The relation between spatial skills and mathematical abilities: The mediating role of mental number line representation

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Abstract

Spatial skills are positively related to mathematical abilities, yet the mechanism of such relation remains unclear. This study examined the mediating role of mental number line representation in the relations between spatial skills and performance in various mathematical tasks. One hundred and nine second-graders were tested on mental rotation, perspective taking, mental number line representation, visuospatial working memory, arithmetic fact retrieval, calculation and word problems. Using structural equation modelling with the bootstrap procedure, we found that after controlling for age, gender and visuospatial working memory, mental number line representation fully mediated the relation between spatial skills and calculation and the relation between spatial skills and word-problem solving. However, it was not a significant mediator between spatial skills and arithmetic fact retrieval. This study highlights the important role of mental number line representation in explaining the relation between spatial skills and mathematical abilities.

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Introduction

Spatial skills are positively related to mathematical performance (Battista, 1990; Casey, Nuttall, Pezaris, & Benbow, 1995; Guay & McDaniel, 1977; Hawes, Moss, Caswell, & Poliszczuk, 2015; Kyttälä & Lehto, 2008; Mix et al., 2016). This relation is evident from early childhood into adulthood (Gathercole & Pickering, 2000; Kyttälä, Aunio, Lehto, Van Luit, & Hautamäki, 2003; Tolar, 2009; Yurt & Sunbul, 2014). However, the mechanism of this relation is not well-articulated. In the present study, we examine the mediating role of mental number line representation in the relation between spatial skills and mathematical abilities. Understanding the mechanism of this relation can help educators target important skills in mathematical learning.

Overview of Spatial Skills

Spatial skills are defined as the ability to produce, retrieve, retain, and manipulate symbolic and non-linguistic information such as shape, size, and location (Hegarty & Waller, 2005; Linn & Peterson, 1985). In a meta-analysis of spatial training literature, Uttal et al. (2013) proposed that spatial skills can be dynamic or static. Dynamic spatial skills involve transformation or movement whereas static ones do not. Examples of dynamic spatial skills are mental rotation (i.e., the capacity to mentally rotate two-dimensional or three-dimensional objects; Hegarty & Waller, 2005; Linn & Peterson, 1985; Shepard & Metzler, 1988), spatial visualization (i.e., the ability to mentally manipulate simple figures to form complex configurations, or visualize three-dimensional objects from two-dimensional representations, and vice versa; Linn & Peterson, 1985; Manger & Eikeland, 1998; Mix & Cheng, 2012), and perspective taking (i.e., the ability to mentally visualize a viewpoint different from one's own perspective; Frick, Möhring, & Newcombe, 2014; Mix & Cheng, 2012). Examples of static spatial skills are the ability to perceive objects, paths, or spatial configurations surrounded by distracting information (Uttal et al., 2013) and the ability to understand abstract spatial principles such as verticality and horizontal invariance (Mix & Cheng, 2012; Uttal et al., 2013). To the best of our knowledge, most – if not all – studies on the relations between spatial skills and mathematical abilities are concerned with dynamic skills (e.g., mental rotation, spatial visualization, and perspective taking; Battista, 1990; Casey, Dearing, Vasilyeva, Ganley, & Tine, 2011; Casey et al., 2015; Casey et al., 1995; Cheng & Mix, 2013; Delgado & Prieto, 2004; Guay & McDaniel, 1977; Hegarty & Kozhevnikov, 1999; Johnson, 1998; Kyttälä et al., 2003; Kyttälä & Lehto, 2008; Markey, 2010; Reuhkala, 2001; van den Heuvel-Panhuizen, Elia, & Robitzsch, 2015). These studies have found a strong association between dynamic spatial skills and mathematical abilities. Although dynamic spatial skills (as well as static spatial skills) can be further divided into intrinsic (i.e., intra-object relations) or extrinsic (i.e., inter-object relations; Uttal et al., 2013), we focus on the broad category of dynamic spatial skills that have been frequently studied in the literature to reveal a general picture of the mechanism underlying the spatial-math association.

Relation between Spatial Skills and Mathematical Abilities

Positive relations between spatial skills and mathematical abilities have been reported across age groups. Several studies have found that adolescents and adults with higher mental rotation ability perform better in mental arithmetic (Kyttälä & Lehto, 2008; Reuhkala, 2001), word problems (Delgado & Prieto, 2004; Hegarty & Kozhevnikov, 1999; Kyttälä & Lehto, 2008), geometry concepts (Battista, 1990; Delgado & Prieto, 2004; Kyttälä & Lehto, 2008), and the mathematics component of the SAT (Casey et al., 1995). Similar patterns have also been observed for school-aged children. For instance, Casey et al. (2015) found that performance in mental rotation with two- and three-dimensional objects among first-grade girls was the strongest predictor of their fifth-grade performance in geometry, measurement, numbers, and algebra. Other studies have also revealed that school-aged children’s spatial visualization is significantly correlated with their performance in measurement (Casey et al., 2011), arithmetic word-problem solving (Hegarty & Kozhevnikov, 1999), and overall mathematics achievement (Guay & McDaniel, 1977; Johnson, 1998; Markey, 2010). The relation between spatial skills and mathematical abilities has even been observed among kindergarteners who have not yet received formal schooling. For example, the mental rotation ability of kindergarteners was positively related to their early mathematical skills, such as concepts of comparison, classification, correspondence and seriation, use of numerals, structured counting, resultative counting, and general knowledge of numbers (Kyttälä et al., 2003).

