The short-run and long-run equilibria for commuting with autonomous vehicles

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Abstract

Recent empirical studies indicated that using autonomous vehicles (AVs) can reduce commuters' value of time. In this context, this paper investigates how variation in value of time for AVs will reshape the commuting dynamics in the short-run and the implication on AV-related policies in the long run. We find that in the short-run, the adoption of AV can create more congestion delay since delay becomes cheaper for commuters. In the long-run, a number of external factors such as ownership cost and safety concerns may affect commuters' preference for AVs as against to traditional vehicles (TVs). This will influence the AV penetration, which in turn affects the daily commuting equilibrium. Multiple long-run equilibria with different AV penetrations may exist, depending on the additional cost/benefit of AVs with respect to TVs. Government subsidies may be needed to drive the system from inefficient long-run equilibrium to a more efficient one.

Keywords: Short-run equilibrium, long-run equilibrium, autonomous vehicles (AV), AV penetration, bottleneck model.

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1 Introduction

With the rapid development of technologies, autonomous vehicles (AVs) are expected to be ready for substituting traditional non-autonomous vehicles (TVs) in the near future (Cantarella and Di Febbraro, 2017). There is a growing interest towards planning and operation issues for autonomous vehicles systems. In particular, Fajardo et al. (2011) examined automated intersection control in the context of autonomous vehicles. Zhu and Ukkusuri (2015) explored the intersection control problem in a connected and autonomous traffic environment within a dynamic traffic assignment context. Levin and Rey (2017) proposed a new protocol for reservation-based intersection control with AVs. The implementation of AV zones or lanes was studied by Chen et al. (2016, 2017); mixed AV and TV lanes by Wu et al. (2020). Under Vickrey's single-bottleneck setting, van den Berg and Verhoef (2016) and Lamotte et al. (2017) examined the dynamic traffic equilibrium in the peak hour with autonomous vehicles. They have explored the improved capacity from driver-less cars. Parking issues for AVs and the associated behavior patterns have been considered very recently by Liu (2018). This study envisions the adoption of autonomous vehicles in the future and examines both the short-run traffic equilibrium under given AV penetration among the travel demand and the long-run travel equilibrium with an endogenous AV penetration.

Different from Lamotte et al. (2017) and Liu (2018), for the short-run traffic equilibrium, this paper considers a mixed traffic environment with both AVs and TVs. One can expect that it will take years to turn the existing stock of TVs into AVs where mixed traffic is present (Cantarella and Di Febbraro, 2017). Thus, it is urgent to understand the travel behavior and equilibrium characteristics with mixed TV and AV in the immediate future.

This paper further incorporates the prediction that AVs will reduce value of time for commuters during trips. Recent empirical efforts (Steck et al., 2018) showed that value of time can be substantially reduced by using AVs. This is because a range of activities (e.g., work, rest, and entertainment) can be performed during the trip if the AV is highly automated. This morning commute equilibrium with different values of time is studied by van den Berg and Verhoef (2011, 2016). In particular, while van den Berg and Verhoef (2016) focuses on the effects of road capacity variation and market competition with AVs, this paper focuses on the effects of AV penetration on commuting equilibrium. We explicitly quantify the total congestion delay against the AV penetration, identify the optimal AV penetration that minimizes the total travel cost at the user equilibrium, establish the relationship between queuing delay and AV penetration, and examine the relative efficiency of system optimum with AV tolling.

More importantly, this study considers the long-run commuting equilibrium, where the

AV penetration is endogenously determined and it evolves over time. Recently, in a different context, Li et al. (2018) examined a mixed traffic equilibrium with both human-driven and autonomous vehicles and its day to day evolution, which focuses on short-run static mixed traffic equilibrium. Noruzoliaee et al. (2018) have characterized the market penetration of autonomous vehicles (AVs) in urban transportation networks and quantified the traffic equilibrium with AVs. However, they have not considered traffic dynamics and evolution of AV penetration.

Following Liu (2018), the point-queue bottleneck model (Vickrey, 1969) is adopted to capture the essentials of the traffic dynamics and the trade-off between congestion delay and schedule delay. For a recent review on the bottleneck model, one can refer to e.g., Small (2015). As one may notice, since AVs reduce the value of time for commuters, the commuting equilibrium with both AVs and TVs is similar to that with heterogeneous commuters where value of time varies. There is a branch of studies examining the commuting equilibrium with heterogeneous commuters in the context of the bottleneck model (e.g., Arnott et al., 1994; Lindsey, 2004; van den Berg and Verhoef, 2011; van den Berg, 2014; Xiao et al., 2011; Nie and Liu, 2010; Liu and Nie, 2011; Liu et al., 2014, 2016). However, it is noteworthy that the physical meaning and interpretation of the commuting equilibrium with heterogeneous vehicle types is totally different from that with heterogeneous values of time.

As mentioned, besides modeling the short-run dynamic commuting equilibrium with given AV and TV demands, this study further explores the long-run equilibrium where the ownerships of AVs and TVs among the population might change, i.e., AV penetration is endogenous in the long-run equilibrium. Recently, Masoud and Jayakrishnan (2017) discussed a shared ownership program under which households will share the ownership and ridership of a set of autonomous vehicles. However, dynamic commuting equilibrium with mixed traffic is not considered in their study. In the long-run, besides the direct cost components considered in the short-run traffic equilibrium, many factors could affect adoption of AVs, and thus will affect AV penetration. As discussed in Fagnant and Kockelman (2015) and Talebian and Mishra (2018), safety issues, parking options, technology costs (as well as economies of scale), perception of AVs, security and privacy issues may all affect the adoption of AVs. However, the literature has not provided an analytical framework to model the long-run commuting equilibrium where both the direct cost components in the short-run traffic equilibrium and other factors are considered. This paper fills the mentioned gap at a strategic level. Particularly, a cost/benefit term related to the AV demand among the population is adopted to capture the above mentioned factors in Fagnant and Kockelman (2015). Note that overall this term is a cost if positive and is a benefit if negative (against the cost of TVs). There are many studies that have modeled both the long-run and short-run equilibrium in transport (e.g., Peer and Verhoef, 2013). However, as far as the author knows, there is no study theoretically examining the commuting equilibrium in both long-run and short-run contexts for AVs. Only recently, Chen et al. (2016) proposed a computational work on the evolution of AV market penetration and adoption of AV lanes. However, their study focused on static traffic case and dedicated facilities for AVs (without direct flow interaction between AVs and TVs).

This paper contributes to the literature in several ways. Firstly, this paper re-visits the short-run dynamic commuting equilibrium with AVs and reduced value of time, and further explicitly establishes efficiency of user equilibrium and system optimum against the AV penetration. Secondly, this paper takes into account factors other than direct travel cost components and examines the long-run endogenous AV penetration and commuting equilibrium. Thirdly, this paper identifies analytical conditions for the inefficient stable long-run equilibria, which correspond to realistic scenarios reflecting status quo. The effectiveness of the subsidy scheme is discussed with the aim to steer the system to the optimal long-run equilibrium. Overall, this paper sheds light on future AV adoptions, car ownership, and traffic management.

The rest of the paper is organized as follows. Section 2 revisits the dynamic commuting equilibrium with a reduced value of time for AVs, and analyzes the system efficiency against the AV penetration, and proposes and evaluates the first-best tolling scheme with mixed vehicle types. Section 3 discusses the long-run commuting equilibrium with endogenous AV penetrations, examines the evolution of AV penetration, and discusses the effect of government subsidy. Numerical illustrations are presented in Section 4. Finally, Section 5 concludes the paper and provides discussions for future research.

2 Short-run equilibrium with AVs

In this section, we start with a thumbnail description of the short-run commuting equilibrium in the rush-hour with autonomous vehicles (AVs) and non-autonomous traditional vehicles (TVs). For ease of presentation, later we refer to commuters traveling with AVs and TVs as type a (for AVs) and b (for TVs), respectively. In the short-run, we consider that the total travel demand is fixed, and the AV and TV ownerships among the travel demand are also given. The short-run equilibrium is then the joint traffic equilibrium for the AV commuters and TV commuters. It is assumed that all vehicles are owned privately by individual commuters. It is worth mentioning that in the future AVs might be publicly owned and operated to provide Mobility As a Service (see e.g., Fagnant and Kockelman, 2014, for shared autonomous vehicles services), which is not considered here.

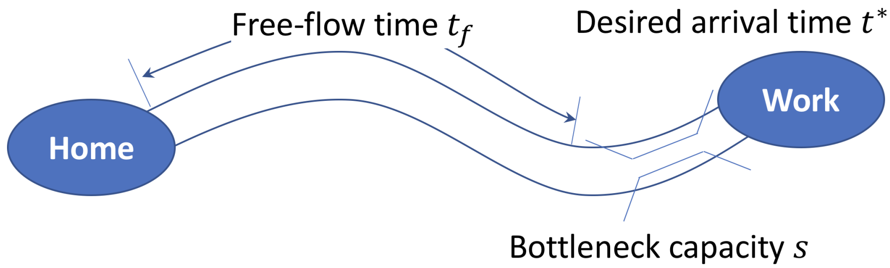


Figure 1: The city network.

We consider a stylized home-work city depicted in Figure 1 with a highway connecting home and workplace. There is a highway bottleneck close to the workplace (city center), which has a capacity of s. The free-flow travel time between home and bottleneck is a constant t_f. Every day there is a total number of N commuters traveling from home to the city center. All commuters have a desired arrival time t^* at the workplace. Early or late arrivals at the workplace will be penalized. Also, all commuters drive, either with AVs or TVs.

In the short-run, the total travel demand N is fixed, and car-ownership among the N commuters is given. Without loss of generality, we assume $N > 0$ throughout the paper. Let N_a and N_b be the numbers of commuters traveling with AVs and TVs, respectively. Then, we have $N_a + N_b = N$. The short run equilibrium is the departure/arrival equilibrium of the N_a AV commuters and N_b TV commuters.

Based on the above setting, for commuters ($i = a$ for those with AVs and $i = b$ for those with TVs) departing at time t, the travel cost can be written as

$$c_i(t) = \alpha_i \cdot T(t) + \beta \cdot [t^* - t - T(t)]^+ + \gamma \cdot [t + T(t) - t^*]^+ . \tag{1}$$

where $T(t)$ is the travel time experienced by the commuters , α_i is the value of time when the commuters are driving the AVs ($i = a$) or TVs ($i = b$), β and γ are the penalties for a unit time of early and late arrivals at the destination for commuters, and $[\cdot]^+ = \max\{0, \cdot\}$. It is assumed that $\gamma > \alpha_b > \alpha_a > \beta$. Note that $\gamma > \alpha_b > \beta$ is a standard assumption in the literature, which is consistent with many empirical evidences. $\alpha_b > \alpha_a$ means that AV self-driving time is less costly than the driving time when commuters operate a TV. This is expected in the future since AVs can drive themselves, and commuters have more flexibility during commuting (e.g., the commuters might sleep, work, or have other entertainment activities), see e.g., Steck et al. (2018). Moreover, $\alpha_a > \beta$ means that while commuters can have flexibility in AVs, in-vehicle travel delay is still more expensive than the early arrival penalty.[1]

In Eq.(1), the travel time $T(t)$ consists of the free-flow travel time t_f and the queuing delay at the highway bottleneck, i.e., $T(t) = t_f + \frac{q(t)}{s}$ where $q(t)$ is the queue length experienced by commuters departing from home at time t and s is the bottleneck capacity, and the free-flow time for other road sections is zero. Note that the queue length at the bottleneck at time t is equal to $q(t - t_f)$.

2.1 Commuting equilibrium revisited

Suppose that both types of commuters depart simultaneously during a time duration with a positive length, then $\frac{dc_a(t)}{dt} = \frac{dc_b(t)}{dt} = 0$ must hold for this duration as the equilibrium condition. It can be readily verified from Eq.(1) that this condition can never be satisfied. Instead, the isocost curves can be constructed as shown in Figure 2, where commuters have different values of time.

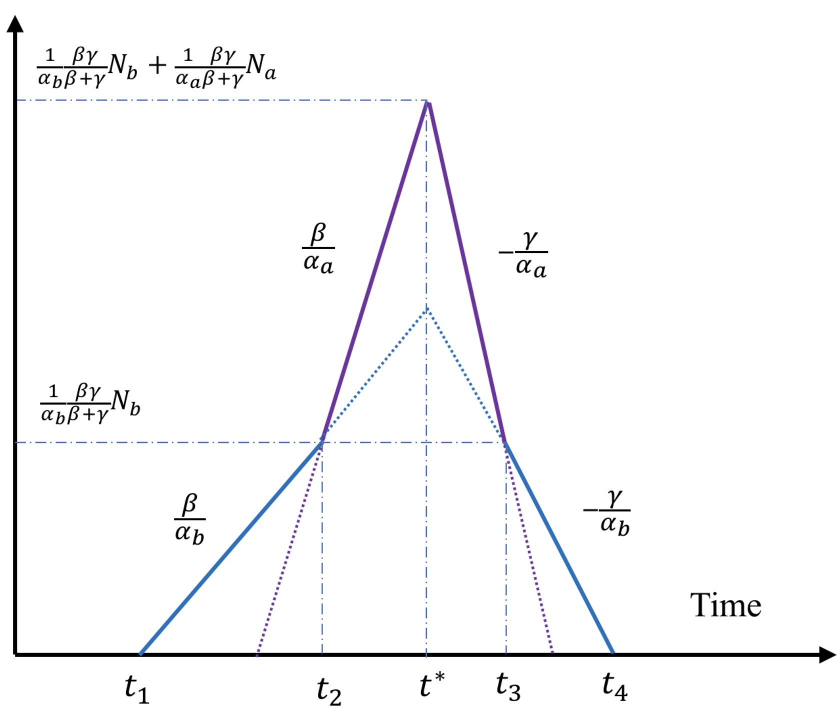


Figure 2: The isocost curves for the case with both AVs and TVs.

