Improved one-phase model of uniform corrosion for predicting volume of rust

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Abstract: One-phase models of rebar corrosion have been widely used by practicing engineers to predict the volume of rust and the time to cracking of the concrete cover, but the models have only considered the deformation of concrete and not the deformation of steel or the rust layer. However, neglect of the deformation of the rust layer may result in inaccurate predictions. This study therefore proposes an

adjustment factor to take into account the effect of the deformation of the rust layer on predicting the volume of rust with a conventional one-phase model of uniform corrosion. Furthermore, the effects of the material properties of the rust layer and corrosion-induced expansive pressure on the adjustment factor are investigated. To avoid inaccurate predictions of the volume of rust, the proposed adjustment factor should be applied in the conventional one-phase model of corrosion when the Young's modulus of rust is less than 460 MPa.

Notation

E_{rust} is the Young's modulus of rust

K_{rust} is the bulk modulus of rust

d_f is the free-expansion deformation of the steel-rust composite

d_c is the expansion deformation of the steel-rust under restraining pressure

d_s is the sum of the deformation of the steel and rust layer caused by the restraining pressure

d_{s1} is the deformations of the rebar

d_{s2} is the deformations of the rust

P_{rust} is the corrosion-induced expansive pressure on the surrounding concrete

D is the initial diameter of the rebar,

d_r is the thickness of the corroded steel bar that takes into consideration the deformation of the rust layer

E_{steel} is the Young's modulus of the steel

ν_{steel} is the Poisson's ratio of the steel

P_{rust1} is the restraining pressure of the inside of rust layer

P_{rust2} is the restraining pressure of the outside of rust layer

ν_{rust} is the Poisson's ratio of rust

V_{deform} is the volume of rust associated with the deformation of the rust layer

β is the ratio of the volume of rust to consumed steel

d_0 is the thickness of the porous zone.

V_{rust} is the total volume of rust

$V_{rust,1}$ is the volume of rust obtained from a conventional one-phase model of uniform corrosion

$V_{deform1}$ is the deformation volume of rust associated with the conventional one-phase model

η is the adjustment factor

$P_{rust,c}$ is the critical expansive pressure

Keywords: Corrosion/ Cracks & cracking/ Modelling/ Cement/cementitious materials/ Durability-related properties

1. Introduction

The corrosion of steel bars in reinforced concrete (RC) members has been identified as one of the most predominant structural deterioration problems worldwide (Hu et al., 2018, Zhang et al., 2018). The durability, serviceability, and strength of RC structures can be severely affected by corrosion-induced damage (Ye et al., 2018, Aliabdo et al., 2018, Zhang et al., 2017a). When water and oxygen are available, the initiation of rebar corrosion takes place once the free chloride content or the pH in the pore solution of concrete at the surface of the steel rebars reaches the threshold limit (Zhu et al., 2018, Liu et al., 2017, Al-Alaily and Hassan, 2018). The penetration process of chloride and carbonation oxide can be affected by the components of concrete (Elgalhud et al., 2018, Silva et al., 2017) and the crack width on the cover surface (Zhang et al., 2017b, Lu et al., 2017). As the volume of rust is about two to six times that of corroded steel, the expansion of rust would exert pressure onto the surrounding concrete. This may eventually lead to cracking, delamination and spalling of the concrete cover. The deterioration process and service life of RC structures are therefore influenced by the volume of rust.

A number of models on rebar corrosion (El Maaddawy and Soudki, 2007, Wang and Liu, 2004, Berra et al., 2003, Guzmán et al., 2011, Šavija et al., 2013, Yang et al., 2018) have been developed to analyze the residual service life of RC structures. These models often assume that the distribution of rust and expansive pressure are uniformly distributed at the steel-concrete interface. Furthermore, only the deformation of concrete was considered in those models, but not the steel bars and the rust layer (Chernin and Val, 2011, Zhang et al., 2017c). These one-phase models of uniform corrosion have been widely used in practice due to their simple formulation and short computational time.

