Virial expansion for a three-component Fermi gas in one dimension: The quantum anomaly correspondence

Jeff Maki[1] and Carlos R. Ordóñez[2,3]

[1]*Department of Physics and Center of Theoretical and Computational Physics, University of Hong Kong, Hong Kong, China*

[2]*Physics Department, University of Houston, Houston, Texas 77024-5005, USA*

[3]*ICAB, Universidad de Panama, Panama, Republica de Panama*

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In this paper we explore the transport properties of three-component Fermi gases confined to one spatial dimension, interacting via a three-body interaction, in the high temperature limit. At the classical level, the three-body interaction is scale invariant in one dimension. However, upon quantization, an anomaly appears which breaks the scale invariance. This is very similar to the physics of two-component fermions in two spatial dimensions, where the two-body interaction is also anomalous. Previous studies have already hinted that the physics of these two systems are intimately related. Here we expand upon those studies by examining the thermodynamic properties of this anomalous one-dimensional system in the high temperature limit. We show there is an exact mapping between the traditional two-body anomalous interaction in two dimensions, to that of three-body interaction in one dimension. This result is valid in the high temperature limit, where the thermodynamics can be understood in terms of few-body correlations.

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I. INTRODUCTION

Symmetry is an important tool in understanding any physical system. For this reason, it is not surprising that when a classical symmetry is unexpectedly broken upon quantization, a phenomenon known as a quantum anomaly, it can create quite a stir among physicists [1]. In cold atom experiments, one such anomaly to be predicted and observed was the scale anomaly in two-dimensional Fermi gases [2–4].

The two-dimensional Fermi gas with short-ranged two-body interactions (henceforth simply called the anomalous two-dimensional Fermi gas) is classically scale invariant [5]. If this symmetry were present under quantization, it would drastically reduce the complexity of the energetics and dynamics [5–9]. However, this is not truly the case here upon quantization, as a new energy scale will enter the problem. For the case of attractive interactions, this new energy scale is simply the two-body bound state [2,10]. The presence of this new energy scale explicitly breaks the scale symmetry of the classical model. In this case, the breaking of scale invariance is logarithmically weak, and there have been numerous theoretical and experimental studies examining to what extent scale symmetry and the quantum anomaly are present in the physics of two dimensional quantum gases [2–4,6–8,11,12].

Recently, a number of additional anomalous systems have been identified: bosons with three-body interactions in one spatial dimension [13], three-component fermions with three-body interactions in one spatial dimension [14,15], and the one-dimensional quantum gas with a derivative coupling [16]. These systems, which we will simply call anomalous one-dimensional quantum gases, are classically scale invariant, but upon quantization, a bound state appears. For the case of a one-dimensional quantum gas with three-body interactions, previous studies have shown that the coupling constant varies logarithmically with the bound state energy—just as the two-dimensional quantum anomaly. This result has been recently used to study a number of thermodynamic and dynamic properties of these one-dimensional anomalous quantum gases [13,14,17–20]. One particular facet of these systems was noted in Ref. [14], namely the logarithmic breaking of scale invariance led to a mapping between the physics of two-dimensional fermions and that of these anomalous three-component fermions in one dimension, which we call the anomaly correspondence. In particular it was shown that the third virial coefficient δb_3 for the anomalous one-dimensional Fermi gas is directly related to its two-dimensional counterpart.

Our goal is to explicitly test this analogy by computing the thermodynamic and transport properties of these three-component fermions in the large temperature limit. First, it is necessary to check whether the thermodynamics of the system obey the anomaly correspondence. To check this we focus on the virial coefficients and Tan's contact [21], which have been shown to be related to the two-dimensional anomalous Fermi gas [14]. Once the thermodynamic properties have been examined, we proceed to calculate the bulk viscosity.

