Jointly modeling area-level crash rates by severity: A Bayesian multivariate random-parameters spatio-temporal Tobit regression

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This study investigates the inclusion of spatio-temporal correlation and interaction in a multivariate random-parameters Tobit model and their influence on fitting areal crash rates with different severity outcomes. The spatial correlation is specified via a multivariate conditional autoregressive (MCAR) prior, whereas the temporal correlation is specified by a linear time trend. A spatio-temporal interaction is formulated as the product of a time trend and a spatial term with an MCAR prior. A multivariate random-parameters spatio-temporal Tobit model is developed for *slight injury* and *killed or serious injury* crash rates using one year of crash data from 131 traffic analysis zones in Hong Kong. The proposed model is estimated and assessed in the Bayesian context using Markov chain Monte Carlo simulation. The model estimation results show that spatial and temporal effects and their interactive effects are significant (independent) and that the spatial and interactive effects have strong correlations across injury severities. Intersection density and commercial land use are found to have heterogeneous effects on safety. The values of the deviance information criterion, mean absolute deviance, and mean squared prediction error indicate that the proposed model outperforms a multivariate random-parameters Tobit model and a multivariate random-parameters spatial Tobit model in terms of model fit. These findings highlight the importance of appropriately accommodating spatio-temporal correlation and interaction for the joint analysis of areal crash rates by severity.

Keywords: areal traffic safety; crash rates by severity; spatio-temporal correlation; unobserved heterogeneity; multivariate random-parameters Tobit model.

1. Introduction

Since the concept of safety-conscious planning emerged, a great deal of research has been devoted to developing and applying sophisticated methodological approaches to the analysis of traffic crashes at the area/macro level (Abdel-Aty et al. 2013). Such crash prediction models (also known as safety performance functions) greatly aid in ecological analysis (i.e., identifying factors contributing to area-wide crash occurrences) (Dong et al. 2017; Wu and Loo 2017), hotspot identification (i.e., finding the most promising areas for improving safety) (Dong et al. 2016b), and incorporating safety considerations into long-term transportation planning (Aguero-Valverde 2013). Most of these studies aimed to establish explicit relationships between total crash frequencies in certain areal units (e.g., states, counties, traffic analysis zones [TAZs]) and macro-structural factors, such as road infrastructure (Quddus 2008a; Xu and Huang 2015), traffic characteristics (Guo et al. 2015, 2017), socio-economic indexes (Hadayeghi, Shalaby, and Persaud 2010; Wang and Kockelman 2013; Huang et al. 2016a), and environmental conditions (Aguero-Valverde and Jovanis 2006), and many of them developed total crash frequency prediction models.

Over the past decade, with the growing demand for comprehensive evaluations of safety performance and the development of cost-effective safety improvement projects, numerous methods have been proposed to analyze crash frequencies/rates at different degrees of injury severity (Dong et al. 2016a;

El-Basyouny and Sayed 2009; Ma, Kockelman, and Damien 2008; Xu, Wong, and Choi 2014; Yasmin and Eluru 2018). Occurrences of crashes with severe outcomes (e.g., fatalities) are rare within the limited observational periods, and thus there may be many zero observations in the collected crash data. Although zero-inflated count models have been developed to deal with crash frequencies with a preponderance of zeros, some researchers have challenged the rationality of the perfectly safe state assumed in a transportation system (Lord, Washington, and Ivan 2005, 2007). For such cases, crash rate analysis may be a good alternative to conventional frequency analysis because crash rates are usually modeled by Tobit-based regressions, which are appropriate for handling censored data (e.g., crash rates that are left-censored at zero) (Anastasopoulos, Tarko, and Mannering 2008). Moreover, in proactive road-safety practices, as a standardized measure of the relative safety performance of an areal unit, knowledge of the crash rate facilitates traffic agencies in their assessment of the traffic-safety implications of candidate transportation network schemes. Crash-rate models can even be used to determine the effects of traffic volume on crash exposure and crash risk (Zeng et al. 2017b).

Given the potential correlation between various crash severities, it is necessary to model crash rates simultaneously by severity. Multivariate Tobit regression and its random-parameters version have been advocated for this purpose (Anastasopoulos et al. 2012; Anastasopoulos 2016; Zeng et al. 2017a). To capture the effects of unobserved factors shared by successive periods, temporal correlation has been incorporated into the multivariate Tobit model (Zeng et al. 2018a). However, none of these multivariate models has been applied in areal safety analysis or has accounted for the underlying spatial correlation. As Washington, Karlaftis, and Mannering (2011) noted, models omitting spatial correlation underestimate the variance of parameters, which may lead to misidentification of the factors that contribute to crash occurrence.

Although a number of studies have explored multivariate spatial modeling (Barua, El-Basyouny, and Islam 2014, 2016; Huang et al. 2017; Wen et al. 2018; Zeng et al. 2018b), multivariate temporal modeling (Zeng et al. 2018a), and univariate spatio-temporal modeling (Aguero-Valverde and Jovanis 2006; Castro, Paleti, and Bhat 2012; Cheng et al. 2018a; Dong et al. 2016b; Wang, Quddus, and Ison 2013), multivariate spatio-temporal analysis is scarce in the traffic-safety literature. Ma, Chen, and Chen (2017) investigated the multivariate spatio-temporal modeling of crash frequencies by injury severity on freeway segments. Their empirical analysis revealed significant spatial and temporal effects and heterogeneous correlations in the crash panel data, and that the multivariate spatio-temporal model outperformed the multivariate random-effects model and multivariate spatial models. More recently, Cheng et al. (2018b) proposed three multivariate spatio-temporal models for analyzing crash frequencies by transportation mode in 58 Californian counties. Their findings suggest the existence of not only spatial and temporal correlations but also their interactive effects, which are likewise found in univariate spatio-temporal models. However, no study has accounted for the unobserved heterogeneity in the safety effects of observed factors (Zeng et al. 2018a), which is also an important characteristic of crash data (Mannering, Shankar, and Bhat 2016). Random-parameters modeling is among the most popular methods of capturing unobserved heterogeneity (Wang, Huang, and Zeng 2017), along with Markov switching (Malyshkina, Mannering, and Tarko 2009) and finite mixture/latent class approaches (Zou, Zhang, and Lord 2013). To the best of our knowledge, no reported research has used a multivariate random-parameters spatio-temporal approach to jointly model area-wide crash rates by injury severity.

