a target number. The children were asked to estimate the location of the target number on the number line without measurement tools. The target numbers involved 3, 7, 19, 52, 103, 158, 240, 297, 346, 371, 438, 475, 502, 586, 613, 690, 721, 760, 835, 874, 907, and 962.

Rational number concept. The rational number concept task assessed the children's understanding of fractions and decimals using seven different sub-tasks, including both nonsymbolic-to-symbolic and symbolic-to-nonsymbolic transformation of fractions and decimals, simple fraction addition and multiplication, and decimal comparison. For example, one item in the nonsymbolic-to-symbolic transformation of fractions was to present the children with a square divided into four equal parts, two of which were shaded. The children were asked to write down the fractions that matched the shaded portion. The maximum scores were 44.

Phonological processing. It was assessed using a syllable recall task. The participants listened to a series of Cantonese syllables at a pace of one syllable per sound. Their task was to recall the syllables in the exact order after listening. After one practice trial, the participants were given a total of 15 trials arranged in five difficulty levels. The first item consisted of three syllables, and the number of syllables increased by one in every three trials. The participants got one mark for each correctly recalled syllable as well as each correct recall order.

Visual-spatial skills. This was measured using the Corsi Block task (Corsi, 1972). The experimenter showed nine identical black boxes positioned on a blackboard. On each trials, the experimenter tapped a particular sequence of boxes at the pace of one box per second. Afterwards, the participants tapped the same boxes that the experimenter showed, in the same sequence. The participants were first given a practice trial. The formal test included 10 trials arranged in five difficulty levels (i.e., two trials for each level), which started from the sequence length of two blocks. When the participants succeeded on both items, they moved on to the next higher sequence length, which increased by one. The task was terminated when the participants failed both responses in the same difficulty level. One point was given to each correct answer.

Working memory. The backward digit span was adopted to test the working memory at the verbal level (Alloway, & Alloway, 2010; Alloway, Gathercole, & Elliott, 2010). The experimenter verbally presented a sequence of digits at the pace of one digit per second. The children's task was to reproduce the digit sequence in the backward order. One practice trial was given. On the formal test, the 12 experimental trials were arranged in six difficulty levels. The first two items involved two digits, and the number of digits in each item increased by one in every two trials. If the participants failed both items at a particular difficulty level, the task was terminated. One mark was awarded for each correct answer.

Data Analysis

To evaluate the children's performance on the number line estimation task, a linear regression was calculated in which the subjective estimates were predicted by the objective magnitudes. The fit of this linear model (i.e., R^2) was used to measure the linearity of representation on the number line estimation tasks (Siegler & Booth, 2004). In the dot enumeration task, there was somewhat of a ceiling effect on the children's accuracy (accuracy across waves was 86%, 89%, 92% and 94%), thus children's reaction time was used to measure their performance in the task (Butterworth, 2003). Thus, in this task, smaller values represent better performance. In other tasks, the total scores of correct responses were used in the analyses.

Missing data in this period were due mainly to failure to locate the participants for the later waves. We did analyses to test whether students remained in the study were different from those who did not. Significant differences were only found in the dot enumeration task at T1; students who remained in the study showed faster responses at T1 in the dot enumeration task ($p = .042$). In all of the other variables, students who remained in the study were not different from those who left.

The analyses were conducted using Mplus 6 because it allows for the use of all available data through the Full Information Maximum Likelihood (FIML) function that handle missing data in longitudinal investigations (Muthén & Muthén, 1998–2010). This resulted in a sample size of 211 for all analyses. Indicators to evaluate the fitness of the model to the observed data included the χ^2-test, the Comparative Fit Index (CFI), the Tucker-Lewis Index (TLI), the Root Mean Square Error of Approximation (RMSEA), and the Standardized Root Mean Square Residual (SRMR). When a χ^2 test has a nonsignificant result, it indicates a good model fit. For both the CFI and TLI, values of .95 or greater usually indicate an adequate model fit. For RMSEA and SRMR, values of .05 or less indicate an adequate model fit (Hu & Bentler, 1999).