Other studies have examined the relation between perspective taking and mathematical abilities. Guay and McDaniel (1977) tested whether children’s ability to visualize the shape of three-dimensional objects from various viewpoints was related to their performance on the mathematics subtest of the Iowa Test of Basic Skills. Interestingly, a significant relation was observed in grades 5 to 7, but not in grades 2 to 4. Another study focusing on kindergarteners found that in both the Netherlands and Cyprus, children’s perspective-taking ability was significantly related to their mathematical performance (van den Heuvel-Panhuizen et al., 2015).

Recent intervention studies have proven that spatial skills training can bring about significant improvement in mathematics. Lowrie, Logan, and Ramful (2017) conducted a 10-week classroom intervention program for elementary school children on spatial visualization, mental rotation, and spatial orientation skills. The intervention not only improved the children’s spatial abilities but also enhanced their mathematics performance compared to the control group, which received standard mathematics instruction. Likewise, a group of 6- to 8-year-old children who received a 40-minute mental rotation training session demonstrated significant improvement in solving missing-term problems (e.g., 4 + ___ = 12), whereas children in the control group did not demonstrate any improvement in any mathematical tasks (Cheng & Mix, 2013). The authors speculated that better mental rotation ability leads to more flexible rearrangement of the missing terms when solving such problems.

Mechanisms of the Link between Spatial Skills and Mathematical Abilities

There appears to be a strong link between spatial skills and mathematical abilities, yet the mechanisms of this link remain unclear. From a neuropsychological perspective, both mental rotation and mathematical tasks activate the intraparietal sulcus in the parietal lobe, suggesting that the parietal neural networks that support spatial transformation may also contribute to arithmetic transformation (e.g., calculation) (Hubbard, Piazza, Pinel, & Dehaene, 2009). Apart from shared neuropsychological processes, researchers have attempted to study math anxiety as a possible link between spatial skills and mathematical ability. There is evidence of spatial impairment in math-anxious individuals (Ferguson, Maloney, Fugelsang, & Risko, 2015). These individuals generally report a worse sense of direction, as well as more spatial and general anxiety, and perform worse in mental rotation and perspective-taking tasks. Importantly, regression analyses have suggested that spatial skills are one of the most robust predictors of math anxiety, which in turn is strongly negatively correlated with math achievement. Another potential linkage between spatial abilities and mathematics performance, which is the focus of our investigation here, is the mental number line.

Mental Number Line Representation

People represent quantities spatially in the form of a horizontal, mental number line. Numbers are arranged in ascending order along this mental number line, with smaller numbers placed towards the left side (Galton, 1880, 1881). This explains why people respond faster to small numbers with their left hands and to large numbers with their right hands (Dehaene, Bossini, & Giraux, 1993). This left-right bias is known as the spatial-numerical association of response code (SNARC) effect, and it is reversed in populations that read and write numbers from right to left (Shaki, Fischer, & Petrusic, 2009). Since numbers are spatially represented, comparing numbers that are far apart (e.g., 3 vs. 9) is easier than comparing numbers that are close to each other (e.g., 3 vs. 4) (i.e., the distance effect; Moyer & Landauer, 1967). Moreover, Fischer and Shaki (2014) found that addition elicits bias to the right whereas subtraction elicits bias to the left. Therefore, people tend to overestimate the sum of addition problems and underestimate the difference of subtraction problems. Neuropsychological studies have also suggested that people with spatial neglect display impairment regarding magnitude judgments (Vuilleumier, Ortigue, & Brugger, 2004). These studies provide convergent evidence for spatial representation of numbers in the form of a mental number line.

Mental number line representation can be measured by the number line estimation task (Laski & Siegler, 2007). A horizontal number line is marked with a start point (e.g., 0) and an end point (e.g., 100). Participants are given a target number and asked to estimate its position on the number line. Developmental studies have shown that such spatial representation of numbers emerges early in childhood (Hoffmann, Hornung, Martin, & Schiltz, 2013; van Galen & Reitsma, 2008) and develops from a logarithmic pattern (i.e., larger numbers are more compressed) to a linear pattern (i.e., all numbers are evenly spaced) over time (Siegler & Booth, 2004). Furthermore, children’s accuracy in the number line estimation task increases with age (Booth & Siegler, 2006, 2008).

