

Short-term Reactive Power Reserve Optimization Based on Trajectory Sensitivity

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Abstract – An increasing concern is paid to short-term voltage stability with the growth of penetration of induction motor loads. Reactive power reserve(RPR) of power system is critical to improve voltage stability. A definition of short-term voltage stability-related RPR(SVRPR) is proposed. Generators vary their contributions to voltage stability with their location and system condition, etc. Voltage support coefficient based on the second-order trace sensitivity method is proposed to evaluate SVRPR's contribution to short-term voltage stability. The evaluation method can account for the generator's reactive support in transient process and the contingency severity. Then an optimization model to improve short-term voltage stability is built. To deal with multiple contingencies, contingency weight taking into account both its probability and severity is proposed. The optimization problem is solved by primal dual interior point method. Testing on IEEE_39 bus system, it is indicated that the method proposed is effective. Short-term voltage stability is improved significantly by the way of SV-RPR optimization. Hence, the approach can be used to prevent the happening of voltage collapse during system's contingency.

Keywords: Reactive power reserve, Short-term voltage stability, Contingency, Optimization

1. Introduction

The main contribution of RPR is maintaining voltage security when all kinds of disturbances occur in power system. Generally speaking, voltage collapse will happen when reactive power demand of power system can't be satisfied because of restrictions of var sources output capability and network transmission. To avoid voltage collapse or voltage dip, it is necessary to preserve enough RPR in power system [1]. Appropriate RPR profile can be obtained by proper reactive power management. Hence, RPR optimization by reactive power dispatch is very significant to operators to improve voltage security.

However, a generator's reactive support has different properties in its short- and long-term dynamics because the response of the generator's automatic excitation system is different when considering different durations [2]. Considering short-term dynamics, a generator can begin a forced excitation and produce large amounts of reactive power to support a drop in system voltage; thus, we cannot neglect the forced excitation response in this case. Considering long-term dynamics, the RPR supply is primarily restricted by the rated excitation, and thus, the most important factor is the reactive power that can be

provided over the long-term. Therefore, the RPR concerning short- and long-term voltage stabilities should be separately defined and examined.

There have been some studies on long-term voltage stability-related RPR(LV-RPR). Definitions of generator RPR can be classified into two categories: technical RPR [3,4] and effective RPR [5,6]. However voltage stability depends on system RPR which is based on all of the RPRs in the system. There are usually two ways to evaluate system RPR. One is simply adding all generator effective reactive reserves up as system RPR [7]. The other one is summing generator technical reactive reserves with weighting factors [8,9]. The latter one has been draw more attention since it can put it into consideration that the contribution of RPR to voltage stability varies with the generator location. Based on the above system reactive reserves, many LVRPR optimization methods have been proposed [8,10]. But the accuracy and applicability of the optimization model still need to be improved.

The aforementioned research on RPR evaluation and optimization are concerned with long-term or static voltage stability, while so far no RPR definitions directly related to short-term stability has been proposed. Evaluation of RPR with respect to contingencies were proposed, but just on quasi-steady state time scale [3,11]. However, along with large scale and long distance transmission system's input as well as increasing proportion of wind farm and motor in power system, short-term voltage security is becoming a predominant problem [12]. A clustering based method to group dynamic contingencies is proposed and an approach

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to identify dynamic voltage control area and the most effective candidate locations for placing dynamic reactive sources [13]. However, large amount of information from contingencies considering various system scenarios are needed in this approach, and the results are more suitable for dynamic reactive source planning. Hence it is urgent to present the definition and evaluation of SVRPR to aid the system operator in operation.

To assess SVRPR' contributions to short-term voltage stability during contingency, dynamic information of power system must be considered, especially for var sources such as generators. Time domain simulation is well-adapted to component models and can reflect component dynamic characteristics comprehensively. In recent years, researchers have done heuristic works of applying trace sensitivity to study power system's stability [14]. Most of the researches focus on the first-order trajectory sensitivity. However, a power system is a complex nonlinear system. When the system suffering large disturbances, the first-order trajectory sensitivity cannot reflect control variables' effect to state variables accurately [15].

