Are the Acuities of Magnitude Representations of Different Types and Ranges of Numbers Related? Testing the Core Assumption of the Integrated Theory of Numerical Development

Abstract

The current study tested whether the magnitude representation acuities of different types and ranges of numbers, as proposed in the integrated theory of numerical development (Siegler, 2016), are significantly related to each other. A sample of 123 kindergarteners was assessed four times over the period from kindergarten to fourth grade on magnitude representation acuities of different types and ranges of numbers (nonsymbolic numerical magnitude in kindergarten, small whole-number magnitude in grade 1, large whole-number magnitude in grade 2, rational number magnitude in grade 4). The children were also evaluated for their mathematics achievement, intelligence, working memory capacity, reading skills, attention level, and multiplication skills. The results showed that the magnitude representation acuities of different types and ranges of numbers were significantly related to each other, and these numerical magnitude representation acuities were either directly or indirectly related to children's mathematics achievement in grade 4. The findings from this work provide empirical support to the core assumption of the integrated theory of numerical development and highlight the significance of numerical magnitude representations at an early developmental stage to the acquisition of more advanced numerical magnitude representations in later elementary school years.

Keywords: numerical magnitude, mathematics, development

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Children's mathematical development has become a hot topic in the field of developmental psychology. A search using the keyword "math" in PsycArticles yielded a total of 10382 results between the period of 2010 to 2019, a figure doubling that of the previous decade. Researchers' attention towards children's numerical development is supported for a number of reasons. First, being members of the numerate society, our mathematics skills are related to important outcomes in our lives, such as educational attainment, financial status, and psychological well-being (Parsons & Bynner, 2005; Ritchie & Bates, 2013). Second, mathematics achievement in adolescence is strongly predicted by numerical skills at the age of 4 (Watts, Duncan, Siegler, & Davis-Kean, 2014), suggesting that early numerical skills play a vital role in our mathematical development. Third, interventions targeting improvement in children's numerical skills have been shown to be effective in boosting children's arithmetic performance and mathematics achievement (e.g., Fuchs et al., 2013; Hyde, Khanum, & Spelke, 2014; Siegler & Ramani, 2009).

While there has been increasing attention paid to the field of numerical development, researchers have different foci within the field. Some researchers concentrated on the core systems of our nonsymbolic numerical magnitude representations (Feigenson, Dehaene, & Spelke, 2004; Hyde, 2011), while others focused on comparing the relations between the acuities of varying forms of numerical magnitude representations (symbolic vs. nonsymbolic) and children's mathematics achievement (DeSmedt, Noël, Gilmore, & Ansari, 2013; Schneider, Beeres, Coban, Merz, Schmidt, Stricker, & DeSmedt, 2016). Most recently, the topic of rational numbers has become increasingly popular, probably based on its strong relation with mathematics achievement (Siegler et al., 2012; Torbeyns, Schneider, Xin, & Siegler, 2014) as well as the difficulties faced by children, adolescents, and adults in processing them (Kloosterman, 2010; Rittle-Johnson, Siegler, & Alibali, 2001; Siegler & Lortie-Forgues, 2015). These difficulties may stem from the nature of rational numbers itself (e.g., fractions, for example, consist of two whole numbers and lead people to automatically activate their primitive representation of whole numbers, Vamvakoussi, 2015), as well as certain culturally contingent factors (e.g., inadequate understanding of rational numbers among teachers; see Siegler & Lortie-Forgues, 2017, for a summary). Although these topics seem rather diverse, they can be captured by a single unifying theme - the representation of numerical magnitude.

An Integrated Theory of Numerical Development

Synthesizing the existing literature on numerical development, Siegler and colleagues (Siegler, 2016; Siegler, Thompson, & Schneider, 2011) proposed the integrated theory of numerical development. According to this theory, the core of children's numerical development lies in their continuously improving understanding of numerical magnitude. The central assumption of this theory is that the numerical magnitude of all rational numbers can be located on the mental number line, which covers an increasingly larger range over time. Starting with small-whole numbers, the mental number line expands to the right to cover larger whole numbers, interstitially to cover fractions and decimals, and to the left to cover negative numbers. Numbers seem to be represented differently in varying developmental stages, and children's response patterns in the number line estimation task reflect such changes. Within a particular number range, children tend to exhibit a logarithmic response pattern for whole numbers (smaller whole numbers are separated further apart than larger whole numbers) initially. With increasing exposure to whole numbers within that number range, the representation of numbers becomes increasingly linear (equal spacing between all consecutive whole numbers). This change in response patterns observed in the number line estimation task is known as the log-to-linear shift (Siegler, Thompson, & Opfer, 2009; but see Barth & Paladino, 2011, for an alternative explanation of the response pattern based on proportional judgment). To truly understand numbers, especially fractions and decimals, we need to be aware of the fact that all forms of rational numbers have magnitudes that can be represented on particular points on the number line. As this numerical magnitude understanding is crucial to our numerical development, our numerical magnitude understanding should be causally related to our mathematics skills, and interventions that lead to improvement in numerical magnitude understanding should also bring about improvement in our mathematics achievement.