[1] If this is not the case, it means that commuters prefer staying in an AV rather than being early in the office. While this might be possible in some certain circumstances, we consider that in general people have more flexibility in an office or surrounding areas than in a car, and thus $\alpha_a > \beta$.

The time points in Figure 2 are given as follows:

$$t_1 = t^* - \frac{\gamma}{\beta+\gamma}\frac{N}{s}; t_2 = t^* - \frac{\gamma}{\beta+\gamma}\frac{N_a}{s}; t_3 = t^* + \frac{\beta}{\beta+\gamma}\frac{N_a}{s}; t_4 = t^* + \frac{\beta}{\beta+\gamma}\frac{N}{s}. \quad (2)$$

The type a commuters will arrive at work within $[t_2, t_3]$ while the type b commuters will arrive within $[t_1, t_2]$ and $[t_3, t_4]$. This is because type a commuters (with AVs) value travel delay relatively less against schedule delay penalties than type b commuters.

We can further derive the equilibrium departure rates (from home) for both types of commuters by letting $\frac{dc_i(t)}{dt} = 0$, which are

$$r_i^e = \frac{\alpha_i}{\alpha_i - \beta}s; r_i^l = \frac{\alpha_i}{\alpha_i + \gamma}s. \quad (3)$$

where e and l represent early and late arrival (at work), respectively. It can be readily verified that $r_a^e > r_b^e > s > r_a^l > r_b^l$.

With the above results, the time-dependent equilibrium flow pattern can be plotted in Figure 3, where the purple and blue solid lines represent the departure from home for type a and b commuters, respectively, the purple and blue dashed line represent the arrivals at the highway bottleneck for type a and b commuters, respectively, and the black solid line represents the cumulative departure from the highway bottleneck (also the arrival at the city center or the workplace). The time points in Figure 3 are given in Eq.(2).

It is shown in Figure 3 that the two types of commuters travel in different time windows – AV users travel around the desired arrival time t^* where the queuing time is longer and TV users travel in the earlier and later windows where the schedule delay cost is larger. Such a separation is motivated by the divergent preferences towards congestion delay and schedule delay for AVs and TVs.

Based on the commuting equilibrium, the equilibrium individual travel costs for both types of commuters can be derived as follows:

$$c_a = \alpha_a t_f + \frac{\beta\gamma}{\beta+\gamma}\left(\frac{N_a}{s} + \frac{\alpha_a}{\alpha_b}\frac{N_b}{s}\right); c_b = \alpha_b t_f + \frac{\beta\gamma}{\beta+\gamma}\frac{N_a + N_b}{s}. \quad (4)$$

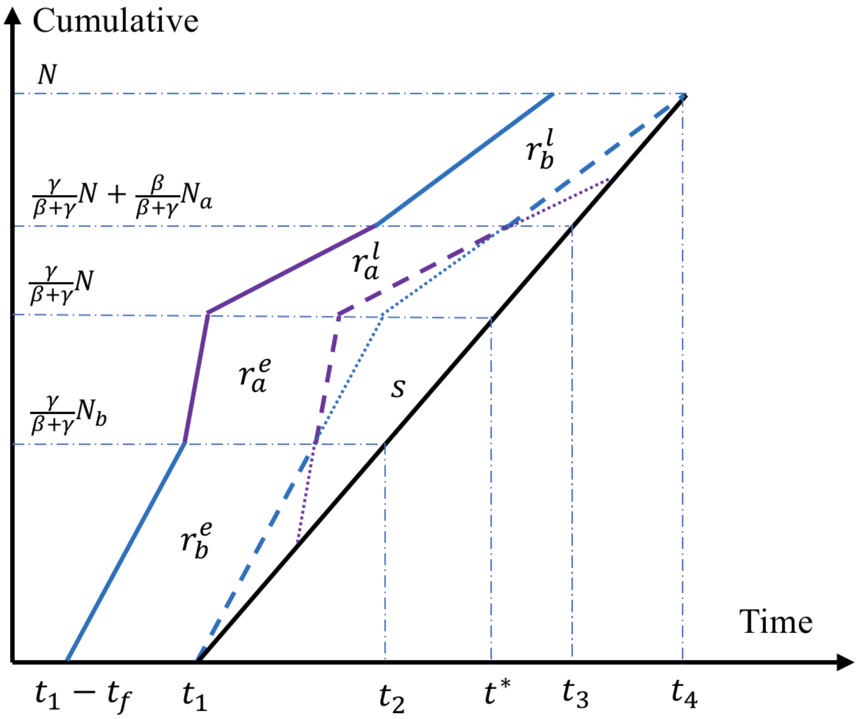


Figure 3: The departure/arrival pattern at User Equilibrium.

The above commuting equilibrium where AV and TV users have different VOTs is similar to that established in van den Berg and Verhoef (2011, 2016). While van den Berg and Verhoef (2016) examined the effects of road capacity variation with AVs, the capacity issue is not considered in this paper; instead, this paper primarily focuses on the effects of AV penetration on commuting equilibrium.

As new contributions to the short-run commuting equilibrium with AVs, the following subsections will (1) explicitly analyze the effects of AV penetration on system efficiency, (2) identify the optimal AV penetration that minimizes the total travel cost at the user equilibrium, (3) establish the relationship between queuing delay and AV penetration, and (4) examine the relative efficiency of system optimum with AV tolling.

2.2 Effects of AV penetration on system efficiency

Denote $x = \frac{N_a}{N}$ the penetration proportion of AVs for any given $N > 0$, and thus $N_a = xN$ and $N_b = (1-x)N$. The effects of AV penetration on short-term individual travel costs are summarized in Lemma 1:

Lemma 1. *The short-run individual travel cost by AV increases with the AV penetration proportion and that by TV is invariant with the penetration.*

Proof. It follows from Eq.(4) that

$$\frac{\partial c_a}{\partial x} = \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\left(1-\frac{\alpha_a}{\alpha_b}\right) > 0; \frac{\partial c_b}{\partial x} = 0 \tag{5}$$

□

Lemma 1 implies that in the short run where the AV technologies and supporting facilities are fixed, the more people use AVs, the higher the cost is for AV users. This is brought by the competition effect among the same user type who intend to travel in similar time windows. In contrast, the AV penetration does not affect the travel cost of users who intend to travel in other time windows, i.e., TV users.

We now proceed to examine the effect of AV penetration on system cost. Given the individual travel costs in Eq.(4), the total travel cost of all commuters is

$$TC = \sum_i c_i \cdot N_i. \tag{6}$$

The total travel cost TC can be written as a function of x. Now we examine the efficiency of the equilibrium against the AV penetration x.

Proposition 1. *The optimal AV penetration proportion to minimize* TC *in Eq.*(6) *is*

$$(x)^* = \begin{cases} 0.5 + 0.5 \cdot \frac{\alpha_b t_f}{\frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}} \geq 0.5 & t_f < \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\frac{1}{\alpha_b} \\ 1 & t_f \geq \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\frac{1}{\alpha_b} \end{cases}. \tag{7}$$

Proof. Based on Eq.(6) and $N_a = xN$ and $N_b = (1-x)N$, we can rewrite TC as a function of x as follows:

$$TC = N\left[\alpha_a t_f x + \alpha_b t_f (1-x) + \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\left(1-\left(1-\frac{\alpha_a}{\alpha_b}\right)(1-x)\,x\right)\right]. \tag{8}$$

By taking the first-order and the second order derivatives of Eq.(8) with respect to x, we have

$$\frac{dTC}{dx} = N\left[\alpha_a t_f - \alpha_b t_f - \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\left(1-\frac{\alpha_a}{\alpha_b}\right)(1-2x)\right], \tag{9}$$

and

$$\frac{d^2TC}{dx^2} = N\left[2\frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\left(1-\frac{\alpha_a}{\alpha_b}\right)\right] > 0. \tag{10}$$

Eq.(10) suggests that TC is strictly convex with respect to x. Moreover, $\left(\frac{dTC}{dx}\right)_{x=0} < 0$ holds since $\alpha_a < \alpha_b$.

When $t_f < \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\frac{1}{\alpha_b}$, it can be verified that $\left(\frac{dTC}{dx}\right)_{x=1} > 0$. Therefore, TC is minimized at $x = 0.5 + 0.5 \cdot \frac{\alpha_b t_f}{\frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}}$ such that $\frac{dTC}{dx} = 0$ (interior optimum). Since $\frac{\alpha_b t_f}{\frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}} \geq 0$, we have $0.5 + 0.5 \cdot \frac{\alpha_b t_f}{\frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}} \geq 0.5$.

When $t_f \geq \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\frac{1}{\alpha_b}$, it can be verified that $\frac{dTC}{dx} \leq 0$, and the equality at most holds at $x = 1$. Therefore, TC is minimized at $x = 1$ (corner optimum). □

Proposition 1 and its proof have several implications. Firstly, the optimal penetration proportion of AVs to maximize the efficiency of the traffic equilibrium is always greater than or equal to 50%. Secondly, when free-flow travel time is relatively large ($t_f \geq \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\frac{1}{\alpha_b}$), the cost saving from reduced value of time for AVs is relatively significant, i.e., a larger t_f indicates a larger $(\alpha_b - \alpha_a)\, t_f$. Therefore, we should always let all commuters to use AVs. Thirdly, when free-flow travel time is relatively small ($t_f < \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\frac{1}{\alpha_b}$), to let all commuters use AVs is non-optimal. This is because when x increases, travel cost c_a for AV commuters in Eq.(1) increases. As a result, the benefit of using AVs against TVs decreases.

We now take a further look at the congestion delay cost as part of the total travel cost. Here we focus on the congestion delay because the total free-flow time is fixed to be $N \cdot t_f$. The total congestion delay is:

$$TCD = \left(\frac{0.5}{\alpha_a}\frac{\beta\gamma}{\beta+\gamma}\frac{N_a}{s} + \frac{1}{\alpha_b}\frac{\beta\gamma}{\beta+\gamma}\frac{N_b}{s}\right) N_a + \left(\frac{0.5}{\alpha_b}\frac{\beta\gamma}{\beta+\gamma}\frac{N_b}{s}\right) N_b \tag{11}$$

Similar to that of TC, we examine how TCD changes with the penetration proportion of AVs.

Proposition 2. *The total congestion delay TCD in Eq.(11) increases with the AV penetration proportion.*

Proof. Given that $N_a = xN$ and $N_b = (1-x)N$, we rewrite TCD as a function of x as follows:

$$TCD = 0.5\frac{\beta\gamma}{\beta+\gamma}\frac{N^2}{s}\left(\frac{x^2}{\alpha_a} + \frac{1-x^2}{\alpha_b}\right) \tag{12}$$

By taking the first-order derivative of Eq.(12) with respect to x, we have

$$\frac{dTCD}{dx} = 0.5\frac{\beta\gamma}{\beta+\gamma}\frac{N^2}{s}\left(\frac{2x}{\alpha_a} - \frac{2x}{\alpha_b}\right). \tag{13}$$

Since $\alpha_a < \alpha_b$ and $x \geq 0$, we have $\frac{dTCD}{dx} \geq 0$ and the equality only holds when $x = 0$. This completes the proof. □

Proposition 2 indicates that while an AV allows flexibility for the commuter during the trip and reduces the cost for a unit of travel time, it will result in more congestion delays. More intuitively speaking, when delay is cheaper for commuters, they will queue more to seek less schedule delay cost.

The implication of Proposition 2 is important. The external cost of congestion is not fully captured in this paper – in reality, larger congestion delay normally means more energy consumption and more crowded road/urban environment. When taking all these complications into account, the adoption of AVs can yield inefficient outcomes for society. The implication of this proposition raises the need for future research and practice to comprehensively examine all benefits and costs from the adoption of AVs for the society.

2.3 Congestion tolling and system optimum

In this section, a time-dependent toll will be developed to achieve the system optimum. At the system optimum, the following conditions should hold: (1) the queuing delay should be completely eliminated (i.e., minimum queuing delay is equal to zero); (2) the schedule delay cost should be minimized (travels are concentrated around t^*). To satisfy these two conditions, firstly, the departure rates of both types of commuters from home should be equal to the bottleneck capacity s (and then there is zero queuing delay); and secondly, the arrival times (at work) for commuters should be within $[t_1, t_4]$.

Suppose $\tau(t)$ is the toll we impose on commuters arriving at the bottleneck at time t, then the following design of $\tau(t)$ can drive the system to optimum.

$$\tau(t) = \begin{cases} 0 & t \in (-\infty, t_1) \\ \beta\left(t - t^* + \frac{\gamma}{\beta+\gamma}\frac{N}{s}\right) & t \in [t_1, t^*) \\ \gamma\left(t^* - t + \frac{\beta}{\beta+\gamma}\frac{N}{s}\right) & t \in [t^*, t_4] \\ 0 & t \in [t_4, +\infty) \end{cases}, \tag{14}$$

Note that since there is a positive free-flow travel time, the toll experienced by the commuters departing from home at time t is equal to $\tau(t + t_f)$.[2] The total toll revenue is $TTR = 0.5 \cdot \frac{\beta\gamma}{\beta+\gamma}\frac{N^2}{s}$, and the total system cost is $TSC = \alpha_a t_f N_a + \alpha_b t_f N_b + 0.5 \cdot \frac{\beta\gamma}{\beta+\gamma}\frac{N^2}{s}$.