As the Young's modulus of steel rebars (~200 GPa) is much higher than that of concrete (about 20 – 30 GPa), the deformation of the rebars in the one-phase models can be neglected as it is found to be less than 1% for typical RC slab structures by Zhang and Su (Zhang and Su, 2017). On the other hand, the influence of elastic deformation of the rust layer on the prediction of the volume of rust depends on the Young's modulus and Poisson's ratio of rust as well as corrosion-induced expansive pressure. It is worth mentioning that there is a reciprocal relationship between elastic deformation due to rust and the bulk/Young's modulus of rust. When the Young's modulus of rust is low, the influence of the deformation of the rust layer on the total volume of rust will be very high. Previous studies (Su and Zhang, 2015, Lu et al., 2011, Molina et al., 1993, Balafas and Burgoyne, 2010) have shown that the influence of the deformation of the rust layer on calculating the volume of rust could be neglected if the Young's modulus of rust E_{rust} or the bulk modulus of rust K_{rust} is high, for instance, $E_{rust} \geq 500$ MPa (Su and Zhang, 2015), $E_{rust} \geq 1$ GPa (Lu et al., 2011), $K_{rust} \geq 4$ GPa (Molina et al., 1993) or $K_{rust} \geq 300$ MPa (Balafas and Burgoyne, 2010). Experimental studies on the properties of rust (Suda et al., 1993, Care, 2008, Liu and Su, 2018, Zhao et al., 2012b) have shown that E_{rust} can actually vary from 10 MPa to 300 MPa which is much less than the minimum recommended threshold limit of 500 MPa. Thus deformation of the rust layer should not be neglected in conventional one-phase models of uniform corrosion (Chen and Leung, 2018).

In this study, an adjustment factor is derived to improve the prediction of the volume of rust with a conventional one-phase model of uniform corrosion. A numerical parametric study is carried out to investigate the effects of the material properties of rust and corrosion-induced expansive pressure on the adjustment factor. Furthermore, the range of Young's modulus values of rust which is applied in the modified model is also discussed.

2. Research significance

The traditional method for calculating the rust volume with its mass and density does not consider the deformation of rust caused by the confinement of surrounding concrete before cover cracking. Neglecting the deformation of the rust layer would be incorrect for the calculation of rust volume in corrosion models. In this paper, an adjustment factor is originally derived to improve the prediction of the volume of rust with considering the deformation of rust for conventional one-phase model of uniform corrosion. Furthermore, a parametric study is conducted to investigate the effects of the material properties of the rust and corrosion-induced expansive pressure on the adjustment factor. It is found that the proposed adjustment factor should be applied when the Young's modulus of rust is less than 460 MPa.

3. Adjustment factor for volume of rust

In this section, the volume of rust derived from using a three-phase model of uniform corrosion is first discussed. The adjustment factor for the volume of rust obtained from a conventional one-phase model of uniform corrosion will then be derived.

As shown in Fig. 1a, the expansion of rust in a corroded rebar is confined by the surrounding concrete. Owing to the confinement effect, the free-expansion deformation of the steel-rust composite (d_f) is reduced to (d_c). Enforcing the compatibility condition on deformation in the radial direction at the steel-rust-concrete interface, the following equation can be obtained:

$$d_{\mathrm{f}} = d_{\mathrm{s}} + d_{\mathrm{c}} \tag{1}$$

where d_{s} is the sum of the deformation of the steel and rust layer caused by the restraining pressure (P_{rust}), hence,

$$d_{\mathrm{s}} = d_{\mathrm{s1}} + d_{\mathrm{s2}} \tag{2}$$

where d_{s1} and d_{s2} are the deformations of the rebar and rust as illustrated in Figs. 1b and 1c, respectively.

As shown in Fig. 1b, the steel rebar can be treated as a plane strain problem subjected to only external pressure. The deformation of the steel bar can be expressed with Eq.(3) (Ugural, 1994).

$$d_{\mathrm{s1}} = \frac{P_{\mathrm{rust}}(D/2-d_r)(1-\nu_{\mathrm{steel}})}{E_{\mathrm{steel}}} \approx \frac{P_{\mathrm{rust}}D(1-\nu_{\mathrm{steel}})}{2E_{\mathrm{steel}}} \tag{3}$$

where P_{rust} is the corrosion-induced expansive pressure on the surrounding concrete, D is the initial diameter of the rebar, d_{r} is the thickness of the corroded steel bar that takes into consideration the deformation of the rust layer, and E_{steel} and ν_{steel} are the Young's modulus and Poisson's ratio of the steel, respectively.

As $d_{\mathrm{f}} < D$, it is reasonable to assume that the restraining pressure of the inside and outside of the rust layer (P_{rust1} and P_{rust2} as shown in Fig. 1c) are the same and equals to P_{rust}. Therefore, the deformation of the rust layer in the radial direction can be expressed as:

$$d_{s2} = \frac{P_{\text{rust}}(d_f + d_r)}{K_{\text{rust}}} \tag{4}$$

where K_{rust} is the bulk modulus of rust. The bulk modulus of rust can be related to the Young's modulus and Poisson's ratio of rust ν_{rust} in Eq. (5).