On top of proposing these hypotheses concerning the development of numerical magnitude, Siegler (2016) further proposed four major stages of development for our numerical magnitude understanding. Initially, we are born with a nonsymbolic numerical magnitude system (Izard, Sann, Spelke, & Streri, 2009; Xu & Spelke, 2000) that becomes increasingly precise with age. While 3-year-olds children can discriminate numerosities differing in a ratio of 3:4, acuity improves to 5:6 among 6-year-olds (Halberda & Feigenson, 2008). With an increasingly precise representation of nonsymbolic numerical magnitude, we begin associating number symbols with this nonsymbolic numerical magnitude representation (Dehaene, 2005; Wong, Ho, & Tang, 2016) so that these number symbols are linked to their referents and acquire their meanings. We then expand the range of numbers on our mental number line to include increasingly larger whole numbers. In the final step, we acquire the meanings of other forms of numbers (e.g., fractions and decimals) by realizing that although these numbers differ from whole numbers in various important ways (e.g., the absence of unique successors; does not necessarily get larger after multiplication, etc.), all rational numbers have magnitudes and therefore have their corresponding places on the mental number line. As predicted by the integrated theory of numerical development, the acuities of all these numerical magnitude representations are significantly related to our mathematics achievement (i.e., nonsymbolic numerical magnitude: Chen & Li, 2014; small whole-number magnitude: Schneider et al., 2016; large whole-number magnitude: Booth & Siegler, 2006; Friso-van den Bos et al., 2015; rational-number magnitude: Bailey, Hoard, Nugent, & Geary, 2012; Torbeyns et al., 2014). Although children from different cultures seem to differ in terms of their understanding of numerical magnitude, its role in children's mathematics achievement seems to be universal (Siegler & Mu, 2008; Torbeyns et al., 2014). Experimental evidence also supports the relations between the acuities of these numerical magnitude representations and children's arithmetic skills and mathematics achievement (nonsymbolic numerical magnitude: Hyde et al., 2014; small whole-number magnitude: Siegler & Ramani, 2009; large whole-number magnitude: Kucian et al., 2011; rational-number magnitude: Fuchs et al., 2013). These data empirically support one of the assumptions of the integrated theory of numerical development: our understanding of numerical magnitude underlies the core of our numerical development.

The Linkages Between the Magnitude Representations of Different Types and Ranges of Numbers

While Siegler (2016) has posited several major hypotheses regarding the development of numerical magnitude representation, these hypotheses are built upon a core assumption of the integrated theory of numerical development, i.e., "*the development of numerical magnitude knowledge involves representing increasingly precisely an increasingly broad range of numbers on a mental number line*" (Siegler, 2016, pp 353-354). If all types and ranges of numbers are represented on the same mental number line, individual differences in the acuities of magnitude representations of different types and ranges of numbers should be related to each other even though the exact representation acuities for different types and ranges of numbers may differ due to different processing demands (e.g., place-value concept is needed for processing large, but not small, whole numbers). Albeit being stated as a form of conclusion in the integrated theory of numerical development, the longitudinal relations among the acuities of numerical magnitude representations for different types and ranges of numbers remain untested. Evaluating such an assumption is of high theoretical importance as it may affect how researchers in the field conceptualize the construct of numerical magnitude. While this hypothesis seems straightforward, there have been various theoretical accounts and alternative hypotheses suggesting otherwise.

First, the relation between nonsymbolic and symbolic numerical magnitude representations has been debated. A number of researchers have suggested that nonsymbolic numerical magnitude representation provides the basis upon which our symbolic numerical magnitude representation can be developed (Dehaene, 2004; Geary, 2013). Children then learn the meaning of the number symbols through making associations with nonsymbolic numerical magnitude representations. Yet, other researchers have suggested that the two develop independently (Carey, 2001; LeCorre & Carey, 2007; Lyons, Ansari, & Beilock, 2012; Noël & Rousselle, 2011). Empirical evidence tends to be mixed on this issue. While findings from various studies support the relation between nonsymbolic and symbolic numerical magnitude representation acuities (Libertus, Odic, Feigenson, & Halberda, 2016; Toll, VanViersen, Kroesbergen, & VanLuit, 2015; VanMarle, Chu, Li, & Geary, 2014), other studies did not observe the same relation (Desoete, Ceulemans, DeWeerdt, & Pieters, 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). The significance of our nonsymbolic numerical magnitude representation to our symbolic numerical magnitude representation for both small and large numbers, in particular, or our mathematics achievement, in general, remains controversial.

Second, while much empirical evidence indicates that children's representation of symbolic whole-number magnitude becomes increasingly precise and linear over an increasing number range across development (Siegler et al., 2009), there is little empirical evidence showing that individual differences in the acuities of symbolic whole-number magnitude representations of varying number ranges are related to each other. On one hand, Thompson and Opfer (2010) proposed that children may learn the magnitude of large numbers through making analogies with small numbers (e.g., seeing the similarities between how 600 is located between 0 and 1000 and how 6 is located between 0 and 10). On the other hand, the log-to-linear shift observed in many studies (Siegler et al., 2009) suggests that children may not make proper use of their understanding of small whole numbers to grasp larger whole numbers in a spontaneous manner because they can demonstrate logarithmic response patterns in a particular number range even though they do exhibit a linear response pattern in a smaller number range (Siegler & Opfer, 2003). If children do not spontaneously generalize their representation of small-whole numbers to understand large-whole numbers, it is unclear whether a precise representation of small whole-number magnitude actually facilitates the acquisition of large whole-number magnitude in natural development. Given the complexity of our symbolic number system, knowledge pertaining to the symbolic number system, such as children's understanding of place value, may actually play a larger role in the acquisition of large whole-number magnitude. An empirical test on the relation between small and large whole-number magnitude representation seems necessary.

Third, while various researchers have proposed that the understanding of whole-number magnitude may facilitate the acquisition of rational-number magnitude as both *"require the same type of encoding of each number relative to other numbers"* (e.g., the encoding of 400 within the range of 0 to 1000 is similar to encoding 2/5 within the range of 0 to 1; Bailey, Siegler, & Geary, 2014, p. 777), the whole number bias, on the other hand, suggests that the representations of whole-number versus rational-number magnitudes are qualitatively different. The whole-number bias refers to the overgeneralization of whole-number knowledge to rational numbers, resulting in errors (Ni & Zhou, 2005). For example, many children think that 1/4 is greater than 1/3 because 4 is greater than 3, and 0.83 is smaller than 0.273 because 83 is smaller than 273 (Alibali & Sidney, 2015; Rittle-Johnson et al., 2001). This whole number bias is suggested to be based on componential processing, meaning that people process rational numbers through their components (i.e., numerators and denominators in fractions) instead of the integrated magnitude as a whole (Bonato, Fabbri, Umiltà, & Zorzi, 2007; Braithwaite & Siegler, 2018). This is probably because the components, which are usually natural numbers, are more primitive and thus more automatically activated from our long-term memory (Tzelgov, Ganor-Stern, Kallai, & Pinhas, 2014; Vamvakoussi, 2015). Only with adequate experience with rational-number magnitudes would people be able to override the intuitive processing (i.e., focusing on the components) with the analytic processing (i.e., integrating different components to access the magnitude of the rational number; Alibali & Sidney, 2015; Vamvakoussi, Van Dooren, & Verschaffel, 2012). Although whole-number bias does tend to decrease with age (Braithwaite & Siegler, 2018; Durkin & Rittle-Johnson, 2015), signs of whole-number bias were still observed among eighth graders (Braithwaite &Siegler, 2018; Obersteiner, Van Hoof, Verschaffel, & Van Dooren, 2016). The whole number bias suggests that the role of whole-number magnitude understanding in the acquisition of rational numbers may not be consistent with the proposal from the integrated theory of numerical development, and a direct examination of such a relation is therefore needed.