The research of reactive power reserves with respect to contingencies has been studied in [3]. However, the evaluation of reactive power reserve with respect to contingencies didn't deal with short-term time scale. In this paper, a novel definition of SVRPR is proposed. Voltage support coefficient based on trace sensitivity is adopted to evaluate SVRPR's contribution to short-term voltage stability. To improve the accuracy of evaluation of SVRPR, the second-order trace sensitivity is introduced to the calculation of voltage support coefficient. An optimization model covering multiple contingencies is proposed to improve short-term voltage stability. It is solved by primal dual interior point method.

The rest of this paper is organized as follows: Section 2 introduces the definition and assessment of SVRPR. Section 3 introduces a method of SVRPR optimization. The SVRPR optimization performance of the simulation results on IEEE_39 system are presented in Section 4.

2. Definitions and Evaluation of SVRPR

The definitions of SVRPR are established in two-step manner. Generators are the main var sources in power system, in this paper the definition of generator SVRPR is taken as an example. Firstly we propose the definition of SVRPR of individual generator, namely individual SVRPR. The individual SVRPR at different locations in the system should not be directly summed up since they have different contributions to voltage stability. So we define system SVRPR as the sum of weighted individual SVRPR. The weighting factors are decided by individual SVRPR's contribution. The detailed definitions are given as follows.

2.1 Individual SVRPR

The generator's automatic excitation system has different properties in short-term and long-term dynamics. In short-term process, it can start forced excitation and produces large amount of reactive power to support system voltage drop. While in long-term process, it is restricted by rated excitation. So reactive support provided by a generator has different properties in short-term and long-term process. On a short-term time scale, the RPR cannot be simply defined as the difference between the maximum reactive power output and the current output because the maximum reactive power output varies with time during transient processes. Thus, we use the following integral to describe the reactive support of generators over a period:

$$Q_{GRS} = \int_{t_c}^{t_r} k_t(Q_t(t) - Q_{t_0})dt \quad (1)$$

where $Q_t(t)$ is the reactive power generator actually provided in the process; Q_{t_0} is the reactive power generator provided before fault. t_c is the fault clear time; and t_r is the time at the end of the short-term period.

In a transient process, when a generator can provide reactive power more quickly, short-term voltage stability is typically maintained more easily. Different types of generators have different response times to disturbances. To assign larger weights to those generators with faster excitation responses, the attenuating function $K_t = e^{-t}$ is used to measure the contribution of the reactive support at different times. In a transient process, promptly providing reactive power plays a key role in voltage support; in addition, the short circuit current attenuates based on the exponential law. Thus, the coefficient accurately reflects the real situation.

This definition can be developed to other dynamic var sources such as Static Var Compensator(SVC) and Static Var Generator(SVG) similarly.

2.2 System SVRPR

Short-term voltage stability depends on the level of the system SVRPR, which includes all of the individual SVRPRs. However, the contributions of each individual SVRPR vary based on their locations and dynamics. To assess SVRPR appropriately, a novel definition of the system SVRPR is proposed as follows:

$$Q_{RS} = \sum_{i=1}^{n_G} \int_{t_c}^{t_r} k_{vs}(t) \cdot k_t(Q_t^i(t) - Q_{t_0}^i)dt \quad (2)$$

Where $k_{vs}(t)$ is the voltage support coefficient of generator i , and n_G is the number of generators.

The voltage support coefficient $k_{vs}(t)$ is used to evaluate the individual RPR's contribution. The coefficient is defined as follows:

$$k_{vs}(t) = \partial V_j / \partial Q_i \quad (3)$$

It corresponds to the voltage sensitivity of weak bus j with respect to Q_i . This coefficient describes the reactive support capability of generator i to the weak bus j . The computation process of $k_{vs}(t)$ will be introduced in detail in the next section. The weak bus is judged by the transient voltage dip acceptable margin δ_{vd} [16] which is computed by (4):

$$\delta_{vd} = [V_{\min} - (V_{cr} - k_v T_{cr,v})] \times 100\% \quad (4)$$

where V_{\min} is the minimum voltage of the relevant bus, V_{cr} is the threshold value of the transient voltage dip, k_v is the conversion factor of the critical allowed duration to the voltage deviation, and $T_{cr,v}$ is the critical allowed duration of the bus voltage.

