RUNNING HEAD: FRACTION AND DECIMAL

The roles of place-value understanding and nonsymbolic ratio processing system in symbolic rational number processing

Abstract

Background: While it has been widely demonstrated that children's and adolescents' understanding of rational number plays an important role in their mathematics achievement, we have limited knowledge about the cognitive correlates of this understanding.

Aims: The current study aimed at examining whether children's nonsymbolic ratio processing and their understanding of place-value structure of whole numbers play a role in their understanding of fractions and decimals, and whether their roles are different for fractions versus decimal understanding.

Sample: A sample of 124 fourth graders was tested.

Methods: Participants were tested on their symbolic rational number processing, nonsymbolic ratio processing, place-value understanding of whole numbers, mathematics achievement, as well as a series of domain-general and domain-specific cognitive skills related to symbolic rational number processing.

Results: The findings suggest that while the understanding of place-value of whole numbers significantly predicted the understanding of both fractions and decimals, nonsymbolic ratio processing specifically predicted the understanding of fractions, but not decimals.

Conclusions: The findings highlight the roles of place-value understanding and nonsymbolic ratio processing in the acquisition of symbolic rational numbers.

Keywords: fractions, decimals, rational numbers, place-value understanding, nonsymbolic ratio processing system

The roles of place-value understanding and nonsymbolic ratio processing system in symbolic rational number processing

The understanding of rational numbers (e.g., fractions and decimals) has received a lot of attention in the psychological literature recently (e.g., Hurst & Cordes, 2016; Rapp, Bassok, Dewolf, & Holyoak, 2015; Siegler et al., 2012) not only because of its theoretical significance (the understanding of fractions and decimals require an understanding of the core elements of numbers: numerical magnitude; Siegler, Thompson, & Schneider, 2011), but also because of the difficulties faced by many children and adolescents during the learning process (Kloosterman, 2010; Rittle-Johnson, Siegler, & Alibali, 2001) as well as its strong predictive power to both concurrent (Torbeyns, Schneider, Xin, & Siegler, 2015) and future (Bailey, Hoard, Nugent, & Geary, 2012; Siegler et al., 2012) mathematical outcomes.

The theoretical and practical significance of rational number processing has motivated researchers to look for its predictors (Bailey, Siegler, & Geary, 2014; Hansen et al., 2015; Hoof, Verschaffel, & Dooren, 2017; Jordan et al., 2013; Vukovic et al., 2014), with the hope that such information will inform educators about the important elements in the process of learning rational numbers. Despite the fact that these studies examined the understanding of fractions in different grades, similar findings have been obtained from these studies. For example, various domain-general capacities, such as intelligence, working memory, and attention, have been shown to be reliable predictors of children's and adolescents' understanding of fractions (Bailey et al., 2014; Hansen et al., 2015; Jordan et al., 2013; Vukovic et al., 2014). On top of these domain-general cognitive capacities, children's understanding of whole numbers, as reflected by their performance in whole number line and whole number arithmetic, also plays a role in children's and adolescents' mastery of fraction knowledge (Bailey et al., 2014; Hansen et al., 2015; Hoof et al., 2017; Jordan et al., 2013; Vukovic et al., 2014). While these findings have informed us about some of the important elements for learning fractions, there is much room for further exploration within this domain as only a limited number of domain-specific cognitive measures had been included in these studies. In this study, it is proposed that the both the nonsymbolic ratio processing system and the understanding of the place-value structure of whole numbers are important elements that correlate with children's rational number processing.

The nonsymbolic ratio processing system (RPS)

Humans are innately equipped with the cognitive capacity to represent nonsymbolic numerosities. This system, known as the Approximate Number System or the ANS (Dehaene, 2001; Feigenson, Dehaene, &Spelke, 2004), allows us to quickly compare the numerosities of two arrays of objects. With this ANS, we can associate the symbolic whole numbers to the representation of nonsymbolic numerosities, so that the number symbols acquire their meanings (Geary, 2013). Recent studies suggest that ANS is significantly, yet moderately, related to our mathematics outcomes (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2016), probably through providing a basis for the number-numerosity mapping process (Wong, Ho, & Tang, 2016).