To analyze how the adoption level of AVs could affect the efficiency of the tolling, we define the following relative efficiency of the system optimum against the user equilibrium.

$$\theta = \frac{TC - TSC}{TC - (\alpha_a t_f N_a + \alpha_b t_f N_b)}. \tag{15}$$

[2]How to derive the toll is omitted here to save space while interested readers may refer to e.g., van den Berg and Verhoef (2011).

In Eq.(15), $TC - (\alpha_a t_f N_a + \alpha_b t_f N_b)$ is the total reducible cost, and $TC - TSC$ is the total cost reduction at the system optimum against user equilibrium. θ measures the relative efficiency of the first-best tolling to reduce the reducible cost.

Proposition 3. *The relative efficiency of system optimum against user equilibrium satisfies*

$$1 - \frac{0.5}{0.75 + 0.25 \cdot \frac{\alpha_a}{\alpha_b}} \leq \theta \leq 0.5. \tag{16}$$

Proof. Let $x = \frac{N_a}{N}$ (note that we only consider $N > 0$), which is the penetration proportion of AVs. Based on Eq.(6), Eq.(15) and $TSC = \alpha_a t_f N_a + \alpha_b t_f N_b + 0.5 \cdot \frac{\beta\gamma}{\beta+\gamma} \frac{N^2}{s}$, we can establish the following

$$\theta = 1 - \frac{0.5}{1 + \left(1 - \frac{\alpha_a}{\alpha_b}\right)(x^2 - x)}. \tag{17}$$

Note that $0 \leq x \leq 1$. One can readily verify that θ reaches the maximum when $x = 0$ or 1, which is 0.5, and θ reaches the minimum when $x = 0.5$, which is $1 - \frac{0.5}{0.75+0.25\cdot\frac{\alpha_a}{\alpha_b}}$. □

The above results indicate that the relative efficiency of tolling reaches the maximum when all commuters travel with identical vehicle (either AV or TV). This is explained as follows. The efficiency gain of system optimum is from the elimination of congestion delay cost, which is given by

$$\begin{aligned} TCDC &= \left(\frac{0.5}{\alpha_a}\frac{\beta\gamma}{\beta+\gamma}\frac{N_a}{s} + \frac{1}{\alpha_b}\frac{\beta\gamma}{\beta+\gamma}\frac{N_b}{s}\right) N_a \cdot \alpha_a + \left(\frac{0.5}{\alpha_b}\frac{\beta\gamma}{\beta+\gamma}\frac{N_b}{s}\right) N_b \cdot \alpha_b \\ &= \frac{\beta\gamma}{\beta+\gamma}\frac{N^2}{s}\left(\left(1 - \frac{\alpha_a}{\alpha_b}\right)x^2 - \left(1 - \frac{\alpha_a}{\alpha_b}\right)x + 0.5\right). \end{aligned} \tag{18}$$

It follows that

$$\frac{dTCDC}{dx} = \frac{\beta\gamma}{\beta+\gamma}\frac{N^2}{s}\left(1 - \frac{\alpha_a}{\alpha_b}\right)(2x - 1), \tag{19}$$

$$\frac{d^2TCDC}{dx^2} = 2\frac{\beta\gamma}{\beta+\gamma}\frac{N^2}{s}\left(1 - \frac{\alpha_a}{\alpha_b}\right) > 0. \tag{20}$$

It can be readily verified that $TCDC$ reaches the minimum at $x = 0.5$ and the maximum at $x = 0$ or $x = 1$.

3 Long-run equilibrium with AVs

While the previous section analyzes the short-run commuting equilibrium under given vehicle type distribution, this section looks into the long-run equilibrium where both the vehicle type distribution and travel pattern are endogenous. With developments of self-driving technologies, infrastructures, and facilities, costs associated with AV ownership and maintenance may evolve and public concerns for licensing and safety issues are also subject to change. This is to say that the 'functional barriers' and 'psychological barriers' studied in Talebian and Mishra (2018) may be resolved gradually. These are expected to be reflected in commuters' preferences between AV and TV. This section aims to incorporate the external factors and explore how they affect the AV penetration and the commuting equilibrium in the long-run.

The long-run AV penetration depends on the long-run average cost of using AV as opposed to TV. In addition to the daily commuting cost discussed in the previous section, we need to take into account the discrepancy of any additional vehicle type-specific cost associated with the external factors mentioned above. The latter term is driven by commuter's perception of the relative external cost or benefit associated with using AV with respect to TV.

Viewing the usage of TV as the benchmark, the additional cost associated with TV is normalized to zero, and the long-run average cost of using TV is equal to the short-run commuting cost. To reflect the discrepancy between AV and TV, a term δ is introduced to capture commuter's perception of the relative external cost or benefit associated with using AV with respect to TV. In contrast to previous studies (e.g., Talebian and Mishra, 2018) investigating specific adoption barriers of AV, this paper considers a general δ term assuming that the explicit function forms and associated parameters are attainable elsewhere. The term δ can be either positive or negative and is referred to as the "average usage cost of AV" thereafter. When δ is positive, it implies that AV is less advantaged than TV in users' perception, and that traveling with AV imposes additional cost. In contrast, δ being negative means that AV's advantages outweigh disadvantages and that traveling with AV brings additional benefit (when compared with TVs). As such, the long-run average cost (per day) of traveling with each vehicle type is the sum of daily commuting cost and usage cost, i.e.,

$$C_a = c_a + \delta; C_b = c_b, \tag{21}$$

where C_a and C_b represent the long-run average cost associated with AV and TV respectively and c_a and c_b are the daily commuting cost given by Eq.(4).

The evolution of δ can be driven by a variety of factors, such as developments of self-driving technologies, infrastructures, and facilities, financial investments, and commuter's psychological perception of AV. Amongst, 'price' and 'safety' may be the major factors for users. It is generally expected that the increase in AV penetration will have positive externalities on AV usage. Firstly, due to economies of scale, the average costs associated with the research, production, and maintenance of AVs will be reduced, and thus operating an

AV will be less expensive. Secondly, when there are more AVs, larger proportions of surrounding vehicles can be connected and coordinated. This expects to increase the reliability and efficiency of traffic (Talebpour and Mahmassani, 2016). Thirdly, with more AVs, there will be more AV-friendly infrastructures and facilities to support self-driving.[3]

When the AV technology is sufficiently mature and the whole population travel with AV, as an extreme case, it is expected that AV will be more advantaged than TV. To own and to use an AV will be relatively cheaper than a TV as the total production is much larger, and traveling with an AV will be safer as the vehicle is aware of all vehicles around. In another extreme where no one uses AV, the impedance of adopting AV is fairly large due to high manufacturing costs, safety concerns, etc. Without loss of generality, we introduce the following assumption regarding δ.

Assumption 1. *$\delta = \delta(N_a)$ is a continuously differentiable function of N_a, $N_a \in [0, N]$; when $N_a = 0$, $\delta(N_a) > 0$; when $N_a = N$, $\delta(N_a) < 0$.*

In general, the increase of AV penetration expects to reduce the average 'price' of owning and operating AVs due to economies of scale. However, the penetration may have mixed effects on 'safety'. On one hand, when there are more AVs, the traffic can be more efficiently coordinated; on the other hand, the presence of surrounding AVs may psychologically impose anxiety on human drivers, which will negatively influence the driving behavior and cause dangers for both TVs and AVs. When the latter effect dominates, increasing AV penetration may lead to less safe traffic. This may happen when the AV penetration is within in a certain range.

Overall, there is not yet an established relationship between the average usage cost δ and N_a. The increase in N_a may have positive and negative effects. When the positive effects (e.g., economies of scale) dominate, δ decreases with N_a; and otherwise when the negative effects (e.g., safety concerns) prevail. Therefore, in this paper, we consider δ a general function of N_a without assuming any specific forms.

Assumption 1 guarantees the existence of $\delta'(N_a)$ while allowing for flexibility on the $sign\,(\delta'(N_a))$. Although $\delta'(N_a)$ can be either positive or negative, Assumption 1 governs that it cannot always be positive throughout $N_a \in [0, N]$. It is assumed that besides N_a, other factors influencing δ are assumed to be exogenous and given to us.

[3]In addition to 'price' and 'safety', many other factors may play a role, such as energy, legislative, and privacy considerations. The variations of these exogenous factors are out of the scope of this paper.

3.1 Long-run equilibria with endogenous vehicle type and daily commuting choices

In the long run, the choice of a commuter is twofold – the vehicle type choice (between AV and TV) and the daily commuting travel choice (departure time choice). Correspondingly, the long-run equilibrium entails both an equilibrium vehicle type distribution and an equilibrium daily commuting traffic pattern. The former, reflected by the equilibrium AV penetration, is driven by the long-run average cost associated with each vehicle type. The latter, as established in Section 2, is in turn governed by the penetration proportion. Therefore, the endogenous penetration proportion is the key to characterize the long-run equilibrium. Thereafter, the long-run equilibrium refers to the equilibrium AV penetration, namely the demand distribution between N_a and N_b.

In particular, the long-run equilibrium can be characterized by the following equilibrium conditions:

$$N_i \cdot (C_i - \mu) = 0, \forall i \in \{a, b\} \tag{22a}$$

$$C_i - \mu \geq 0, \forall i \in \{a, b\} \tag{22b}$$

$$N - N_a - N_b = 0 \tag{22c}$$

$$N_i \geq 0, \forall i \in \{a, b\} \tag{22d}$$

where μ is equal to the minimum travel cost at the long-run equilibrium. The above conditions can be derived based on the Karush-Kuhn-Tucker (KKT) conditions of the equilibrium assignment problem (one may refer to, e.g., Sheffi, 1985, for detailed derivations). The equilibrium conditions dictate that the long-run equilibrium can be achieved where the long-run average cost of AV and TV is equal (the interior equilibrium) or all commuters travel with one type of vehicle with lower cost (the boundary equilibrium).

It is evident that the feasible set of $\{N_i\}$ defined by Eqs.(22c)-(22d) is non-empty, compact and convex, and that the cost functions $C_i, \forall i \in \{a, b\}$ are continuous. Therefore, there exist at least one equilibrium solution for Eq.(22); however, the uniqueness of the equilibrium is not guaranteed. Depending on $\delta(N_a)$ and the resultant C_a, there can be various cases where the equilibrium governed by Eq.(22) is achieved, and in some cases multiple equilibria may arise.

Since $N_b = N - N_a$, based on Eqs.(4) and (21), the long-run average costs of AV and TV can be respectively rewritten as

$$C_a = \alpha_a t_f + \frac{\beta\gamma}{\beta+\gamma}\left(\frac{N_a}{s} + \frac{\alpha_a}{\alpha_b}\frac{N-N_a}{s}\right) + \delta\left(N_a\right), \tag{23a}$$

$$C_b = \alpha_b t_f + \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}. \tag{23b}$$

The effect of AV penetration on long-run cost can be characterized by taking the derivative of long-run costs with respect to the number of AV users:

$$\frac{dC_a}{dN_a} = \frac{\beta\gamma}{\beta+\gamma}\frac{1}{s}\left(1-\frac{\alpha_a}{\alpha_b}\right) + \delta'\left(N_a\right), \tag{24a}$$

$$\frac{dC_b}{dN_a} = 0. \tag{24b}$$

Eq.(24a) dictates that when AV penetration changes, the changing direction of its long-run average cost is unclear. This is because the first term on the right-hand side, representing the marginal effect of AV penetration on daily commuting cost, is positive, as established in Lemma 1. The second term, representing the marginal effect of AV on the external usage cost, can be either positive or negative according to the assumption we made on $\delta(N_a)$. In contrast, Eq.(24b) prescribes that AV penetration has no effect on the cost of traveling with TV.

When the whole population travel with AV, as an extreme case, $N_a = N$, and the long-run average cost of AV is $C_a^N = \alpha_a t_f + \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s} + \delta\left(N\right)$. It can be readily verified that based on the assumptions of $\delta(N_a)$ we have

$$C_a^N < C_b \tag{25}$$

In another extreme where no one uses AV ($N_a = 0$ and $N_b = N$), the long-run average cost of AV is $C_a^0 = \alpha_a t_f + \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\frac{\alpha_a}{\alpha_b} + \delta(0) > \alpha_a t_f + \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}\frac{\alpha_a}{\alpha_b}$. There is no clear-cut relationship between C_a^0 and C_b. We thus classify all possible scenarios into three cases according to the relationship between these two values: (I) $C_a^0 > C_b$; (II) $C_a^0 < C_b$; (III) $C_a^0 = C_b$. Figure 4 presents some representative scenarios for each case respectively (i.e., 5 scenarios for Case I, 8 scenarios for Case II, and 2 scenarios for Case III). Each subfigure (a)-(f) conveys one or more possible scenario(s), in which the distance between the vertical axes is equal to N, the total number of commuters by AV and TV combined which is fixed. The long-run average cost of AV, C_a, is measured to the right from the left-hand axis, and that of TV, C_b, is measured to the left from the right-hand axis. The long-run average cost of TV is represented by the horizontal line in green color, as given in Eq.(21). We note that the long-run average cost of AV may appear in various forms, but it is tedious to elaborate all possibilities.