2.3 Computation of voltage support coefficient based on second-order trajectory sensitivity

Voltage support coefficient K_{vs} can be expressed as follows:

$$K_{vs} = \frac{\Delta V_L}{\Delta Q_G} = \frac{\partial V_L / \partial V_{G0}}{\partial Q_G / \partial V_{G0}} \quad (5)$$

where V_L is voltage of load bus; Q_G is reactive power of generator; V_{G0} is initial voltage of generator bus. To get the value of K_{vs} , $\partial V_L / \partial V_{G0}$ and $\partial Q_G / \partial V_{G0}$ need to be obtained.

Generator reactive power can be expressed as (6).

$$Q_G = R(V_G, \delta, \theta, E'_d, E'_q) \quad (6)$$

Then $\partial Q_G / \partial V_{G0}$ can be derived as (7).

$$\frac{\partial Q_G}{\partial V_{G0}} = \frac{\partial GR}{\partial E'_d} \frac{\partial E'_d}{\partial V_{G0}} + \frac{\partial R}{\partial E'_q} \frac{\partial E'_q}{\partial V_{G0}} + \frac{\partial R}{\partial V_G} \frac{\partial V_G}{\partial V_{G0}} + \frac{\partial R}{\partial \delta} \frac{\partial \delta}{\partial V_{G0}} + \frac{\partial R}{\partial \theta} \frac{\partial \theta}{\partial V_{G0}} \quad (7)$$

where V_G, δ, E'_d, E'_q are voltage magnitude, power angle, direct axis transient field voltage and quadrature axis transient field voltage of generator respectively.

We can see that sensitivity computation of state variables (δ, E'_d, E'_q) and algebra variables (θ, V) with respect to generator initial voltage V_{G0} are included in the computation of K_{vs} . So trace sensitivity method is used to compute K_{vs} at different time based on simulation data. The second-order trajectory sensitivity is used to obtain more accurate result. It's computation process is illustrated in appendix. By means of trace sensitivity method, the voltage support coefficient of generator takes into account the RPR's voltage support to the concerning bus and the RPR's dynamics.

3. Optimization of SVRPR

3.1 SVRPR optimization model

Based on the definition of SVRPR, the SVRPR optimization model to improve short-term voltage stability is established. In the model, system SVRPR is set as objective to improve short-term voltage stability.

$$\max Q_{RS} \quad (8)$$

In practice, multiple contingencies should be covered in the optimization of SVRPR. In such case, we can take the objective as a weighted sum of Q_{RS} for each contingency:

$$Q_{RS} = \sum_{j=1}^{N_f} \omega_j \cdot Q_{RS}^j \quad (9)$$

where Q_{RS}^j is the system SVRPR of contingency j ; N_f is the number of severe contingencies; ω_j is the weight of contingency j .

The contingency weight ω_j takes into account both its probability and severity.

$$\omega_j = -\delta_{vdj} \rho_j \quad (10)$$

where ρ_j is the probability of contingency j , which is based on the historical fault statistics.

However, the maximization of Q_{RS} is equal to the maximization of ΔQ_{RS} . And for operators, it is more important to find the direction of control means in time than the optimum control. So the sensitivity of system SVRPR with respect to control variables is linearized so as to reduce computation time. Thus the objective of the model is converted as follows:

$$\max(\Delta Q_{RS}) \quad (11)$$

$$\Delta Q_{RS} = \frac{\partial Q_{RS}}{\partial V_G} \Delta V_G + \frac{\partial Q_{RS}}{\partial V_{SVCreff}} \Delta V_{SVCreff} + \frac{\partial Q_{RS}}{\partial V_{SVGref}} \Delta V_{SVGref}$$

where subscripts $SVCreff$ and $SVGref$ indicate var source $SVCreff$ and $SVGref$ respectively.

In this model, control voltage of reactive power source is considered as an optimization variable and the increment of system's reactive power reserve is considered as objective.

The equality constraints of the model correspond to the power flow equations expressed as follows:

$$\begin{cases} P_{Gi} - P_{Li} - V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \\ Q_{Gi} - Q_{Li} - V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \end{cases} \quad (12)$$

where P and Q mean active and reactive power, V and

θ mean bus voltages and angles, subscripts G and L indicate generator and load buses, respectively.