Results

Table 1 presents the means, standard deviations and correlations among measures. It can be seen that dot enumeration reaction time decreased across the four time points, suggesting that the participants became more efficient in enumerating dots as they grew older. A series of independent sample t-tests showed that gender had no effects upon all predictors (i.e., phonological processing, visual-spatial skills and working memory) and outcome variables (i.e., standardized mathematics achievement at T5, whole number magnitude understanding and rational number concept) except students' whole number line estimation at T4 (all other *p*s $> .10$). The linearity of boys' performance was higher than girls at T4 (mean difference = 0.05, $t = 3.438$, $p < .01$) but it did not hold two years later when students were at grade 4 (mean difference = .002, $t = .165$, $p = .869$).

First, three growth models were tested to describe the developmental trajectories of the children's dot enumeration abilities from K3 to P2S2, namely, the linear growth model, quadratic growth model and latent basis growth model. The factor loading on the intercept for the three models was 1. Growth was measured through fixing the loadings of the repeated measures on the slope factor to be equal to 0, 1, 2 and 4 (reflecting the interval of time); for the quadratic model, the loadings were 0, 1, 4 and 16. For the latent basis model, these loadings were assigned 0 and 1 respectively to the first time point (i.e. K3) and the last time point (i.e., P2S2). The factor loadings of the other time points in the latent basis model were allowed to vary freely. The quadratic model did not converge in Mplus, indicating the misspecification to data. The linear model showed a good fit, $\chi 2$ (5, *N* = 211) = 6.094 (*p* =.297), RMSEA=.032, CFI = .979, TLI = .975, SRMR = .066. The latent basis also showed a good model fit, $\chi 2$ (3, *N* = 211) = 2.841 (*p* =.41), RMSEA=.000, CFI = 1.00, TLI = 1.00, SRMR = .038. According to AIC and BIC values, smaller values were considered to be better. The linear growth model was chosen because it has smaller AIC and BIC. In the linear model, the intercept was 3338.9 ms (*SE* = 41.40) and the reaction time decreasing rates across the three years were 146.59 ms (*SE* =14.80). The initial dot enumeration at K3 (T1) was not related to growth rates of dot enumeration over the next three years (*r* = -.305, *p* = .278). Further analysis showed that gender has no effect on the intercept and growth rates of dot enumeration (both *p*s > .08).

Next, the intercept and slope factor scores were extracted from the above linear growth model in Mplus. To address the second research question, regression analyses were used in which the correlation between three dependent variables was considered. In the model, phonological processing, visual-spatial skills and working memory tested at T1 (i.e., K3) were included to control for the domain-general abilities. Whole number magnitude understanding at T4 (i.e., P2S2) was also included to control for children's proximal whole number knowledge. Gender was also included. All reported parameters are standardized results in Mplus.

Table 2 revealed that, in accounting for variations in standardized mathematics achievement, the intercept of dot enumeration ability (β = -.294, *SE* = .084, *p* < .001, 95% CI = [-.458, -.129]) was significant while the growth rate (β = -.107, *SE* = .068, *p* = .114, 95% CI = [-.240, .026]) was not significant. In accounting for variations in whole number magnitude understanding, both the intercept (β = -.295, *SE* = .113, $p < .01$, 95% CI = [-.517, -.073]) and the growth of dot enumeration were significant (β = -.179, *SE* = .078, $p = .022$, 95% CI = [-.332, -.025]). In accounting for variations in rational number concept, only the growth rate of dot enumeration ability was significant (β = -.176, *SE* = .072, $p = .014$, 95% CI = [-.317, -.035]) while the intercept was not (β = -.145, *SE* = .084, $p = .085$, 95% CI = [-.310, .020]). These held even when the children's whole number magnitude understanding in P2S2 were controlled. The R squares for standardized mathematics achievement, whole number magnitude understanding and rational number concept were 61.7%, 83.9% and 67.8%, respectively (all *p*s < .01).

Discussion

The present study followed a group of Hong Kong children's dot enumeration abilities from K3 to P2. Their three mathematics outcomes were assessed two years later at P4. First, the results revealed that their dot enumeration exhibited a linear growth pattern through the study period, which means that their speed of subitizing and counting became faster continuously and linearly. Second, the growth rates in their dot enumeration accounted for variations in whole number magnitude understanding and rational number concept beyond a battery of domain-general measures, initial dot enumeration ability, and their proximal whole number knowledge. The present study further demonstrated the importance of focusing on the developmental progression of early mathematical ability by showing its predictive power for long-term mathematical development.