Fourth, although there is evidence supporting the relations between the acuities of numerical magnitude representations in certain consecutive stages of development (e.g., Libertus et al., 2016; Toll et al., 2015; VanMarle et al., 2014; Mou et al., 2016; Rinne, Ye, & Jordan, 2017; Van Hoof, Verschaffel, & Van Dooren, 2017; Vukovic et al., 2014), it remains questionable whether the acuity of the earliest form of numerical magnitude representation is related to the acuity of the most advanced form of numerical magnitude representation after all the intermediate stages of development and considering potentially confounding variables. The study by Fazio, Bailey, Thompson, and Siegler (2014) may provide a partial answer to this. In that study, the acuities of magnitude representations of nonsymbolic versus symbolic numerical magnitude representations of both whole numbers and fractions among fifth graders were examined. Although the acuities of both symbolic and nonsymbolic numerical magnitude representations uniquely predict mathematics achievement, the relation between the acuities of symbolic and nonsymbolic numerical magnitude just missed the cutoff for statistical significance ($p = .10$). However, limited by the relatively small sample size (n = 53) and the cross-sectional nature of the study, the findings from Fazio et al. (2014) did not provide a complete answer to the proposed longitudinal relation between the acuities of the earliest and most advanced from of numerical magnitude representations. Given various other theoretical accounts, such as the proposal that nonsymbolic and symbolic numerical magnitude representations are initially independent and only become associated with each other at a later stage of development (LeCorre & Carey, 2007; Noël & Rousselle, 2011), the acuity of nonsymbolic numerical magnitude representation may or may not be longitudinally predictive of the acuity of more advanced forms of symbolic numerical magnitude representation, and the longitudinal relations among the acuities of magnitude representations of different types and ranges of numbers need to be examined.

This study was therefore set out to examine the longitudinal relations among the acuities of numerical magnitude representations of different forms and ranges of numbers through a four-year longitudinal study. Children's acuities of magnitude representations of different types and ranges of numbers were tested using different numerical magnitude tasks in the corresponding developmental periods. As various domain-general cognitive measures (e.g., intelligence, working memory, attention), and reading performance have been shown to be significantly related to the acuities of magnitude representations of different types and ranges of numbers (Jordan et al., 2013; Mou et al., 2016; Namkung & Fuchs, 2016; Resnick et al., 2016), the effects of these potentially confounding factors need to be controlled for. A multiplication measure, a proxy of mathematical skills that was presumably less dependent on magnitude understanding because multiplications are mainly learned through rote verbal learning instead of relying on magnitude understanding (Dehaene, 2001; also see Lee & Kang, 2002, and Zhou et al., 2007, for relevant behavioural and neurological findings), was further included as a control variable to exclude the possibility that the performance in various numerical magnitude tasks were related simply because they all involved numbers. The significant relations observed among the acuities of magnitude representations of different types and ranges of numbers, after considering a comprehensive list of control variables, would provide empirical support to the core assumption made in the integrated theory of numerical development: the development of numerical magnitude understanding is about the ability to represent an increasingly broad range of numbers on the mental number line with increasing precision. The relations between the acuities of these numerical magnitude representations and children's mathematics achievement were also examined. The representation acuity of numerical magnitude, as indicated by children's performance in various numerical magnitude measures, is expected to be related to the mathematics topics that are closely related to numbers (e.g., number knowledge, arithmetic), but not those that are less related to numbers (e.g., shapes and space, measures).

Method

Participants and Procedures

The data reported in the current study came from a longitudinal study on children's numerical development. The initial sample consisted of 210 Chinese kindergarteners (mean age = 6 years and 1 month; *SD* = 4 months; 110 of them were male) from 17 different kindergartens in Hong Kong[1]. All the participants spoke Cantonese as their mother tongue. As a result of attrition, 123 of the participants remained in the final sample (mean age = 10 years and 1 month; *SD* = 4 months; 65 of them were male). The attrition was mainly because of parental refusal of participation as well as failure to contact the participants during the follow-up phases. The final sample (FS) did not differ significantly from the drop-out sample (DOS) in the two measures assessed at Time 1, i.e., the nonsymbolic numerical magnitude measure, mean $z_{FS} = .056$, $SD = .794$, mean $z_{DOS} = -.080$, $SD = .834$, $t(208) = 1.19$, $p = .234$, and the working memory measure, mean $z_{FS} = .012$, $SD = 1.11$, mean $z_{DOS} = -.018$, $SD = .813$, $t(207.434) = .228$, $p = .820$. In Hong Kong, most children learn one-digit numbers when they are in kindergarten. Multi-digit numbers, fractions, and decimals are introduced in grades 1, 3, and 4, respectively. Children should have had adequate experience working with the relevant form of numbers by the time the relevant numerical magnitude representation was assessed.