Relation between Spatial Skills and Mental Number Line Representation

If mental number line representation is a bridge between spatial skills and mathematical abilities, it is expected that it will have a relation with both of these. Previous studies have found a strong connection between mental number line representation and spatial skills. Thompson, Nuerk, Moeller, and Kadosh (2013) found that among adults, better mental rotation ability is associated with more accurate mental number line representation. In a longitudinal study by Gunderson, Ramirez, Beilock, and Levine (2012), first- and second-graders' spatial skills predicted their development in mental number line representation over the course of the school year, even after controlling for their initial performance in a number line estimation task and mathematics and reading achievement tests. In another longitudinal study of older children, LeFevre et al. (2013) observed the same pattern, suggesting that spatial skills may facilitate children's development of mental number line representation. The strong connection between mental number line representation and spatial skills can be explained by the fact that mental number line representation is spatial in nature, and better spatial skills can help fine-tune the positions of individual numbers relative to others on the mental number line.

Relation between Mental Number Line Representation and Mathematical Abilities

Apart from its connection to spatial skills, mental number line representation is also strongly related to mathematical abilities. In fact, there is growing evidence for a link between mental number line representation and performance in early numeracy skills, such as counting (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Fischer, Moeller, Bientzle, Cress, & Nuerk, 2011; Muldoon, Towse, Simms, Perra, & Menzies, 2013), number comparison (Sella, Berteletti, Lucangeli, & Zorzi, 2017), numerosity estimation (Booth & Siegler, 2006; Siegler & Booth, 2004), and approximate calculation (Gunderson et al., 2012; Sella, Sader, Lolliot, & Cohen Kadosh, 2016). When low-income preschoolers were given linear number board game training to enhance mental number line representation, they demonstrated significant improvement in number comparison (Booth & Siegler, 2008; Ramani & Siegler, 2008), suggesting that mental number line representation likely underlies early numeracy skills. Moreover, mental number line representation is strongly predictive of concurrent and later mathematics achievement from kindergarten through third grade (Booth

& Siegler, 2006, 2008; Siegler & Booth, 2004). Importantly, such relations remain significant even after controlling for reading and arithmetic achievement and intelligence (Bailey, Siegler, & Geary, 2014; Booth & Siegler, 2006, 2008; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007).

The link between mental number line representation and mathematical abilities is not limited to early numeracy and general mathematics achievement but also includes various mathematical skills examined in the present study (i.e., arithmetic fact retrieval, calculation, and word-problem solving). First, it is linked to arithmetic fact retrieval (Fabbri, 2011; Siegler & Ramani, 2009; Träff, 2013). It has been shown that 10- to 13-year-olds' performance in the number line estimation task uniquely explains the variance in arithmetic fact retrieval. Arithmetic fact retrieval involves the retrieval of an exact answer from long-term memory, in a way that is similar to semantic knowledge (Fabbri, 2011). According to Siegler and Ramani (2009), the mental number line serves as a retrieval structure that facilitates encoding, storage, and retrieval of numerical information through organizing the information around numbers' magnitudes. Therefore, a well-developed and accurate mental number line supports children's acquisition and storage of arithmetic facts around numbers' magnitude, which in turn enhances their ability to quickly retrieve arithmetic facts from long-term memory.

Second, several developmental studies have suggested that accurate mental number line representation is associated with better performance in calculation (Berteletti et al., 2010; Booth & Siegler, 2006, 2008; Halberda, Mazzocco, & Feigenson, 2008; Lefevre et al., 2013; Siegler & Booth, 2004; Siegler & Ramani, 2009). Children's mental number line representation is not only positvely related to their concurrent arithmetic knowledge but also predictive of their learning of novel arithmetic problems, even after controlling for prior arithemtic knowledge, short-term memory regarding numbers, and math achievement test scores (Booth & Siegler, 2008). Morever, showing children number line representations of the magnitudes of addends and sums of arithmetic problems can improve their arithmetic learning (Booth & Siegler, 2008). Kucian et al. (2011) adopted a similar approach in a training programme in which children with developmental dyscalculia received a five-week computer intervention to improve their mental number line representation and calculation performance. The training yielded positive results, as evidenced by the children's enhanced spatial representation of numbers and better performance in solving calculation problems. Improvement in arithmetic skills may have arisen from practice with translating arithemtic problems into ordinal relations on the number line (Kucian et al., 2011). Other researchers have suggested that mental number line representation could facilitate arithmetic learning by constraining a set of possible answers to those of approximately correct magnitude, which in turn reduces the possibility of errors in calculation (Booth & Siegler, 2008).

Finally, to the best of our knowledge, only one study has provided evidence for the link between mental number line representation and word-problem solving. The study found that among various number abilities (i.e., enumeration, number line estimation, and digit comparison), only number line estimation contributed unique variance in word-problem solving performance (Träff, 2013). Word-problem solving involves a complex interplay of the reading process, language comprehension, problem presentation, and calculation processes (Kintsch & Greeno, 1985; Lee, Ng, Ng, & Lim, 2004; Swanson, 2004). Problem presentation, which is the translation of word problems into personally understandable forms and the integration of relevant information into a mathematical, visual, or mental model (Mayer, 1985, 2013), is regarded as the most demanding and critical part of word-problem solving for children (Gonsalves & Krawec, 2014). To establish problem presentation, one must integrate the different relation propositions, number propositions, and question propositions that comprise the word problem (Kintsch & Greeno, 1985). Importantly, this integration process, as well as the final problem presentation, are founded on the mental number line (Träff, 2013). Hence, mental number line representation plays an important role in problem presentation during word-problem solving.