Fundamentally speaking, scale-invariant systems in the normal phase have a vanishing bulk viscosity [22,23]. For this reason the bulk viscosity is an important quantity in understanding the breaking of scale invariance, whether it be explicit or anomalous. Although one is often concerned with the static bulk viscosity, it is useful to consider the spectral function of the bulk viscosity, $\zeta(\omega)$. This quantity has been calculated in the high temperature limit for fermions with two-body interactions in a variety of spatial dimensions [24–28].

In order to calculate the thermodynamic properties of the one-dimensional anomalous Fermi gas, we perform the virial expansion to third order in the fugacity z, following the arguments presented in Ref. [26]. We explicitly calculate the virial coefficients, Tan's contact, and bulk viscosity for the one-dimensional anomalous fermions, and show that they are indeed proportional to their two-dimensional counterparts. This allows us to explicitly verify this mapping between anomalous systems, which was previously based on scaling arguments [14], and to construct a dictionary for the anomaly correspondence.

The remainder of the article is organized as follows: In Sec. II we review the few-body physics of both the two-dimensional and one-dimensional anomalous fermions. We then apply this approach to calculate the shift in the third virial coefficient and Tan's contact in Sec. III. In Sec. IV we then compute the bulk viscosity. We then conclude in Sec. V.

II. REVIEW OF THE FEW-BODY PHYSICS

We begin by reviewing the few-body physics of the three-component anomalous fermions, and how it relates to the standard anomalous paradigm in two spatial dimensions. The Hamiltonian for the anomalous three-component fermions is

$$\begin{aligned} H = &\sum_{\sigma=1}^{3}\sum_{k}\frac{k^2}{2}\psi_\sigma^\dagger(k)\psi_\sigma(k) \\ &+\frac{g}{L^2}\sum_{k_i,l_i}\psi_1^\dagger(k_1)\psi_2^\dagger(k_2)\psi_3^\dagger(k_3)\psi_3(l_3)\psi_2(l_2)\psi_1(l_1), \\ &\delta_{k_1+k_2+k_3,l_1+l_2+l_3}, \end{aligned} \tag{1}$$

where we have set $\hbar$ and m to be unity. The operator $\psi_\sigma(k)$ is the field operator that annihilates a fermion with spin, $\sigma = 1, 2, 3$, and momenta k, while L is the length of the system, and the sum is over all six momenta.

Naively one would expect that g is dimensionless, however, this model is ultraviolet (UV) divergent, and depends on a short distance cutoff Λ. The act of removing this length scale from the problem, will produce the quantum anomaly. To understand this UV divergence, consider the three-body scattering amplitude in the presence of the vacuum $T_3(Q, Q_0)$, where the scattering amplitude is a function of the center-of-mass momentum Q and energy Q_0. In this case the three-body scattering amplitude can be found as the summation of the diagrams shown in Fig. 1. The result is

$$\begin{aligned} T_3^{-1}(Q, Q_0) = &\frac{1}{g} - \frac{2}{\sqrt{3}}\int_{-\infty}^{\infty}\frac{dp}{2\pi} \\ &\times\int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{1}{Q_0 - Q^2/6 - p^2 - q^2 + i\delta}. \end{aligned} \tag{2}$$

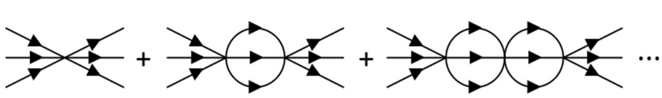

FIG. 1. Feynman diagrams that lead to the three-body scattering amplitude $T_3(Q, Q_0)$. Each line corresponds to a free fermionic propagator, while each vertex is a three-body interaction g.

In order to obtain Eq. (2), we have performed the following coordinate transformation to the following coordinates:

$$k_1 = \frac{Q}{3} - p + \frac{q}{\sqrt{3}} \quad k_2 = \frac{Q}{3} + p + \frac{q}{\sqrt{3}} \quad k_3 = \frac{Q}{3} - \frac{2q}{\sqrt{3}}. \tag{3}$$

This transformation has a nontrivial Jacobian, which gives the factor of $2/\sqrt{3}$.