The inequality constraints considered in the model are expressed as follows:

$$\begin{cases} V_{i \min} \leq V_i \leq V_{i \max}, i = 1, \dots, N \\ V_{SVCg \min} \leq V_{SVCg} \leq V_{SVCg \max}, g = 1, \dots, N_{SVC} \\ V_{SVGh \min} \leq V_{SVGh} \leq V_{SVGh \max}, h = 1, \dots, N_{SVG} \\ Q_{Gi \min} \leq Q_{Gi} \leq Q_{Gi \max}, i = 1, \dots, N_G \\ B_{SVCg \min} \leq B_{SVCg} \leq B_{SVCg \max}, g = 1, \dots, N_{SVC} \\ I_{SVGh \min} \leq I_{SVGh} \leq I_{SVGh \max}, h = 1, \dots, N_{SVG} \end{cases} \quad (13)$$

where V_{SVCg} , V_{SVGh} indicate the given voltage of SVC_g and SVG_h respectively; B_{SVC} is susceptance of SVC ; I_{SVG} is the current of SVG ; N , N_G , N_{SVC} , N_{SVG} are the numbers of buses, generators, SVC and SVG respectively.

3.2 Solving procedure

In fact, the converted model is a ORPF problem. It can be solved by PCPDIPM [17]. In this method, the Lagrange multipliers and the slack variables are used to deal with the equality and inequality constraints, and the logarithmic barrier functions are constructed to guarantee the non-negativity conditions of the slack variables. The Lagrange function without the constraints can be formulated as follows:

$$L = -\Delta Q_{RS} - \rho^T g(x) + \pi^T h_s(x) - \mu \left(\sum_{i=1}^{n_h} \ln s_i \right) \quad (14)$$

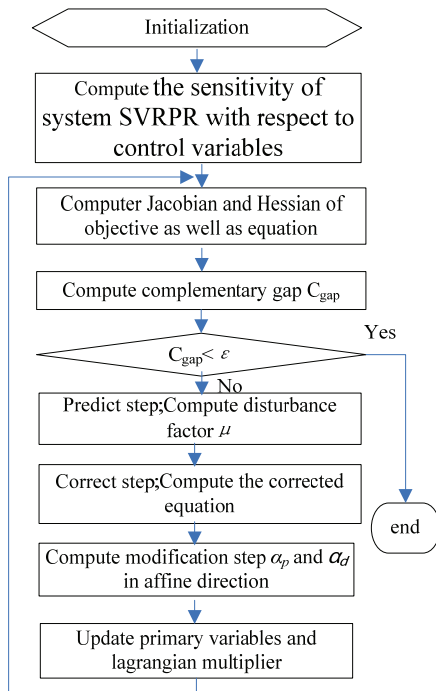


Fig.1. The procedure of optimization based on PDIPM

where $g(x)$ represents equality constrains

$h_s(\cdot)$ represents inequality constraints with slack variables s .

ρ and π are the vectors Lagrange multipliers for the equality and inequality constraints;

s is the vector of slack variables; μ is the vector of barrier parameters n_h is the number of $h_s(x)$.

The solving procedure of the optimization is shown as Fig. 1.

4. Case study

In this section, the proposed SVRPR optimization model is tested on IEEE 39-bus system. It's line diagram is shown as Fig. 2. Parameters of lines and transformers are from reference [18]. Parameters of generators are listed in Table 1.

For a predefined set of faults, the contingencies studied are the 3-phase symmetric faults at the heads of lines 16-15, 17-16, and 21-16. It is assumed that a fault occurs at $t = 0.1$ s and that the head and terminal breakers of the lines then open successively at 0.09s and 0.1 s, respectively. Simulations are carried by PSD-BPA software. The