Mental Number Line Representation as a Mediator

To the best of our knowledge, very few studies have examined the mediating role of mental number line representation in relating spatial skills to mathematical abilities. One exception is a longitudinal study by Gunderson et al. (2012) that found children's mental rotation ability at age 5 was a good predictor of their performance in an approximate symbolic calculation task at age 8, and that this relation was mediated by their mental number line knowledge at age 6. The results suggest that spatial skills support children's development of early numerical knowledge by facilitating their acquisition of a linear, spatial representation of numbers. However, this study only considered children's approximate symbolic calculation. Whether the mediating role of the mental number line can be generalized to the relations between spatial skills and broader domains of mathematical abilities remains unexplored. In particular, we do not yet know whether mental number line representation bridges the relation between spatial skills and various types of school mathematics performance. Hence, the present study sought to extend this finding to other mathematical skills taught daily in the classroom, including arithmetic fact retrieval, calculation, and word-problem solving.

The Present Study

This study aimed to explore the mediating role of mental number line representation in the relation between spatial skills and various types of mathematical abilities. In light of the literature just outlined, we hypothesized that spatial skills serve as a precursor of children's development of accurate mental number line representation, which in turn helps them to develop various types of mathematical abilities, including arithmetic fact retrieval, calculation, and word-problem solving. Dynamic spatial skills, including mental rotation and perspective taking skills, were chosen as the measures of children's spatial skills because of their strong association with performance of mathematical tasks that require complex conceptualization (Mix et al., 2016). The findings offer a clearer picture of how spatial skills are related to mathematical abilities in various domains, thereby providing directions for enhancing children's mathematical performance through spatial training.

Method

Participants

An initial sample of 109 Chinese second-graders (mean age = 7.71 years, *SD* = 0.33 years, 61 boys and 48 girls) were recruited from two primary schools in Hong Kong. Sixteen children did not complete all the tasks due to absence from school, and one child failed to complete the word problem task. These children were excluded, so the final sample consisted of 92 children (mean age = 7.69 years, *SD* = 0.33 years, 50 boys and 42 girls). All children participated with parental consent.

Measures

Mental rotation. The mental rotation task was modified based on a study by Levine, Huttenlocher, Taylor, and Langrock (1999). In each test item, children were shown two separate parts of a shape. They were asked to mentally combine the parts to form a complete target shape and then choose the correct one from four choices (Figure 1). In the original version, half of the target shapes were unilaterally symmetric and half were bilaterally symmetric. In a pilot study, we found that children aged 6 to 8 performed relatively well on the original version (mean = 78.08%, *SD* = 5.52%). Therefore, we increased the level of difficulty in this study by adding asymmetric stimuli that were more challenging than the symmetric stimuli in the original version (Tzuriel & Egozi, 2007). In our sample, children performed significantly worse on asymmetric items compared to unilaterally symmetric items

($t(91) = 11.359$, $p < .001$) and bilaterally symmetric items ($t(91) = 19.964$, $p < .001$), which provided evidence that the modified version was more challenging (mean = 71.39%, *SD* = 13.64%). The version used for the present study included four types of orientations that varied in terms of the relative positioning of the two parts of the shape: direct translation, diagonal translation, direct rotation, and diagonal rotation (Figure 2). Test items varied in terms of symmetry and orientation to form 12 combinations of problem types. Each of these combinations was presented six times, amounting to a total of 72 test items. The order of problem types and the position of target shapes in the choice array were randomized across trials, with the constraints that (1) the target shapes were not in the same position for more than two consecutive trials and (2) the same problem type did not appear twice in a row. Each correct answer scored one point. The Cronbach's inter-item reliability was good ($\alpha = .88$).

Perspective taking. The perspective-taking task (Frick et al., 2014) measured participants' ability to imagine a scene from different viewpoints. In the instruction trials, real three-dimensional objects were presented on an A4-sized piece of white cardboard, as shown in Figure 3. A cylinder and a cone were placed side by side. One toy photographer (introduced as *the mother*) stood at 90° and another (introduced as *Peter*) at a 180° angle to the children's line of sight. Both toy photographers pointed their cameras at the cylinder and cone. In each instruction trial ($n = 4$), a picture of the cylinder and cone taken from a different perspective was presented. Children were asked who (*the mother*, *Peter*, or *none of them*) had taken the picture. They could verify their answers by viewing the objects from the toy photographers' perspectives. The subsequent practice trials ($n = 3$) with feedback served to check participants' understanding of the instructions, and to facilitate the transition from three-dimensional to two-dimensional stimuli. Therefore, no three-dimensional objects were displayed. In each practice trial, a scene in which a toy photographer was taking picture of some objects was displayed, and four alternative pictures were shown. Participants were asked to choose the picture that the toy photographer could have taken. While one picture showed the correct view, the other three were foils that displayed the same objects with different spatial relations or orientations. The items for the test trials (n = 54) were presented on color printouts (Figure 4). Test items varied in terms of angle of viewing (0°, 90° or 180°) and number of objects (1, 2 or 4) to form nine combinations of problem types. Each combination was presented six times with different objects and toy photographers. Each correct answer scored one point. The Cronbach's inter-item reliability was high (α = .93).