From a methodological perspective, the multivariate conditional autoregressive (MCAR) prior is the dominant method for specifying spatial correlation under the multivariate modeling framework (Barua, El-Basyouny, and Islam 2014) because it accommodates the distinct spatial effects of each response variable and the correlations between them. According to previous studies (Aguero-Valverde 2013; Barua, El-Basyouny, and Islam 2014; Wang and Kockelman 2013), the MCAR model outperforms its univariate counterpart in terms of model fit. While more complex methods have been described in the literature (Cheng et al. 2018a; Quddus 2008b), the linear time trend and the product of the spatial term and the time trend are the most widely used formulations of temporal correlation and spatio-temporal interaction, respectively (Aguero-Valverde and Jovanis 2006; Cheng et al. 2018b; Dong et al. 2016b; Meng et al. 2017b). Therefore, incorporating the MCAR prior, linear time trend, and their product into the link function of a multivariate random-parameters Tobit model is expected to adequately capture the spatio-temporal effects in different crash-severity rates simultaneously.

Overall, this study analyzes area-level crash rates by injury severity using a multivariate random-parameters Tobit model with spatio-temporal correlation and interaction. To validate the strength of the proposed interaction model, it is estimated and compared with a multivariate random-parameters Tobit model and a multivariate random-parameters spatial Tobit model in the Bayesian context using the WinBUGS freeware (Lunn et al. 2000). A comprehensive crash dataset collected in 131 selected TAZs in Hong Kong is used in the empirical analysis.

The remainder of this paper is structured as follows. Section 2 describes the collected crash dataset for the model's development. Section 3 presents the formulation and specifications of the candidate models, the criteria for model assessment, and the process of model estimation. In Section 4, models are compared based on goodness-of-fit measures, and parameter estimates are illustrated with reference to the findings of previous research. Conclusions are drawn and directions for further research are proposed in Section 5.

2. Data preparation

A comprehensive crash dataset obtained from the Traffic Information System maintained by the Transport Department of Hong Kong is used for model development and comparison. This dataset includes 131 TAZs (as shown in Fig. 1) defined in the Hong Kong Planning Vision and Strategy zoning system, for which full traffic and geometric information have been recorded. The average area of these TAZs is 0.37 km^2, with the smallest 0.04 km^2 and the largest 2.11 km^2. The traffic crashes reported in 2011 are mapped onto these areas using geographical information system techniques. To investigate the temporal variations of crash rates, the crashes occurring in 2011 in each TAZ are aggregated in 4-h periods, according to the crash times documented in police reports: 07:00-11:00 (morning), 11:00-15:00 (noon), 15:00-19:00 (afternoon), 19:00-23:00 (evening), 23:00-03:00 (midnight), and 03:00-07:00 (dawn). The aggregated crash counts for each 4-h period are further grouped according to injury severity. Three degrees of severity are classified in original crash reports in Hong Kong: slight injury, serious injury, and fatality. As in previous studies (Zeng et al. 2017a, 2018a), rare fatal crashes are combined with serious injury crashes to constitute the class of “killed or serious injury” (KSI) crashes in the current research.

[Insert Fig. 1 near here]

The hourly traffic volumes for more than 100 roadways distributed in the selected road network are available in the Hong Kong Annual Traffic Census system. Global positioning system (GPS) data collected from 480 taxis are used to estimate the traffic volumes within each TAZ during each 4-h period. The GPS data, which are derived from GPS probes mounted in the taxis, include instantaneous travel information on the location, time, direction, speed, and occupancy, and are reported to the traffic control center at 30-s intervals. Due to the full network coverage of the taxi

GPS data, occupied-taxi flow on any road segment can be easily quantified. The linear data projection method (Wong and Wong 2015, 2019) is a commonly used data-scaling method that can compatibly fuse data acquired from different sources for unbiased traffic data estimation using the mean of the scaling factor. It has been applied in various studies to unbiasedly estimate different unobservable traffic data, such as traffic flow (Wong and Wong, 2016a, 2016b), traffic density (Wong, Wong, and Liu 2019), and travel time (Meng et al., 2017a). Such a time-focused data-scaling method is employed in this study to unbiasedly infer the hourly traffic volumes on roadways without fixed detectors, based on the volumes on roadways with both fixed detectors and the taxi GPS data. The annual average 4-h period traffic (AAPT) in each TAZ is calculated as the sum of the traffic volumes on the covered hours and roadways.

The crash rate (number of crashes per million vehicle-kilometers traveled) by injury severity for each 4-h period, $CR_{i,t,k}$, is used as the dependent variable in this study, and is calculated as

$$CR_{i,t,k} = \frac{No\_crash_{i,t,k}}{AAPT_{i,t} \times L_i \times 365/1000,000}, \quad i = 1, 2, \cdots 131, \quad t = 1, 2, \cdots 6, \quad k = 1, 2, \qquad (1)$$

where $No\_crash_{i,t,k}$ is the number of crashes with injury-severity degree k occurring in TAZ i during period t, $AAPT_{i,t}$ is the AAPT within TAZ i during period t, and L_i is the total length of the road segments within TAZ i. Among the zero observations, there were 51 (6.5%) slight injury cases and 322 (41%) KSI cases. The histograms of the slight injury and KSI crash rates are shown in Fig. 2 (a) and (b), respectively.

[Insert Fig. 2 near here]

Traffic volume is used as a risk factor for modeling crash rates by injury severity, along with travel speed, road density, intersection density, land-use pattern, and road-network pattern. The average travel speed of the taxis in each observation is estimated based on their real-time speed information from the GPS dataset. Average travel speed is a reasonable surrogate for the actual travel speed because the findings of Pei et al. (2009) indicate that these two variables are almost equivalent in Hong Kong traffic.