If we are equipped with the capacity to represent nonsymbolic numerosity, are we also equipped with the capacity that allows us to represent nonsymbolic ratio? A recent study from Matthews, Lewis, &Hubbard (2016) has provided an affirmative answer to this question. In this study, undergraduates were asked to compare the ratio of black versus white dots and the ratio of the lengths of black versus white lines. Participants' performance was significantly above chance, and all the ratio comparison tasks converged as a single factor. This factor, labelled as the Ratio Processing System (RPS), significantly predicted undergraduates' performance in symbolic fraction tasks beyond the effect of various control variables, such as ANS and line-length acuity. The findings suggest that we are also equipped with an RPS that allows us to represent nonsymbolic ratios, and the acuity of this RPS is related to adults' symbolic fraction magnitude understanding. The current study aimed at extending these findings by examining whether this RPS is available to children, and whether the acuity of this RPS is related to children's processing of symbolic rational numbers.

The understanding of the structure of the symbolic number system

While the ANS and the RPS may provide a foundation for us to represent nonsymbolic numerosity and ratio, the approximate nature of these systems prevent us from accurately representing any exact amount. Our ancestors have therefore developed the symbolic number system that allows us to represent numerosity and ratio in an exact manner. Yet, with an infinitely amount of numerosity and ratio to be represented, it is impossible for us to have one symbol for each numerosity and ratio. What we can do is to use a finite set of symbols (0 to 9) and combine them in different ways (e.g., 14, ¼, and 0.41), and these rules of combining number symbols become our symbolic number system. While this symbolic number system allows us to represent an infinite amount of numerosity and ratio with a small set of symbols, it may also make the acquisition of numbers difficult. For example, it may be difficult for children to understanding why putting a "1" with and "8" forms the number "18", which has a completely different meaning as 1 + 8 (the concept of place value of whole numbers). The mastery becomes more challenging when it comes to rational numbers as it is hard to understand why 1/2 is bigger than 1/3 while 2 is smaller than 3, and it is also difficult to figure out why 0.27 is smaller than 0.8 while 27 is greater than 8. The understanding of place value and the idea of partitioning (dividing quantities into equal-sized groups, the key idea underlying rational number understanding) have been described by Siemon, Bleckly, and Neal (2012) as two of the "six big ideas of numbers". The understanding of these six "big ideas of numbers" are inter-connected (C. Hurst & Hurrell, 2014). The understanding of place value, for instance, consists of the understanding of positional property of numbers: that the quantity represented by the individual digit is determined by the position it holds within the numeral (C. Hurst & Hurrell, 2014). In fact, the position of the digit convey the idea of measurement unit (e.g., the 2 in 26 means 2 units of ten; Sophian, 2013). Such understanding seems to be fundamental to the understanding of rational numbers, which is basically about how new positions of the digits convey novel measurement units (e.g., the 4 in ¼ means that the unit of measurement is now a quarter instead of one; Sophian, 2013). It is therefore reasonable to hypothesize that the understanding of place value of whole numbers would be an important stepping-stone for the acquisition rational numbers. This possibility was put into test in the current study.

Factor structure and the differential prediction of rational number understanding

Before looking more specifically into how the aforementioned factors correlate with children's symbolic rational number understanding, the factor structure underlying the rational number processing tasks needs to be clarified. Based on the findings from previous studies on both whole numbers and rational numbers, a three factor structure was proposed. First, based on the results of the factor analysis with eight different whole number and fraction tasks, Fazio et al. (2014) came up with two distinct factors underlying the numerical magnitude tasks: one capturing all symbolic tasks and the other capturing all nonsymbolic tasks. Yet, no decimal tasks were involved in this study. Although fractions and decimals can be used interchangeably to represent finer magnitudes, the structure of the two notations seems to drive our attention to different aspects of the number. Fractions, with a bipartite structure, allows us to represent the relation between two quantities, which is a unique aspect of fractions that is absent in decimals and whole numbers, and this unique aspect seems to facilitate the processing of relations (DeWolf, Bassok, & Holyoak, 2015a; H. S. Lee, DeWolf, Bassok, & Holyoak, 2016). The relative simplicity of the one-dimensional structure of decimals, on the other hand, seems to make their magnitudes more accessible (DeWolf, Grounds, Bassok, & Holyoak, 2014; H. S. Lee et al., 2016). The study from DeWolf et al.

(2015b) further highlights the uniqueness of fractions against decimals by demonstrating that fraction and decimal processing are related to algebra performance in different ways: decimal processing is related to algebra performance because of its involvement of finer magnitude representation, fraction processing is related to algebra performance because of its relational nature. Such findings seem to suggest that fractions and decimals are processed differently and thus imply two distinct symbolic rational number factors. Together with the nonsymbolic RPS factor (Matthews et al., 2016), a three-factor model is therefore expected. Such factor structure was examined using Confirmatory Factor Analysis (CFA) in the current study.