$$K_{\text{rust}} = \frac{E_{\text{rust}}}{3(1 - 2\nu_{\text{rust}})} \tag{5}$$

Rust is a granular material and its structure consists of a powder grain aggregate (Ouglova et al., 2006, Liu and Su, 2018). It has been found that the Young's modulus of rust increases with an increase of expansive pressure (Liu and Su, 2018, Ouglova et al., 2006, Xu et al., 2017) or rust thickness (Care, 2008). In this study, for simplification, rust is treated as an elastic material. This assumption has been widely adopted in many analytical models (Zhao et al., 2012a, Su and Zhang, 2015, Zhu and Zi, 2017). The deformation of the rust layer when subjected to expansive pressure is illustrated in Fig. 1c. The volume of rust associated with the deformation of the rust layer (V_{deform}) can be expressed with Eq. (6).

$$V_{\text{deform}} = \pi d_{s2}(D + d_f) \approx \pi d_{s2} D \tag{6}$$

As corrosion progresses, the total volume of rust can be derived from (Su and Zhang, 2015, Zhao et al., 2011): (1) the volume of steel consumed in the corrosion process; (2) penetration of rust into the porous zone which is caused by entrapped or entrained air at the interface between the steel and concrete (Liu and Weyers, 1998), and (3) the expansive pressure induced at the steel/concrete interface. Therefore, the total thickness of the rust can be determined by using:

$$\beta d_{\mathrm{r}} = d_{\mathrm{r}} + d_0 + d_{\mathrm{f}} \quad (7)$$

where β is the ratio of the volume of rust to consumed steel and d_0 is the thickness of the porous zone. It should be noted that researchers have not yet reached consensus on whether rust fills corrosion-induced cracks (Wong et al., 2010, Ožbolt et al., 2012, Tran et al., 2011, Chernin et al., 2012) and if so, how much; thus, the amount of rust in corrosion-induced cracks is not taken into account in this study.

The volume of rust associated with the deformation of the rust layer (V_{deform}) which is called the deformation volume of rust herein can be derived from Eqs. (1) to (7).

$$V_{\mathrm{deform}} = \frac{3\pi D P_{\mathrm{rust}}(1-2\nu_{\mathrm{rust}})(\beta d_{\mathrm{r}} - d_0)}{E_{\mathrm{rust}}} \quad (8)$$

It is worth noting that in Eq. (8), P_{rust}, E_{rust}, ν_{rust}, D, β and d_0 are independent of the deformation of the rust layer except for d_{r}.

The total volume of rust (V_{rust}) obtained from the three-phase model with consideration of the deformation of the rust layer can be written as:

$$V_{\mathrm{rust}} = V_{\mathrm{rust,1}} + V_{\mathrm{deform}} \quad (9)$$

where $V_{\mathrm{rust,1}}$ is the volume of rust obtained from a conventional one-phase model of uniform corrosion; see Eq.(10):

$$V_{\mathrm{rust,1}} = \pi\beta d_{\mathrm{r1}}(D - d_{\mathrm{r1}}) \approx \pi\beta d_{\mathrm{r1}} D \quad (10)$$

where d_{r1} is the thickness of the corroded steel obtained from the conventional one-phase model. As the thickness of the corroded steel determined from the one-phase model is different from that obtained with the three-phase model, the corresponding deformation volume of rust will also change. The deformation volume of rust associated with the conventional one-phase model ($V_{deform1}$) is determined with Eq. (11).

$$V_{\mathrm{deform1}} = \frac{3\pi D P_{\mathrm{rust}}(1-2\nu_{\mathrm{rust}})(\beta d_{\mathrm{r1}}-d_0)}{E_{\mathrm{rust}}} \tag{11}$$

The relationship between d_r and d_{r1} is

$$\frac{d_{\mathrm{r1}}}{d_{\mathrm{r}}} = \frac{V_{\mathrm{rust,1}}}{V_{\mathrm{rust}}} \tag{12}$$

With Eqs. (1) to (12), the total volume of rust with the effect of the deformation of the rust layer can be obtained:

$$V_{\mathrm{rust}} = \eta V_{\mathrm{rust,1}} \tag{13}$$

where η is the adjustment factor for the volume of rust obtained from the conventional one-phase model of corrosion. This factor can be analytically expressed as:

$$\eta = \frac{\beta d_{\mathrm{r1}} - d_0 A}{\beta d_{\mathrm{r1}}(1-\mathrm{A})} \tag{14}$$

where

$$A = \frac{3P_{\mathrm{rust}}(1-2\nu_{\mathrm{rust}})(\beta d_{\mathrm{r1}}-d_0)}{(\beta d_{\mathrm{r1}}-d_0)E_{\mathrm{rust}}} \tag{15}$$