The Development of Dot Enumeration

According to Piaget (1952), the ability to do/perform mathematical procedures is directly proportional to the increase in mental age. In support of the argument, the present findings

provide empirical evidence concerning the specific pattern of change/increase in the most basic numerical processing task over development, i.e., dot enumeration in the range of 1-9.
Dot enumeration speed increased across the 3 years in a linear way. The linear development of dot enumeration further suggests that this ability did not experience slow or flat growth during the late kindergarten to early primary school years; instead, after children have grasped the counting list in the 1-9 range, their efficiency in dealing with representation of this small range is still growing continuously in a linear way.

Our results provide further evidence suggesting that the development of children's abilities is not the same as the static status at a single point in time. The null correlation between initial level and growth rates of dot enumeration further suggests that the development of dot enumeration requires something different from the initial level of dot enumeration assessed in K3, which should be considered as a separate predictor in research.

The Predictive Power of Dot Enumeration

While the most proximal whole number knowledge was controlled, we found that the growth rates of dot enumeration consistently predicted two outcomes touching on whole number knowledge and rational numbers. In other words, it is not only the amount of knowledge the children owned at the beginning of the study that mattered; the changes in the amount of knowledge also mattered for their mathematics achievement two years later. Considering previous evidence (Gray & Reeve, 2014; Gray & Reeve, 2016; Reeve et al., 2012) and our findings, it suggests that dot enumeration, the early representation of small ranges of numerosities, is longitudinally essential in the acquisition of advanced mathematical competences.

A further interesting observation from this study was that, in predicting rational number concept, only the growth rates in dot enumeration were significant, rather than the initial level of dot enumeration at K3. Previous studies have found consistently that rational number concept relies largely on prior whole number knowledge (Hansen et al., 2015; Jordan et al., 2013; Siegler, Thompson, & Schneider, 2011; Vukovic et al., 2014). The present study found that growth rates in dot enumeration were still significant after controlling children's prior whole number magnitude understanding at T4. As Siegler, Thompson, and Schneider (2011) suggested, although fractions and decimals are assumed by most researchers to be different conceptually from whole numbers (Ni & Zhou, 2005), development of fractions and whole numbers should be considered from the perspective of continuity rather than discontinuity. At its core, the development of numerical knowledge involves the processing of number magnitude and extending the number line and number system to include all types of numbers, including fractions and decimals. Empirically, fractions have been found to be an integrative part of the number magnitude system at the behavioral and brain levels (Jacob & Nieder, 2009a, 2009b; Resnick et al., 2016). From this perspective, a main task for students in learning rational numbers is to integrate rational numbers into the existing number system by the means of magnitude representation, a fundamental view in Siegler, Thompson, and Schneider (2011). Therefore, the connection between the growth rate of dot enumeration and the learning of rational number concept is inferred to lie in that both require integrating new number knowledge to existing number system through magnitude understanding; this interpretation is well consistent with the classic theory on magnitude representation of all types of numbers (Dehaene, 1997). This argument is speculative and further empirical investigation is needed.

The test LAMK involves knowledge of shapes and times (e.g., clock), word reading abilities, comprehension of math word problems; it is not a pure number assessment. That is probably why the change of dot enumeration seems to be less important in this case. This is consistent with that Reeve et al. (2012)'s finding that dot enumeration speed merely reflects the number-specific processing rather than the domain-general processing.

Limitation, Contributions and Implications

One of the limitations of the current study is that the present findings maybe dependent on the school curriculum and instructions on mathematics. Readers need to be cautious when they generalize current findings to other instructional contexts. Overall, this study focused on one core mathematics competency, dot enumeration, in a 5-year longitudinal study. From K3 to P2S2, children's dot enumeration speeds increased continuously and linearly. Further, when we controlled the most proximal whole number knowledge and the initial level of dot enumeration at the start of the study, the growth rates over the 3 years showed significant predictive power in two mathematics outcomes assessed two years later, namely whole number magnitude knowledge and rational number concept.