We categorize the land use of each selected TAZ into one of four patterns—residential, commercial, mixed (combined commercial and residential use), and other (e.g., government, institution, or community)—according to the dominant trip purpose (of which the percentage is over 50%) in the area (Guo et al. 2017). We set the residential area as the reference pattern.

In topology theory, global integration is a measure of the accessibility of each roadway in a network (Haq 2001). The global integration of a network is calculated as the mean of the measured accessibility of all roadways within the network. Guo et al. (2015, 2017) found considerable differences in the global integration values of the typical patterns (including grid, deformed grid, and irregular patterns) of Hong Kong's road network. Consequently, the global integration value of each TAZ obtained from space syntax is used to quantify its network pattern. Its definition and a more detailed description can be found in Guo et al. (2015, 2017).

We summarize the definitions and descriptive statistics of the response variables and risk factors for model estimation and comparison in Table 1. We conduct correlation tests and multi-collinearity diagnoses for the risk factors using IBM SPSS. The results indicate significant independence exists between the factors.

[Insert Table 1 near here]

3. Methodology

3.1. Model specification

3.1.1. Multivariate random-parameters Tobit model

Crash rates are continuous numbers that are left-censored at zero. Generally, there are many zero observations in crash rates at severe injury degrees (such as the over 40% observed zero KSI crash rates in the collected crash data). The multivariate Tobit model is the basic method for jointly modeling crash rates by injury severity because it can simultaneously account for the censoring characteristic and heterogeneous correlations across crash-injury severity (Anastasopoulos et al. 2012). To better capture the potential heterogeneous effects of observed factors across observations, Zeng et al. (2017b) advocated a multivariate random-parameters Tobit model. Its formulation is specified as follows[1]:

[1] Note that the model specification is specific to the collected crash data for simplicity. However, it can be easily extended to a more general structure (Zeng et al. 2017a).

$$Y_{i,t,k}^{*}=\beta_0^{i,t,k}+\sum_{m=1}^{8}\beta_m^{i,t,k}x_m^{i,t,k}+\varepsilon_{i,t,k}\,, \tag{1}$$

$$Y_{i,t,k}=\begin{cases}Y_{i,t,k}^{*}, & if\ Y_{i,t,k}^{*}>0\\ 0, & if\ Y_{i,t,k}^{*}\le 0\end{cases},\quad i=1,2,\cdots 131,\quad t=1,2,\cdots 6,\quad k=1,2\,, \tag{2}$$

where $Y_{i,t,k}^{*}$ is a latent variable used to link the observed crash rate $Y_{i,t,k}$ and the risk factors $x_1^{i,t,k},x_2^{i,t,k},\cdots,x_8^{i,t,k}$ for area i, period t, and injury severity level k. As indicated by Eq. (2), $Y_{i,t,k}^{*}$ can be observed only when it is positive. $\beta_0^{i,t,k},\beta_1^{i,t,k},\cdots,\beta_8^{i,t,k}$ are random parameters corresponding to the constant and the risk factors for severity outcome k, and are assumed to follow normal distributions:

$$\boldsymbol{\beta}_{k,m}\sim N(\bar{\beta}_{k,m},\theta_{k,m}^2),\ \boldsymbol{\beta}_{k,m}=\left(\beta_m^{1,1,k},\ \beta_m^{1,2,k},\ \cdots\ \beta_m^{131,6,k}\right),\ k=1,2,\ m=0,1,\cdots,8\,, \tag{3}$$

where $\bar{\beta}_{k,m}$ and $\theta_{k,m}(>0)$ are the mean and standard deviation, respectively, of the random parameters.

The error term $\varepsilon_{i,t,k}$ denotes the unstructured heterogeneous effect, which is assumed to follow a multi-normal distribution with zero mean, i.e.,

$$\boldsymbol{\varepsilon}_{i,t}\sim N_2(\mathbf{0},\boldsymbol{\Sigma}),\ \boldsymbol{\varepsilon}_{i,t}=\begin{pmatrix}\varepsilon_{i,t,1}\\ \varepsilon_{i,t,2}\end{pmatrix},\ \boldsymbol{\Sigma}=\begin{pmatrix}\delta_{1,1} & \delta_{1,2}\\ \delta_{2,1} & \delta_{2,2}\end{pmatrix}. \tag{4}$$

In the variance-covariance matrix $\boldsymbol{\Sigma}$, the diagonal element $\delta_{k,k}(k=1,2)$ is the variance of the error term $\varepsilon_{i,t,k}$, and the off-diagonal element $\delta_{1,2}(=\delta_{2,1})$ is the covariance between $\varepsilon_{i,t,1}$ and $\varepsilon_{i,t,2}$. To assess the magnitudes of the unstructured heterogeneous effects, the standard deviations are calculated as $\sigma_k=\sqrt{\delta_{k,k}}$. The correlation coefficient $\rho=\delta_{1,2}/(\sigma_1\sigma_2)$ describes the correlation between $\varepsilon_{i,t,1}$ and $\varepsilon_{i,t,2}$.