As one can see $T_3(Q, Q_0) = T_3(0, Q_0 - Q^2/6)$, which is required by Galilean invariance. Therefore, we define the parameter $\epsilon = Q_0 - Q^2/6$, the energy of the relative motion of the three particles, and $T_3(Q, Q_0)$ will only be a function of ϵ. Upon performing the integration over the intermediate momenta, one obtains

$$T_3^{-1}(\epsilon + i\delta) = \frac{1}{g} + \frac{\ln\left(\frac{\Lambda^2}{-\epsilon - i\delta}\right)}{2\pi\sqrt{3}}. \tag{4}$$

One can remove the UV dependence we renormalize g via the the Landau pole, $E = -E_B$, defined as the pole of the three-body scattering amplitude at zero center-of-mass momentum. This identification leads to the following expression for the coupling constant:

$$g = -\frac{2\pi\sqrt{3}}{\ln\left(\frac{\Lambda^2}{E_B}\right)}. \tag{5}$$

Equation (5) states the coupling constant is no longer a constant but a function of the new energy scale, the Landau pole, $-E_B$. For attractive interactions, the Landau pole is at low energies, and corresponds to the bound state energy, while for repulsive interactions, the Landau pole is at high energies, and as a result, has no physical significance.

It is important to note that in order to obtain this Hamiltonian physically, it is necessary to set the two-body interaction to be zero. If the two-body interaction is vanishingly small yet attractive, the resulting three-body potential is also attractive. In this case there can be nonuniversal bound states that can arise, and this effective description breaks down [19]. In this work we work with the universal aspects of the given effective field theory, and neglect the contributions from any nonuniversal bound state.

Using Eq. (5), we can eliminate the UV divergence, and express the T matrix in terms of physical quantities:

$$T_3(\epsilon + i\delta) = \frac{2\pi\sqrt{3}}{\ln\left(\frac{E_B}{-\epsilon - i\delta}\right)}. \tag{6}$$

This should be compared to the two-body scattering amplitude:

$$T_2(\epsilon + i\delta) = \frac{4\pi}{\ln\left(\frac{E_B}{-\epsilon - i\delta}\right)}, \tag{7}$$

and the two-body coupling constant:

$$g_{2D} = -\frac{4\pi}{\ln\left(\frac{\Lambda^2}{E_B}\right)}. \tag{8}$$

Assuming that the scattering properties of the two models can be matched, one can easily see

$$g = \frac{\sqrt{3}}{2}g_{2D}. \tag{9}$$

In the following sections, we will exploit this fact to show that the connection between these two anomalous systems runs deeper, leading to a mapping between thermodynamic quantities in the high temperature limit.

III. CALCULATION OF THE PRESSURE AND TAN'S CONTACT

In order to study the thermodynamic and transport properties of this anomalous system, we employ the virial expansion. For full details we refer the reader to Ref. [26]. The main idea is to split the partition function into N-body sectors:

$$Z = \sum_{N=0}^{\infty} z^N \mathrm{Tr}_N[e^{-\beta H}], \tag{10}$$

where $z = e^{\beta\mu}$ is the fugacity, $\beta = 1/T$, μ is the chemical potential, and Tr_N denotes the trace over the N-body sector of the Hilbert space.

In the high temperature limit, $z \ll 1$. This allows one to expand the partition function in terms of the fugacity, and to consider only a small number of few-body contributions to the partition function. For our purposes, we will work up to $O(z^3)$, or equivalently to $N = 3$, as this is the first nontrivial order where interaction effects appear.