In Figure 4, we present some representative appearances of C_a for each case. Each dotted curve represents one possible C_a. While there might be numerous other appearances of C_a (e.g., with more than four intersections with C_b) that are not shown, the scenarios covered in Figure 4 are representative to illustrate how boundary and interior equilibria look like, respectively. The general trend is that the long-run average cost of AVs is less than TVs when the whole population use AVs. Some scenarios involve oscillations of C_a, which is the sum of daily travel cost and the additional usage cost. The oscillations reflect the uncertain tradeoffs between positive and negative externalities of AV penetration. When negative externalities play the major role, C_a increases with N_a.

Wherever the equilibrium conditions Eq.(22) are satisfied is an equilibrium point, each of which refers to a long-run equilibrium allocation of (N_a, N_b). Specifically, where C_a intersects with C_b ($C_a = C_b$) yields an interior equilibrium; and where all commuters use one type of vehicle with strictly lower cost (i.e., $N_i = N$ and $N_j = 0$ where $C_i < C_j$, $i, j \in \{a, b\}$, $i \neq j$) characterizes a boundary equilibrium. Each equilibrium point is marked by a blue or red circle in Figure 4 (the color of the circle indicates the stability of the equilibrium, which will be examined in the next section). In line with the above discussion, there exist at least one equilibrium in each possible scenario and there might be multiple equilibria in some scenarios, depending on the appearance of C_a.

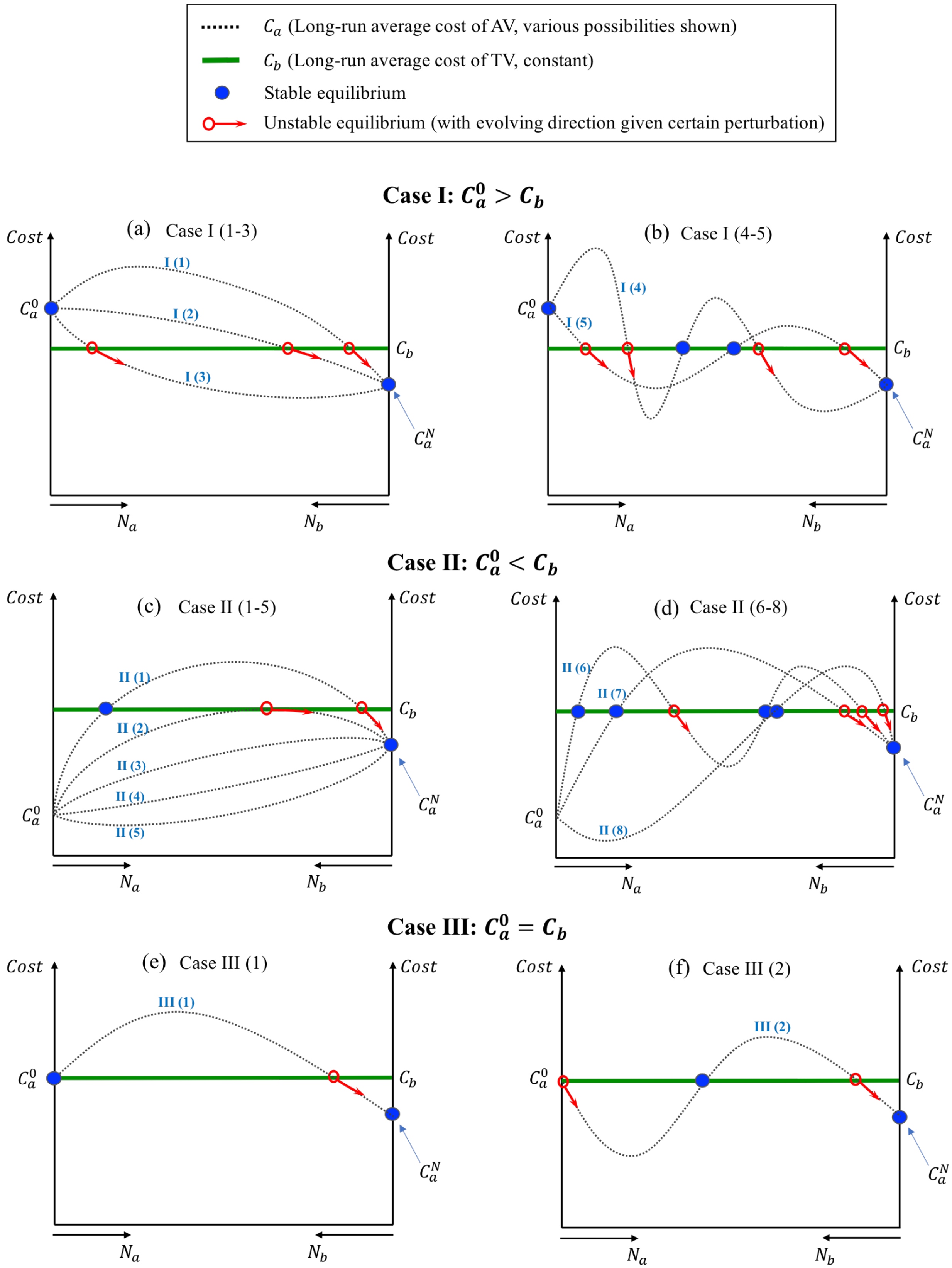


Figure 4: Representative long-run equilibria with endogenous AV penetration.

3.2 Evolution of AV penetration and stability of long-run equilibria

Section 3.1 discusses possible long-run equilibria that may arise with different forms of long-run average costs. We now proceed to envisage the evolution of AV penetration among population and the associated effect on long-rum equilibria. Let k denote the calendar time (which is assumed to be continuous) and $\dot{N}_a = \frac{dN_a}{dk}$, which represents how N_a evolves over time. Note that N_a is suffice to characterize the demand distribution between AV and TV given $N_a + N_b = N$ and $\dot{N}_b = -\dot{N}_a$. To examine the stability of the dynamical system, we assume that commuters swap from TV to AV at a rate proportional to the cost difference between C_a and C_b, following the standard "proportional swap" principle in the literature (e.g., Smith, 1984), such that

$$\dot{N}_a = \rho \cdot (N - N_a) \cdot [C_b - C_a]^+ - \rho \cdot N_a \cdot [C_a - C_b]^+ , \tag{26}$$

where $\rho > 0$ and $[\cdot]^+ = \max\{0, \cdot\}$.[4] Equation (26) describes how the number of AV users evolve with the long-run cost difference between AV and TV. When the AV cost is smaller than TV, users shift from TV to AV at the rate proportional to the number of TV users and cost difference. When the AV cost is greater than TV, users shift backwards TV at the rate proportional to the number of AV users and cost difference.

The parameter ρ represents the proportional swap rate, which reflects the sensitivity of users with respect to the long-run cost difference. In the present paper, we assume the parameter is exogenously given; however, in practice, the sensitivity needs dedicated calibration analysis.

The evolutions of N_a and N_b (i.e., AV penetration) in the long-run will influence the short-run commuting equilibrium analyzed in Section 2.1. According to the equilibrium established in Figure 2 and Figure 3, when AV users increase, more travelers (AV users) will travel around the desired arrival time t^*, leading to longer queuing time for AV users. It follows from Lemma 1 that the AV short-run cost increases with the AV penetration; and the short-run travel cost of TV users is invariant, as they always travel in the early and late time windows and are not affected by the AV queues.

We introduce the following Lyapunov Theorem that defines the stability of the dynamical system:

[4] There is a branch of studies examining the day-to-day evolution of traffic dynamics, e.g., Cascetta and Cantarella (1991), Guo and Liu (2011), Watling and Cantarella (2013), Xu et al. (2014), Guo et al. (2015), where dynamical systems have been developed. This study, while focuses on evolution of AV penetration, follows a similar dynamical modeling framework to the literature.

Definition 1. *The Lyapunov Theorem (Smith, 1984):*

The dynamical system Eq.(26) is stable if there is a continuously differentiable scalar function $V(N_a)$, defined on $[0, N]$, such that

(1) $V(N_a) \geq 0$, $\forall N_a \in [0, N]$;

(2) $V(N_a) = 0$ if and only if N_a is an equilibrium, and

(3) $\frac{dV(N_a)}{dN_a} \cdot \dot{N}_a < 0$ if N_a is not an equilibrium.

Following Smith (1984), we introduce the following Lyapunov function

$$V(N_a) = (N - N_a) \cdot \left([C_b - C_a]^+\right)^2 + N_a \cdot \left([C_a - C_b]^+\right)^2. \tag{27}$$

It can be readily verified that (1) $V(N_a) \geq 0$ for all $N_a \in [0, N]$, and (2) $V(N_a) = 0$ if and only if N_a is an equilibrium. In addition, since both C_a and C_b are continuously differentiable, $V(\cdot)$ is continuously differentiable. To verify condition (3) of Definition 1, we examine $sign\{\frac{dV(N_a)}{dN_a} \cdot \dot{N}_a\}$ given a nonequilibrium N_a.

To ease the presentation, we denote Δ_C the discrepancy between the long-run average costs of AV and TV, and thus we have

$$\Delta_C = C_a - C_b = -\frac{\beta\gamma}{\beta + \gamma}\left(1 - \frac{\alpha_a}{\alpha_b}\right)\frac{N - N_a}{s} + \delta\left(N_a\right). \tag{28}$$

Taking the derivative with respect to N_a on both sides of Eq.(28), we obtain the marginal effect of AV penetration on the cost difference:

$$\frac{d\Delta_C}{dN_a} = \frac{\beta\gamma}{\beta + \gamma}\left(1 - \frac{\alpha_a}{\alpha_b}\right)\frac{1}{s} + \delta'\left(N_a\right). \tag{29}$$

Given that N_a is not an equilibrium solution, $\Delta_C \neq 0$. When $\Delta_C > 0$, we have $\dot{N}_a = -\rho \cdot N_a \cdot \Delta_C < 0$, and $V(N_a) = N_a \cdot (\Delta_C)^2$. It then follows that $\frac{dV(N_a)}{dN_a} = \Delta_C \cdot \left(\Delta_C + 2N_a \cdot \frac{d\Delta_C}{dN_a}\right)$.

When $\Delta_C < 0$, $\dot{N}_a = -\rho \cdot (N - N_a) \cdot \Delta_C > 0$, and $V(N_a) = (N - N_a) \cdot (\Delta_C)^2$. It follows that $\frac{dV(N_a)}{dN_a} = \Delta_C \cdot \left(-\Delta_C + 2 \cdot (N - N_a) \cdot \frac{d\Delta_C}{dN_a}\right)$. Based on the above, we thus have

$$\frac{dV(N_a)}{dN_a} \cdot \dot{N}_a = \begin{cases} -\rho \cdot N_a \cdot (\Delta_C)^2 \cdot \left(\Delta_C + 2N_a \cdot \frac{d\Delta_C}{dN_a}\right) & \Delta_C > 0 \\ -\rho \cdot (N - N_a) \cdot (\Delta_C)^2 \cdot \left(-\Delta_C + 2 \cdot (N - N_a) \cdot \frac{d\Delta_C}{dN_a}\right) & \Delta_C < 0 \end{cases} \tag{30}$$

Regarding the stability of the dynamical system Eq.(26), we have the following proposition:

Proposition 4. *An AV penetration N_a is a stable equilibrium if any of the following is valid* *(i)* $C_a > C_b$ *and* $N_a = 0$*,*

(ii) $C_a < C_b$ *and* $N_a = N$*, or*

(iii) $C_a = C_b$ *and* $\dot{C}_a \cdot \dot{N}_a > 0$*.*

Proof. When (i) $C_a > C_b$ and $N_a = 0$, $\Delta_C > 0$. It is readily verified that the equilibrium condition Eq.(22) is satisfied, and thus $C_a > C_b$ and $N_a = 0$ secure an equilibrium solution. $\forall N_a \neq 0$ in the neighborhood $\{\mathcal{B} : \|N_a - 0\| < \epsilon\}$, $\epsilon > 0$,

$$\lim_{\epsilon \to 0} \frac{dV(N_a)}{dN_a} \cdot \dot{N}_a = \lim_{N_a \to 0} -\rho \cdot N_a \cdot (\Delta_C)^2 \cdot \left(\Delta_C + 2N_a \cdot \frac{d\Delta_C}{dN_a} \right) < 0.$$

When (ii) $C_a < C_b$ and $N_a = N$, $\Delta_C < 0$. Similarly, it can be verified that the equilibrium condition Eq.(22) is satisfied, and thus $C_a < C_b$ and $N_a = N$ secure an equilibrium solution. $\forall N_a \neq N$ in the neighborhood $\{\mathcal{B} : \|N_a - N\| < \epsilon\}$, $\epsilon > 0$,

$$\lim_{\epsilon \to 0} \frac{dV(N_a)}{dN_a} \cdot \dot{N}_a = \lim_{N_a \to N} -\rho \cdot (N - N_a) \cdot (\Delta_C)^2 \cdot \left(-\Delta_C + 2 \cdot (N - N_a) \cdot \frac{d\Delta_C}{dN_a} \right) < 0.$$

When (iii) $C_a = C_b$ and $\dot{C}_a \cdot \dot{N}_a > 0$ prevail, they combined secure an equilibrium solution. We denote the equilibrium AV penetration as $\widehat{N}_a$. Given $\dot{C}_a \cdot \dot{N}_a > 0$, $\frac{dC_a}{dN_a} > 0$, and it follows that $\frac{d\Delta_C}{dN_a} = \frac{dC_a}{dN_a} - \frac{dC_b}{dN_a} = \frac{dC_a}{dN_a} > 0$. $\forall N_a \neq \widehat{N}_a$ in the neighborhood $\{\mathcal{B} : \|N_a - \widehat{N}_a\| < \epsilon\}$, $\epsilon > 0$,

$$\begin{aligned} &\lim_{\epsilon \to 0} \tfrac{dV(N_a)}{dN_a} \cdot \dot{N}_a \\ &= \lim_{N_a \to \widehat{N}_a} \begin{cases} -\rho \cdot N_a \cdot (\Delta_C)^2 \cdot \left(\Delta_C + 2N_a \cdot \frac{d\Delta_C}{dN_a} \right) < 0 & \Delta_C > 0 \\ -\rho \cdot (N - N_a) \cdot (\Delta_C)^2 \cdot \left(-\Delta_C + 2 \cdot (N - N_a) \cdot \frac{d\Delta_C}{dN_a} \right) < 0 & \Delta_C < 0 \end{cases} \end{aligned}$$

Based on Definition 1, the dynamical system Eq.(26) is stable in the neighborhood of the equilibria governed by condition (i), (ii), or (iii). □

Proposition 4 elucidates the stability of the long-run equilibria established in Section 3.1. Recall that we present in Figure 4 representative equilibria and highlight the stable equilibria by blue circles. One could verify that each stable equilibrium point falls into one of the three situations (i)-(iii) in Proposition 4.