The deformation volume and the total volume of rust vary throughout the corrosion process. The total volume of rust leading to cover cracking is a key parameter for predicting the service life of RC structures. This study focuses on evaluating the deformation volume and the total volume of rust associated with the critical expansive pressure, $P_{\mathrm{rust,c}}$ at which unstable cracks propagate across the thickness of the cover (Su and Zhang, 2015). It should be noted that the critical expansive pressure that causes cracking of the concrete cover is influenced by the material properties of concrete (i.e. Young's modulus, tensile strength and tensile stress-strain relationship) and the geometrical properties of the concrete cover (i.e. cover thickness and reinforcing bar diameter) (Zhang and Su, 2017) but not the deformation of the rust layer and rebars. This means that $P_{\mathrm{rust,c}}$ obtained from the one-phase and three-phase models of corrosion is the same.

In this study, $P_{\mathrm{rust,c}}$ is determined from a three-phase model of uniform corrosion by considering the following conditions: (1) the force equilibrium in the tangential direction, (2) volume expansion in the steel-rust-concrete interface, (3) deformation compatibility in the steel-rust-concrete interface, (4) the force equilibrium in the radial direction, and (5) bilinear relationship of the tension softening curve of concrete (Su and Zhang, 2015). After considering the critical pressures associated with wide ranges of tensile capacities of concrete, reinforcement diameters and cover thicknesses, $P_{\mathrm{rust,c}}$ is determined by the least squares method and expressed as (Zhang and Su, 2017):

$$P_{\mathrm{rust,c}} = -0.00338 f_{\mathrm{t}} D c + 0.11308 f_{\mathrm{t}} c + 0.00118 D^2 c - 0.03689 D c +$$

$$0.02319D^2 f_{\mathrm{t}} - 0.68993 f_{\mathrm{t}} D + 3.9058 f_{\mathrm{t}} - 0.10141 D^2 + 0.22599c + 3.0511D - 15.418 \quad (16)$$

where f_{t} is the tensile strength of concrete and c is the thickness of the concrete cover.

To obtain the adjusted total volume of rust, the adjustment factor, η, should be evaluated first from Eqs. (14) and (15) in which the expansive pressure P_{rust}, and the thickness of the corroded steel d_{r1}, can be calculated from a conventional one-phase model. Then, the total volume of rust can be obtained from Eq. (13).

4. Results and discussion

4.1. Validation

Lu et al. (Lu et al., 2011) studied the influence of the Young's modulus and Poisson's ratio of rust on the volume of corroded steel. Zhao et al. (Zhao et al., 2011) developed an analytical model to simulate corrosion-induced cover cracking. When cracking took place throughout the entire cover, the deformation of the rust layer obtained by using their analytical model and through experiments is discussed in (Zhao et al., 2012c). The input parameters of the model are listed in Table 1. The corrosion-induced expansive pressure values and the thickness of the corroded steel are taken from our previous works (Zhang and Su, 2017, Su and Zhang, 2015). Fig. 2 is a comparison of the adjustment factor determined from this study and that obtained by Lu et al. (Lu et al., 2011) and Zhao et al. (Zhao et al., 2011, Zhao et al., 2012c). It can be observed that the analytical results here are in good agreement with the results in Lu et al. (Lu et al., 2011) and Zhao et al. (Zhao et al., 2011, Zhao et al., 2012c).

Table 1 Input parameters of model

	f_t (MPa)	c (mm)	D (mm)	d_0 (μm)	β	v_{rust}	E_{steel} (GPa)	v_{steel}
Ref. (Lu et al., 2011)	3.3	35	16	16	3	0.35	200	0.3
Ref. (Zhao et al., 2011, Zhao et al., 2012c)	1.96	27	16	20	2.6	0.25	200	0.3

4.2.Parametric study

In this section, the effects of the Young's modulus and Poisson's ratio of rust and corrosion-induced expansive pressure on the adjustment factor, η, are examined. The input parameters are the same as those in the three-phase model (Su and Zhang, 2015, Zhang and Su, 2017).

4.2.1 Effect of bulk modulus of rust

The variations in the adjustment factor, η, from the Young's modulus of rust, E_{rust}, are provided in Fig. 3. It can be observed that with an increase in the Young's modulus of rust, the adjustment factor first substantially decreases and then is asymptotically reduced to 1, which is in agreement with Lu et al. (Lu et al., 2011) and Su and Zhang (Su and Zhang, 2015).

4.2.2 Effect of Poisson's ratio of rust

The effect of the Poisson's ratio of rust, ν_{rust}, on the adjustment factor, η, is shown in Fig. 4. It can be observed that a larger Poisson's ratio of rust results in a lower adjustment factor. This finding is supported by the analytical results in Lu et al. (Lu et al., 2011) and Balafas and Burgoyne (Balafas and Burgoryne, 2011). The reason is that a larger Poisson's ratio of rust will result in a higher bulk modulus of rust and fewer errors in the prediction of the deformation of the rust layer.