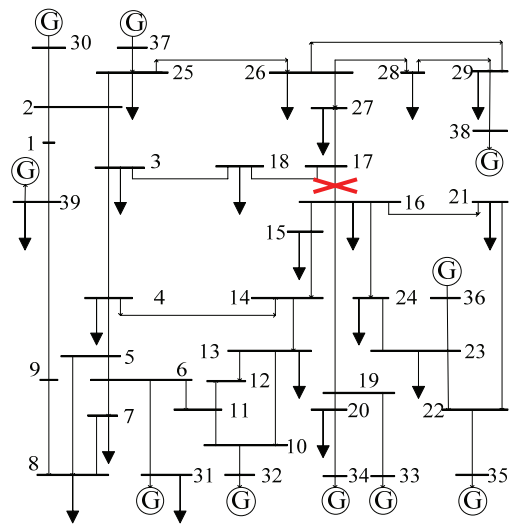


Fig. 2 Structure of IEEE39 system

Table 1. Parameters of generators

Gen.	E_{MWS}	R_a	X_d	X_q	X_d'	X_q'	$Td0'$	$Tq0'$	X_s	D
30	4200	0	1	0.61	0.31	0.31	10.2	1.5	0.125	0
31	3030	0	2.95	2.82	0.697	0.697	6.561	1.5	0.304	0
32	3580	0	2.495	2.37	0.531	0.531	5.7	1.5	0.35	0
33	2860	0	2.62	2.58	0.436	0.436	5.69	1.5	0.295	0
34	2600	0	6.7	6.2	1.32	1.32	5.4	0.44	0.54	0
35	3480	0	2.54	2.41	0.5	0.5	7.3	0.4	0.224	0
36	2640	0	2.95	2.92	0.49	0.49	5.66	1.5	0.322	0
37	2430	0	2.9	2.8	0.57	0.57	6.7	0.41	0.28	0
38	3450	0	2.106	2.05	0.57	0.57	4.79	196	0.298	0
39	50000	0	0.2	0.19	0.06	0.06	7	0.7	0.3	0

Table 2. Normalized voltage support coefficients of generators

fault	Generator								
	30	31	32	33	34	35	36	37	38
16-15	0.46	0.25	0.27	0.67	0.31	0.735	0.41	0.399	0.4
17-16	0.16	0.47	0.68	1.00	0.42	0.91	0.58	0.12	0.17
21-16	0.48	0.57	0.79	0.99	0.45	0.81	0.72	0.34	0.393

Table 3. Comparison of the transient voltage dip acceptable margin before and after optimization

fault	bus	Before optimization δ_{vd}	After optimization δ_{vd}
6-15 from	17	-63.04%	12.68%
17-16 from	15	-52.42%	0.83%
21-16 from	15	-68.28%	9.85%

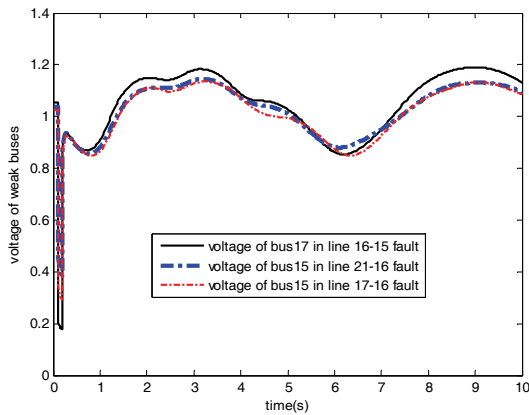


Fig. 3. Multiple fault optimized voltage curves of weak buses

probabilities of all the faults are set as 20%.

As an example, the voltage support coefficients of generators in the fault of line 17-16 are listed in Table 2; these values correspond to the summation of K_{vs} during the transient process and have been normalized. The sensitivity of generator 33 is shown to be the largest, and the sensitivities of generator 30,37,38, and 39 are far smaller than those of the other generators. The results coincide with the fact that the electric distances between generators 30,37,38, and 39 and bus 15 become large; thus, their voltage support capacity for bus 15 becomes weaker, as shown in Fig. 2.

Multiple faults considered, the optimized weak bus voltages of the three faults are shown in Fig. 3. From the curves, the voltages of the weak buses are found to all be stable. Concurrently, Table 3 shows that the voltage dips of the weak buses in all three faults have been considerably mitigated. As an example of one single fault, the voltage of bus 15 in the fault of line 17-16 is shown in Fig. 4. However, if only an individual fault is considered, then short-term voltage stability cannot be guaranteed in other faults. For example, for the SVRPR that only includes the fault in line 16-15, the optimized weak bus voltages of the three faults are shown in Fig. 5. The curves

Table 4. Comparison of transient voltage dip acceptable margin

fault	bus	The first-order TS δ_{vd}	The second-order TS δ_{vd}
16-15 from	17	11.98%	12.68%
17-16 from	15	0.72%	0.83%
21-16 from	15	9.56%	9.85%