Parental consent was obtained through the kindergartens before the initial assessment. The participants were then assessed in their own kindergartens at Time 1 when they were 6-years-olds (mean age = 6 years and 1 month). They were assessed three more times (Time 2: middle of grade 1; mean age = 6 years and 6 months; Time 3: end of grade 2, mean age = 8 years and 1 month; Time 4: end of grade 4, mean age = 10 years and 1 month; *SD* = 4 months for all four time points) in their homes. The measures were conducted at different time points: measures of nonsymbolic numerical magnitude representation acuity and working memory at Time 1 (kindergarten); measures of small-number magnitude representation acuity and intelligence at Time 2 (Grade 1); measures of large-number magnitude representation acuity, word reading, multiplication skills, and demographic

[1]There was no effect of kindergarten on participants' performance in any of the measures (*p*s > .1 in all Kruskal-Wallis tests). The effect of elementary school on participants' performance was not assessed due to the small number of participants per school (i.e., < 2).

information at Time 3 (Grade 2); and measures of rational-number magnitude representation acuity, mathematics achievement, and attention rating at Time 4 (Grade 4). The measures were conducted with appropriate timing (i.e., after the relevant concept had been formally introduced in school). Each assessment took roughly two hrs to complete. All the assessments were conducted either by the author or by trained psychology majors. Souvenirs and/or supermarket coupons were given to the participants after each assessment as tokens of appreciation.

Measures

All the numerical magnitude measures were computerized measures, while the mathematics achievement and control measures were conducted either verbally or using paper and pencil.

Nonsymbolic numerical magnitude representation acuity. Participants' acuity of nonsymbolic numerical magnitude representation was assessed employing two nonsymbolic tasks - the nonsymbolic comparison task and nonsymbolic arithmetic task. Both tasks rely on the same nonsymbolic numerical magnitude system (Barth et al., 2006; Feigenson et al., 2004; McCrink & Spelke, 2010; Xu & Spelke, 2000), and the ratio effect observed for both tasks supports such a claim (Barth et al., 2006; Halberda & Feigenson, 2008; McCrink & Spelke, 2010). The nonsymbolic numerical comparison task was adopted from Piazza, Izard, Pinel, Le Bihan, and Dehaene (2004). For each item, participants were presented with two arrays of dots, and they had to identify the array with more dots by pressing the corresponding keys ("F" when the left array was larger; "J" when otherwise). One array always consisted of 16 dots (which appeared on either side in a pseudo-random manner), while the other array consisted of 10 to 22 dots (resulting in 10 different ratios from 1.6 to 1.063). There were five practice trials with feedback followed by 50 experimental trials.

The larger array was presented on the left side for half of the trials, and on the right for the other half of the trials.

The nonsymbolic arithmetic task consisted of three conditions: addition, subtraction, and multiplication. The nonsymbolic addition condition was constructed based on the a similar task from Gilmore, McCarthy, and Spelke (2010), while the nonsymbolic subtraction and multiplication conditions were developed by the author based on similar logic. For all three conditions, the participants first saw an array of dots in the lower-left corner. The dots were then covered up by a rectangular shade. With the addition condition, another array of dots moved into the shade, while in the subtraction condition, a number of the dots in the original array moved away from the shade. In the multiplication condition, one, two, or three identical shades came from the original shade, meaning that the array was multiplied by 2, 3 and 4, respectively (see Figure 1)[2]. Animations illustrating the aforementioned operations, with the shades faded away to reveal the transformed array at the end, were shown to the participants together with experimenters' explanation before each condition so as to ensure their understanding. The participants had to judge whether the final array in the shade(s) was more or less numerous than the comparison array presented on the right side and then respond by pressing the corresponding keys ("F" when the left array was more numerous; "J" when otherwise). There were three practice trials with feedback followed by 24 experimental trials for each of the three conditions. In both nonsymbolic tasks, all the dots presented varied in size. The average dot size was positively related to numerosity in half of the trials and negatively related to numerosity in another half. These two types of trials were intermixed, which prevented participants from judging numerosity based on total occupied area. No time limit was imposed on the nonsymbolic numerical magnitude tasks,

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[2] This approach is expected to make the idea of multiplication more explicit to the participants as compared to the procedures used by McCrink and Spelke (2010).

but both the instruction (they were explicitly told not to count) and reaction times (mean reaction time ≤ 2.2 s; *SD* ≤ .8s) suggested that the participants did not count during these tasks. As indicated by Inglis and Gilmore (2014), performance in these tasks was reflected by accuracy[3]. Cronbach's αs were .63 and .68 for the two nonsymbolic tasks.

Small whole-number magnitude representation acuity. The number comparison task, adopted from the Dyscalculia Screener from Butterworth (2003), was utilised to determine participants' acuity of symbolic numerical magnitude representation of small whole numbers. Participants were shown two Arabic numerals ranging from 1 to 9, and they had to pick the one with a larger numerical value. They pressed the "F" key when the numeral on the left was larger, and "J" when it was otherwise. The larger number was presented on the left side for half of the trials, and on the right on the other half of the trials. There were four practice trials with feedback followed by 36 experimental trials. As the overall accuracy was high (mean accuracy >95%), the reaction time (in seconds), which is a commonly used index that captures individual variation in the number comparison task (De Smedt et al., 2013), was employed as the indicator of participants' symbolic numerical magnitude-processing skills. The Cronbach's α of the reaction time was .95.

Large whole-number magnitude representation acuity. The whole number-line task, modified from Siegler and Opfer (2003), was adopted to assess participants' representation acuity of numbers ranging from 0 to 1000. The participants saw a number line on the computer screen with 0 on the left and 1000 on the right. They also saw a number above the line, and they had to locate the number to the right position on the number line. There were three practice trials (i.e., 500, 250, 750) with feedback, and the feedback focused on participants' performance (i.e., whether their estimates appeared to be accurate)

[3]Replacing accuracy with inverse efficiency (accuracy divided by reaction time) render the correlations with large number magnitude and rational number magnitude non-significant.

instead of giving them the correct answers (e.g., 500 should be over here). There were 22 experimental trials (items: 3, 7, 19, 52, 103, 158, 240, 297, 346, 371, 438, 475, 502, 586, 613, 690, 721, 760, 835, 874, 907, and 962) with an oversampling over the small number range. Such oversampling serves as increase the sensitivity of the measure as children make larger errors on numbers over this range (Siegler & Booth, 2004). Similar to many other studies (Schneider et al., 2018), participants' performance was indicated by the percentage absolute error (PAE) calculated using the following formula: $PAE = \frac{|\text{estimate} - \text{target number}|}{\text{range}}$. Cronbach's α of this task was .85.