4.2.3 Effect of expansive pressure

The effect of expansive pressure, P_{rust}, on the adjustment factor, η, is shown in Fig. 5. It can be seen that the adjustment factor increases with an increase in the expansive pressure. This is reasonable as higher expansive pressure means greater deformation of the steel bars and rust layer (Zhao et al., 2011, Balafas and Burgoryne, 2011). Thus, the volume of rust will be greatly underestimated in the conventional one-phase model of corrosion.

5. Condition for use of proposed adjustment factor

According to the parametric study in Section 3.2, the Young's modulus of rust and the critical expansive pressure both have a significant influence on the adjustment factor. Unfortunately, the range of the Young's modulus of rust is still unknown and a wide range of values (from 10 MPa to 200 GPa) have been reported in the literature

(Suda et al., 1993, Care, 2008, Liu and Su, 2018, Zhao et al., 2012b, Zhao et al., 2012a, Karin, 2002, Ouglova et al., 2006, Sanz et al., 2017, Chen and Leung, 2015). In this section, the range of the Young's modulus values of rust that requires the use of the proposed adjustment factor for accurately calculating the volume of rust is examined.

According to Zhang and Su (Zhang and Su, 2017), the minimum critical value of the expansive pressure that could cause unstable cracking of the cover of typical RC structures is 5 MPa which can develop when f_t = 2.8 MPa, c = 25 mm and D = 20 mm. On the other hand, when f_t = 4.3 MPa, c = 80 mm and D = 12 mm, the maximum critical value of the expansive pressure will be produced. Fig. 6 shows the effect of the Young's modulus of rust on the adjustment factor under the considered minimum and maximum critical values of expansive pressure. It can be seen that there are two cases as follows.

(1) When 460 MPa ≤ E_{rust}, the adjustment factor is less than 1.05 for a typical range of critical values of expansive pressure. This means that when 460 MPa ≤ E_{rust}, the conventional one-phase model can be used to calculate the volume of rust of general RC structures without introducing significant errors.

(2) When E_{rust} < 460 MPa, the adjustment factor can be greater than 1.05 for some RC structures, which means that the adjustment factor should be used when

calculating the volume of rust.

Therefore, it is recommended that the proposed adjustment factor should be used when the Young's modulus of rust is less than 460 MPa for general RC structures.

6. Conclusion

In this study, an adjustment factor is proposed to improve the accuracy of the prediction of the volume of rust which corresponds to cracking of the concrete cover obtained from a conventional one-phase model of uniform corrosion. The effects of the material properties of rust (e.g. Young's modulus and Poisson's ratio) and critical value of the expansive pressure on the adjustment factor are investigated. It is found that the Young's modulus of rust has a significant influence on the adjustment factor.

This study also finds that the conventional one-phase model of corrosion can be generally used for accurately calculating the volume of rust when $E_{rust} \geq 460$ MPa for typical RC structures. However, when $E_{rust} < 460$ MPa, the proposed adjustment factor should be used to improve the accuracy of the prediction of the volume of rust determined from the conventional one-phase model of corrosion.

It is worth noting that although this study focuses on the effect of the deformation of the rust layer based on predictions of the volume of rust with a conventional one-phase model of uniform corrosion, a similar adjustment factor for the prediction of the volume of rust with one-phase models of non-uniform corrosion can also be derived.