This study, with a focus on the changes in dot enumeration, suggests that practitioners may focus on children's growth in mathematics abilities. Even in the primary school years, a simple number-processing concept like dot enumeration may demonstrate a continuous and likely linear growth rather than a flat one. A flat growth may indicate children's deficits in some underlying skills, for example, slow development of magnitude understanding. Furthermore, it is suggested that the developmental growth of dot enumeration be incorporated into students' math appraisal/assessment or criteria distinguishing children with mathematics difficulties from those without, in addition to the initial level. In Zhang et al. (2018), it is argued that students with low achievement can be classified as two distinct categories, namely, mathematics learning difficulty and persistent low achievement. For the two categories, school instruction should be tailored to children's needs. The discrimination of the two categories can be revealed by focusing on growth rates. For example, in children with mathematics learning difficulties, their growth rates during instruction are relatively flat while children with low mathematics achievement show improvement in response to instruction to some extent although these two categories of students have similar initial status of mathematics achievement. The more accurately we can identify different types of learning difficulties, the more possible it is for researchers and practitioners to provide individually tailored instruction. Focusing on developmental growth in dot enumeration is also meaningful in evaluating which dot-enumeration intervention programs are effective.

References

Alloway, T. P., & Alloway, R. G. (2010). Investigating the predictive roles of working memory and IQ in academic attainment. *Journal of Experimental Child Psychology, 106*(1), 20-29. https://doi.org/10.1016/j.jecp.2009.11.003

Alloway, T. P., Gathercole, S. E., & Elliott, J. (2010). Examining the link between working memory behaviour and academic attainment in children with ADHD. *Developmental Medicine & Child Neurology*, *52*(7), 632-636. https://doi.org/10.1111/j.1469-8749.2009.03603.x

Aunio, P., Aubrey, C., Godfrey, R., Pan, Y., & Liu, Y. (2008). Children's early numeracy in England, Finland and People's Republic of China. *International Journal of Early Years Education, 16*(3), 203-221. https://doi.org/10.1080/09669760802343881

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, *17*(5), 775-785. https://doi.org/10.1111/desc.12155

Bartelet, D., Vaessen, A., Blomert, L., & Ansari, D. (2014). What basic number processing measures in kindergarten explain unique variability in first-grade arithmetic proficiency?. *Journal of experimental child psychology*, *117*, 12-28. https://doi.org/10.1016/j.jecp.2013.08.010

Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical estimation in preschoolers. *Developmental Psychology, 46*(2), 545-551. https://doi.org/10.1037/a0017887

Butterworth, B. (2003). *Dyscalculia screener*. London: NFER Nelson Publishing Company Ltd.

Camos, V. (2003). Counting strategies from 5 years to adulthood: Adaptation to structural features. *European Journal of Psychology of Education, XVIII*, 251-265.

Corsi, P. M. 1972. Human memory and the medial temporal region of the brain. Dissertation Abstracts International, 34 (02), 891B. (University Microfilms No. AAI05–77717).

Chan, W. W. L. (2014). Understanding and processing numbers among Chinese children. *Psychology & Neuroscience, 7*(4), 583 -591. http://dx.doi.org/10.3922/j.psns.2014.4.18

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Elman J., Bates E., Johnson M., Karmiloff-Smith A., Parisi D., Plunkett K. (1998). *Rethinking innateness: A connectionist perspective on development*. Cambridge, MA: MIT press.

Fuson, K. C., Richards, J., & Briars, D. J. (1982). The acquisition and elaboration of the number word sequence. In C. Brainerd (Ed.), *Progress in cognitive development research: Vol 1. children's logical and mathematical cognition* (pp. 33-92). New York: Springer-Verlag.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, Mass.: Harvard University Press.

Gray, S. A., & Reeve, R. A. (2014). Preschoolers' dot enumeration abilities are markers of their arithmetic competence. *PloS One, 9*(4), e94428. https://doi.org/10.1371/journal.pone.0094428

Gray, S. A., & Reeve, R. A. (2016). Number-specific and general cognitive markers of preschoolers' math ability profiles. *Journal of Experimental Child Psychology*, *147*, 1-21. https://doi.org/ 10.1016/j.jecp.2016.02.004

Hansen, N., Jordan, N. C., Fernandez, E., Siegler, R. S., Fuchs, L., Gersten, R., & Micklos, D. (2015). General and math-specific predictors of sixth-graders' knowledge of fractions. *Cognitive Development*, *35*, 34-49. https://doi.org/ 10.1016/j.cogdev.2015.02.001

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling: A Multidisciplinary Journal*, *6*(1), 1-55. https://doi.org/ 10.1080/10705519909540118

Iuculano, T., Tang, J., Hall, C. W., & Butterworth, B. (2008). Core information processing deficits in developmental dyscalculia and low numeracy. *Developmental Science, 11*(5), 669-680. https://doi.org/ 10.1111/j.1467-7687.2008.00716.x