In addition to the three situations (i)-(iii), there is one more situation that entails a long-run equilibrium, i.e., (iv) $C_a = C_b$ and $\dot{C}_a \cdot \dot{N}_a < 0$. Note that conditions (i)-(iv) are complete in terms of characterizing the long-run equilibria such that they cover all possible situations that satisfy the generic equilibrium condition Eq.(22); at the same time, they are mutually exclusive. However, different from the previous three conditions, the equilibrium governed by condition (iv) is unstable. Consider any equilibrium AV penetration N_a that satisfies condition (iv). When there is a small perturbation that changes the AV penetration, the dynamical system will deviate from the current equilibrium. If perturbation increases the penetration by a small amount such that $\dot{N}_a > 0$ ('positive perturbation'), the penetration will continue to increase as the long-run average cost of AV falls below that of TV given $\dot{C}_a < 0$; and vice versa if the direction of perturbation is reversed. In Figure 4, red circles signifies those equilibria in line with condition (iv) and the associated arrows represent the evolving direction of AV penetration given positive perturbations.

The physical meanings of equilibrium conditions (i)-(iv) are significant in the sense that each equilibrium showcases a representative scenario in the evolution process of AV adoption among population. When condition (i) stands, $C_a > C_b$ and $N_a = 0$. The long-run cost of AV is larger than that of TV and very few people commute with AV. This represents the status quo in practice – significantly high cost, lack of supporting infrastructure, and widespread safety concerns associated with self-driving – the AV penetration is almost zero in the population.

In contrast, condition (ii) ($C_a < C_b$ and $N_a = N$) signifies another extreme where the whole population commute with AV and the long-run cost of AV is reduced to below that of TV. As discussed above, given the economy of scale of AV manufacturing, the development of self-driving technologies, facilities and infrastructures, and the elimination of safety concerns, this may be realized some day in the future. Conditions (i) and (ii) both yield stable equilibrium because in both scenarios the whole population is stick to the option with significantly lower cost and no one has the incentive to swap.

Condition (iii) represents an intermediate scenario where part of the population commute with AV. $C_a = C_b$ and $\dot{C}_a \cdot \dot{N}_a > 0$ imply that the two options have identical cost but the cost of AV increases with the penetration (primarily because of the increasing commuting time as discussed in Section 3.1). It is a stable equilibrium because if someone swaps from TV to AV, it will marginally increase the cost of AV, and evidently the higher cost of AV will prevent her/him from doing so; and vice versa for swapping from AV to TV.

Proposition 4 implies that scenario (i)-(iii) each corresponds to a stable equilibrium of the dynamical system Eq.(26), and requires external interventions to break the equilibrium status when needed.

As a complement to condition (iii), condition (iv) describes the scenario where the two options have identical cost but the average cost of AV decreases with the penetration (primarily because of the decreasing usage cost as discussed in Section 3.1). As discussed above, where $C_a = C_b$ and $\dot{C}_a \cdot \dot{N}_a < 0$ corresponds an unstable equilibrium. Intuitively, if someone swaps from TV to AV, it will marginally reduce the cost of AV. The lower cost of AV will encourage more commuters to swap from TV to AV and thus the AV penetration will continue to grow, namely, such an equilibrium is unstable.

3.3 Discussions on the equilibrium efficiency and AV subsidy

Section 3.2 identifies three cases that may arise with a stable equilibrium. This section examines the efficiency of the equilibrium from the system's perspective. Denote LTC the system-wide long-run total cost, which is the sum of long-run cost of all AV users and TV users, i.e.,

$$LTC = N_a \cdot C_a + N_b \cdot C_b. \tag{31}$$

Let $LTC_{(m)}$, $m \in \{i, , ii, iii\}$ represents the long-run total cost at the equilibrium governed by condition (i)-(iii) respectively. When condition (i) stands, $C_a > C_b$, $N_a = 0$, and thus $LTC_{(i)} = 0 \cdot C_a + (N - 0) \cdot C_b = N \cdot C_b$; when condition (ii) stands, $C_a < C_b$, $N_a = N$, and thus $LTC_{(ii)} = N \cdot C_a^N + 0 \cdot C_b = N \cdot C_a^N$; when condition (iii) stands, $C_a = C_b$, $\dot{C}_a \cdot \dot{N}_a > 0$, and thus $LTC_{(iii)} = N_a \cdot C_b + (N - N_a) \cdot C_b = N \cdot C_b$. Given the relationship $C_a^N < C_b$ established in Eq.(25), we have

$$LTC_{(i)} = LTC_{(iii)} > LTC_{(ii)}. \tag{32}$$

Eq.(32) shows that equilibrium condition (ii) yields the lowest long-run total cost among the three conditions and it is actually the minimum LTC that the system can achieve. The other two conditions, whilst each lead to a stable equilibrium, incur larger LTC. Based on the comparison, the equilibrium under condition (ii) is featured as an "efficient equilibrium", and "inefficient equilibrium" under condition (i) or (iii). From the system's point of view, the efficient equilibrium (system optimum) is preferable than inefficient ones (system non-optimum) as it leads to lowest deadweight loss.

As established in Section 3.2, the evolution of AV penetration may incur multiple equilibrium statuses, including both stable equilibrium and unstable equilibrium. When the system reaches an unstable equilibrium, a small perturbation will destroy the equilibrium and the system will evolve to other status automatically. However, when it hits a stable equilibrium, the system being disturbed (by small perturbations) will rebound to the current equilibrium, and thus is more likely to stabilize at the current status. If the stable equilibrium is an efficient one, it leads to the lowest LTC and it is the best scenario can be achieved from the system's point of view. If it is an inefficient equilibrium, however, the system cost is not the optimal but the system will stabilize at the current equilibrium. This, to some extent, explains the current situation where purchase price is high, safety concern is present, and AV adoption rate is very low. Without external intervention that is able to reduce LTC or δ, the system is unlikely to move elsewhere.

To prevent the system from staying at an inefficient equilibrium and further steer it to the optimum, external intervention is necessary. With such intention, the government may consider to provide subsidies that confer the advantage. The design of a subsidy scheme should aim to bridge the discrepancy between the long-run average costs of AV and TV. The most common subsidy in the transportation sector is the ones provided to the public transport users or operators in order to lower their costs or supplement their income (Frankena, 1981; Wang et al., 2017). In the same spirit, the recipient of the AV subsidy could be the AV users so that the subsidy will compensate part of the cost C_a. Alternatively, the subsidy could be provided to upstream stakeholders, expecting that the spillover effect will reduce the usage cost δ imposed on AV users. Relevant stakeholders may include those involved in the development of self-driving technologies, AV manufacturing and maintenance, and/or AV infrastructure and facility upgrades, etc. By subsidizing the expenses of these stakeholders, the aggregate cost of manufacturing and supporting AVs will be lower and self-driving will be safer, which in turn reduces the average usage cost of each individual user.

Without loss of generality, the following analysis will focus on the subsidy to be provided to each individual user to achieve the system optimum. In reality, when considering subsidizing relevant stakeholders, the amount of subsidy should guarantee that the spillover effect on AV user's cost is significant enough to bridge the gap between AV and TV costs.

In terms of the time of implementation, the subsidy scheme should take effect when the system reaches an inefficient equilibrium until it hits an unstable equilibrium from where the system itself will evolve towards the efficient equilibrium.

Therefore, given any AV penetration N_a, the amount of subsidy to be provided per capita should be

$$\phi(N_a) = max\{C_a(N_a) - C_b, 0\} + \varepsilon, \tag{33}$$

where $max\{C_a(N_a) - C_b, 0\}$ represents the lower bound of the subsidy and $\varepsilon > 0$ represents a buffer amount exceeding the minimum. Theoretically, ε could be infinitely small but it must be positive in order to ensure the system will evolve to the optimum. The value of ε has influences on the convergence speed of the system, for which sensitivity test is done in Section 4.2.3.

Figure 5 illustrates the subsidy required to conquer the stable equilibrium governed by condition (i) or (iii), and to steer the system to an unstable equilibrium. The lower bound of the subsidy is represented by blue arrows which exactly cover the gap between C_a (black curve) and C_b (green line). If the government intends to provide a constant subsidy to cover the whole period, the amount should be able to bridge the maximum gap between the costs, i.e., $max\{C_a\} - C_b$. The provided constant subsidy will translate the C_a curve downwards to the dotted curve, which is entirely below C_b meaning that adoption of AV will be almost automatic.

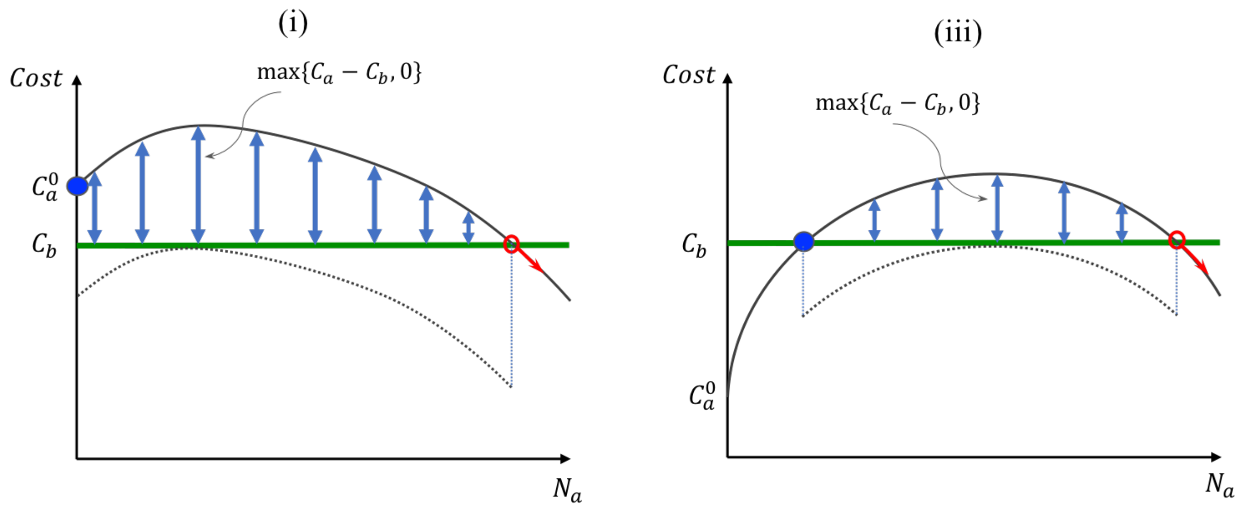


Figure 5: The AV subsidy takes effect from the stable equilibrium governed by condition (i) or (iii).

4 Numerical studies

This section presents some numerical experiments to illustrate the proposed model and analysis. We start with the major common numerical settings. Following Liu et al. (2015), the value of time α_b is 9.91(\$/h), the early arrival penalty β is 4.66(\$/h), and the late arrival penalty γ is 14.48(\$/h). Empirical studies suggest that commuters' value of time will be reduced by around 30% when traveling with AV as oppose to TV. We let $\alpha_b = 0.7 \cdot \alpha_b = 6.94$(\$/hour). It is assumed that the free-flow travel time is $t_f = 0.25$(hour), the total demand is $N = 10000$(veh/hour), and the capacity of the highway bottleneck is $s = 3000$(veh/hour). Based on these settings, we numerically establish the short-run and long-run equilibria with AVs and explore their characteristics respectively.