3.1.2. Multivariate random-parameters spatial Tobit model

The multivariate random-parameters Tobit model is capable of accommodating unstructured heterogeneous effects and their correlations; however, it ignores the spatial correlation among areal units derived from the shared effects of unobserved confounding factors, and the correlations of spatial effects among crash rates with different severity outcomes in the same area. To rectify this, a residual term, $\phi_{i,k}$, with an MCAR prior can be added to the link function (Aguero-Valverde 2013):

$$Y_{i,t,k}^{*} = \beta_0^{i,t,k} + \sum_{m=1}^{8} \beta_m^{i,t,k} x_m^{i,t,k} + \varepsilon_{i,t,k} + \phi_{i,k} . \tag{5}$$

Proximity structure is an important element for the specification of an MCAR prior (Barua, El-Basyouny, and Islam 2014). As it is the most prevalent structure, the 0-1 first-order neighbor is used in the present research. Specifically, if TAZs i and j share a common border, then the adjacent weight between them $\omega_{i,j} = 1$; otherwise, $\omega_{i,j} = 0$. Given the 0-1 first-order neighbor structure, the MCAR prior can be expressed as follows:

$$\mathbf{\Phi}_i \sim N_2(\bar{\mathbf{\Phi}}_i, \mathbf{\Omega}_s / n_i), \quad \mathbf{\Phi}_i = \begin{pmatrix} \phi_{i,1} \\ \phi_{i,2} \end{pmatrix}, \quad \bar{\mathbf{\Phi}}_i = \begin{pmatrix} \bar{\phi}_{i,1} \\ \bar{\phi}_{i,2} \end{pmatrix}, \quad \mathbf{\Omega}_s = \begin{pmatrix} \delta_{1,1}^{s} & \delta_{1,2}^{s} \\ \delta_{2,1}^{s} & \delta_{2,2}^{s} \end{pmatrix}, \tag{6}$$

where $n_i = \sum_{i \neq j} \omega_{i,j}$ is the number of TAZs that are adjacent to TAZ i, $\bar{\phi}_{i,k} = \sum_{i \neq j} \phi_{j,k} \omega_{i,j} / n_i$, $\mathbf{\Omega}_s$ is the variance-covariance matrix for spatial correlation, where $\delta_{1,1}^{s}$ and $\delta_{2,2}^{s}$ reflect the spatial variances of slight injury and KSI crash-rates, respectively, and $\delta_{1,2}^{s} (= \delta_{2,1}^{s})$ reflects the spatial covariance between them. To assess the magnitudes of the spatial effects, the standard deviations are calculated as

$\sigma_k^s = \sqrt{\delta_{k,k}^s}$. To measure the correlation between the spatial effects, the correlation coefficient is calculated as $\rho_s = \delta_{1,2}^s / (\sigma_1^s \sigma_2^s)$.

3.1.3. Multivariate random-parameters spatio-temporal Tobit model

When multiple observations are made for each area over the period, temporal correlations may exist because of unobserved or unobservable time-dependent factors and factors with time-dependent safety effects that are not explicitly specified in the model (Cheng et al. 2018c; Huang, Chin, and Haque 2009; Zeng, Sun, and Wen 2017). The interactive effects of the independent spatial and temporal correlations should also be taken into account, because there may be variations in the spatial or temporal effects across the time or spatial units (Dong et al. 2016b). The multivariate random-parameters spatio-temporal Tobit model is developed by incorporating a linear time trend and interaction term into the link function of the multivariate random-parameters spatial Tobit model (Cheng et al. 2018b), which can be expressed as follows:

$$Y_{i,t,k}^* = \beta_0^{i,t,k} + \sum_{m=1}^{8} \beta_m^{i,t,k} x_m^{i,t,k} + \varepsilon_{i,t,k} + \phi_{i,k} + \tau_t(\alpha_k + \varphi_{i,k}) . \quad (7)$$

where τ_t represents the period t and α_k is the severity-specific scalar parameter for the linear time trend over all TAZs. $\varphi_{i,k}$ is the spatial component of the spatio-temporal interaction and is also specified via an MCAR prior:

$$\mathbf{\Psi}_i \sim N_2(\bar{\mathbf{\Psi}}_i, \mathbf{\Omega}_a / n_i), \quad \mathbf{\Psi}_i = \begin{pmatrix} \varphi_{i,1} \\ \varphi_{i,2} \end{pmatrix}, \quad \bar{\mathbf{\Psi}}_i = \begin{pmatrix} \bar{\varphi}_{i,1} \\ \bar{\varphi}_{i,2} \end{pmatrix}, \quad \mathbf{\Omega}_\text{a} = \begin{pmatrix} \delta_{1,1}^a & \delta_{1,2}^a \\ \delta_{2,1}^a & \delta_{2,2}^a \end{pmatrix}, \quad (8)$$

where $\bar{\varphi}_{i,k} = \sum_{i \neq j} \varphi_{k,j} \omega_{i,j} / n_i$. As the specification of the spatial correlation, $\sigma_k^a = \sqrt{\delta_{k,k}^a}$ is calculated to measure the magnitude of the spatio-temporal interactive effect for the crash rate at severity level $k(=1,2)$, and $\rho_a = \delta_{1,2}^a / (\sigma_1^a \sigma_2^a)$ is computed to measure the correlation in the spatio-temporal interaction.

3.2. Assessment criteria

The three most prevalent criteria in the context of Bayesian inference, the deviance information criterion (DIC), mean absolute deviance (MAD), and mean squared prediction error (MSPE), are used to assess the goodness of fit of the above models. DIC is a Bayesian generalization of Akaike's information criterion that penalizes larger-parameter models. It thus provides a combined measure of model complexity and fitting (Spiegelhalter et al. 2002). The DIC is calculated as

$$DIC = \bar{D} + pD, \tag{9}$$

where $\bar{D}$ is the posterior mean deviance, which can be used as a measure of the fitness or adequacy of models, and pD is the effective number of parameters that can be used to measure model complexity. In general, models with lower DIC values are preferable, and models with over 10 differences can be ruled out because of high DIC (Spiegelhalter et al. 2005).

To evaluate the goodness of fit of the models in terms of the crash rates at each crash severity level, the severity-specific MAD and MSPE are calculated as follows (Zeng and Huang 2014):

$$MAD_k = \frac{1}{6\times 131}\sum_{t=1}^{6}\sum_{i=1}^{131}\left|Y_{i,t,k} - \lambda_{i,t,k}\right|, \tag{10}$$

$$MSPE_k = \frac{1}{6\times 131}\sum_{t=1}^{6}\sum_{i=1}^{131}(Y_{i,t,k} - \lambda_{i,t,k})^2, \tag{11}$$

$$\lambda_{i,t,k} = \begin{cases} Y^*_{i,t,k}, & if\ Y^*_{i,t,k} > 0 \\ 0, & if\ Y^*_{i,t,k} \le 0 \end{cases}, \tag{12}$$

where $\lambda_{i,t,k}$ represents the expected crash rate for area i, period t, and injury severity level k.