If fractions and decimals are indeed processed differently (focusing on relations between quantities versus finer magnitudes), their correlates may also be different. Current evidence does not allow us to answer this question as most of the studies focus only on fraction processing (Bailey et al., 2014; Hansen et al., 2015; Jordan et al., 2013; Vukovic et al., 2014). The correlates of decimal processing remain largely unexplored. By examining fourth graders' magnitude understanding of fractions and decimals separately and with the effects of domain-general (intelligence, working memory, and attention) and domain-specific (whole number magnitude processing) cognitive predictors of symbolic rational number processing, as well as the early arithmetic performance, controlled, the current study allowed us to compare the correlates of fraction versus decimal processing. As the understanding of both fractions and decimals involves the mastery of the rules behind the symbolic number system (i.e., how the position of the digit conveys the measurement unit), the mastery of the earlier forms of these rules (i.e., place-value understanding of whole numbers) should predict the understanding of both fractions and decimals. The nonsymbolic RPS, on the other hand, involves the representation of the relation between two quantities, and it is thus expected to be more strongly related to the understanding of fractions (which also involves a two-dimensional structure) compared to the understanding of decimals (which involves one-dimensional magnitude).

Method

Participants

A sample of 210 participants was recruited from 17 kindergartens for the current study. These kindergartens were located in 11 different districts (out of 18) in Hong Kong. The involved districts were rather diverse in terms of district average household income (ranging from district with 2^{nd} highest average household income to district with lowest average household income). All the participants were Cantonese-speaking Chinese. They were assessed three more times (Time 2: middle of Grade 1, Time 3: end of Grade 1, Time 4: end of Grade 4) after the initial assessment (Time 1) in their kindergartens. Due to attrition, the final sample consisted of 124 fourth-graders. Participants in the final sample did not differ from the dropped-out sample in any measures assessed in the early stages (i.e., Times 1 to 3, $ts < 1.9$; $p > .06$). Binary logistic regression suggested that the variables measured at the first three time points did not predict attrition, $\chi 2(6) = 6.709$, $p = .349$, indicating that the data was missing at random (MAR). Participants had a mean age of 10 years and 1 month (S.Ds. = 4 months) at Time 4, at which the rational number processing tasks were conducted. By that time, participants should have received adequate instructions on both fractions and decimals (introduced at Grades 3 and 4 respectively). Both fractions and decimals are usually taught through partitioning of objects (e.g., dividing a pizza into four equal parts, dividing a large square into 100 small squares) and quantities (e.g., dividing eight candies into four equal portions) in the local context. Counting is usually involved in the instructional process.

Measures

A total of 15 measures had been conducted. Except for mathematics achievement and decimal comparison, all the numerical measures were computerised measures. Others were conducted in verbal or paper-and-pencil format.

Symbolic rational number processing.

Fraction comparison. In the fraction comparison task, which was modified from Fazio et al. (2014), participants were shown two fractions, and they had to judge which of the fraction was numerically larger. They were to press the F key when the fraction on the left was larger and the J key otherwise. Similar to Fazio et al. (2014), there were four ratio bins (i.e., 1.15 to 1.28, 1.28 to 1.43, 1.48 to 1.65, 2.46 to 2.71) with 10 items in each ratio bin. The numerators ranged from 5 to 21. To ensure that the participants pay attention to both the numerators and the denominators, the larger fraction had either (a) a larger numerator and an equal denominator, (b) an equal numerator and a smaller denominator, (c) a larger numerator and a larger denominator, (d) a larger numerator and a smaller denominator, or (e) a smaller numerator and a smaller denominator.

Fraction number line. The fraction number line task was modified from Fazio et al. (2014). The participants were presented with a number line with 0 on the left end and 1 on the right end. They also saw a fraction above the number line. Their task was to locate the fraction to the corresponding position on the number line by moving the mouse. A practice trial was provided to ensure participants' understanding. There were 24 experimental trials, with items being evenly distributed on the number line.