Visuospatial working memory (VSWM). A VSWM task was created for this study based on the Spatial Addition subtest from the Wechsler Memory Scale – Fourth Edition (WMS – IV) (Wechsler, 2009) and the static VSWM task created by Kyttälä et al. (2003). Similar to the static VSWM task by Kyttälä et al. (2003), children were presented a 5 × 5 grid during each test trial. Within the grid, some squares were marked with either an 'X' or an 'O', and the rest were blank (Figure 5). A stimulus stayed on a projector screen for four seconds and then disappeared. Children were told to recall the positions of 'X's by marking a cross on an empty grid paper. Each correctly marked square scored one point, which accumulated to form a hit score. With reference to the Spatial Addition subtest from the WMS – IV (Wechsler, 2009), an additional rule was set to increase the demand on the VSWM. No marking was required for the 'X's that located beneath an 'O'. Each correctly rejected square was given one point, and the total was the correct rejection score. The hit score and correct rejection score were added to form the VSWM score. If children answered with a number of crosses exceeding the number of 'X's in the visual stimulus, that test trial was considered failed and given 0 points. This differentiated children who randomly marked 'X's. The difficulty of test items was manipulated by increasing the number of 'X's and 'O's in the grid. The easiest item contained 5 'X's and 3 'O's whereas the most difficult one contained 9 'X's and 5 ‘O’s. Before the test trials (n = 30) began, children were given two practice trials with feedback. The Cronbach’s inter-item reliability was good (α = .83).

Mental number line representation. The number line estimation task by Laski and Siegler (2007) was used to measure mental number line representation. Children were introduced to a 25-cm horizontal line with 0 and 100 at the left and right ends, respectively. The experimenter explained that each number between 0 and 100 had its own position on the number line. For each test item (n = 20), children were asked to mark the position of a target number on the number line with a vertical stroke using a pen. The order of the 20 target numbers (3, 8, 12, 17, 24, 29, 33, 39, 42, 48, 52, 57, 61, 64, 72, 79, 81, 84, 90 and 96) was randomized. To examine the accuracy of the estimates, the percentage absolute error (PAE) was calculated by the following formula: PAE = |estimate – estimated quantity|/scale of the estimates. For instance, if a child estimated the number 60 at a position that corresponded to 70, the PAE would be 10% (|70 – 60|/100). An average PAE value was calculated from all test items and then transposed by the formula: score = 1 – average PAE, such that a higher score indicated a more accurate mental number line representation. The Cronbach’s inter-item reliability was good (α = .84).

Arithmetic fact retrieval. Arithmetic fact retrieval was assessed by two subtests that consisted of single-digit addition and subtraction fact problems. Each subtest contained 60 questions. Children were given one minute to complete as many questions as possible. The score was obtained by totaling the number of correct answers in both subtests.

Calculation. The calculation task comprised 24 addition and subtraction items. In light of the local mathematics curriculum guide for grade 2, all operands and answers involved two-digit numbers only. Six problems were presented in conventional format (e.g., 64 + 18 = ___), whereas 18 problems were presented in fill-in-the-blank format (e.g., 12 +

___ = 39). Children received one point for each correct answer. The Cronbach's inter-item reliability was good (α = .82).

Word problems. A total of 12 word problems involving change, combine, and comparison relationships were presented (e.g., Tom's pen costs $36, which is $13 more than Jerry's pen. How much is Jerry's pen?). In light of the local mathematics curriculum guide for grade 2, all operands and answers involved two-digit numbers only. The items were chosen with reference to local textbooks and supplementary exercises to ensure an appropriate level of difficulty. Children were asked to write mathematical sentences, and one point was awarded for each correctly written mathematical sentence. The Cronbach's inter-item reliability was good (α = .73).

Procedure

All tasks were conducted in groups of 25 to 30 children in paper-and-pencil format. Altogether, three sessions were scheduled on three different days within two weeks. Each session lasted approximately 50 minutes. The duration of each session, which was comparable to a typical test or examination session for elementary school students in Hong Kong, was deemed to be appropriate by the teachers. Regarding the workload of tasks, all tasks except the number line estimation task were divided into three parts of equal length and difficulty. Children attempted one part in each session to avoid fatigue. The order of the three parts was counterbalanced. Children completed the number line estimation task once in each session, and an average PAE was calculated to obtain a more reliable measure.