To evaluate the N-body partition function we need the matrix elements of the evolution operator $e^{-\beta H}$. In general this is an impossible task. However, at the few-body level, we can obtain an analytic result by employing the following identity:

$$e^{-\beta H} = \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-\beta E} \mathrm{Im}\left[\frac{1}{E - H - i\delta}\right]. \tag{11}$$

At first and second order in the fugacity, the Hamiltonian is simply the noninteracting Hamiltonian H_0. At $O(z^3)$ we will need to include the effect of interactions. The exact propagator at the three-body level can be evaluated [26], and is related to the three-body scattering amplitude defined in Eq. (6):

$$e^{-\beta H} = \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-\beta E} \mathrm{Im}\left[\frac{1}{E - H_0 - i\delta} + \frac{1}{E - H_0 - i\delta} T_3 \frac{1}{E - H_0 - i\delta}\right], \tag{12}$$

where T_3 is the scattering amplitude operator which has matrix elements that only depend on the center-of-mass energy and momentum: $T_3(E - Q^2/6 - i\delta)$.

The partition function, and subsequently the pressure, is evaluated by tracing Eqs. (11) and (12) over all three particle states; see Appendix A for explicit expressions. From there, one can use the relationship between the partition function and the pressure:

$$\beta PL = \ln(Z), \tag{13}$$

to obtain the following expression for the pressure:

$$P = \frac{3T}{\lambda_T}\left[z - \frac{1}{2\sqrt{2}} z^2 + \frac{z^3}{3\sqrt{3}} + \frac{z^3}{6\pi} \int_{-\infty}^{\infty} d\epsilon\, e^{-\beta\epsilon} \mathrm{Im}\left[\frac{T_3(\epsilon - i\delta)}{-\epsilon + i\delta}\right]\right]. \tag{14}$$

The first three terms of Eq. (14) are the noninteracting contributions to the pressure, while the last term is due to the three-body interactions.

We now define the virial coefficients b_n via

$$P = \frac{\nu T}{\lambda_T^d} \sum_n z^n b_n. \tag{15}$$

Here ν is the spin degeneracy, which is 3 for the one-dimensional case and 2 for the two-dimensional case, and d is the dimension. With Eq. (15), we can identify the last term as the shift in the virial coefficient:

$$\delta b_3 = \frac{1}{6\pi} \int_{-\infty}^{\infty} d\epsilon\, e^{-\beta\epsilon} \mathrm{Im}\left[\frac{T_3(\epsilon - i\delta)}{-\epsilon + i\delta}\right]. \tag{16}$$

With our explicit expression for $T_3(\epsilon - i\delta)$, one can show that the imaginary part of $T_3(\epsilon - i\delta)$ has the form,

$$\mathrm{Im}[T_3(\epsilon - i\delta)] = 2\pi\sqrt{3} E_B \pi \delta(\epsilon + E_B) + \frac{2\pi^2\sqrt{3}}{\ln^2\left(\frac{E_B}{\epsilon}\right) + \pi^2} \theta(\epsilon), \tag{17}$$

which leads to the following expression for the virial coefficient:

$$\delta b_3 = \frac{1}{\sqrt{3}}\left[e^{\beta E_B} - \int_0^{\infty} \frac{d\epsilon}{\pi} \frac{e^{-\beta\epsilon}}{\epsilon} \frac{\pi}{\ln^2\left(\frac{E_B}{\epsilon}\right) + \pi^2}\right]. \tag{18}$$

Both Eqs. (17) and (18) refer to the case of attractive interactions, where the Landau pole E_B coincides with the three-body bound state. In the case of repulsive interactions, one can neglect the contribution to $\mathrm{Im}[T_3(\epsilon - i\delta)]$ from the Landau pole.