3.3. Model estimation

Due to the complexity of the multivariate models, the parameters and hyper-parameters are estimated using Bayesian methods available in WinBUGS (Lunn et al. 2000). Bayesian inference, which does not require easily calculable likelihood functions, is conducted using well-established Markov chain Monte Carlo (MCMC) techniques. The Gibbs sampler and the Metropolis-Hastings algorithm are the most widely used MCMC simulation algorithms.

A specification of the prior distributions of the (hyper-) parameters is required to obtain the Bayesian estimates. The prior distributions reflect prior knowledge about the (hyper-) parameters; in the absence of sufficient knowledge, non-informative (vague) prior distributions are usually specified (Zeng and Huang 2014). Specifically, a diffused normal distribution $N(0,10^4)$ is used as the prior of the means of the random parameters $\bar{\beta}_{k,m}$ $(k=1,2; m=0,1,\cdots,8)$ and the scalar parameters α_k $(k=1,2)$. A diffused gamma distribution $gamma(0.001, 0.001)$ is used as the prior of the precision of the random parameters, $1/\theta^2_{k,m}$ $(k=1,2; m=0,1,\cdots,8)$ (Zeng et al. 2017a). A Wishart prior, $W(\mathbf{P}, r)$, is used for $\boldsymbol{\Sigma}^{-1}$, $\boldsymbol{\Omega}_s^{-1}$, and $\boldsymbol{\Omega}_a^{-1}$, where $\mathbf{P} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ represents the scale matrix and $r = 2$ is the degrees of freedom (Barua, El-Basyouny, and Islam 2014). For each model, a chain of 200,000 iterations of the MCMC simulation is constructed, and the first 150,000 iterations are excluded as a burn-in. The ratios of the Monte Carlo errors are monitored relative to the standard deviations of the estimates and the MCMC trace plots for the model parameters are visually inspected to assess the MCMC convergence. Note that a random parameter is simplified as fixed across observations when the Bayesian estimate of its variance is insignificant at the 95% level (Zeng et al. 2017b).

4. Results analysis

4.1. Model comparison

Table 2 shows the Bayesian goodness-of-fit measures and hyper-parameter estimates for the models. Comparing the multivariate random-parameters spatial Tobit model with the multivariate random-parameters Tobit model, we can see that the former fits the crash dataset much better than the latter, as the spatial model yields a significantly lower DIC (3,340 for the spatial model versus 3,925 for the aspatial model) and lower MAD and MSPE for both the slight injury and KSI crash rates. These results are reasonable, because the magnitudes of the spatial effects for both the slight injury and KSI crash rates are very large ($\sigma_1^s = 50.4$ and $\sigma_2^s = 5.24$). Previous findings have shown that the inclusion of spatial effects can greatly improve model-fit (Wang and

Kockelman 2013; Zeng et al. 2019). Moreover, the correlation between the spatial effects for the two crash severities is very high ($\rho_s = 0.99$), which shows the necessity of using the MCAR prior for multivariate spatial modeling. By specifying the dependency in spatial effects, the multivariate spatial model benefits from borrowing strength across crash severities, thereby reducing model misspecification (Aguero-Valverde 2013; Li et al., 2019). The strong correlation between the spatial effects may be attributed to missing factors such as terrain features and various socio-economic attributes, which are spatially clustered (although widely spread) but temporally stationary, and which are shared by the crash-severity levels (Wang and Kockelman 2013). After accounting for spatial correlation, the magnitudes of the unstructured heterogeneous effects decrease by 21% (from 0.68 to 0.54) and 13% (from 0.38 to 0.33) for the slight injury and KSI crash rates, respectively. Barua, El-Basyouny, and Islam (2016) noted that a proportion of the unstructured heterogeneous effects found in multivariate models may be derived from the effects of unobserved factors shared by adjacent areas; that is, spatial correlation.

[Insert Table 2 near here]

The results for the multivariate spatio-temporal model indicate that the scalar parameters of the linear temporal trends for the crash rates at the two severity levels are both statistically significant at the 95% level, suggesting a non-negligible temporal correlation in the crash dataset. The estimates of the scalar parameters imply that the slight injury crash rates increase but the KSI crash rates decrease across the divided periods. The different change directions need more scrutiny in further research. The significant spatial-temporal interaction can also be seen in the Bayesian estimates of σ_1^a and σ_2^a with their 95% Bayesian credible intervals away from zero. ρ_a is expected to be 0.99, which indicates a strong correlation between the crash severities in the spatio-temporal interaction. It accounts for the safety effects of missing factors that are spatially clustered and temporally dependent and are shared across crash severities. Examples of such missing factors include weather and lighting conditions. Incorporating temporal correlation and spatio-temporal interaction into the multivariate random-parameters spatial Tobit model considerably improves the goodness of fit, as indicated by the lower DIC (= 3,291) of the spatio-temporal model compared with the spatial model. Moreover, the spatio-temporal model yields lower MAD and MSPE for KSI crash-rates and equivalent counterparts for slight injury crash rates.

It is interesting that the correlation in the unstructured heterogeneous effects is statistically insignificant in all three of the models, perhaps because the correlations among the crash severities are mainly in their spatial effects and spatio-temporal interactive effects (Barua, El-Basyouny, and Islam 2014; Fountas et al., 2019). Notably, the magnitudes of the unstructured heterogeneous effects are significant, which indicates that there are unobserved factors with significant safety effects that are not spatially/temporally correlated and that do not interact in the crash data.