Data Analyses

All data analyses were conducted using R (R Core Team, 2017) with the lavaan package (Rosseel, 2012). Before testing the hypothesis, the data was screened for univariate outliers. Eight data points that were ≥ 3 *SD* away from the means were excluded. Using the Mahalanobis distance in SPSS, no multivariate outliers were found. Since only children with a complete data set were included in modelling, the final sample consisted of 85 participants (mean age = 7.69 years, *SD* = 0.32 years, 44 boys and 41 girls). Using structural equation modelling, we first examined the total effect of spatial skills on mathematical abilities, and then the indirect effects by introducing mental number line representation as a mediator. Because of the relatively small sample size (*N* = 85) and positively skewed data (skewness = 183.3 $p < .001$), we ran structural equation models with a bootstrap procedure, which enabled us to estimate model fit indices and parameters by producing multiple data sets based on our sample. Such a procedure does not have a normality assumption and thus can be used with data that are significantly skewed. The results of bootstrap analysis provide means of the model fit indices and parameters based on all the datasets. This procedure renders valid results even in small samples of *N* = 50 (Briggs, 2006; Preacher & Hayes, 2004; Yung & Chan, 1999).

Results

Descriptive Statistics

Table 1 presents descriptive statistics, including means, standard deviations, and maximum possible scores. Significant gender differences were observed in mental number line representation, arithmetic fact retrieval, and calculation ($ps < .05$), indicating that boys performed better than girls in these tasks. As a result, gender was included as a dichotomous measured predictor in later analyses. Table 2 presents the correlations among the variables. There were significant correlations between most variables. Importantly, the hypothesized mediator (mental number line representation) correlated significantly with the hypothesized predictors (mental rotation: $r = -.328$, $p < .01$; perspective taking: $r = -.297$, $p < .01$) and outcome variables (arithmetic fact retrieval: $r = -.245$, $p < .05$; calculation: $r = -.458$, $p < .001$; word problems: $r = -.473$, $p < .001$). Given that mental number line representation also correlated significantly with age ($r = -.235$, $p < .05$), age was included as a control variable in subsequent analyses. However, mental rotation did not correlate with calculation (r = .190, p = .08), and arithmetic fact retrieval did not correlate with mental rotation (r = .071, p = .52), perspective taking (r = .173, p = .11), or VSWM (r = .176, p = .11) respectively.

Mental Number Line Representation as a Mediator between Spatial Skills and Mathematical Abilities

We hypothesized that mental number line representation mediate the relations between spatial skills and various types of mathematical abilities, including arithmetic fact retrieval, calculation, and word-problem solving. To test this hypothesis, we first constructed a structural equation model predicting our outcome variables, namely arithmetic fact retrieval, calculation, and word-problem solving, using spatial skills as the predictor (Figure 6). In addition to age and gender, VSWM was also included as a control variable because the literature suggests that it correlates with spatial skills (Just & Carpenter, 1985; Lehmann, Quaiser-Pohl, & Jansen, 2014) and mathematical abilities (Gathercole & Pickering, 2000; Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Swanson & Sachse-Lee, 2001). Models are considered to fit the data well if they meet the following criteria: insignificant Chi-squared (χ^2) test, comparative fit index (CFI) > .95, and standardized root mean square residual (SRMR) < .08 (Hu & Bentler, 1999). Further, a root mean square error of approximation (RMSEA) below .10 is considered to be good (Aron, Coups, & Aron, 2013). A bootstrap analysis with 5000 replications was performed. The findings were significant if zero was not included in the confidence intervals (Preacher & Hayes, 2008; Shrout & Bolger, 2002). The fit indices suggested that the model fit the data well, χ^2 (8, N = 85) = 11.955, p = .153, CFI = .962, RMSEA = .076, SRMR = .060. Mental rotation and perspective taking formed a latent variable, “spatial skills”, for which the parameter loadings to its indicators were significant (mental rotation: b = 1.000, CI [1.000, 1.000]; perspective taking: b = .947, CI [.242, 2.012]). We found that spatial skills significantly predicted word-problem solving (b

= .129, CI [.001, .248]) but not arithmetic fact retrieval (*b* = .206, CI [-0.354, .945]) or calculation (*b* = .121, CI [-.030, .334]). However, contemporary views on mediation analysis consider a significant total effect non-essential for tests of mediation (Hayes, 2009; Shrout & Bolger, 2002). In the present study, it was possible that mediation exists between spatial skills and various types of mathematical abilities, but the total effect is not significant because of the relatively small sample size (*N* = 85). Therefore, we proceeded with the tests of indirect effects despite an insignificant total effect of spatial skills on arithmetic fact retrieval and calculation.