Rational-number magnitude representation acuity. Both fraction and decimal number-line tasks were employed to more comprehensively capture the construct of rational-number magnitude representation as stated in the integrated theory of numerical development (Siegler, 2016). In the fraction number-line task, participants were shown a number line ranging from 0 to 1. For each item, they saw a fraction (ranging from 0 to 1) above the number line, and their task was to locate the fraction to its corresponding position on the number line using the mouse. The items were 1/19, 2/25, 1/9, 3/16, 1/5, 2/9, 4/15, 1/3, 6/17, 9/22, 3/7, 4/9, 1/2, 7/12, 3/5, 11/18, 2/3, 12/17, 3/4, 4/5, 11/13, 7/8, 10/11, and 22/23. The decimal number-line task was similar to the fraction number-line task except that the participants saw decimals instead of fractions above the number line. The decimals had similar numerical values as the fractions in the fraction number-line task, but they were rounded to different decimal places, with one-third of the items being rounded to the tenth (.1, .2, .4, .5, .6, .7, .8, .9), hundredth (.08, .22, .33, .44, .61, .66, .75, .85), and thousandth decimal places (.053, .188, .267, .353, .429, .583, .875, .957), respectively. The rounding to different decimal places prevented children from using the whole number strategy (i.e., to ignore the "0." and treat the decimal numbers as integers). To familiarize the participants with the task, the same number line was present on the instruction page so the participants could try moving the cursor on the number line. There was a total of 24 experimental trials in both tasks, and the items were evenly distributed on the number line. Both measures yielded robust reliabilities (Cronbach's α = .91 for fraction number line and .89 for decimal number line). Performance in these two tasks was indicated by PAE.

Mathematics achievement. Participants' mathematics achievement was evaluated using the Learning and Achievement Measurement Kit 3.0 (LAMK 3.0; Hong Kong Education Bureau, 2015). The LAMK 3.0 is a series of achievement tests developed by the local education bureau to assess students' achievement in three major subjects (i.e., Chinese, English, and mathematics). The fourth-grade mathematics version employed in this study covered all major topics in fourth-grade mathematics curriculum, including numbers (e.g., number knowledge, eight items; arithmetic, 14 items; numerical magnitude, six items), measures (six items), shapes and space (10 items), and data handling (six items). Participants were given 45 minutes to complete all 50 items. The raw score was utilised to indicate participants' mathematics achievement. The Cronbach's α was .91 for this task.

Control measures.

Working memory. The backward digit span was employed to assess participants' working-memory capacity. The experimenter orally presented a series of digits at a pace of one digit per second. After listening, the participants had to recite the digits backward. The participants had to recall all digits correctly to gain one mark. A practice trial was presented to ensure participants' understanding. A total of 10 items was arranged in five difficulty levels, from two digits per item in level 1 to six digits per item in level 5. The task was discontinued when the participant failed both items in a level. Raw scores were used in the analyses. The Cronbach's α was .50.

Intelligence. Participants' nonverbal intelligence was assessed using Raven's Standard Progressive Matrices (Raven, 1976). For each item, participants were shown a visual pattern with a missing piece. They had to choose one out of six-to-eight pieces that could fill the missing space in the visual pattern. There was a total of 36 items in the short form (Sets A to C). The Cronbach's α was .84. Raw scores were converted into scaled scores based on the local norm.

Reading. The word reading subtest of the Hong Kong Test of Specific Learning Difficulties in Reading and Writing for Primary School Students, second edition (HKT-P(II)) (Ho et al., 2007) was used to assess participants' reading skills. The HKT-P(II) is a locally standardized assessment tool for the diagnosis of dyslexia in the local setting. In the word-reading subtest, the participants are asked to read aloud a list of 150 two-character Chinese words. Participants received one mark for each correctly read word, and the task was discontinued if they failed to score in 15 consecutive items. Raw scores were used in the analyses. The Cronbach's α was .98 for this task.

Multiplication. The author constructed a whole-number multiplication task as a measure of mathematical skills that was relatively independent of numerical magnitude understanding. The tasks consisted of 10 whole-number multiplication problems (ranging from single-digit to two-digit multiplications). The items were presented both visually (on question cards) and verbally (experimenter read out the items). Participants were allowed to work out the answers using paper and pencils. No time limit was imposed. Raw scores were used in the analyses. The Cronbach's α of this task was .64.

Attention. The Chinese version of the inattention and hyperactivity subscale of the Strengths and Difficulties Questionnaire (Lai et al., 2010) was utilised to assess participants' attention levels. Parents rated the participants based on their level of sustained attention and hyperactivity on a 3-point scale from 0 (not true) to 2 (certainly true). There were five items in total. Examples of items include "easily distracted, concentration wanders" and "sees tasks through to the end, good attention span". Sum of the ratings of all items indicated participants' level of inattention and hyperactivity. Reliability (α = .76) and validity (significantly discriminate the clinical sample from the community sample) of the subscale have been demonstrated in a local validation study (Lai et al., 2010). The Cronbach's α from the current study was .67.

Results

Descriptive statistics and correlations

Before conducting the analysis, the data were first converted into standardized scores. The standardized scores of the nonsymbolic numerical comparison task (M = 36.15, SD = 4.28) and nonsymbolic arithmetic task (M = 49.56, SD = 4.43) were averaged to form a nonsymbolic numerical magnitude composite, and the standardized scores of the PAE of the fraction number line (M = 17.28%, SD = 11.13%) and decimal number line (M = 12.70%, SD = 8.65%) were averaged to form the rational-number magnitude composite. The outliers ($z > 3$ or < -3) were then identified and Winsorized (replaced with a z value of 3 or -3) to reduce the influences of extreme values (Tabachnick & Fidell, 2007). A total of 9 outliers had been Winsorized. Table 1 features the descriptive statistics, reliabilities, and correlations among the variables. While the reliabilities of the nonsymbolic numerical magnitude tasks (α = .63 and .68), backward digit span (α = .50), the inattention and hyperactivity subscale of SDQ (α = .67), and the multiplication task (α = .64) were relatively low, all other measures were reliable. The reaction time for the number comparison task, the PAE of the number-line tasks, and the inattentiveness ratings are multiplied by -1 so that positive values indicate better performance. All the numerical magnitude measures were significantly correlated (except for the one between nonsymbolic arithmetic and fraction number line, $r = .11$, $p = .22$), with magnitudes ranging from $r = .18$ to r = .47, and all numerical measures correlated significantly with children's mathematics achievement ($.25 \leq r \leq .50$). The correlations provided preliminary support to the integrated theory of numerical development: the acuities of all numerical magnitude representations were significantly correlated with each other and with children's mathematics achievement.