4.2. Parameter interpretation

To comprehensively justify the model validity, estimates of the risk factors' coefficients are interpreted. Because of the considerable outperformance of the multivariate spatio-temporal interaction model, for brevity only the model parameters that are significant at least at the 90% level, as summarized in Table 3, are discussed in this section. The results indicate that intersection density has heterogeneous effects on both the slight injury and KSI crash rates, while commercial land use has heterogeneous effects on KSI crash rates only. Traffic volume and average speed are also found to have significant (fixed) effects on KSI crash rates only. With respect to the other significant factors, despite the consistent plus and minus signs, there are obvious discrepancies in the magnitudes of the coefficients for the two crash severities. These findings show the necessity of modeling crash rates at different severity levels to quantify the distinctive effects of the contributing factors at each level.

[Insert Table 3 near here]

The effects of intersection density on the slight-injury and KSI crash-rates follow two normal distributions, with means of 6.82 and 1.04 and standard deviations of 2.50 and 0.76, respectively. The results indicate that more slight-injury and KSI crashes can be expected to occur on the majority (99.6% and 91.3%) of TAZs with more intersections per kilometer of roadway. These findings are consistent with those in the literature, and with engineering intuition: because of complicated maneuvers that take place at intersections, they are among the most hazardous locations in a roadway network (Guo, Wang, and Abdel-Aty 2010). In addition, frequent signal changes may lead to more rear-end crashes occurring at signalized intersections (Wang and Abdel-Aty 2006). Nevertheless, a higher intersection density results in lower slight injury and KSI crash rates for a minority (0.4% and 8.7%) of TAZs.

Regarding land use, the *Commercial* variable results in a normally distributed parameter for KSI crash rates. Given the estimated -3.29 mean and 5.42 standard deviation of the parameter, the KSI crash rates are lower in 69.3% of commercial areas but higher in the other 30.7%, relative to residential areas. The estimates of the coefficients for slight injury crash-rates are consistent with the general findings for KSI crash rates: commercial areas are safer than residential areas. The fewer crash occurrences in Hong Kong's commercial areas may be partially attributable to the presence of additional pedestrian overpasses (as displayed in Fig. 3) (Guo et al. 2017), which separates motor vehicles and street-crossing pedestrians. Moreover, there are many ground-floor shops along the streets in Hong Kong's residential areas, as shown in Fig. 4. These shops attract large numbers of surrounding residents, which may lead to an increase in pedestrian-related crashes. The negative coefficients of *other* areas for both the slight injury and KSI crash rates imply that the safety performance of other areas (e.g., government, institution, or community) is better than in residential areas. These results are in line with our previous findings in Zeng et al. (2018b).

[Insert Fig. 3 near here]

[Insert Fig. 4 near here]

The results show that KSI crash rates significantly decrease with increasing traffic volume, which is generally consistent with the findings in many previous studies on roadway segment-level crash rate analysis (Anastasopoulos et al. 2012; Zeng et al. 2017a, 2017b, 2018a). According to the cognitive or behavioral compensatory theory (Hockey 1997), drivers tend to invest mental effort and prioritize the driving task when driving in heavy traffic, by stopping secondary tasks (e.g., conversing with passengers, listening to music), thus making fewer mistakes leading to crashes.

It is interesting that lower KSI crash rates are significantly related to higher average speeds. This finding may be somewhat counterintuitive; however, several microstudies on Hong Kong traffic safety (Huang et al. 2016b; Zeng et al. 2016, 2017a, 2018a) have revealed that roadways designed for higher speeds are usually well planned, constructed, managed, and maintained, and they thus have improved safety performance.

The negative signs of the *Integration* variable mean that lower slight injury and KSI crash rates are expected in more accessible TAZs (i.e., those with higher global integration). These results are concordant with the findings of Guo et al. (2015), who argued that a TAZ with high global integration tends to be a grid network in which the visual field of drivers is greater than that in networks of other (e.g., deformed grid and irregular) patterns. Therefore, drivers have more time to perceive the roadway environment and react to avoid a crash.

We also find that crash rates at both severity degrees are negatively associated with roadway density. This finding may also be attributable to the better accessibility of areas with higher road density (Guo et al. 2015). Additionally, dense roadways usually appear in areas with high travel-demand (e.g., central business districts). The built environments of these areas, in which high standards of road-infrastructure configuration and maintenance have the potential to decrease crash risk, are usually favorable to drivers.

5. Conclusions

This study investigates multivariate spatio-temporal analysis for classifying area-wide crash rates by injury severity. A multivariate random-parameters spatio-temporal Tobit model is specified within a Bayesian paradigm that can capture independent spatial and temporal effects and their interactive effects, the correlation across crash severities in these structured random effects, and unobserved heterogeneity. A one-year dataset (including crash, traffic, road facility, and land-use information on 131 TAZs in Hong Kong) is adopted to demonstrate the model, which is calibrated and assessed by Bayesian methods via the WinBUGS program.

The independent spatial effects (specified via an MCAR prior), temporal effects (specified by a linear time trend), and spatio-temporal interactive effects (formulated as the product of a time trend and a spatial term with the MCAR prior) are all statistically significant at the 95% level. The values of DIC, MAD, and MSPE show that the proposed model has substantially better goodness of fit than other possible models, including a multivariate random-parameters Tobit model and a multivariate random-parameters spatial Tobit model. The results confirm the importance of simultaneously accounting for spatio-temporal correlation and interaction under a multivariate modeling framework. The parameter estimates show that intersection density has heterogeneous effects on crash rates at both degrees of severity and that commercial land use has heterogeneous effects on KSI crash rates only. The results align with previous findings, further supporting the validity of the model.

In summary, the present empirical analysis demonstrates the superiority of the multivariate random-parameters spatio-temporal Tobit model and the significance of spatio-temporal correlation and interaction and unobserved heterogeneity in area-wide crash data. This indicates that the proposed model may be suitable for use by planners and engineers implementing proactive road-safety planning. Notably, while the proposed model can be applied to any number (≥2) of dependent variables, only two levels of injury severity are present in our collected dataset, resulting in the bivariate models seen in the empirical analysis. Field data with more (≥3) severity levels could be used to further assess the proposed model's performance. As Aguero-Valverde and Jovanis (2006) noted, despite its applicability and popularity, the linear time-trend may be a restrictive assumption. Exploring a flexible structure may provide a deeper insight into the temporal correlation. Future research could explore other types of spatio-temporal interaction formulations (Cheng et al. 2018b), although this could lead to a more complex model structure. Finally, from a transportation-planning perspective, investigating the effects of additional socioeconomic (e.g., employment), demographic (e.g., age cohorts), and environmental factors (e.g., rainfall and light level) on zonal crash rates by severity is fully warranted.