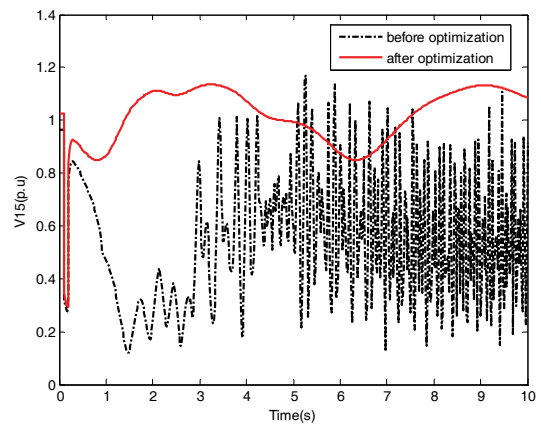


Fig. 4. Voltage curve of bus 15 in line 17-16's head fault

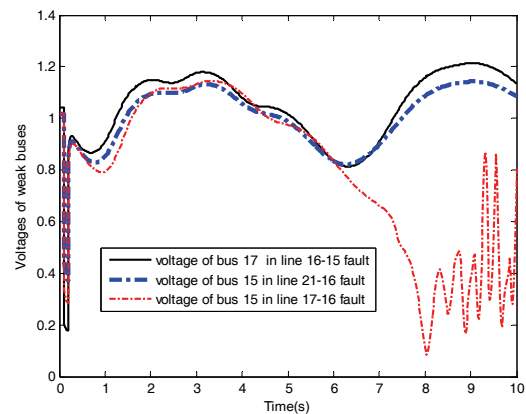


Fig. 5. Single fault optimized voltage curves of weak buses

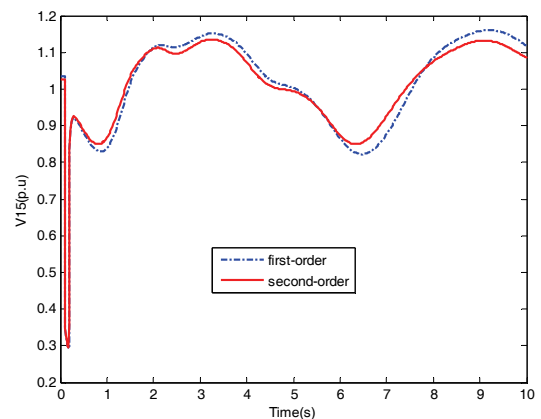


Fig. 6. Optimized voltage curve of bus 15 in line 17-16's fault

indicate that voltage instability occurs in the fault in line 17-16. It follows that SVRPR optimization with multiple contingencies is essential.

In comparison, we also compute K_{vs} based on the first-order trace sensitivity. The transient voltage dip acceptable margins of weak buses are listed in Table 4. The improvement is less than using the second-order trace sensitivity. As an example, voltage of bus 15 in line 17-16's head fault is shown in Fig. 6. Comparing the curves, optimization based on the second-order trace sensitivity can improve short-term voltage stability more.

5. Conclusion

This paper focuses on a method for reactive reserve optimization to improve short-term voltage stability effectively. Conclusions of the investigation are summarized as follows:

The definition and three proposed weighting factors of short-term reactive reserve are appropriate and effective. The evaluation method considered both supporting capacity of one bus to the other and demand of reactive power reserve according to severity of faults.

The proposed optimization method of short-term reactive power reserve can improve short-term voltage stability effectively. Compared with the first-order TS, the second-order TS can indicate the short-term voltage support capability more accurately.

In future research, the scalability of the approach for large-scale power system still needs to be explored. To simplify the computational complexity, the power system is expected be divided into appropriate voltage regions according to short-term voltage response. By this way, short-term voltage problems of each region can be improved respectively with less computation expense.

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Appendix

The second-order trajectory sensitivity is computed as follows:

Differential algebraic equation of power system can be simplified as follows.

$$\dot{x} = f(x, y, \lambda) \tag{A1}$$

$$0 = g(x, y, \lambda) \tag{A2}$$

where $x \in (\delta, \omega, E_d', E_q')$, $y \in (\theta, V)$ are vectors of state

variables and algebraic variables respectively, λ is control variable.