Mediation analyses

To examine whether the magnitude representation acuities of different types and ranges of numbers were related to each other, a mediation analysis (a combination of several regression analyses) was conducted using the PROCESS Statistical Package for the Social Sciences (SPSS) macro (Hayes, 2013). Structural equation modelling was not used because of the limited sample size. Instead of examining indirect effects through the Sobel test, the bootstrapping procedure with bias-corrected confidence intervals was applied because the latter does not assume normal distribution of the indirect effect (Hayes & Scharkow, 2013). The bootstrapping procedure employed in the current study involved selecting 5,000 bootstrap samples with replacement. The point estimates for the indirect effects were calculated within each sample. The 95% confidence intervals were then calculated based on the sampling distributions of these estimates (Hayes & Scharkow, 2013). The indirect effects were considered statistically significant if 0 lies outside the 95% confidence interval.

In the first mediation model, nonsymbolic numerical magnitude representation acuity was the independent variable, rational-number magnitude representation acuity was the dependent variable, and small and large whole-number magnitude representation acuities served as the mediators (see the upper half of Figure 2). The effects of intelligence, working memory, reading, attention, and multiplication were controlled for in the model. All the regression coefficients reported subsequently are standardized. The results suggested that the numerical magnitude representation acuity at each stage was significantly predicted by the numerical magnitude representation acuity in the previous stage as stated in the integrated theory of numerical development (nonsymbolic to small-whole number: $\beta = .296$, $p = .007$, 95% CI = .083 to 509; small-whole number to large-whole number: $\beta = .189$, $p = .047$, 95%

CI = .002 to .375; large-whole number to rational number: β = .336, $p < .001$, 95% CI = .204 to .468). The total indirect effect from nonsymbolic numerical magnitude representation acuity to rational-number magnitude representation acuity was also significant (β = .152, 95% CI = .047 to .309), as were the indirect effects through only small whole-number magnitude representation acuity (β = .047, 95% CI = .010 to .119), only large whole-number magnitude representation acuity (β = .087, 95% CI = .009 to .215), and through both small and large whole-number magnitude representation acuities (β = .019, 95% CI = .002 to .064; see Table 2 for a summary of the indirect paths). Nonverbal intelligence and multiplication significantly predicted rational-number magnitude representation acuity (β = .151, $p = .03$, 95% CI = .014 to .289 for nonverbal intelligence; β = .160, $p = .024$, 95% CI = .022 to .299 for multiplication), but not small and large whole-number magnitude representation acuities (*ps* > .07), while word reading predicted small whole-number magnitude representation acuity (β = .189, $p = .03$, 95% CI = .017 to .362), but not large and rational-number magnitude representation acuities (*p*s>.1). Working memory and attention were not predictive of any of the magnitude measures (*p*s > .07). The variables together accounted for 44.9% of the variance in rational-number magnitude representation acuity. The results supported the core assumption of the integrated theory of numerical development by demonstrating that the acuities of magnitude representations of different types and ranges of numbers were significantly related to each other.

To further determine whether the magnitude representation acuities of various types and ranges of numbers were significantly related to children's mathematics achievement, another mediation analysis was carried out. Children's mathematics achievement was now the dependent variable, while their rational-number magnitude representation acuity became the third mediator (see Figure 2). Again, the effects of intelligence, working memory, reading, attention, and multiplication were controlled for in the model. The results suggested that all the acuities of numerical magnitude representations were either directly (large whole-number magnitude representation acuity: β = .160, *p* = .037, 95% CI = .010 to .310; rational-number magnitude representation acuity: β = .216, *p* = .027, 95% CI = .026 to .407) or indirectly (nonsymbolic numerical magnitude representation acuity: β = .115, 95% CI = .020 to .254; small whole-number magnitude representation acuity: β = .095, 95% CI = .030 to .200; see Table 2 and Figure 2) related to children's mathematics achievement. Word reading (β = .234, *p* <.001, 95% CI = .102 to .366), attention rating (β = .149, *p* = .023, 95% CI = .021 to .277), and multiplication skills (β = .187, *p* = .012, 95% CI = .041 to .333), but not nonverbal intelligence (*p* = .20) or working memory (*p* = .26), significantly predicted children's mathematics achievement. All the variables together accounted for 55.7% of the variance in children's mathematics achievement. The results confirmed the significance of numerical magnitude representations in children's mathematics learning.

As the mathematics achievement test utilised in the current study consisted of a variety of topics (e.g., number knowledge, numerical magnitude, arithmetic, shapes and space, measures, data handling), further mediation analyses were conducted to establish the relations between different numerical magnitude measures and performance for different mathematics topics. Table 3 shows the results. The total indirect effects from nonsymbolic numerical magnitude representation acuity to the mathematics topics were significant for number knowledge (β = .119, 95% CI = .025 to .267), numerical magnitude (β = .146, 95% CI = .036 to .298), and arithmetic (β = .120, 95% CI = .024 to .260), while their indirect effects on topics such as shapes and space (β = .027, 95% CI = -.052 to .138), measures (β = .025, 95% CI = -.064 to .140), and data handling (β = .068, 95% CI = -.018 to .172) were not statistically significant. The direct effects from acuities of certain numerical magnitude representation to various mathematics topics were also significant (e.g., from rational-number magnitude representation acuity to number knowledge, β = .268, *p*

= .015, 95% CI = .053 to .483; from small whole-number representation acuity, β = .176, *p* = .033, 95% CI = .014 to .338, and rational-number magnitude representation acuity, β = .335, *p* = .003, 95% CI = .118 to .552, to numerical magnitude; and from large whole-number magnitude representation acuity to arithmetic, β = .241, *p* = .005, 95% CI = .074 to .408). These findings support the hypothesis that the acuities of numerical magnitude representation are significantly related to topics that are closely related to numbers (e.g., number knowledge, numerical magnitude, and arithmetic), but not to those topics that are distally related to numbers (e.g., shapes and space, measures, data handling).