Jacob, S. N., & Nieder, A. (2009a). Notation-independent representation of fractions in the human parietal cortex. *The Journal of Neuroscience, 29*(14), 4652-4657. https://doi.org/10.1523/JNEUROSCI.0651-09.2009

Jacob, S. N., & Nieder, A. (2009b). Tuning to non-symbolic proportions in the human frontoparietal cortex. *European Journal of Neuroscience, 30*(7), 1432-1442. https://doi.org/10.1111/j.1460-9568.2009.06932.x

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology, 116*(1), 45-58. https://doi.org/ 10.1016/j.jecp.2013.02.001

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice, 22*(1), 36-46. https://doi.org/10.1111/j.1540-5826.2007.00229.x

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *The American Journal of Psychology, 62*(4), 498-525.

Laski, E. V., & Siegler, R. S. (2007). Is 27 a big number? correlational and causal connections among numerical categorization, number line estimation, and numerical magnitude comparison. *Child Development, 78*(6), 1723-1743. https://doi.org/10.1111/j.1467-8624.2007.01087.x

Liu, Y., Lin, D., & Zhang, X. (2016). Morphological awareness longitudinally predicts counting ability in Chinese kindergarteners. *Learning and Individual Differences, 47*, 215-221. https://doi.org/10.1016/j.lindif.2016.01.007

Muthén, L. K., & Muthén, B. O. (1998-2010). *Mplus user's guide (vol. 6)*. Los Angeles, CA: Muthén & Muthén.

Ni, Y., & Zhou, Y. D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist, 40*(1), 27-52. https://doi.org/10.1207/s15326985ep4001_3

Piaget, J. (1952). *The child's conception of number*. C. Gattegno & F. M. Hodgson, Trans. New York: Routledge & Kegan Paul Ltd.

Petscher, Y., Quinn, J. M., & Wagner, R. K. (2016). Modeling the co-development of correlated processes with longitudinal and cross-construct effects. *Developmental Psychology*, *52*(11), 1690-1704. https://doi.org/ 10.1037/dev0000172

Reeve, R., Reynolds, F., Humberstone, J., & Butterworth, B. (2012). Stability and change in markers of core numerical competencies. *Journal of Experimental Psychology: General*, *141*(4), 649-666. https://doi.org/10.1037/a0027520

Reigosa-Crespo, V., Valdés-Sosa, M., Butterworth, B., Estévez, N., Rodríguez, M., Santos, E., . . . Lage, A. (2012). Basic numerical capacities and prevalence of developmental dyscalculia: The Havana survey. *Developmental Psychology*, *48*(1), 123-135. https://doi.org/10.1037/a0025356

Resnick, I., Jordan, N. C., Hansen, N., Rajan, V., Rodrigues, J., Siegler, R. S., & Fuchs, L. S. (2016). Developmental growth trajectories in understanding of fraction magnitude from fourth through sixth grade. *Developmental Psychology*, *52*(5), 746-757. https://doi.org/10.1037/dev0000102

Rodic, M., Zhou, X., Tikhomirova, T., Wei, W., Malykh, S., Ismatulina, V., . . . Lemelin, J. (2015). Cross-cultural investigation into cognitive underpinnings of individual differences in early arithmetic. *Developmental Science*, *18*(1), 165-174. https://doi.org/10.1111/desc.12204

Rojas, R., & Iglesias, A. (2013). The language growth of Spanish-Speaking English language learners. *Child Development, 84*(2), 630-646. https://doi.org/10.1111/j.1467-8624.2012.01871.x

Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. *Developmental Science, 14*(2), 280-291.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, *75*(2), 428-444. https://doi.org/10.1111/j.1467-8624.2004.00684.x

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation evidence for multiple representations of numerical quantity. *Psychological Science, 14*(3), 237-250. https://doi.org/10.1111/1467-9280.02438

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62*(4), 273-296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Siegler, R. S. (2016). Continuity and change in the field of cognitive development and in the perspectives of one cognitive developmentalist. *Child Development Perspectives, 10*(2), 128-133. https://doi.org/10.1111/cdep.12173

Sophian, C. (2000). Perceptions of proportionality in young children: Matching spatial ratios. *Cognition*, *75*(2), 145-170. https://doi.org/ 10.1016/S0010-0277(00)00062-7

Sophian, C., & Madrid, S. (2003). Young children's reasoning about many-to-one correspondences. *Child Development*, *74*(5), 1418-1432. https://doi.org/ 10.1111/1467-8624.00615

Starkey, P., & Cooper, R. G. (1995). The development of subitizing in young children. *British Journal of Developmental Psychology*, *13*(4), 399-420. https://doi.org/10.1111/j.2044-835X.1995.tb00688.x

The Education Bureau HKSAR. (2006). *Guide to the pre-primary curriculum*. Hong Kong: Government Printing Department.