Equation (18) is no more than the famed Beth-Uhlenbeck formula [29]. Moreover, comparing to the two-dimensional case [10], we can identify the following relation:

$$\delta b_3 = \frac{1}{\sqrt{3}} \delta b_2, \tag{19}$$

where δb_2 is the shift in the second virial coefficient for the anomalous two-dimensional Fermi gas. This relationship was obtained previously in Ref. [14]

The presence of E_B in the pressure will lead to a nonzero contact. The contact can be defined using the following relations [14]:

$$\begin{aligned} PL &= 2\langle H\rangle + 2C_3, \\ C_3 &= LE_B \frac{\partial P}{\partial E_B} \\ &= \frac{g^2}{2\pi\sqrt{3}} \int dx \langle \psi_1^\dagger(x)\psi_2^\dagger(x)\psi_3^\dagger(x)\psi_3(x)\psi_2(x)\psi_1(x)\rangle. \end{aligned} \tag{20}$$

From Eqs. (15) and (20), we can write down the definition of the contact as

$$\tilde{C}_3 = \frac{C_3}{L} = \frac{3T}{\lambda_T} z^3 E_B \frac{\partial \delta b_3}{\partial E_B}, \tag{21}$$

and similarly for two dimensions:

$$\tilde{C}_2 = \frac{C_2}{L} = \frac{2T}{\lambda_T^2} z^2 E_B \frac{\partial \delta b_2}{\partial E_B}. \tag{22}$$

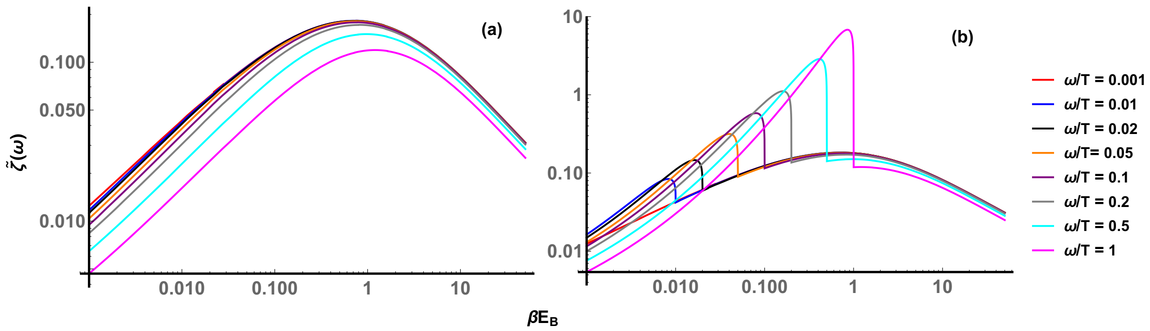

FIG. 2. The bulk viscosity as a function of the Landau pole energy E_B. $\tilde{\zeta} = \zeta(\omega)(4z^3/(2\pi\sqrt{3})^2)^{-1}\lambda_T/\sqrt{3}$. (a) For repulsive three-body interactions. (b) For attractive three-body interactions, where we include the presence of the bound state energy. The plots are identical for frequencies greater than βE_B. The structure in (b) is identical to the two-dimensional bulk viscosity evaluated in Ref. [26].

Using the relationship between the virial coefficients, Eq. (19), one can then show

$$\tilde{C}_3 = \frac{\sqrt{3}}{2} z\lambda_T \tilde{C}_2, \tag{23}$$

where $\tilde{C}_2 = C_2/L^2$ is the two-dimensional contact density. The first factor comes from the fact that the spatial dimensions are different, and as a result, the dimensions of the contact will be different. The second factor is due to the relation between the coupling constants, Eq. (9).