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A list of Figures

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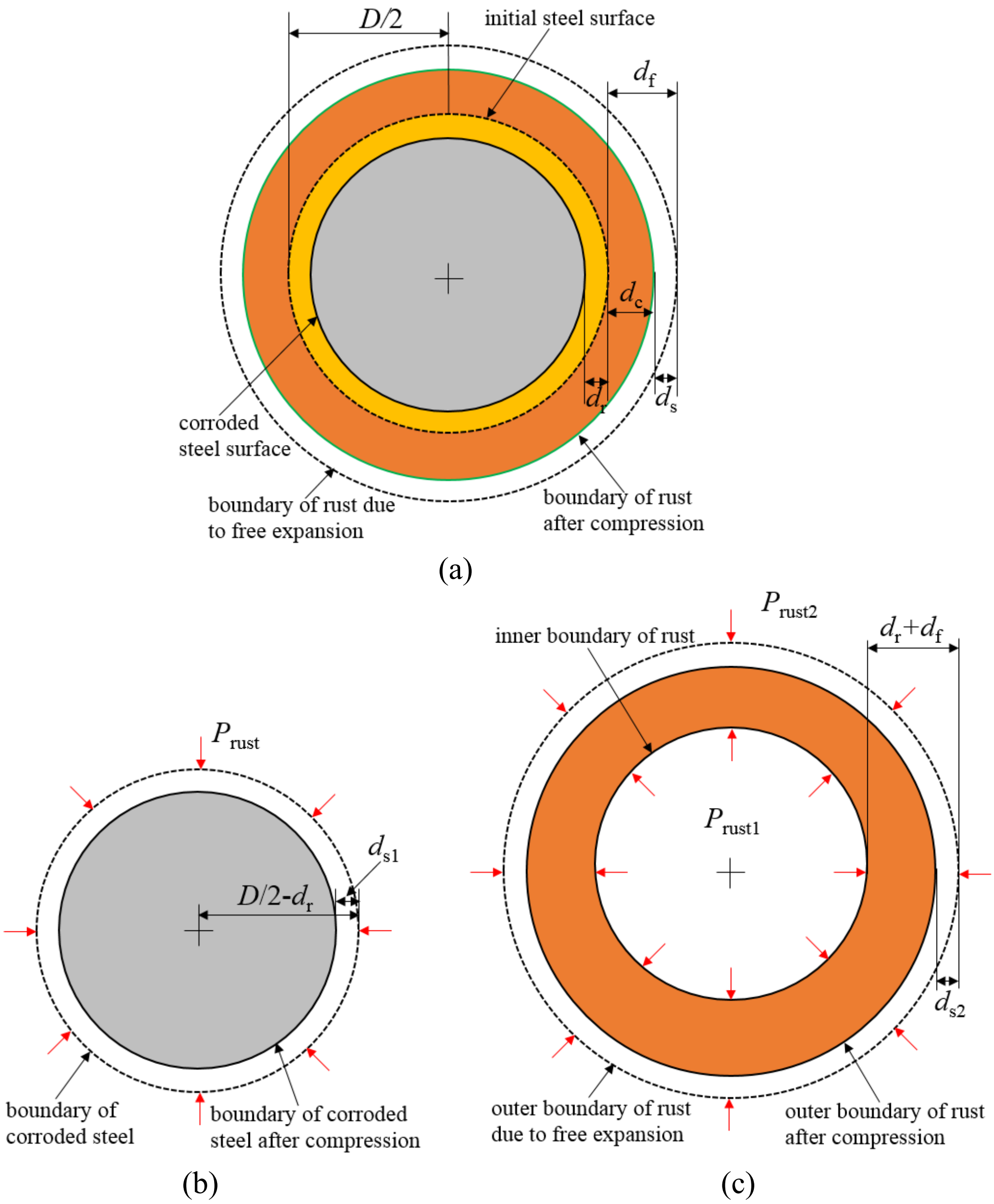

Fig 1. Deformation at steel-concrete interface in three-phase model of corrosion: (a) compatibility condition on deformation in radial direction; (b) deformation of steel bar; (c) deformation of rust layer.

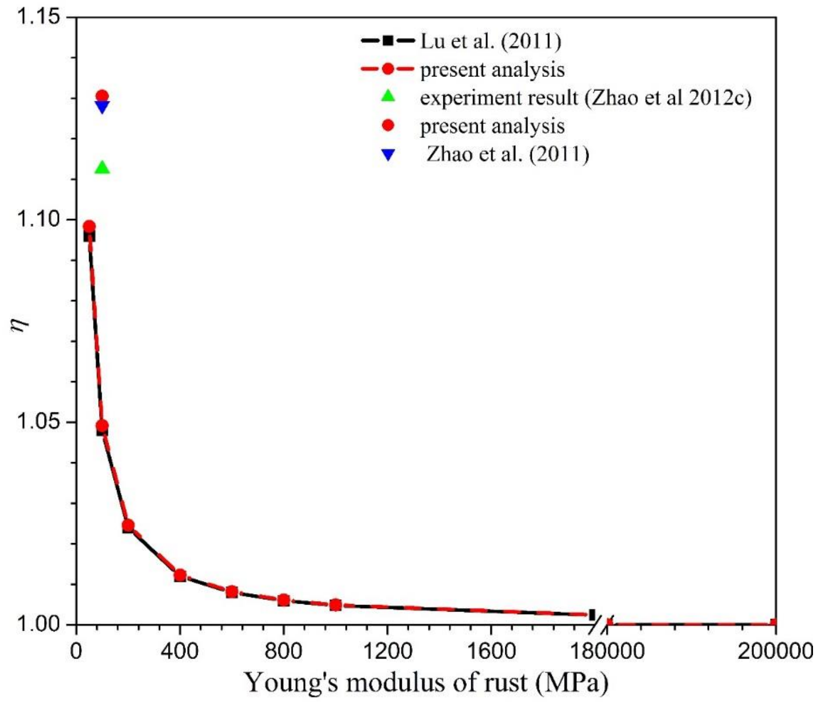

Fig. 2. Comparison of adjustment factor for volume of rust from different methods