Vukovic, R. K., Fuchs, L. S., Geary, D. C., Jordan, N. C., Gersten, R., & Siegler, R. S. (2014). Sources of individual differences in children's understanding of fractions. *Child Development, 85*(4), 1461-1476. https://doi.org/ 10.1111/cdev.12218

Wong, R. S. M., Ho, F. K. W., Wong, W. H. S., Tung, K. T. S., Chow, C. B., Rao, N., . . . Ip, P. (2018). Parental involvement in primary school education: Its relationship with children’s academic performance and psychosocial competence through engaging children with school. *Journal of Child and Family Studies, 27*(5), 1544-1555. https://doi.org/10.1007/s10826-017-1011-2

Zhang, X., & Lin, D. (2017). Does growth rate in spatial ability matter in predicting early arithmetic competence? *Learning and Instruction, 49*, 232-241. doi:10.1016/j.learninstruc.2017.02.003

Zhang, X., Räsänen, P., Koponen, T., Aunola, K., Lerkkanen, M. K., & Nurmi, J. E. (2018). Early Cognitive Precursors of Children's Mathematics Learning Disability and Persistent Low Achievement: A 5-Year Longitudinal Study. *Child Development,* online. https://doi.org/10.1111/cdev.1312

Table 1

Means, standard deviations and correlations among measures

Variables	N	Mean	*SD*	1	2	3	4	5	6	7	8	9	10	11
1. Phonological processing (T1)	210	84.67	19.38	—										
2. Visual-spatial skills (T1)	210	5.81	1.69	.198**	—									
3. Working memory (T1)	210	5.35	1.23	.226**	.163*	—								
4. Dot enumeration (T1)	211	3381.97	721.51	-.038	-.079	-.062	—							
5. Dot enumeration (T2)	179	3170.14	686.58	-.111	-.257**	-.164*	.172*	—						
6. Dot enumeration (T3)	168	3030.92	597.91	-.126	-.290**	-.289**	.275**	.332**	—					
7. Dot enumeration (T4)	155	2777.71	561.74	-.183*	-.345**	-.245**	.124	.328**	.379**	—				
8. Whole number magnitude understanding(T4)	155	0.89	7.49	.097	.314**	.232**	.045	-.163**	-.034	-.344**	—			
9. Standardized mathematics achievement (T5)	124	40.06	8.29	.275**	.414**	.316**	-.229*	-.335**	-.429**	-.441**	.420**	—		
10. Whole number magnitude understanding (T5)	124	0.93	.06	.129	.209*	.077	-.151	-.242**	-.251**	-.382**	.227**	.454**	—	
11. Rational number concept (T5)	124	35.42	7.31	.340**	.377**	.244**	-.149	-.253**	-.260**	-.439**	.364**	.792**	.466**	—

$^{*}p < .05$; $^{**}p < .01$.

Table 2

Regression analyses predicting children's standardized mathematics achievement, whole number magnitude understanding and rational number concept

	Standardized mathematics achievement (T5)		Whole number magnitude understanding (T5)		Rational number concept (T5)	
	β	*SE*	β	*SE*	β	*SE*
Gender	.048	.076	-.044	.104	-.003	.088
Phonological processing (T1)	.124	.073	.051	.071	.221	.069
Visual-spatial skills (T1)	.163	.084	.024	.093	.181	.085
Working memory (T1)	.100	.058	-.067	.076	.045	.062
Prior whole number magnitude understanding (T4)	$.244^{**}$	.094	.119	.128	$.211^{*}$	.104
Dot enumeration intercept	$-.294^{***}$	.084	$-.295^{**}$	.113	$-.145^{+}$	.084
Dot enumeration growth	-.107	.068	$-.179^{*}$	.078	$-.176^{*}$	.072

$^{+}p < .10$; $^{*}p < .05$; $^{**}p < .01$; $^{***}p < .001$.