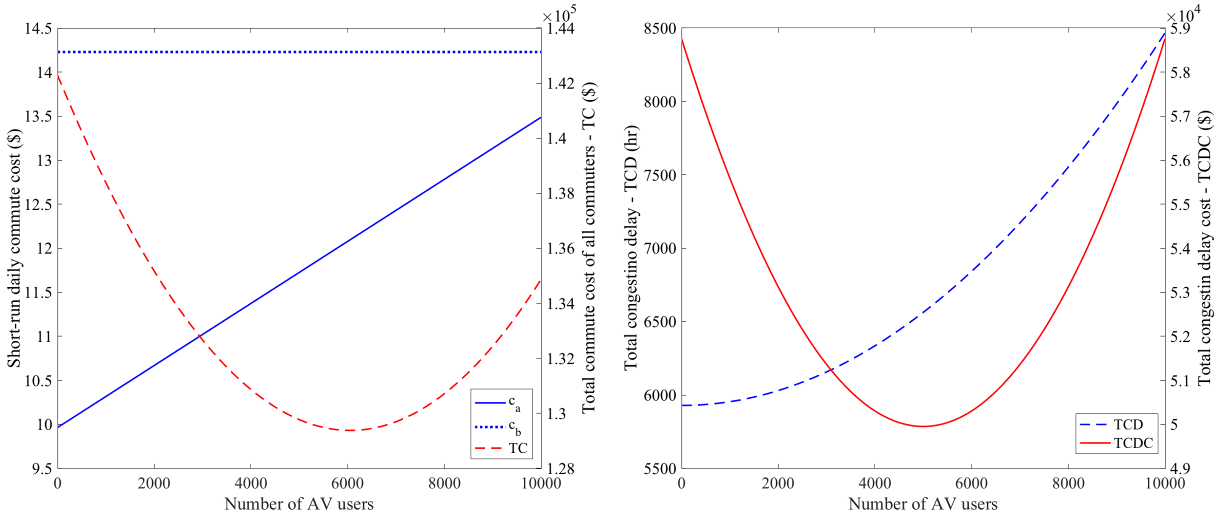
4.1 Short-run commuting equilibrium

We firstly look at the short-run commuting equilibrium. Following Section 2, we assume in the short-run, a commuter's vehicle type is fixed and so is the number of AV users among population. Given any AV penetration rate, commuters optimize their departure time choices to minimize their daily commuting travel cost. In equilibrium, the departure/arrival pattern is shown in Figure 3. Figure 6 presents how the key cost terms change with the number of AV users (AV usage). The left panel conveys the individual travel costs (c_a and c_b) and the total travel cost (TC). The right panel contains the total congestion delay (TCD), and the total congestion delay cost ($TCDC$).

When the AV penetration rate increases from zero to one, the individual travel cost of AV uses represented by the solid blue line in Figure 6 increases from 9.97($) to 13.49($), and that of TV users (the dotted blue line) remains to be 14.23($). In accordance with Lemma 1, the growth of AV penetration leads to the increase of commuting cost of AV users, but has no direct effect on the commuting cost of TV users. This is due to the competition among the same user type (AV users) who intend to travel in similar time windows. However, it does not affect the travel cost of other type of users (TV users) who intend to travel in other time windows.

The total travel cost, represented by the dashed red curve decreases with the AV penetration when it is relatively small; when it exceeds a certain value, the total cost increases with the penetration. This is because increasing AV penetration reduces the total travel time cost (enabled by a reduced value of time) but increases the individual cost of AV users (due to the competition). The former effect dominates when AV penetration is small while the latter is more significant when the penetration is large. The cut-off AV usage is 6054 where the penetration rate is greater than 0.6 (as anticipated in Proposition 1).

Figure 6 also shows that the total congestion delay time (the dashed blue curve) monotonically increases with the AV penetration. When the time is multiplied by the value of time of AV and TV users respectively, the total congestion delay cost (the solid red curve) has a minimum at 5000, where the penetration rate is 0.5 (in line with Proposition 2).



(a) c_a, c_b, and TC

(b) TCD and $TCDC$

Figure 6: The changes of (a) individual travel costs and total cost, (b) total congestion delay and total congestion delay cost with respect to the number of AV users at the short-run commuting equilibrium.

4.2 Long-run evolution of the AV penetration

This section incorporates the AV usage cost δ function and explores the evolution of the AV penetration in the long run. Following the analysis in Section 3, we introduce the $\delta(N_a)$ function defined in the interval $N_a \in [0, N]$; when $N_a = 0$, $\delta(N_a) > 0$; when $N_a = N$, $\delta(N_a) < 0$. Section 3.1 described numerous scenarios of the long-run equilibria with various functional forms of δ. In reality, the barrier of adopting AVs is currently tremendous since the purchase price of AV is high and the safety concern of self-driving is significant. It is thus more relevant to focus on the scenarios reflecting such concerns. The numerical analysis will therefore only consider scenarios that fall into Case I in Figure 4, with the understanding that the equilibria yielded from Case II and III can be reviewed as special cases of those from Case I and thus are expected to exhibit similar properties.

A particular δ function can be either monotonic or non-monotonic. We introduce two particular functions to represent each and to illustrate the evolution of the dynamical system established in Section 3.2 as well as the stability of long-run equilibria. The exponential function

$$\delta_{exp}(N_a) = 10\Bigg\{1 - exp\Bigg[-10000\left(\frac{1}{N_a} - \frac{1}{10000}\right)\Bigg]\Bigg\} - 3 \tag{34}$$

is designated as the monotonic form, which is an adopted version from the well-known Newell model (Newell, 1961). The non-monotonic function is represented by a polynomial function

$$\delta_{poly}(N_a) = -1.8 \times 10^{-10} N_a^3 + 2.8 \times 10^{-6} N_a^2 - 1.28 \times 10^{-2} N_a + 18.88 \tag{35}$$

In the following sections, we will present numerical results with the two functions, respectively. It would be interesting to identify and calibrate the "true" functions for δ, which is beyond the scope of this study.

4.2.1 With the monotonic δ_{exp} function

Figure 7 depicts the long-run average cost of AV and TV users, which includes both the commuting cost and usage cost. The usage cost of a TV is normalized to zero and thus the long-run average cost is identical to the daily commuting cost, i.e., 14.23($). It is represented by the dashed red line in Figure 7. The usage cost of an AV thus reflects the discrepancy between the two vehicle types. Given the monotonic function Eq.(34), the change of usage cost with the number of AV users is represented by the downward-sloping dotted blue curve. The combined cost of AV users, representing by the solid red curve, steadily increases from 16.97($) to the maximum 17.48($), and then drops to 10.49($) when the AV penetration increases from zero to one. When the number of AV users is 5735, the long-run average cost of the two vehicle types is identical. Following the long-run equilibrium condition Eq.(22), the scenario presented in Figure 7 involves two boundary equilibria at $E_{exp}^1(N_a^1 = 0, C_{eq}^1 = 14.23)$ and $E_{exp}^3(N_a^3 = 10000, C_{eq}^3 = 10.49)$, and one interior equilibrium at $E_{exp}^2(N_a^2 = 5735, C_{eq}^2 = 14.23)$. Since the boundary equilibria E_{exp}^1 and E_{exp}^3 satisfy condition (i) and (ii) of Proposition 4 respectively, they are expected to be stable equilibrium statuses. The interior equilibrium E_{exp}^2 arises at which the AV cost decreases with penetration – condition (iii) of Proposition 4 is violated. Thus, E_{exp}^2 is expected to be an unstable equilibrium.

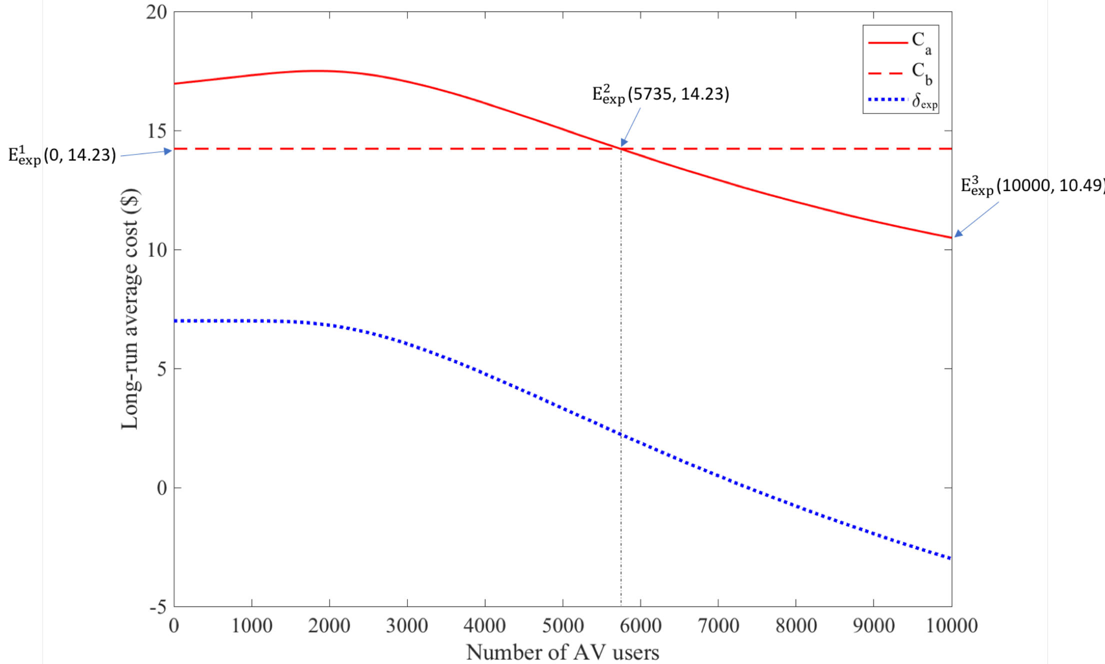


Figure 7: Long-run average costs with the monotonic δ_{exp} function.

Figure 8 and Figure 9 present the evolutions of N_a and $(C_a - C_b)$ of the dynamical system Eq.(26) starting from different initial N_a values. The parameter ρ is set to be 0.001 (extensive sensitivity analysis shows that the value of ρ only affects the convergence speed – it influences neither the convergence nature nor the stability of any equilibrium status). It is shown in Figure 8 and Figure 9 that the evolution processes starting from an initial $N_a \in [0, 5735)$ converge to the lower boundary equilibrium $E^1_{exp}(0, 14.23)$ where the average cost of AV is higher than TV (cost difference is positive) and no one uses AV. Those starting from an initial $N_a \in (5735, 10000]$ evolve to the upper boundary equilibrium $E^3_{exp}(10000, 10.49)$ where the average cost of AV is below TV (cost difference is negative) and the whole population commute with AV. Such an observation validates the anticipation from Proposition 4 that E^1_{exp} and E^3_{exp} are stable equilibria. The two intervals $[0, 5735)$ and $(5735, 10000]$ are the 'attraction domains' of the two equilibrium points E^1_{exp} and E^3_{exp} respectively (for the definition of 'attraction domain', one may refer to Bie and Lo, 2010).

The dynamical system starting exactly from the interior equilibrium ($N^2_a = 5735$) stays at this point. We find that any small deviation in the initial value leads to other statuses. For example, with an initial $N_a = 5700 < N^2_a$, the system evolves to E^1_{exp}; with $N_a = 5800 > N^2_a$, it evolves to E^3_{exp}. In line with the analysis in Section 3.2, the numerical results demonstrate that the interior equilibrium E^2_{exp} is unstable since a small perturbation will destroy the equilibrium, from which the system will evolve to other status automatically.

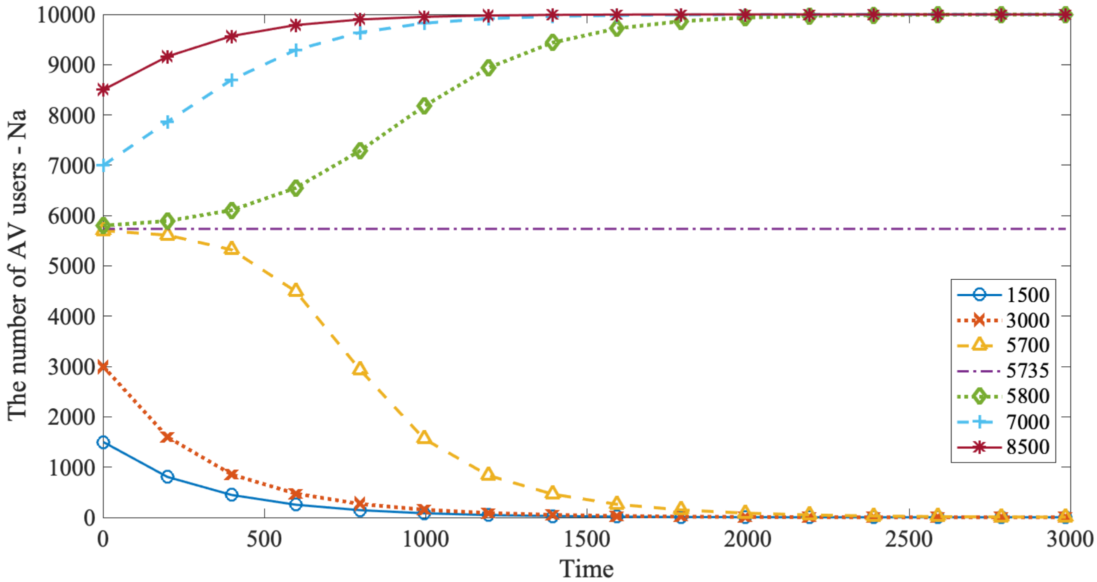


Figure 8: Evolution of AV usage with different initial N_a values (with the monotonic δ_{exp} function).