Discussion

This study examined the integrated theory of numerical development (Siegler, 2016; Siegler et al., 2011) by testing its core assumption: the development of numerical magnitude understanding can be reflected by the representation of an increasingly broad range of numbers on the mental number line with increasing precision. A sample of 123 kindergarteners was followed for four years until they were in the fourth grade. They were assessed on the acuities of magnitude representations of different types and ranges of numbers at the appropriate time. The results indicated that the acuities of magnitude representations of different types and ranges of numbers were significantly related to each other, and the acuities of these numerical magnitude representations were either directly or indirectly related to children's mathematics achievement. Theoretical and practical implications are discussed in the following.

Relations Between the Acuities of Magnitude Representations of Different Types and Ranges of Numbers

According to the integrated theory of numerical development, Siegler et al. (Siegler, 2016; Siegler et al., 2011) proposed that the growth of numerical magnitude representation is the core of numerical development. This growth can be described in four major stages: the refinement of the nonsymbolic numerical magnitude representations, the connection between small whole-number magnitude representations and the corresponding nonsymbolic numerical magnitude representations, the representations of large whole-number magnitude, and, finally, the extension of whole-number magnitude representation to the representation of rational-number magnitudes. If different types and ranges of numbers are encoded on the same mental number line, the measures of these numerical magnitude representations should at least be related to each other. This assumption, although seemingly straightforward, has either remained untested (cf. Fazio et al., 2014) or led to contradictory findings (Desoete et al., 2012; Sasanguie et al., 2013). The current results demonstrated that the acuities of magnitude representations of different types and ranges of numbers were significantly related to each other with reasonable magnitudes (β between consecutive forms of numerical magnitudes ranged from .19 to .34). The use of different measures for varying types and ranges of numbers further eliminated the possibility that the observed relations were explained by the commonality in the measurement method. Furthermore, numerical magnitude representation acuity, as indicated by children's performance in different numerical magnitude measures, predicted performance in mathematics topics that involve numbers (e.g., number knowledge, arithmetic) but not those of which the relevance to numbers is minimal (e.g., shapes and space), thus providing both convergent and discriminant validity to the construct. It should be noted that for topics such as number knowledge (e.g., identifying the highest common factor and lowest common multiple between two numbers) and arithmetic, the processing of numerical magnitude should not be assumed (e.g., arithmetic problems can be solved through learned procedures, such as reciting the multiplication table). The significant relations observed between numerical magnitude representation acuity and these topics suggest that numerical magnitude representation plays a role in solving these problems even though numerical magnitude information is not explicitly asked for, and the fact that the relation survived even after another aspect of numerical skills (i.e., multiplication skills) had been controlled for, thereby suggesting that it is numerical magnitude, but not other aspects of numerical understanding, that contributes to the mastery of these topics. With a reasonable sample size, three important domain-general cognitive skills, reading skills, and multiplication as control variables, as well as a bootstrapping procedure with 5,000 samples, the current findings are robust, providing tentative support to the assumption that the magnitude representations of different types and ranges of numbers are represented on the same mental number line.

While the results as a whole are theoretically interesting because they support the core assumption of the integrated theory of numerical development, the connections between each consecutive stage of numerical magnitude development are also intriguing. First, the association between symbolic and nonsymbolic numerical magnitude representations has been heatedly debated. Certain researchers believe that our innate nonsymbolic numerical magnitude representations constitute the basis upon which our symbolic numerical magnitude representations can be developed (Dehaene, 2004; Geary, 2013). Others have suggested that the two developed independently (LeCorre & Carey, 2007; Lyons et al., 2012). A significant relation between nonsymbolic and symbolic numerical magnitude representation acuities was observed in this study, and this remained true even after the potentially confounding factors were controlled for. The findings are in line with those that found positive evidence for the relation between symbolic and nonsymbolic numerical magnitude representations (Libertus et al., 2016; Toll et al., 2015; van Marle et al., 2014), but contrasted with those of Fazio et al. (2014). It is worth noting that although the relation between symbolic and nonsymbolic numerical magnitude representation acuities failed to reach significance in Fazio et al. (2014), the magnitude (i.e., $\beta = .25$) is comparable to the values observed in the current study ($\beta = .30$). Therefore, the insignificant relation observed in

Fazio et al. (2014) can be because of the small sample size (*N* = 53) in that study. Furthermore, the current study extended the previous findings by demonstrating that children's nonsymbolic numerical magnitude reprensetation acuity was either directly or indirectly related to the magnitude representation acuity of symbolic numbers in different number ranges as well as their mathematics achievement. Children's rational-number magnitude representation acuity and their mathematics achievement in grade 4, for example, was both significantly and indirectly predicted by their nonsymbolic numerical magnitude representation acuity in kindergarten. These relations would not be expected if nonsymbolic numerical magnitude representation develops independently from symbolic numerical magnitude representation (LeCorre & Carey, 2007; Lyons et al., 2012). These findings also illustrate why nonsymbolic numerical magnitude representation acuity sometimes fails to predict arithmetic skills when symbolic numerical magnitude factors have been taken into account (Lyons, Price, Vaessen, Blomert, & Ansari, 2014). Instead of playing no roles in our arithmetic skills, the effect of nonsymbolic numerical magnitude representation on our mathematics achievement seems to be fully mediated through our symbolic numerical magnitude representations.

Second, while it has been assumed that understanding of small whole-number magnitude representation paves the way for understanding large whole number magnitude, such a relation has remained largely untested. The findings that second graders demonstrate a linear response pattern in a 0-100 number line but a logarithmic response pattern in a 0-1,000 number line (Siegler & Opfer, 2003) suggest that children's understanding of small whole numbers may not be spontaneously generalized to large whole numbers (note that the difference in familiarity over different number ranges alone does not explain such different response patterns as a change in response pattern can occur without exposure to numbers within the unfamiliar number range; Opfer & Siegler, 2007). The current findings, on the contrary, indicate that the relation between small and large whole-number magnitude understanding does exist. Children who were faster in comparing single-digit numbers in grade 1 were also more accurate in locating two- and three-digit numbers on the 0-1,000 number line. These findings are in agreement with the hypothesis that we develop our understanding of large whole numbers by bootstrapping our understanding of small whole numbers (Thompson & Opfer, 2010). Children may gain a better understanding of large whole numbers (e.g., the relative magnitude of 600 and 1,000) by analogizing them with small whole numbers (e.g., relative magnitude of 6 and 10), and progressively aligning numbers within different number ranges facilitates children in making such an analogy (Thompson & Opfer, 2010).