Decimal comparison. The decimal comparison task, which was modified from Desmet, Grégoire, and Mussolin (2010), consisted of 10 trials[1]. In each trial, participants had to decide which of the decimals presented was numerically larger, or they had equal numerical values. They then circled the right sign (> / < / =) on the answer booklet. The items varied in

[1] Due to practical constraints, the current version of decimal comparison was conducted on paper and consisted of fewer items in order to save time. Despite these features, the items did vary in a number of ways that capture the individual difference in decimal understanding (e.g., items with incongruent length and digit value, items with zeros; Desmet et al., 2010). Furthermore, both the reliability index (α = .713) and the factor loading to the decimal factor (β = .772, similar to the loadings of other comparison tasks) suggested that the change in these superficial features did not result in any significant impacts on the results.

difficulty. For instance, some of the items were incongruent in both length (the number with more digits was smaller in magnitude) and digit value (the number with larger digit value was smaller in magnitude; e.g., 0.1 vs. 0.02), and some of the items had zeros either in the middle (0.78 vs. 0.078) or at the end (e.g., 0.4 vs. 0.400).

Decimal number line. The decimal number line task was similar to the fraction number line task except that the fractions were replaced by decimal numbers. There was a practice trial followed by 24 experimental trials. The decimal numbers had approximately the same numerical values as the items in the fraction number line task. One third of the items were rounded to the tenth digit (e.g., 0.4), the hundredth digit (e.g., 0.57), the thousandth digit (e.g., 0.182) respectively.

Nonsymbolic RPS.

Nonsymbolic ratio comparison. In the nonsymbolic ratio comparison task, participants were shown two arrays of dots. Within each array, some of the dots were in yellow, while others were in blue. The participants had to select the array in which they had a larger chance of getting a blue dot if they randomly drew a dot from the array, ignoring the size of the dots. They were to press the F key if the left array provided them with a larger chance of getting a blue dot, and the J key otherwise. Participants were explicitly told not to count the dots. The yellow dots and the blue dots occupied the same total area in each pair of dots. There were 40 items in this task, and the ratios involved were the same as those used in the fraction comparison task.

Nonsymbolic ratio number line. The nonsymbolic ratio number line was modified from Fazio et al. (2014). In this task, the participants were presented with a number line. On the left end, they also saw an array of 99 yellow dots and 1 blue dot, and they saw an array of 99 blue dots and 1 yellow dot on the right end. They saw a third array in the middle, showing a ratio of yellow versus blue dots. Participants were asked to judge if they had to randomly draw one dot from the middle array (ignoring the size of the dots), what would be the likelihood that the drawn dot would be blue in colour. They had to locate the chance on the number line, with the left side indicating a tiny chance and the right side indicating a huge chance. A practice trial was first given to the participants to ensure that they understood the task, and it was followed by 24 experimental items. The total area occupied by the yellow dots was the same as that occupied by the blue dots within each array. The ratios involved in the items were the same as those from the fraction number line task.

Place-value understanding. The strategic counting task, adopted from Chan, Au, and Tang (2014), was used to assess participants' understanding of the place-value structure of whole numbers when they were in Grade 1. The task was developed based on the UDSSI (unitary multi-digit, decades and ones, sequence-tens and ones, separate-tens and ones, and integrated sequence-separate-tens) model concerning the development of place value understanding (Fuson et al., 1997). The model suggested that children's understanding of the place value concept could be observed through their counting processes. For example, children who have a more advanced understanding of place value (e.g., separate-tens and ones) would be able to count in tens and hundreds instead of in ones (which reflected unitary multi-digit conception).

The participants were introduced with different ways of grouping some little squares, with each long rectangle representing 10 small squares, and each large square representing 100 small squares. Afterwards, different numbers of small squares were grouped in different ways, and the participants' task was to count the total number of small squares presented in each item. The total number of small squares ranged from 10 to 999. As some of the items involved non-canonical grouping (which was novel to the participants as the usual teaching in the local context involves only canonical grouping), the participants had to have a robust understanding of the place-value system (i.e., they should know how to count in tens and hundreds and then combine them in a flexible manner) in order to get the item correct. The task was shown to be a reliable (Cronbach's $\alpha = .84$) and valid (significantly correlated with other place-value measures and children's mathematics achievement) measure of children's place-value understanding (Chan et al., 2014).