Subsequently, mental number line representation was introduced as a mediator into the structural equation model (i.e., spatial skills as a predictor of number line representation, and number line representation as a predictor of the three mathematical outcomes; see Figure 7). Again, the findings were evaluated by bootstrap analysis with 5,000 replications. Results showed that the fit indices were satisfactory, χ^2 (11, *N* = 85) = 14.742, *p* = .195, CFI = .973, RMSEA = .063, SRMR = .053. The parameter loadings from the latent variable "spatial skills" to its indicators were significant (mental rotation: *b* = 1.000, CI [1.000, 1.000]; perspective taking: *b* = .955, CI [.308, 1.764]). The paths representing the indirect effects from spatial skills to calculation and word problems through mental number line representation were all significant (spatial skills to mental number line representation: *b* = .097, CI [.016, .182]; mental number line representation to calculation: *b* = .728, CI [.199, 1.287]; mental number line representation to word problems: *b* = .389, CI [.094, .682]). Mental number line representation significantly mediated the relation between spatial skills and calculation (*b* = .070, CI [.004, .165]) as well as that between spatial skills and word problems (*b* = .038, CI [.003, .075]). Considering the insignificant direct paths from spatial skills to calculation (*b* = .036, CI [-.101, .231]) and word problems (*b* = .093, CI [-.009, .234]), we concluded that the mediating effect of mental number line representation was complete. In contrast to our prediction, the path representing the indirect effect from mental number line representation to arithmetic fact retrieval was insignificant (*b* = 1.659, CI [-.709, 4.024]), indicating that mental number line representation is not a significant mediator for the relation between spatial skills and arithmetic fact retrieval (*b* = .160, CI [-.078, .472]. Additionally, the three measures of mathematical abilities were interrelated. Arithmetic fact retrieval was found to be predictive of calculation (*b* = .076, CI [.032, .127]), while calculation was correlated with word-problem solving (*b* = 1.795, CI [.318, 3.026]). Regarding the control variables, visuospatial working memory significantly correlated with spatial skills (*b* = 60.378, CI [16.335, 98.296]), and mental number line representation was predicted by age (*b* = 1.224, CI [.088, 2.298]) and gender (*b* = .847, CI [.136, 1.534]). In the original model without mediators, the predictors altogether explained 14.0% and 27.8% of the variance in calculation and word problems, respectively. After mental number line representation was introduced into the model as a mediator, the predictors altogether explained 31.9% and 35.5% of the variance in calculation and word problems, respectively.

Discussion

The present study aimed to examine the mediating role of mental number line representation in the relation between spatial skills and mathematical abilities. We hypothesized that mental number line representation mediates the relations between spatial skills and various types of mathematical abilities, including arithmetic fact retrieval, calculation, and word-problem solving. The structural equation model with the bootstrap procedure showed that after controlling for age, gender, and visuospatial working memory, mental number line representation fully mediated the relation between spatial skills and calculation and the relation between spatial skills and word-problem solving. However, mental number line representation was not found to be a significant mediator for the relation between spatial skills and arithmetic fact retrieval. The results suggest that mental number line representation plays an important role in explaining the relations between spatial skills and some kinds of mathematical abilities.

Mental Number Line Representation as a Bridge between Spatial Skills and Mathematical Abilities

We hypothesized that mental number line representation would mediate the relation between spatial skills and various types of mathematical abilities. First, our findings show that mental number line representation fully mediated the relation between spatial skills and calculation. This finding is consistent with Gunderson et al. (2012), who showed that children's mental rotation ability at age 5 predicted their performance in an approximate symbolic calculation task at age 8, and such relation was mediated by their mental number line representation at age 6. The present study extended this relation from *approximate* symbolic tasks to *exact* symbolic calculation. It is plausible that good spatial skills facilitate children's development of an accurate and spatially meaningful number line, which in turn supports calculation performance. Mental number line representation has to do with numerical magnitude representation, which is fundamental to arithmetic proficiency (Booth & Siegler, 2008; Fuchs et al., 2013; Siegler & Ramani, 2009). Some children may use the number line as an arithmetic tool by translating arithmetic problems into ordinal relations on the number line (Kucian et al., 2011). Moreover, accurate mental number line representation helps with the detection of implausible answers (Siegler & Lortie-Forgues, 2014) and constrains possible answers to those of approximately correct magnitude, hence reducing errors in calculation (Booth & Siegler, 2008).

Second, we also found that mental number line representation fully mediated the relation between spatial skills and formulation of mathematical sentence for word-problem solving. This is consistent with Träff (2013), who found that children's number line estimation contributes unique variance in word-problem solving. Word-problem solving involves an interplay of reading processes, language comprehension, problem presentation, and calculation processes (Kintsch & Greeno, 1985; Lee et al., 2004; Swanson, 2004). Children who perform poorly in word-problem solving usually struggle with problem presentation (Gonsalves & Krawec, 2014), which is the translation of word problems into personally understandable forms and integration of relevant information into a mathematical, visual, or mental model (Mayer, 1985, 2013). To help children in problem presentation, some researchers have proposed using the number line to illustrate the spatial relationships among different parts of a problem (Gonsalves & Krawec, 2014). For instance, to solve the word problem "Tom's pen costs $36, which is $13 more than Jerry's pen. How much is Jerry's pen?", children can start from 36 on the number line, and then draw an arrow down 13. By translating the word problem onto the number line, children can better organize their mathematical thought process and decrease the amount of text that they have to interpret to work out the mathematical sentence "36 – 13" (Gonsalves & Krawec, 2014). It is plausible that good spatial skills facilitate children's development of the mental number line, which serves as a problem presentation tool for word-problem solving.