By Implicit trapezoidal integration method, differential Eq. (A1) is differenced as (A3).

$$x_{k+1} = x_k + \frac{\Delta t}{2} [f(\cdot)^k + f(\cdot)^{k+1}] \tag{A3}$$

where Δt is the time step of integral.

In every interval (t^k, t^{k+1}), correction equation of differential algebraic Eq. (A2) and (A3) can be expressed as follows.

$$\begin{bmatrix} \frac{\Delta t}{2} f_x^{k+1}(\cdot) - I & \frac{\Delta t}{2} f_y^{k+1}(\cdot) \\ g_x^{k+1}(\cdot) & g_y^{k+1}(\cdot) \end{bmatrix} \begin{bmatrix} \Delta x^{k+1} \\ \Delta y^{k+1} \end{bmatrix} = \begin{bmatrix} x^{k+1} - x^{k+1} - \frac{\Delta t}{2} [f(\cdot)^k + f(\cdot)^{k+1}] \\ -g^{k+1}(\cdot) \end{bmatrix} \tag{A4}$$

where I is an unit matrix.

The calculation process is mainly divided into two stages: The first-order and second-order trajectory sensitivity.

A. The first-order trajectory sensitivity

Both the state and the algebraic variables are dependent on the control variable λ . To calculate the trajectory sensitivity, the sensitivity of the state variables and the algebraic variables to the control variable under steady state must be obtained.

Differential algebraic Eq. (A1) and (A2) can be derivatived as (A5).

$$\begin{bmatrix} f_x(\cdot) & f_y(\cdot) \\ g_x(\cdot) & g_y(\cdot) \end{bmatrix} \begin{bmatrix} x_{\lambda 0} \\ y_{\lambda 0} \end{bmatrix} = \begin{bmatrix} -f_\lambda(\cdot) \\ -g_\lambda(\cdot) \end{bmatrix} \tag{A5}$$

Initial values of the first-order trajectory sensitivity can be obtained from (A5). Then (A1) and (A2) are derivatived as (A6) and (A7) in every step.

$$dx_\lambda/dt = f_x(\cdot)x_\lambda + f_y(\cdot)y_\lambda + f_\lambda(\cdot) \tag{A6}$$

$$0 = g_x(\cdot)x_\lambda + g_y(\cdot)y_\lambda + g_\lambda(\cdot) \tag{A7}$$

By Implicit trapezoidal integration method, (A6) is differenced as (A8).

$$0 = \left[\frac{\Delta t}{2} f_x^{k+1}(\cdot) - 1 \right] x_\lambda^{k+1} + \frac{\Delta t}{2} f_y^{k+1}(\cdot) y_\lambda^{k+1} + x_\lambda^k + \frac{\Delta t}{2} [f_x^k(\cdot)x_\lambda^k + f_y^k(\cdot)y_\lambda^k + f_\lambda^k(\cdot) + f_\lambda^{k+1}(\cdot)] \tag{A8}$$

Then (A7) and (A8) can be expressed as (A9) in every interval (t^k, t^{k+1}).

$$\begin{aligned} & \begin{bmatrix} \frac{\Delta t}{2} f_x^{k+1}(\cdot) - I \frac{\Delta t}{2} f_y^{k+1}(\cdot) \\ g_x^{k+1}(\cdot) & g_y^{k+1}(\cdot) \end{bmatrix} \begin{bmatrix} x_\lambda^{k+1} \\ y_\lambda^{k+1} \end{bmatrix} \\ & = \begin{bmatrix} -x_\lambda^k - \frac{\Delta t}{2} [f_x^k(\cdot)x_\lambda^k + f_y^k(\cdot)y_\lambda^k + f_\lambda^k(\cdot) + f_\lambda^{k+1}(\cdot)] \\ -g_\lambda^{k+1}(\cdot) \end{bmatrix} \end{aligned} \quad (A9)$$

Substituting λ with V_{G0} , the first-order trajectory sensitivity of system variables with respect to V_{G0} can be computed by (A9) in every step.

B. The second-order trajectory sensitivity

Based on the results of the first-order trajectory sensitivity, the second-order trajectory sensitivity can be further calculated.