Mathematics achievement. Participants' mathematics achievement was assessed by the Learning and Achievement Measurement Kit 3.0 (LAMK; Hong Kong Education Bureau, 2015). The LAMK is a series of achievement tests constructed by the local Education Bureau as curriculum-based achievement assessments. The fourth-grade mathematics test was used in the current study. The test covered all the topics being taught in the local fourth-grade mathematics curriculum, including number knowledge (e.g., multiples and factors, eight items), numerical magnitude (e.g., converting equivalent fractions and comparing fractions and decimals, six items), fraction arithmetic (four items), word problems (10 items), measures (e.g., finding out the area of figure, six items), shapes and space (e.g., properties of quadrilaterals, 10 items), and data handling (e.g., graph reading, six items). Participants were given 45 minutes to complete the 50 items in the test.

Control variables. A total of eight measures were conducted as control measures. An arithmetic task (Wong, Ho, & Tang, 2014) was included as a proxy of children's mathematics achievement in Grade 1. The measures on whole number magnitude included the nonsymbolic comparison task (Piazza et al., 2010), the whole number line task (Siegler & Booth, 2004), and the whole number comparison task (Butterworth, 2003). Nonverbal intelligence, working memory, reading skills, and the attention level of the participants were measured by the short form (sets A, B, and C) of Raven's standard progressive matrices (Raven, 2006), backward digit span, a locally standardized word reading test ("The Hong Kong Test of Specific Learning Difficulties in Reading and Writing for Primary School Students-Second Edition," n.d.), and the inattention and hyperactivity subscale of the Chinese version of the Strengths and Difficulties Questionnaire[2] (Lai et al., 2010) respectively (see Supplementary materials for details of these measures).

Scoring.

Except for all the number line tasks (using percentage absolute errors, or PAE, obtained through the formula $\frac{|estimate-target|}{range}$, as the indicator. For instance, if a child marked on a location of 0.4 when he/she was asked to estimated 0.25 out of a 0 to 1 number line, then his/her PAE would be $\frac{|0.4-0.25|}{1}$ = 0.15), the whole number comparison task (using reaction time as the indictor), and the Raven's Standard Progressive Matrices (using scaled score as the indicator), performance was measured in terms of accuracy.

Procedures

Parental consents were obtained before the participants were first tested in their own kindergartens at Time 1. From Time 2 onwards, the participants were tested in their own homes. While the symbolic rational number tasks, the nonsymbolic ratio processing tasks and the mathematics achievement test were conducted at Time 4, the place-value understanding measure was conducted at Time 3. The control measures were conducted at either Time 1 (working memory), Time 2 (intelligence), Time 3 (arithmetic, number-related cognitive measures and reading), or Time 4 (inattention rating). All the tests were administered by either trained psychology undergraduates or the author. Each testing took approximately two hours. As a token of appreciation for the participants' effort, either souvenirs or supermarket coupons were gifted to them after each assessment.

Results

The data was first standardized and screened for the outliers. Data points which were 3

[2] This subscale measured the level of inattention and hyperactivity, with a higher score indicated a higher level of inattention

S.D. beyond the corresponding means were winsorized (replaced by a Z score of 3 or -3)[3]. The means, standard deviations, reliabilities (Cronbach's α), as well as the correlations among the variables, were presented in Table 1. Participants' performance in all symbolic rational number and nonsymbolic ratio processing tasks were significantly above chance (ts > 15, ps < .001), suggesting that they have basic understanding of symbolic rational number magnitude and the ability to process nonsymbolic ratios. Among the number line tasks, participants seemed to perform better when fractions (PAE = 12.94) instead of decimals (PAE = 17.37) or nonsymbolic ratio (PAE = 17.22) were involved. The poor performance in the decimal number line task could be explained by the fact that some of the errors made by the participants resulted in huge PAE (e.g., locating .9 to the position of .009, resulting in an error of .891). This kind of errors was rare in the fraction number line task. The worse performance in the nonsymbolic ratio number line task, on the other hand, could be due to the lack of symbols in this task, resulting in relatively imprecise judgements. All the symbolic rational number tasks were correlated ($|r|$s > .26). Children's mathematics achievement correlated with all the variables included in the current study ($|r|$s > .21, ps < .05).