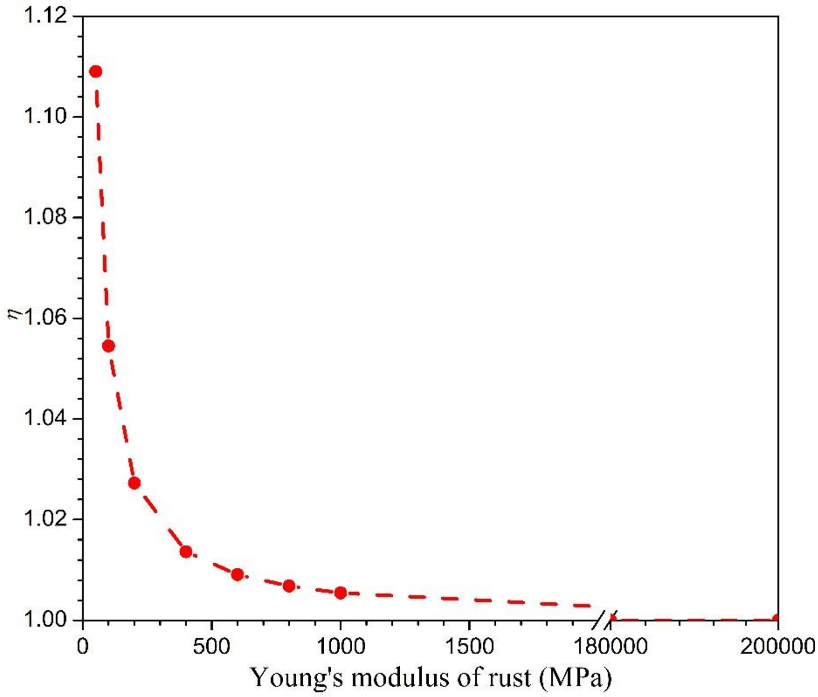

Fig. 3. Effect of E_{rust} on η: ($v_{\mathrm{rust}} = 0.3$, $\beta = 3$, $f_t = 3.3$ MPa, $c = 25$ mm, $D = 16$ mm, $d_0 = 16$ μm, $E_{\mathrm{steel}} = 200$ GPa, $v_{\mathrm{steel}} = 0.3$)

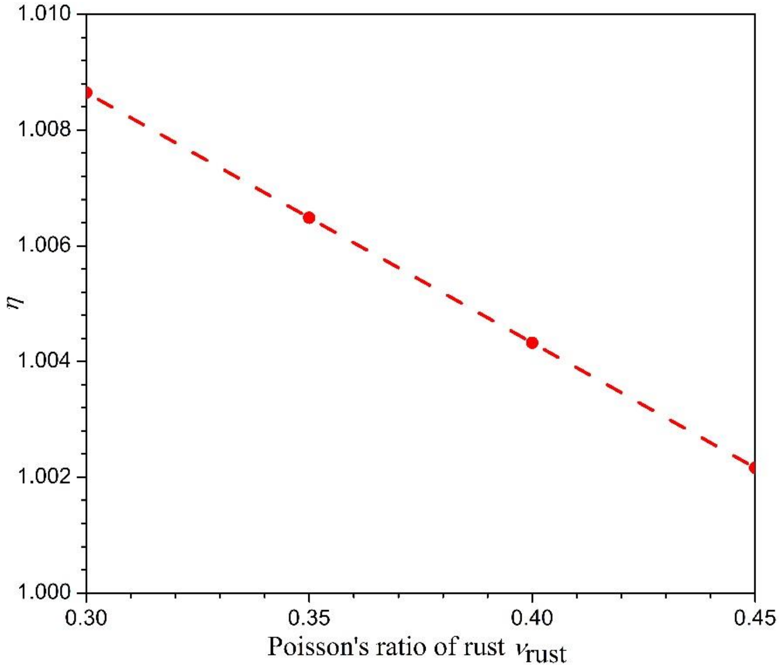

Fig. 4. Effect of ν_{rust} on η: (E_{rust} = 1000 MPa, β = 3, f_t = 3.3 MPa, c = 51 mm, D = 16 mm, d_0 = 16 μm, E_{steel} = 200 GPa, ν_{steel} = 0.3)