Acknowledgments

This study was jointly supported by 1) the Joint Research Scheme of National Natural Science Foundation of China/Research Grants Council of Hong Kong under Grant Nos. 71561167001 and N_HKU707/15; 2) the Natural Science Foundation of China under Grant Nos. 51578247, 713711921, 71671100, and 71801095; 3) the Natural Science Foundation of Guangdong Province under Grant No. 2017A030310161; 4) the Research Fund for the Fok Ying Tong Education Foundation of Hong Kong under Grant No. 142005; and 5) Fundamental Research Funds for the Central Universities under Grant Nos. 2017BQ026 and 2016zzts050.

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Table 1. Descriptive statistics of the variables.

Variable	Description	Mean	S.D.	Min.	Max.	Percent
Response variables						
Slight	Slightly injured crash count per million vehicle-kilometers traveled	7.63	25.7	0	415	
KSI	Killed and seriously injured crash count per million vehicle-kilometers traveled	1.16	4.50	0	82.9	
Risk factors						
AAPT	Annual average 4-h period traffic (10^3 veh)	5.98	10.5	0.02	135	
Speed	Estimated average speed (km/h)	26.8	12.6	7.76	78.0	
Integration	Global integration	1.15	0.43	0.48	3.62	
Road_dens	Roadway length (km) per km^2	50.1	15.0	21.2	109	
Inter_dens	Number of intersections per km	3.56	1.47	1.21	11.2	
Land use	Residential (reference)					58.7
	Commercial					19.1
	Mixed					13.0
	Other					9.2

Table 2. Model comparison results.

	Multivariate random-parameters Tobit model	Multivariate random-parameters spatial Tobit model	Multivariate random-parameters spatio-temporal Tobit model
DIC	3,925	3,340	3,291
MAD_1	0.51	0.41	0.40
MAD_2	0.23	0.20	0.20
$MSPE_1$	0.48	0.30	0.28
$MSPE_2$	0.11	0.08	0.08
σ_1	**0.68 (0.33, 1.31)**[a]	**0.54 (0.31, 0.92)**	**0.53 (0.31, 0.85)**
σ_2	**0.38 (0.26, 0.53)**	**0.33 (0.23, 0.44)**	**0.33 (0.23, 0.44)**
ρ	0.20 (-0.42, 0.72)	0.03 (-0.46, 0.52)	0.03 (-0.48, 0.51)
σ_1^s	—	**50.4 (43.8, 57.9)**	**90.3 (78.6, 103)**
σ_2^s	—	**5.24 (4.27, 6.27)**	**13.6 (11.5, 16.0)**
ρ_s	—	**0.99 (0.98, 1.00)**	**0.99 (0.98, 1.00)**
α_1	—	—	**4.49 (3.58, 5.27)**
α_2	—	—	**-0.56 (-1.12, -0.18)**
σ_1^a	—	—	**11.5 (9.42, 13.5)**
σ_2^a	—	—	**2.60 (2.13, 3.17)**
ρ_a	—	—	**0.99 (0.98, 1.00)**

[a] Estimated mean (95% Bayesian credible interval) for the parameter. Boldface indicates statistical significance at 95% level.

Table 3. Parameter estimates in the multivariate random-parameters spatio-temporal Tobit model[a].

Variable	Slight injury			Killed and serious injury		
	Mean	95% BCI[b]	90% BCI	Mean	95% BCI	90% BCI
Constant	59.2	**(54.5, 67.1)**[c]	**(54.8, 66.8)**	-8.13	**(-11.7, -4.84)**	**(-11.5, -4.97)**
AAPT	-0.10	(-0.21, 0.04)	(-0.20, 0.01)	-0.04	**(-0.08, -0.01)**	**(-0.07, -0.02)**
Speed	-0.05	(-0.12, 0.01)	(-0.11, +0.00)	-0.02	**(-0.04, -0.003)**	**(-0.03, -0.01)**
Integration	-4.94	**(-7.87, -1.62)**	**(-7.70, -2.78)**	-0.79	(-1.54, 0.13)	**(-1.46, -0.13)**
Road_dens	-0.30	**(-0.37, -0.24)**	**(-0.36, -0.25)**	-0.03	**(-0.05, -0.01)**	**(-0.04, -0.02)**
Inter_dens	6.82	**(4.92, 8.00)**	**(5.04, 7.92)**	1.04	**(0.60, 1.32)**	**(0.65, 1.29)**
S.D. of Inter_dens	2.50	**(2.34, 2.66)**	**(2.37, 2.64)**	0.76	**(0.71, 0.81)**	**(0.72, 0.80)**
Commercial	-26.5	**(-38.0, -17.4)**	**(-37.3, -18.4)**	-3.29	**(-5.62, -1.28)**	**(-5.31, -1.52)**
S.D. of Commercial	—	—	—	5.45	**(4.66, 6.33)**	**(4.78, 6.18)**
Other	-17.4	**(-22.9, -11.7)**	**(-21.8, -12.2)**	-2.56	**(-3.59, -1.61)**	**(-3.42, -1.74)**

[a] *Mixed land-use* is excluded as none of their effects on crash rates at the two severity degrees is significant at 90% level.

[b] Bayesian credible interval.

[c] Boldface indicates statistical significance at the corresponding level.