Factor structure of rational number processing

Although it was expected that fraction, decimal, and nonsymbolic ratio tasks formed three distinct factors, other factor structures (e.g., all symbolic rational number and nonsymbolic ratio tasks converged to a single factor) remained possible. To confirm the factor structure underlying the symbolic rational number and nonsymbolic ratio processing tasks, various CFAs were conducted. A total of five possible factor structures were proposed. The first possibility was that all the six symbolic rational number and nonsymbolic ratio tasks converged to a single factor. The second possibility was that all the symbolic tasks converged to a symbolic rational number factor, while the two nonsymbolic ratio tasks formed a

[3] The results remained similar when listwise deletion was employed.

nonsymbolic ratio factor. The symbolic rational number factor maybe further divided in different ways based on either the format of numerical stimuli involved (i.e., purely symbolic versus symbolic-nonsymbolic mapping; the third possibility) or the notation of rational numbers involved (i.e., fractions versus decimals; the fourth possibility). Finally, the symbolic rational number and nonsymbolic ratio tasks may also be divided based on the task nature (i.e., comparison tasks vs. number line tasks).

The fit indices of the five models and the model evaluation criteria were listed in Table 2. Among all the five models, only model 4 (three factor structure: fraction, decimal, and nonsymbolic ratio) fit the data well, $\chi^2(6) = 1.601$, CFI = 1.000, RMSEA = .000, SRMR = .016. The corresponding factor loadings were presented in Table 3. As the correlations between factor scores (Bartlett scores) and the composite scores (averaging the standardized scores of the relevant component tasks) were very high (rs > .96), the composite scores were used in the following analyses for easier interpretation. Similar results were obtained when factor scores were used (see Supplementary Table).

Predicting fraction and decimal magnitude understanding

To examine the contributions of place-value understanding and the nonsymbolic RPS to the understanding of fraction and decimal magnitude, two sets of hierarchical regressions were conducted, with fraction and decimal composites being the dependent variables respectively. The domain-general predictors were put in block 1, and the early whole number predictors were put in block 2. Arithmetic, which served as a proxy of children's early mathematics achievement, was put in block 3. Place-value understanding and the nonsymbolic ratio composite were put in block 4. The results suggested that both place-value understanding and nonsymbolic RPS uniquely predicted fraction magnitude understanding ($\beta = .244$, $p = .010$ for place-value, $\beta = .299$, $p < .001$ for nonsymbolic RPS). Among the controlled variables, only nonsymbolic comparison turned out to be significant ($\beta = .164$, p

= .036). The model accounted for 42.4% of the variance in fraction magnitude understanding (see Table 4 for details).

A slightly different pattern emerged for the prediction of decimal magnitude understanding. While place-value understanding remained as a significant predictor ($\beta = .281$, $p = .004$), the nonsymbolic RPS failed to predict decimal magnitude understanding ($\beta = .049$, $p > .250$). Other than place-value understanding, children's word reading skills ($\beta = .182$, $p = .035$) also turned out to be a significant predictor of decimal magnitude understanding. The model accounted for 39.8% of the variance in decimal magnitude understanding.

Predicting mathematics achievement

To provide readers with more practical information concerning to what extent different kinds of rational number understanding contribute to children's mathematics achievement in Grade 4, another hierarchical regression was conducted, with children's mathematics achievement being the dependent variable. The first four blocks of predictors were the same as the previous hierarchical regressions conducted, and fraction and decimal magnitude composites were put in block 5. Fraction magnitude understanding significantly predicted children's mathematics achievement in Grade 4 ($\beta = .256$, $p = .002$), while the predictive power of decimal magnitude understanding was marginally significant ($\beta = .159$, $p = .051$). The two variables together accounted for 6.8% of the variance in children's mathematics achievement in Grade 4. After accounting for the fraction and decimal magnitude understanding, place-value understanding ($\beta = .182$, $p = .027$) and arithmetic ($\beta = .200$, $p = .015$) were still significant predictors of children's mathematics achievement, while nonsymbolic RPS was not ($\beta = -.060$, $p > .250$). Word reading was also a significant predictor of children's mathematics achievement ($\beta = .142$, $p = .048$). The variables together accounted for 60.8% of the variance in children's mathematics achievement in Grade 4.

To further examine whether the covariance between rational number magnitude and mathematics achievement was due to the involvement of rational number items in the mathematics achievement test, the aforementioned regression model was repeated after removing these rational number items from the mathematics achievement variable. The overall results were largely similar, except that decimal magnitude understanding was no longer a significant predictor of mathematics achievement (β = .118, p = .169). Other predictors, such as fraction magnitude understanding (β = .256, p = .004), place-value understanding (β = .209, p = .017) and arithmetic (β = .221, p = .011) remained to be significant predictors of mathematics achievement. The fraction magnitude understanding alone accounted for 4.4% of the variance in children's mathematics achievement. The whole model accounted for 56.0% of the variance in children's mathematics achievement.