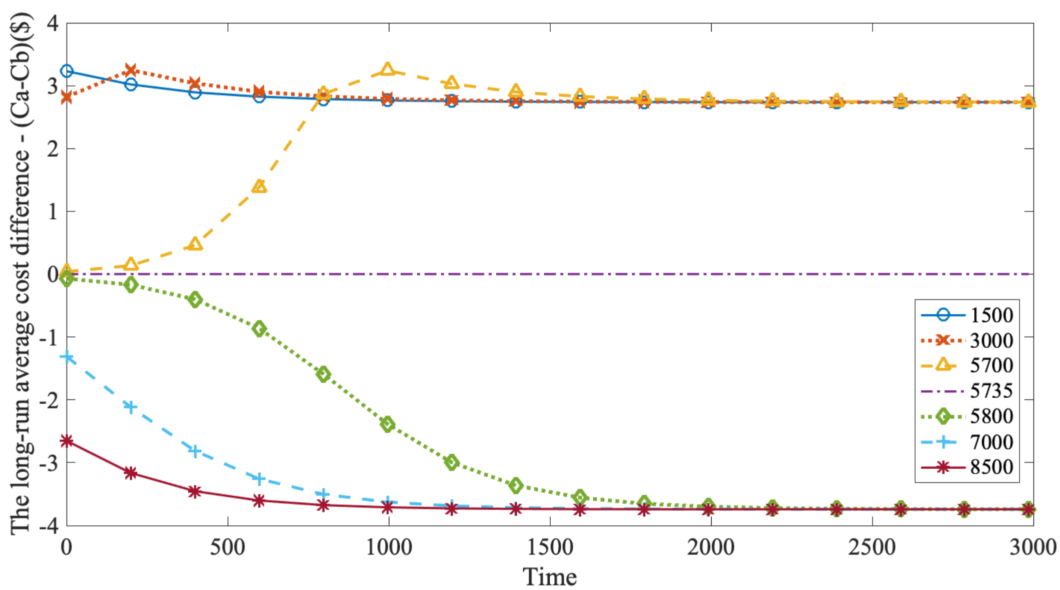


Figure 9: Evolution of long-run average cost difference with different initial N_a values (with the monotonic δ_{exp} function).

4.2.2 With the non-monotonic δ_{poly} function

We now proceed to present the numerical results where the non-monotonic δ_{poly} function given in Eq.(35) prevails. Figure 10 shows how the long-run average costs of AV and TV users change with the AV penetration. It is shown that the average usage cost δ_{poly} (represented by the dotted blue curve) oscillates up and down around zero leading the combined AV cost C_a (solid red curve) oscillates in a parallel manner, with a range around C_b (dashed red line). Such a system yields five equilibrium points, i.e., $E^1_{poly}(N^1_a = 0, C^1_{eq} = 14.23)$, $E^2_{poly}(N^2_a = 1849, C^2_{eq} = 14.23)$, $E^3_{poly}(N^3_a = 5220, C^3_{eq} = 14.23)$, $E^4_{poly}(N^4_a = 8331, C^4_{eq} = 14.23)$, and $E^5_{poly}(N^5_a = 10000, C^5_{eq} = 2.47)$. It is evident that E^1_{poly}, E^5_{poly}, and E^3_{poly} satisfy condition (i), (ii), and (iii) of Proposition 4, respectively. Thus, they are expected to be stable equilibrium that will attract the system to evolve from other statuses. The other two interior equilibria with descending AV cost (E^2_{poly} and E^4_{poly}) are expected to be unstable equilibrium.

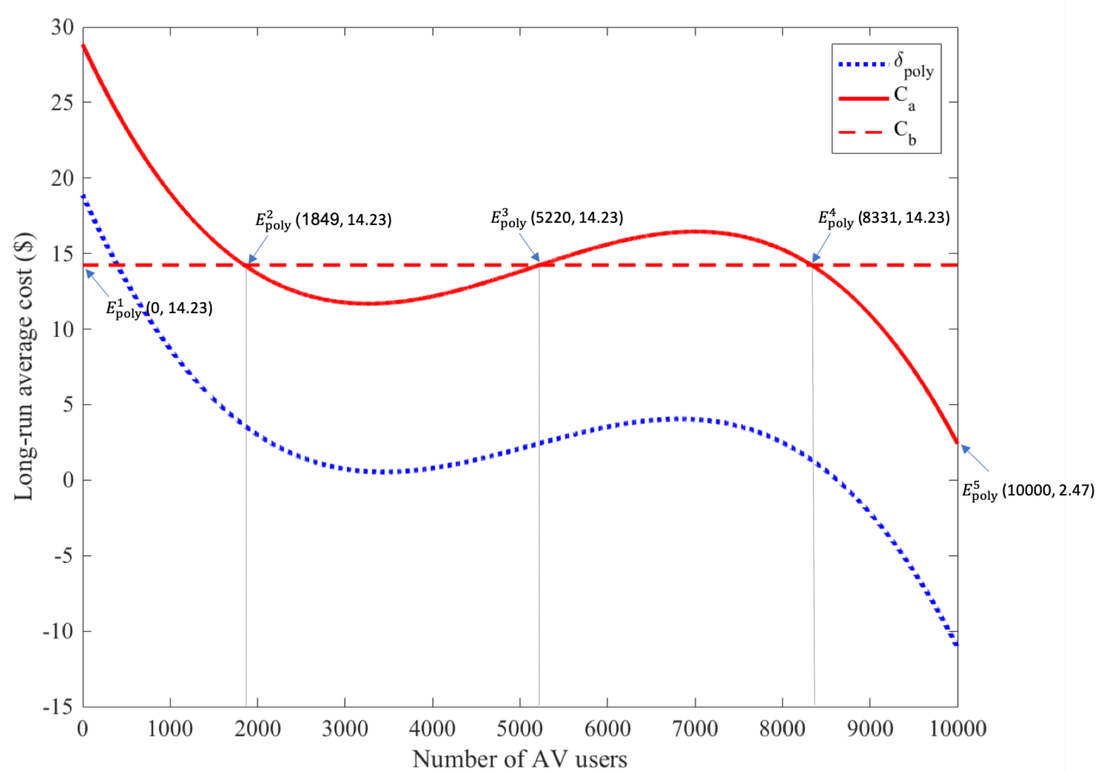


Figure 10: Long-run average costs with the non-monotonic δ_{poly} function.

The non-monotonic δ_{poly} function is implemented in the dynamical system to examine the existence and stability of the equilibria. It is shown in Figure 11 and Figure 12 that the dynamical system indeed entails five equilibrium points as listed above. The system will evolve to E^1_{poly}, E^3_{poly}, and E^5_{poly} when the initial N_a value falls in the interval of $[0, 1849)$, $(1849, 8331)$, and $(8331, 10000]$ respectively. This endorses the stability of these three equilibria and identifies their attraction domains respectively. The other two equilibria $E^2_{poly}(N^2_a = 1849)$ and $E^4_{poly}(N^4_a = 8331)$ act as the watersheds of the attraction domains. When starting from exactly E^2_{poly} or E^4_{poly}, the system will stay; a small deviation in the initial N_a will lead the system to one of the three stable equilibrium statuses (E^1_{poly}, E^3_{poly}, and E^5_{poly}), implying that E^2_{poly} and E^4_{poly} represent an unstable equilibrium status each.

As a remark, extensive sensitivity analysis is conducted to test the robustness of the results involving various parameter values and different forms of the δ function. We find that the general observation – the stability of the long-run equilibrium is governed by Proposition 4 – is robust to the variations. The parameter values will influence where equilibrium arises and how fast the system converges to equilibrium.

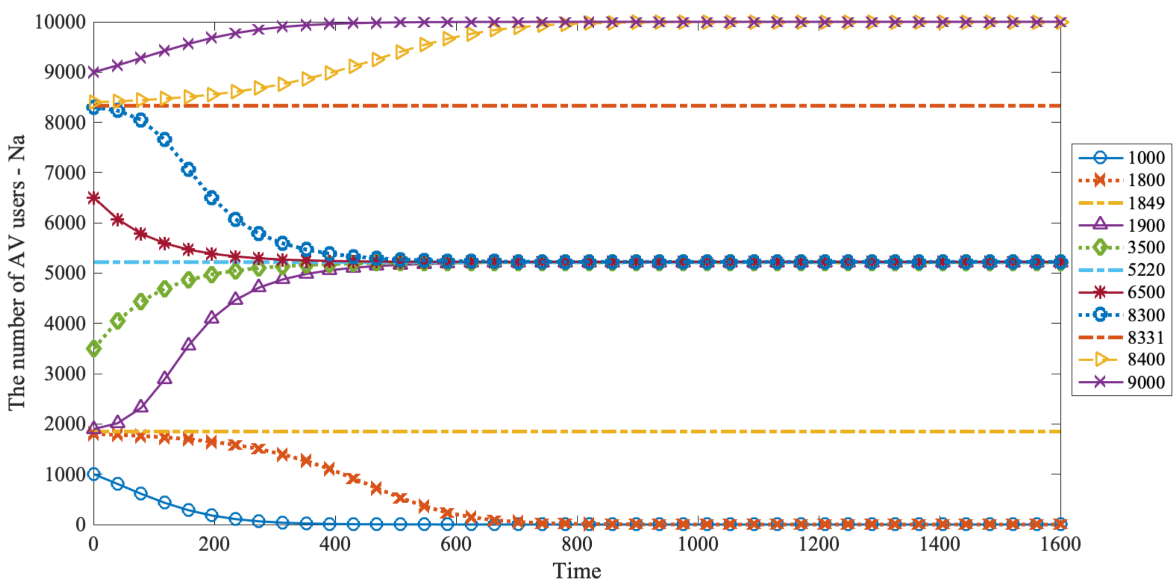


Figure 11: Evolution of AV usage with different initial N_a values (with the non-monotonic δ_{poly} function).

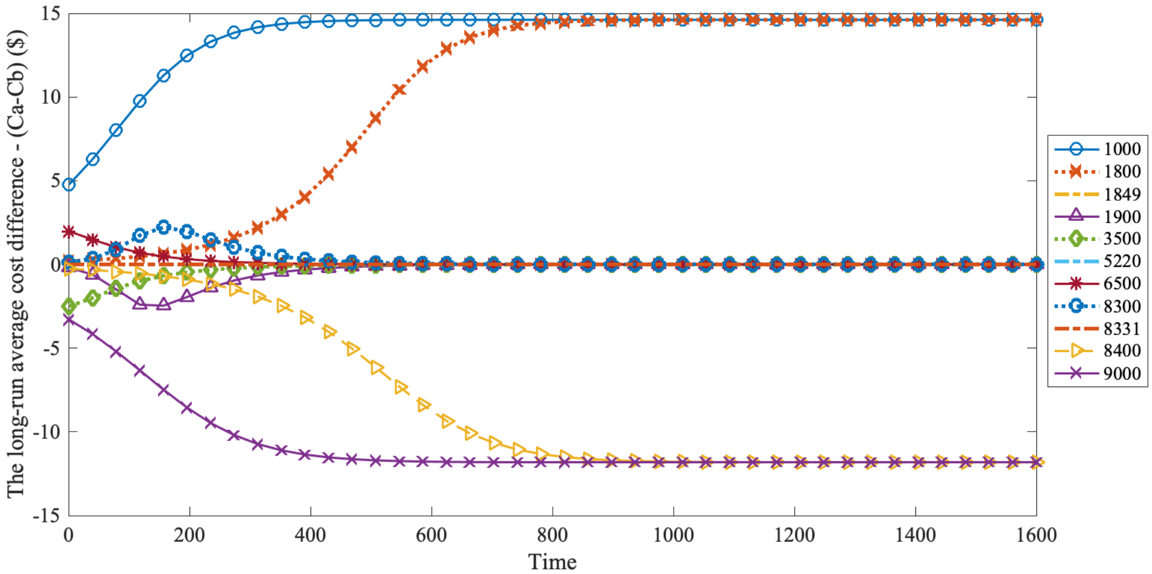


Figure 12: Evolution of long-run average cost difference with different initial N_a values (with the non-monotonic δ_{poly} function).

4.2.3 Effect of subsidy

As analytically examined in Section 3.3, discrepancy exists in the levels of efficiency yielded from different equilibrium points. In terms of the system-wide long-run total cost LTC defined in Eq.(31), the equilibrium confronted with condition (ii) of Proposition 4 yields the lowest LTC and thus is the most efficient equilibrium (system optimum). However, the attraction domain of this equilibrium does not cover the whole feasible domain. Instead, in many cases the dynamical system converges to other equilibrium status and is stuck in an inefficient equilibrium. The status quo in practice can be interpreted by an inefficient equilibrium status where the cost of AV is much higher than TV and the AV penetration is almost zero in the population (such as E^1_{exp} and E^1_{poly}). If we consider commuters as lower-price seekers, they do not have the incentive to adopt the AV. Thus, small increments in the AV penetration are unstable – the system is likely to rebound to the initial equilibrium and stabilize thereafter.

To swerve the system from the inefficient equilibrium and steer it to the optimum, external intervention is necessary. We examine the effect of subsidy scheme proposed in Section 3.3, the amount of which is given in Eq.(33). In the numerical analysis, the buffer amount ε is set to be 0.1 to retrieve the effect of a fully tailored subsidy scheme with minimum amount. Based on the dynamical system with δ_{poly}, the tailored subsidy scheme is presented in Figure 13. Comparing with Figure 10, the subsidy is in effect only when the long-run average cost of AV exceeds that of TV and the amount is equal to to cost difference (plus a small buffer). Figure 14 and Figure 15 present the effect of the subsidy scheme. It is shown that provided with the minimum subsidy, the dynamical system is able to converge to the system optimum E^5_{poly} with the lowest LTC, regardless of the initial status.