We can explicitly check Eq. (23) by evaluating the contact from Eq. (14):

$$\begin{aligned}\tilde{C}_3 = \frac{C_3}{L} &= \frac{z^3}{\lambda_T^3}\int_{-\infty}^{\infty}\frac{dx}{\pi}e^{-x}\mathrm{Im}[T_3(x - i\delta, \beta E_B)] \\ &= \frac{z^3}{\lambda_T^3}2\pi\sqrt{3}\bigg[\beta E_B e^{\beta E_B} \\ &\quad + \int_0^{\infty} dx e^{-x}\frac{1}{\ln^2(\beta E_B/x) + \pi^2}\bigg].\end{aligned} \tag{24}$$

This result is consistent with a perturbative calculation performed in Ref. [20] which evaluated the contact for the ground state at zero temperature. Equation (24) ought to be compared to the contact density of the anomalous two-dimensional Fermi gas:

$$\begin{aligned}\tilde{C}_2 = \frac{C_2}{L^2} &= \frac{z^2}{\lambda_T^2}4\pi\bigg[\beta E_B e^{\beta E_B} \\ &\quad + \int_0^{\infty} dx e^{-x}\frac{1}{\ln^2(\beta E_B/x) + \pi^2}\bigg].\end{aligned} \tag{25}$$

Upon comparison one obtains Eq. (23), the relationship between the contact densities.

IV. THE BULK VISCOSITY SPECTRAL FUNCTION

With the contact identified, we now turn to the bulk viscosity. The bulk viscosity can be defined as

$$\begin{aligned}\zeta(\omega) &= \frac{\mathrm{Im}[\chi(\omega)]}{\omega}, \\ \chi(\omega) &= \frac{i}{ZL}\int_0^{\infty} dt e^{i(\omega+i\delta)t}\mathrm{Tr}[e^{-\beta(H-\mu N)}[\Pi(t), \Pi(0)]],\end{aligned} \tag{26}$$

where $\Pi(t)$ is the spatially integrated stress-energy tensor. It is important to note that in one-dimensional systems, the stress-energy tensor will satisfy

$$\Pi = PL = 2\langle H\rangle + 2C_3. \tag{27}$$

Substituting Eq. (27) into Eq. (26), and noting that the thermal average of the commutator between H and C_3 is defined to be zero when the system is in equilibrium, one obtains

$$\chi(\omega) = 4\frac{i}{ZL}\int_0^{\infty} dt e^{i(\omega+i\delta)t}\mathrm{Tr}[e^{-\beta(H-\mu N)}[C_3(t), C_3(0)]]. \tag{28}$$

Since the contact is a three-body operator, the first non-vanishing term of $\chi(\omega)$ will be of order $O(z^3)$. We can then perform the trace over the three-body sector of the Hilbert space, and obtain an expression using Eqs. (12) and (20). For explicit details on how to evaluate the trace, we refer the reader to Appendix B, here we quote the final expression:

$$\begin{aligned}\chi(\omega) = -4z^3\left(\frac{1}{2\pi\sqrt{3}}\right)^2\frac{\sqrt{3}}{\lambda_T}\int_{-\infty}^{\infty}\frac{d\epsilon}{\pi}\int_{-\infty}^{\infty}\frac{d\epsilon'}{\pi}\frac{e^{-\beta\epsilon} - e^{-\beta\epsilon'}}{\epsilon - \epsilon' + \omega + i\delta}, \\ \mathrm{Im}[T_3(\epsilon - i\delta)]\mathrm{Im}[T_3(\epsilon' - i\delta)].\end{aligned} \tag{29}$$

Substituting this into Eq. (26), we obtain the bulk viscosity:

$$\begin{aligned}\zeta(\omega) &= 4z^3\left(\frac{1}{2\pi\sqrt{3}}\right)^2\frac{\sqrt{3}}{\lambda_T}\frac{1 - e^{-\beta\omega}}{\omega} \\ &\quad \times \int_{-\infty}^{\infty}\frac{d\epsilon}{\pi}e^{-\beta\epsilon}\mathrm{Im}[T_3(\epsilon - i\delta)]\mathrm{Im}[T_3(\epsilon + \omega - i\delta)].\end{aligned} \tag{30}$$

The bulk viscosity for various frequencies as a function of βE_B is shown in Fig. 2, for both repulsive Fig. 2(a), and attractive Fig. 2(b) three-body interactions.