Initial values of the second-order trajectory sensitivity can be computed by (A10), which is the derivation of (A5)

$$\begin{aligned} & \begin{bmatrix} f_x(\cdot) & f_y(\cdot) \\ g_x(\cdot) & g_y(\cdot) \end{bmatrix} \begin{bmatrix} x_{\lambda\lambda 0} \\ y_{\lambda\lambda 0} \end{bmatrix} \\ & = - \begin{bmatrix} f_{xx}(\cdot)x_{\lambda 0}^2 + 2f_{xy}(\cdot)x_{\lambda 0}y_{\lambda 0} + f_{yy}(\cdot)y_{\lambda 0}^2 + f_{\lambda\lambda}(\cdot) + f_{x\lambda}(\cdot)x_{\lambda 0} + f_{y\lambda}(\cdot)y_{\lambda 0} \\ g_{xx}(\cdot)x_{\lambda 0}^2 + 2g_{xy}(\cdot)x_{\lambda 0}y_{\lambda 0} + g_{yy}(\cdot)y_{\lambda 0}^2 + g_{\lambda\lambda}(\cdot) + g_{x\lambda}(\cdot)x_{\lambda 0} + g_{y\lambda}(\cdot)y_{\lambda 0} \end{bmatrix} \end{aligned} \quad (A10)$$

In every interval (t^k, t^{k+1}), the second-order trajectory sensitivity of system variables to the control variable can be obtained by (A11), which is the derivation of (A9).

$$\begin{aligned} & \begin{bmatrix} \frac{\Delta t}{2} f_x^{k+1}(\cdot) - I \frac{\Delta t}{2} f_y^{k+1}(\cdot) \\ g_x^{k+1}(\cdot) & g_y^{k+1}(\cdot) \end{bmatrix} \begin{bmatrix} x_{\lambda\lambda}^{k+1} \\ y_{\lambda\lambda}^{k+1} \end{bmatrix} = - \begin{bmatrix} \alpha \\ g_{\lambda\lambda}^{k+1}(\cdot) \end{bmatrix} \\ & \begin{bmatrix} \frac{\Delta t}{2} [(g_\lambda^{k+1})^T f_{xx}^{k+1}(\cdot) + (g_\lambda^{k+1})^T f_{yy}^{k+1}(\cdot) + f_{\lambda\lambda}^{k+1}(\cdot)] \frac{\Delta t}{2} [(g_\lambda^{k+1})^T f_{xx}^{k+1}(\cdot) + (g_\lambda^{k+1})^T f_{yy}^{k+1}(\cdot) + f_{\lambda\lambda}^{k+1}(\cdot)] \\ [(g_\lambda^{k+1})^T g_{xx}^{k+1}(\cdot) + (g_\lambda^{k+1})^T g_{yy}^{k+1}(\cdot) + 2 \cdot g_{x\lambda}^{k+1}(\cdot)] & [(g_\lambda^{k+1})^T g_{xy}^{k+1}(\cdot) + (g_\lambda^{k+1})^T g_{y\lambda}^{k+1}(\cdot) + 2 \cdot g_{y\lambda}^{k+1}(\cdot)] \end{bmatrix} \\ & \begin{bmatrix} x_\lambda^{k+1} \\ y_\lambda^{k+1} \end{bmatrix} \end{aligned} \quad (A11)$$

where

$$\begin{aligned} \alpha = & \frac{\Delta t}{2} [(x_\lambda^k)^T f_{xx}^k(\cdot)x_\lambda^k + (y_\lambda^k)^T f_{yy}^k(\cdot)y_\lambda^k + (x_\lambda^k)^T f_{yx}^k(\cdot)y_\lambda^k + (y_\lambda^k)^T f_{xy}^k(\cdot)x_\lambda^k \\ & + f_{x\lambda}^k(\cdot)x_\lambda^k + f_{y\lambda}^k(\cdot)y_\lambda^k + f_x^k(\cdot)x_{\lambda\lambda}^k + f_y^k(\cdot)y_{\lambda\lambda}^k + f_{\lambda\lambda}^k + f_{\lambda\lambda}^{k+1}] + x_{\lambda\lambda}^k \end{aligned}$$

Thus substituting λ with V_{G0} , the second-order trajectory sensitivity of state variables with respect to V_{G0} can be computed by (A11) in every step.

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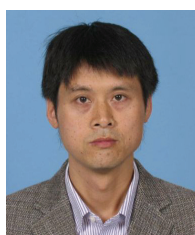
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