In contrast to our prediction, mental number line representation was not found to mediate the relation between spatial skills and arithmetic fact retrieval. One possible reason for this is that the children in our study did not need to rely on mental number line representation for the retrieval of arithmetic facts. Although a previous study suggested that mental number line helps retrieve arithmetic facts from long-term memory (Siegler & Ramani, 2009), the children in our sample may have already overlearned arithmetic facts and thus could simply recall them from memory without relying on any spatial-related strategies. Future studies may verify this by examining children's strategies in retrieving arithmetic facts.

In particular, it is worthwhile to explore whether the mediating role of mental number line representation in the relation between spatial skills and arithmetic fact retrieval depends on the strategies adopted.

Implications

The findings of the present study have both theoretical and practical implications. Theoretically, the findings provide additional support to the mediating role of mental number line representation in the relation between spatial skills and mathematical abilities, beyond approximate calculation task to more sophisticated mathematical abilities such as calculation and formulation of mathematical sentences for word-problem solving. Practically, the results provide additional support to the view that spatial skills training can potentially improve children's mathematical abilities (Cheng & Mix, 2013), possibly through facilitating their development of an accurate and spatially meaningful mental number line representation. This is important because spatial skills are malleable and can be improved with practice (Hawes, LeFevre, Xu, & Bruce, 2015; Uttal et al., 2013). In classrooms, it is recommended that educators use the number line as an instructional method for teaching calculation and word problems. For example, researchers have found that showing children accurate number line representations of the magnitudes of addends and sums could improve their arithmetic learning (Booth & Siegler, 2008). Explicit training on using the number line as an arithmetic tool has also helped children connect the number line knowledge to calculation, yielding promising improvements in calculation (Kucian et al., 2011). Similarly, the number line can serve as a problem presentation tool for word-problem solving (Gonsalves & Krawec, 2014). Through translating a word problem onto the number line, children can better organize their mathematical thought process and reduce the amount of text they need to interpret to work out a mathematical sentence. Further investigation is necessary to explore the efficacy of spatial skills training in improving mathematical abilities and to determine what kinds of mathematical abilities are more responsive to spatial skills training.

Limitations and Future Research Directions

There are a few limitations of the present study. First, performance on word problems was fairly high. As a result, the models were analyzed using the bootstrapping procedure. Still, this limitation may have constrained possible relations between spatial skills and mathematical abilities. Therefore, in future studies, the level of difficulty of the task should be adjusted to an appropriate level. Second, this study was correlational in nature. It did not allow causal inferences regarding the relationships among variables. Third, it is important to keep in mind the relatively small sample size (N = 85) in this study, although the use of bootstrapping method can produce legitimate power for mediation analyses even in small samples of N = 50 (Briggs, 2006; Preacher & Hayes, 2004; Yung & Chan, 1999). Future studies should replicate the current results with a larger sample. Fourth, according to the typology proposed by Uttal et al. (2013), mental rotation and perspective taking may constitute different subtypes of dynamic spatial skills, namely dynamic-intrinsic and dynamic-extrinsic, respectively. Hence, it is worthwhile to further explore the unique association, if any, between mathematical abilities and each dynamic spatial skill subtype using multiple measures for each subtype. Lastly, it should be noted that both calculation and word-problem solving are basic mathematical abilities. Further study is needed to examine whether mental number line representation continues to play a mediating role in the relation of spatial skills and more advanced mathematical abilities (such as fractions and algebra) among older children.

Conclusions

The present study showed that after controlling for age, gender, and VSWM, mental number line representation fully mediated the relation between spatial skills and calculation as well as the relation between spatial skills and word-problem solving. The results highlighted the important role of mental number line representation in bridging the gap between spatial skills and mathematical abilities. Higher spatial skills may facilitate children’s development of an accurate and spatially meaningful mental number line representation, which in turn supports their development of various types of mathematical abilities. The findings here provide a fuller picture of how spatial skills are associated with mathematical abilities in different domains, thereby offering new insights into how children may be helped to enhance their mathematical performance through spatial training.

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Table 1

Descriptive statistics.

	Mean	S.D.	Max.
1. Age	7.69	0.32	---
2. Mental rotation	51.40	9.82	72
3. Perspective taking	36.74	10.95	54
4. Visuospatial working memory	138.71	17.01	210
5. Mental number line representation	93.47	1.76	100
6. Arithmetic fact retrieval	33.94	15.96	120
7. Calculation	19.88	4.25	24
8. Word problems	10.09	2.14	12

Table 2

Correlations among the variables (N = 85).

	1	2	3	4	5	6	7	8
1. Age	-							
2. Mental rotation	0.031	-						
3. Perspective taking	0.057	0.559***	-					
4. Visuospatial working memory	0.176	0.408***	0.239*	-				
5. Mental number line representation	0.235*	0.328**	0.297**	0.216*	-			
6. Arithmetic fact retrieval	-0.044	0.071	0.173	0.176	0.245*	-		
7. Calculation	0.185	0.190	0.215*	0.233*	0.458***	0.427***	-	
8. Word problems	0.113	0.378***	0.431***	0.315**	0.473***	0.234*	0.478***	-

$^{*}p < .05$, $^{**}p < .01$, $^{***}p < .001$