From Eq. (30), one can show that the bulk viscosity is an even function of frequency, $\zeta(\omega) = \zeta(-\omega)$, and has the following large frequency limit:

$$\zeta(\omega \to \infty) = \frac{4\pi}{\ln^2\left(\frac{\omega}{E_B}\right)\omega}\tilde{C}_3. \tag{31}$$

TABLE I. The anomaly correspondence. Here we show the relation between various thermodynamic quantities for the anomalous one-dimensional Fermi gas and the anomalous two-dimensional Fermi gas. Here we note g is the contact interaction, δb is the virial coefficient, C is Tan's contact, and $\zeta(\omega)$ is the bulk viscosity.

g_3/g_2	$\sqrt{3}/2$
$\delta b_3/\delta b_2$	$1/\sqrt{3}$
$\tilde{C}_3/\tilde{C}_2$	$\sqrt{3}/2\, z\lambda_T$
$\zeta(\omega)/\zeta_2(\omega)$	$4\sqrt{3}/2\, z\lambda_T$

One can also integrate Eq. (30) to obtain the following sum rule:

$$\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\zeta(\omega) = -\frac{4}{2\pi\sqrt{3}}E_B\frac{\partial}{\partial E_B}\tilde{C}_3. \tag{32}$$

Again as a comparison we note the bulk viscosity for 2D systems [26] is given by

$$\zeta_2(\omega) = z^2\left(\frac{1}{4\pi}\right)^2\frac{2}{\lambda_T^2}\frac{1-e^{-\beta\omega}}{\omega} \times \int_{-\infty}^{\infty}\frac{d\epsilon}{\pi}e^{-\beta\epsilon}\mathrm{Im}[T_2(\epsilon-i\delta)]\mathrm{Im}[T_2(\epsilon+\omega-i\delta)], \tag{33}$$

or, equivalently,

$$\zeta(\omega) = 4\frac{\sqrt{3}}{2}z\lambda_T\zeta_2(\omega). \tag{34}$$

V. CONCLUSIONS

In this article we have explicitly confirmed that the thermodynamic properties of the anomalous one-dimensional Fermi gas is directly related to that of the anomalous two-dimensional Fermi gas. Thermodynamic properties like the virial coefficient, Tan's contact, and bulk viscosity, can all be related to one another, thanks to the mapping between the anomalous two-body physics in two dimensions and its three-body counterpart in one dimension. The mapping is summarized in Table I.

This anomaly correspondence is an excellent tool in understanding the physics of anomalous systems at high temperatures, because the dominant contribution to the physics comes from the few-body sector. However, this mapping can not fully reproduce the entirety of the physics in both systems. The presence of an extra dimension will allow for the possibility of new phenomena which may have no counterpart for one-dimensional systems. For example, a one-dimensional system will not have a shear viscosity, but a two-dimensional system will.

Equivalences between quantum gases in different dimensions, and with different interactions have been previously discussed in the literature [30–32]. The emphasis of this work, and the emphasis of Refs. [14] and [15], is in the structural similarities between the anomalous one- and two-dimensional systems that occur due to their identical few-body structures.

In the future we will explore this anomaly correspondence to see whether this mapping will be exact when many-body effects are important. To do this, it is necessary to examine the many-body properties of the anomalous one-dimensional Fermi gas, which is the subject of an upcoming work.