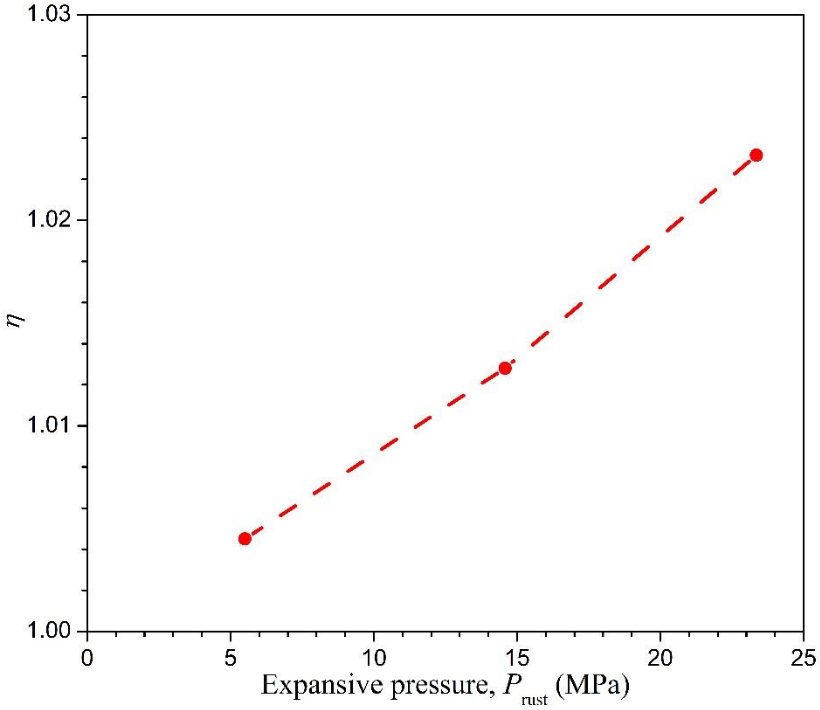

Fig. 5. Effect of P_{rust} on η: (E_{rust} = 1000 MPa, ν_{rust} = 0.3, β = 3, d_0 = 16 μm, E_{steel} = 200 GPa, ν_{steel} = 0.3)

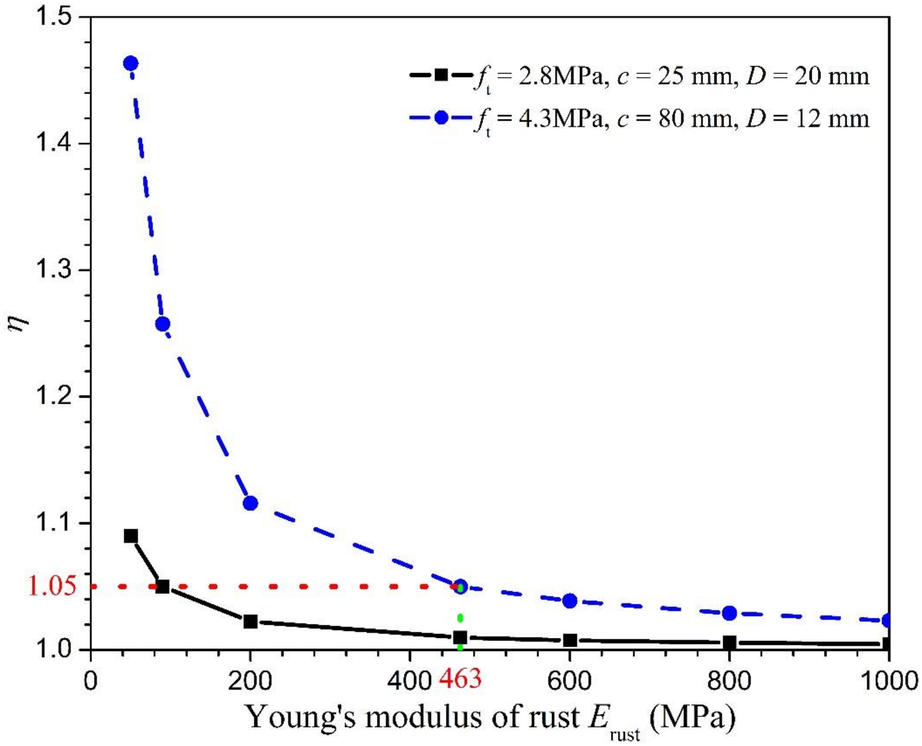

Fig. 6. Effect of E_{rust} on η ($v_{rust} = 0.3$, $\beta = 3$, $d_0 = 16$ μm, $E_{steel} = 200$ GPa, $v_{steel} = 0.3$)