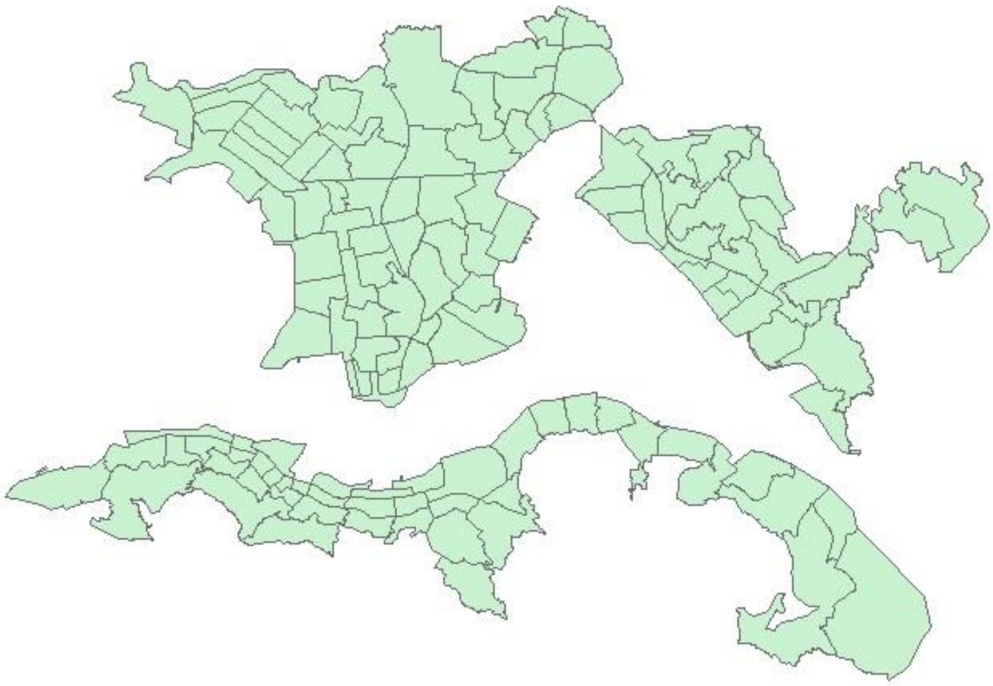
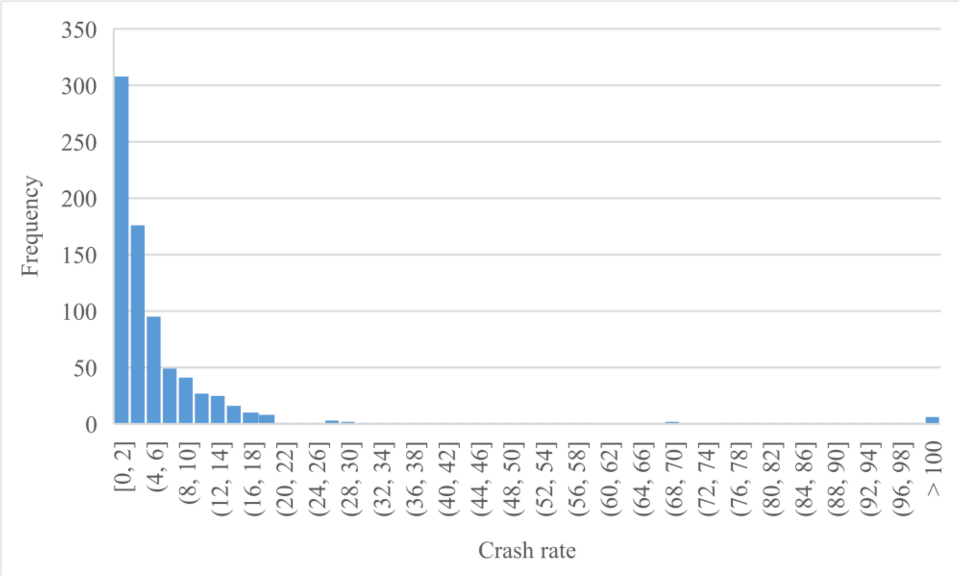
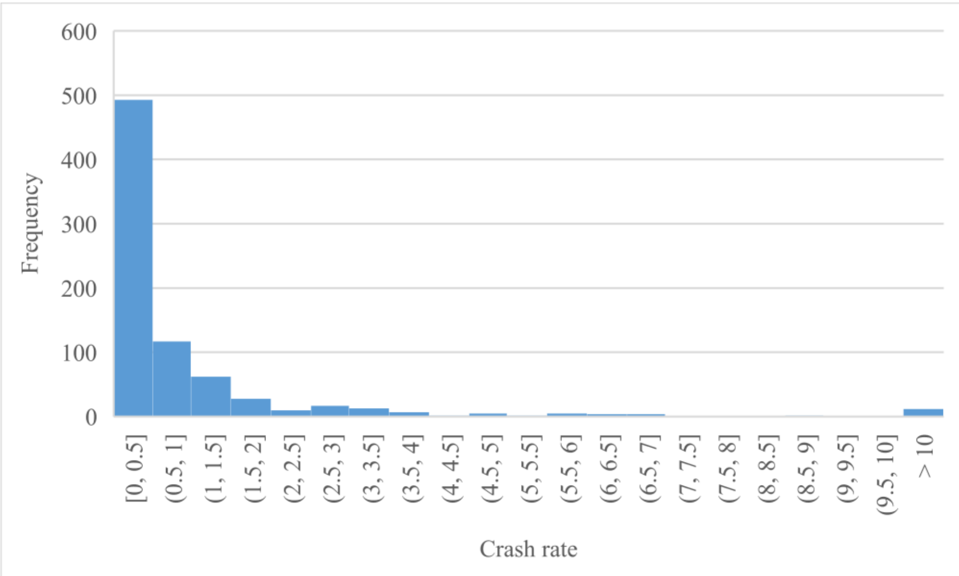


Fig. 1. Selected traffic analysis zones in Hong Kong.



(a) Slight injury crash rate



(b) KSI crash rate

Fig. 2. Histograms of the slight injury and KSI crash rates.

Fig. 3. A typical scene in a commercial area in Hong Kong

Fig. 4. A typical scene in a residential area in Hong Kong