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APPENDIX A: EXPLICIT FORMS FOR THE PARTITION FUNCTION

In this Appendix we write down the explicit form of the partition function. We first note that the partition function can be written as

$$Z = zZ_1 + z^2\left(\frac{Z_1^2}{2}+\delta Z_2\right) + z^3\left(\frac{Z_1^3}{3!}+Z_1\delta Z_2+\delta Z_3+Z_3|_{\mathrm{int}}\right). \tag{A1}$$

The various contributions to the partition function are given by

$$\begin{aligned} Z_1 &= \sum_k e^{-\beta k^2/2},\\ \delta Z_2 &= -\frac{1}{2}\sum_k e^{-\beta k^2},\\ \delta Z_3 &= \frac{1}{3}\sum_k e^{-3\beta k^2/2},\\ Z_3|_{\mathrm{int}} &= \frac{1}{L^2}\sum_Q\sum_{p,q}\frac{2}{\sqrt{3}}e^{-\beta\frac{Q^2}{6}}\int_{-\infty}^{\infty}\frac{d\epsilon}{\pi},\\ &\left[e^{-\beta\epsilon}\mathrm{Im}\left[\frac{T_3(\epsilon-i\delta)}{(\epsilon-p^2-q^2)^2}\right]\right]. \end{aligned} \tag{A2}$$

APPENDIX B: CALCULATION OF THE RETARDED CONTACT-CONTACT CORRELATOR

In this section we write down an explicit form for the trace of the following quantity:

$$A = \mathrm{Tr}_3[e^{-\beta H}[C_3(t), C_3(0)]]. \tag{B1}$$

This quantity is related to the retarded contact-contact correlator by

$$\chi(\omega) = 4\frac{iz^3}{L}\int_0^{\infty}dt\, e^{i(\omega+i\delta)t}A. \tag{B2}$$

It is important to note that A can be rewritten as

$$\begin{aligned} A = \mathrm{Tr}_3[&e^{(-\beta+it)}C_3(0)e^{-iHt}C_3(0)\\ &- e^{(-\beta-it)H}C_3(0)e^{iHt}C_3(0)]. \end{aligned} \tag{B3}$$

It is still possible to use the identities in Eqs. (11) and (12) to express the evolution operators in terms of the propagator. One can then show that the trace of the contact-contact commutator is given by

$$A = \left(\frac{g^2}{2\pi\sqrt{3}L^4}\right)^2 \sum_{Q} \sum_{p,q,p',q'} \sum_{k,l,k',l'} \int_{-\infty}^{\infty} \frac{d\epsilon}{\pi} \int_{-\infty}^{\infty} \frac{d\epsilon'}{\pi},$$
$$e^{-\beta \frac{Q^2}{6}} e^{i(\epsilon-\epsilon')t} (e^{-\beta\epsilon} - e^{-\beta\epsilon'}) \left(\frac{2}{\sqrt{3}}\right)^4,$$
$$\left(\mathrm{Im}\left[\frac{T_3(\epsilon - i\delta)}{(\epsilon - p^2 - q^2 - i\delta)(\epsilon - p'^2 - q'^2 - i\delta)}\right],\right.$$
$$\left.\mathrm{Im}\left[\frac{T_3(\epsilon' - i\delta)}{(\epsilon - k^2 - l^2 - i\delta)(\epsilon - k'^2 - l'^2 - i\delta)}\right]\right). \tag{B4}$$

Noting that

$$\frac{g}{L^2}\frac{2}{\sqrt{3}} \sum_{p,q} \frac{1}{\epsilon - p^2 - q^2 - i\delta} \approx 1, \tag{B5}$$

when $\Lambda \to \infty$. One obtains the final result,

$$A = \left(\frac{1}{2\pi\sqrt{3}}\right)^2 \sum_{Q} e^{-\beta\frac{Q^2}{6}} \int_{-\infty}^{\infty} \frac{d\epsilon}{\pi} \int_{-\infty}^{\infty} \frac{d\epsilon'}{\pi} e^{i(\epsilon-\epsilon')t},$$
$$(e^{-\beta\epsilon} - e^{-\beta\epsilon'})\mathrm{Im}[T_3(\epsilon - i\delta)]\mathrm{Im}[T_3(\epsilon' - i\delta)]. \tag{B6}$$

Substituting Eq. (B6) into Eq. (B2) and performing the integrations over the center-of-mass momentum and time, one obtains Eq. (29).

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