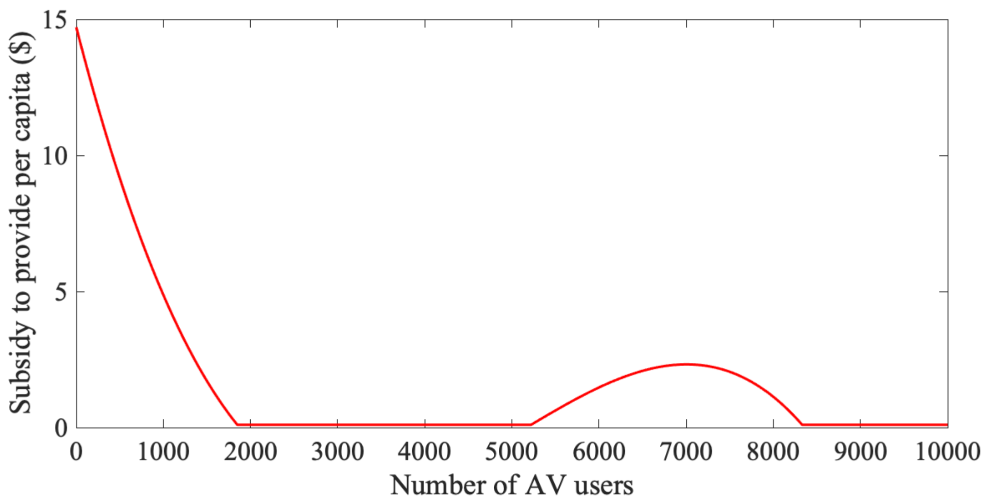


Figure 13: A fully tailored subsidy scheme for the dynamical system with δ_{poly}.

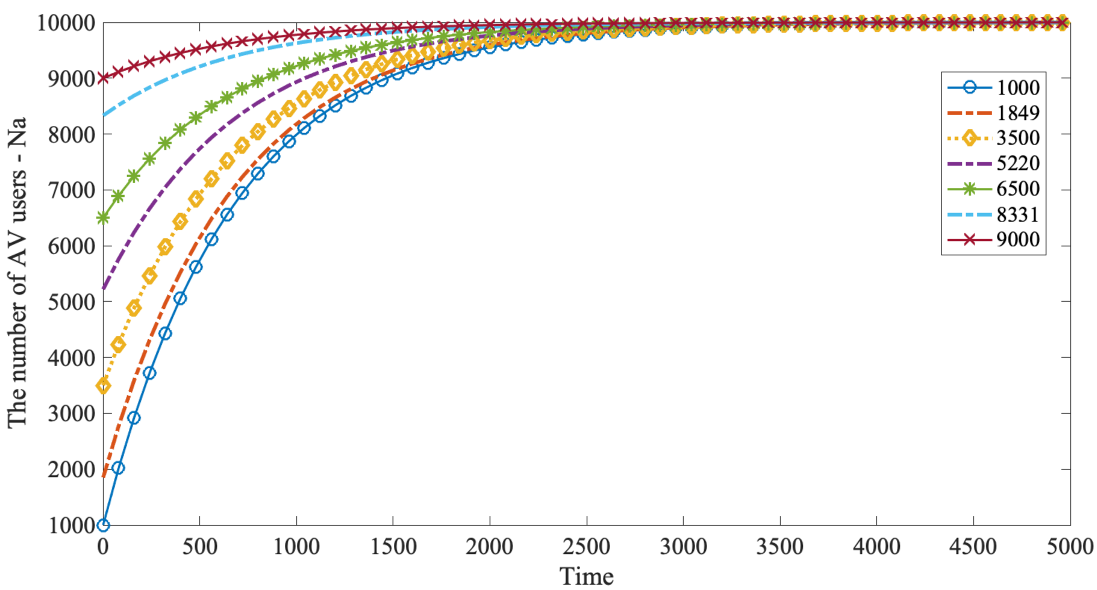


Figure 14: Evolution of AV usage with AV subsidy.

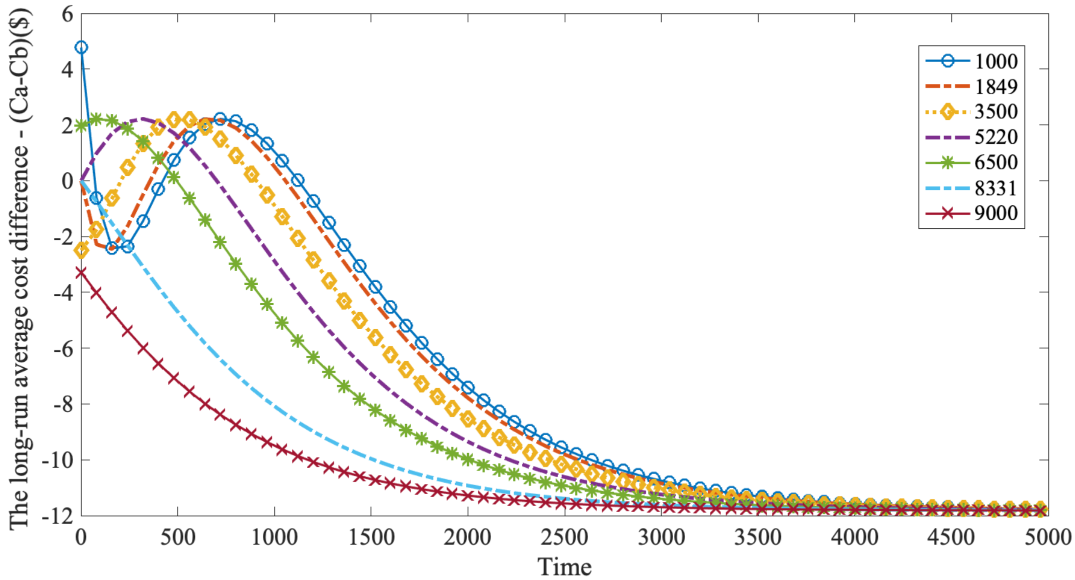


Figure 15: Evolution of long-run average cost difference with AV subsidy.

In the subsidy formula Eq.(33), a buffer term (a small positive constant ε) is added to ensure that the subsidy is strictly greater than the cost difference. The value of ε determines the convergence speed of the system. A sensitivity test is conducted to examine the effect. Figure 16 shows the evolution of N_a from the same initial point but with various values of ε. The benchmark ε is 0.1, in line with previous examples. When ε takes the value of 0.025, 0.05, 0.1, 0.2, and 0.4, the social optimum is achieved after 10000, 7200, 3680, 2080, and 1440 iterations, respectively. It is shown that increasing the amount of buffer term (AV subsidy) will speed up the convergence to social optimum, but the marginal effect decreases with the buffer value.

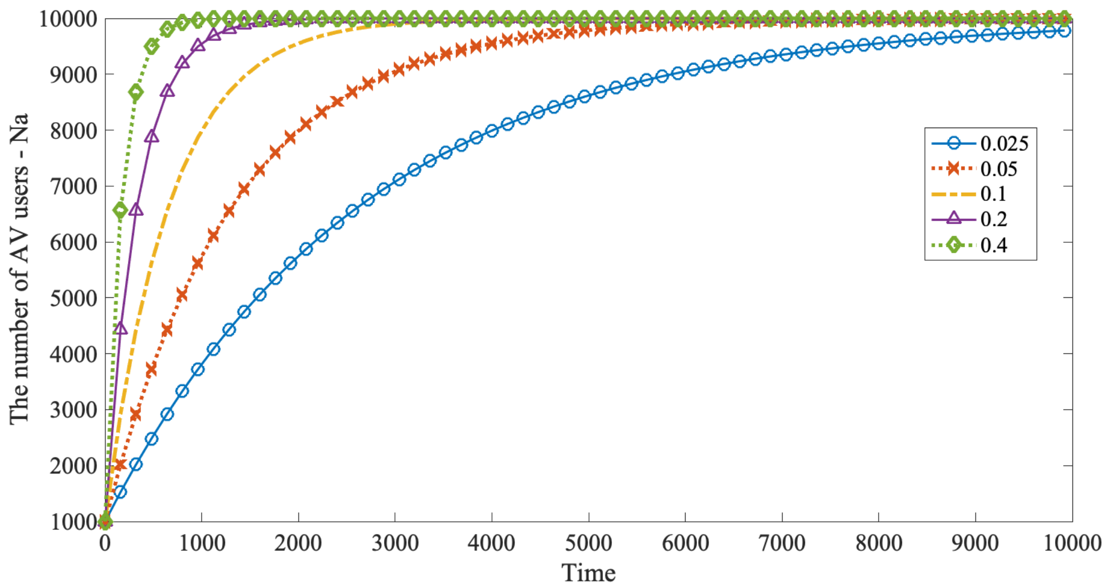


Figure 16: Convergence of AV usage with various ε.

5 Conclusions

To envisage the adoption of autonomous vehicles in both short run and long run, this paper investigates the short-run traffic equilibrium under a given AV penetration and explores the long-run travel equilibrium with an endogenous AV penetration. When examining the long-run equilibrium with AVs, this paper takes into account factors other than travel cost components. This paper is the first analytical framework in the literature to consider both short-run and long-run equilibria with AVs, which sheds light on future AV adoptions, car ownership, and traffic management.

Different from many previous studies focusing on a pure AV context, the short-run model considers a more general situation where the traffic is a mixture of AVs and TVs. This will reflect the reality for years before the whole stock of TVs is replace by AVs. In light of recent empirical studies, the model further incorporates the prediction that the value of travel time can be substantially reduced by using AVs. Base on this, we establish the short-run morning commute equilibrium where commuters optimize their departure time choices. We find that the two types of commuters travel in different time windows. AV users travel around the desired arrival time t^* where the queuing time is longer and TV users travel in the earlier and later windows where the schedule delay cost is larger. Such a separation is motivated by the divergent preferences towards congestion delay and schedule delay.

The observation from the short-run analysis has important implications. Firstly, due to the competition among the same type of users who intend to travel in a same time window, the more people use AV, the higher the cost is for AV users. In contrast, the AV penetration does not affect the travel cost of TV users who intend to travel in other time windows. Secondly, while an AV allows flexibility for the commuter during the trip and reduces the cost for a unit of travel time, it will result in more congestion delays. This is to say, when delay is cheaper for commuters, they will queue more to seek less schedule delay cost. Thirdly, the optimal AV penetration (efficient in the short run) is determined by the tradeoff between in-vehicle travel time cost and schedule delay cost.

The short-run analysis assumes the vehicle type that each commuter owns is fixed and thus the AV penetration rate is exogenously given. When considering a longer time period, however, commuters may replace their vehicles and may shift to another type of vehicle in the meantime. Therefore, the long-run analysis takes into account the endogenous vehicle type choice, which depends on the long-run average cost of using AV as oppose to TV. This includes not only the daily commuting cost but also an ownership-related external cost. The average usage cost δ term is introduced to capture commuter's perception of the relative external cost or benefit associated with using AV with respect to TV, which is driven by the number of AV users. In this context, the long-run equilibrium condition is established with the existence of equilibrium guaranteed. Multiple equilibria may arise depending on the relationship between C_a and C_b. By examining the dynamical system where the AV penetration evolves over time with the change of C_a, the conditions for an equilibrium to be a stable one is established, which falls into three different scenarios. One of the scenario reflects the status quo in practice, where the combined cost of an AV is higher than TV, and the AV penetration rate is very low. There exists other more efficient equilibrium with lower total cost system-wise. However, because the status quo is a stable equilibrium, external intervention is needed to drive it elsewhere.

We also examine the effect of subsidy provided to AV users or relevant stake holders as a compensation for their expenses or income. In order to prevent the system from staying at an inefficient equilibrium and further steer it to the optimum, the amount of subsidy should effectively cover the cost difference whenever the long-run average cost of AV exceeds that of TV. Numerical experiments show that even with the minimum subsidy, the dynamical system is able to converge to the system optimum with the lowest system cost, regardless of the initial status.

The analysis of this paper can be fruitfully extended in the following avenues. Firstly, the road capacity may be improved when AVs come into play. It is generally expected that when there is a larger proportion of AVs in the mixed traffic, the vehicles could be better coordinated and thus yield a larger effective road capacity. Future studies could investigate the traffic equilibrium where the road capacity is driven by the AV penetration (Lamotte et al., 2017; van den Berg and Verhoef, 2016), or even stochastic (Lindsey, 2009; Xiao et al., 2015). However, it should be noted that how AVs and TVs are mixed on road may significantly affect the capacity even with the same AV penetration. Secondly, this study focuses on the split of commuters into AV and TV while treating the total demand as fixed. In reality, not only commuters have other transport options, public transit operators could also upgrade their services to compete with the driving mode. Following the notion of Zhang et al. (2016, 2018); Zhang and Liu (2019), future studies could examine the responses of both transit operators and commuters in a multi-modal system. Thirdly, parking is usually an important concern in daily commuting with private cars and may influence travel choices when the availability is limited (Arnott et al., 1991; Zhang et al., 2008; Yang et al., 2013). It is expected that autonomous vehicles can search for parking in larger areas with much less human effort. Liu (2018) and Zhang et al. (2019) studied the departure time and parking location choices when all commuters travel with AV. We expect that more insights can be generated by examining the problem with endogenous vehicle type choices and mixed traffic characteristics to reflect the immediate future.

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