Third, the relation between the acuities of whole-number magnitudes and rational-number magnitudes also deserves attention because of the complicated relations between whole numbers and rational numbers. While some researchers consider the representation of rational numbers as qualitatively different from that of whole numbers (Leslie, Gelman, & Gallistel, 2008; Wynn, 1995), the integrated theory of numerical development posits that both types of numbers share a central property - they all have magnitudes and can be located on the mental number line (Siegler, 2016). From this point of view, a more precise representation of whole-number magnitude may enable a child to use this symbolic numerical magnitude system to understand other kinds of symbolic numbers, such as fractions and decimals. The current findings provide support to the integrated theory of numerical development by showing that both small and large whole-number magnitude representation acuities significantly predicted future understanding of rational numbers. Similar findings have been observed in other studies (Bailey, Siegler, & Geary, 2014; Mou et al., 2016; Vukovic et al., 2014).

Limitations

While confirming the relations among different numerical magnitude representation acuities, the current findings can be further strengthened if the relevant autoregressor effects have been controlled for. Furthermore, readers should note that the current findings do not confirm the mechanisms involved in the process. With the integrated theory of numerical development, Siegler (2016) proposed that association and analogy are the two major processes facilitating the development of numerical magnitude. Association may explain how symbolic numbers acquire their meanings through connecting with the corresponding nonsymbolic numerical magnitude representation (Dehaene, 2004; Wong et al., 2016), while analogy may describe how children bootstrap their small whole-number magnitude understanding to larger number ranges (Thompson & Opfer, 2010), as well as how children bootstrap their whole-number magnitude understanding to acquire rational-number magnitude (Bailey et al., 2014). Future studies are necessary for confirming the aforementioned proposed mechanisms. A related issue is that while analogy has been put forth as the mechanism through which our small whole-number magnitude representations are bootstrapped to understand large whole numbers, this process does not seem to occur without purposefully aligning numbers within different ranges (Thompson & Opfer, 2010). It remains to be explored whether such an analogy can be made gradually owing to children's increasing knowledge about the symbolic number system, or whether other mechanisms are involved during this spontaneous bootstrapping process. Finally, two issues concerning the measures need to be noted. The first one concerns about the number line tasks. Opfer, Thompson and Kim (2016) had demonstrated that providing anchors to participants before the number line tasks altered their response patterns. Although participants in this study were only informed about their performance (i.e., whether their responses were close to the correct answers) but not the anchors (i.e., the exact locations of the target numbers), it is still possible that such feedback might have led to a slight improvement in the number line estimation performance. Despite such a possibility, the key findings of the current study (i.e., the interrelations among the acuities of the magnitude representations of different types and ranges of numbers) should not be affected. The second issue concerns about the reliability of the measures. The reliabilities of a number of the measures in the current study (nonsymbolic numerical magnitude measures, working memory, inattention rating and multiplication) were relatively low, and such low reliabilities may result in an underestimation of the relevant correlations (Goodwin & Leech, 2006). Readers should be aware of this when interpreting the current findings. Future studies may also increase the reliability of the measures through increasing the total number of trials or the use of adaptive testing procedure (Lindskog, Winman, Juslin, & Poom, 2013). Modelling techniques such as Ratcliff diffusion modelling, which enables researchers to take accuracy, reaction time, and their interactions in a binary response choice task into account (Ratcliff, Thompson, & McKoon, 2015), may also help improve the accuracy in estimating participants' acuities of numerical magnitude representation.

Theoretical and Practical Significance

The present results have both theoretical and practical significance. Theoretically, they bolster the core assumption underlying the integrated theory of numerical development - the development of numerical magnitude understanding can be reflected by the representation of an increasingly broad range of numbers on the mental number line with increasing precision. This is the first piece of longitudinal evidence linking the acuities of magnitude representations of four different types and ranges of numbers, and the interrelations among the acuities of these numerical magnitude representations survived even after a comprehensive set of control variables were included in the model. As the study was conducted in the Asian context, the current findings further support the integrated theory of numerical development by indicating that it can be generalized to other cultures. In particular, the findings highlight the role of an early form of numerical magnitude representation (nonsymbolic numerical magnitude) in the development of later forms of numerical magnitude representations. The nonsymbolic numerical magnitude representation acuity also indirectly predicted children's mathematics achievement four years later through other forms of numerical magnitude representations. By dividing the mathematics achievement test into different subtopics, the current findings further suggest that nonsymbolic numerical magnitude representation acuity is related to various mathematic topics (e.g., number knowledge, arithmetic), but not others (e.g., shapes and space, measures). All these lines of evidence support the importance of nonsymbolic numerical magnitude representation in children's mathematics learning. While several nonsymbolic numerical magnitude interventions have been proven successful (Hyde et al., 2014; Park & Brannon, 2013), these interventions only target a narrow set of outcomes (both studies focus on symbolic whole-number arithmetic only). With the current findings, it would be interesting to examine whether the effect of such interventions can be generalized to other mathematical domains (e.g., the understanding of rational-number magnitudes and general mathematics achievement).

Conclusions

Arising from a four-year longitudinal study with 123 kindergarteners, the current findings support the integrated theory of numerical development by demonstrating that the acuities of magnitude representations of different types and ranges of numbers are either directly or indirectly related to each other. In addition, our results highlight the significance of early forms of nonsymbolic numerical magnitude representation in developing later forms of symbolic numerical magnitude representations. Future intervention studies could further investigate whether nonsymbolic numerical magnitude plays a causal role in the development of advanced symbolic numerical magnitude representations.

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