Discussion

The current study examined the correlates of fourth graders' symbolic rational number processing. After controlling for the effects of domain-general cognitive capacities, the whole number magnitude knowledge, as well as their arithmetic skills, children's place-value understanding in Grade 1 significantly predicted their fraction and decimal magnitude understanding at Grade 4, while the nonsymbolic RPS contributed only to the understanding of fractions, but not decimals, at Grade 4. Fraction magnitude understanding uniquely contributed to children's concurrent mathematics achievement in Grade 4, even after all the rational number items from the test were removed. Theoretical and practical implications were discussed.

The possible factor structure of rational number processing

The current literature on numerical cognition has suggested some theoretical possibilities concerning the potential factor structures behind our rational number understanding. The findings from Fazio et al. (2014), for instance, suggest that whole and rational number tasks can be categorized into two major factors: symbolic versus nonsymbolic. On the other hand, various studies have shown that fractions and decimals may be processed differently. Fractions, due to the two-dimensional structure, seem to facilitate the processing of relations, while the one-dimensional decimals enable more efficient processing of magnitude (DeWolf et al., 2015a, 2014; H. S. Lee et al., 2016). The results of the CFA on the six rational number and nonsymbolic ratio tasks in the current study seem to resemble those from Fazio et al. (2014) in some ways and those from DeWolf et al. (2015a) and Lee et al. (2016) in other ways. First, there is a dividing line separating the nonsymbolic tasks from the symbolic ones. The convergence of all nonsymbolic tasks into a single nonsymbolic ratio factor echoes with the nonsymbolic RPS factor observed in Matthews et al.'s (2016) study. Second, the symbolic rational number construct can be further divided based on notations (i.e., fractions versus decimals). This finding echoes with the differences observed in the processing of fractions versus decimals from previous studies (DeWolf et al., 2015a; H. S. Lee et al., 2016), and the use of different analytic method provides further converging evidence to confirm such differences.

The roles of fraction and decimal processing in children's mathematics achievement

The strong relation between children's rational number processing and their mathematics achievement is one of the driving forces for investigation on children's rational number processing. In the current study, the fraction and decimal magnitude factors demonstrated strong correlations with children's mathematics achievement (*r*s = .59 and .51 respectively). These figures converge with those from previous studies (r ranging from .55 to .87, Siegler et al., 2012; Torbeyns et al., 2014) and confirm the strong relation between rational number processing and our mathematics achievement. The current findings further extend the previous findings by showing that such relation is observed as early as in Grade 4.

On top of replicating the strong relations between rational number processing and mathematics achievement, the current findings revealed the differential roles of the two rational number magnitude factors to children's mathematics achievement. While both fraction and decimal magnitude factors contributed to children's mathematics achievement when all the items were included (accounting for 6.8% of the variance in total), only fraction magnitude factor predicted mathematics achievement when the rational number items in the test were removed (accounting for 4.4% of the variance). It seems that while decimal magnitude understanding is more specifically related to the processing of rational numbers, the role of fraction understanding to mathematics learning seems to be broader. Given that a lot of mathematical problems involve the processing of relations (e.g., identifying the relations between different quantities within a word problem; Wong, 2018), the bipartite nature of fractions, which directs people's attention to the relational aspects of quantities (DeWolf et al., 2015a; H. S. Lee et al., 2016), may explain the relation between fraction understanding and mathematics learning in general. Such speculation remains to be confirmed by future studies.

The significance of place-value understanding and nonsymbolic RPS to symbolic rational number processing

The major goal of the current study was to explore the correlates of children's fraction versus decimal processing, with the nonsymbolic RPS and the understanding of place-value structure of whole numbers being the foci. Children's understanding of place-value of whole numbers was related to the understanding of both fraction and decimal magnitudes, while their nonsymbolic RPS was related to their understanding of fraction, but not decimal, magnitude. This differential prediction pattern seems reasonable when we trace back to the nature of fractions versus decimals. The place-value structure of whole numbers is the first symbolic number rule that children need to acquire in order to understand how the position of the digit conveys the measurement unit. A robust mastery of this basic rule should set up a stronger foundation for the mastery of more advanced combination rules within the symbolic number system (i.e., how new positions convey new measurement units). As the acquisition of both fractions and decimals require the learners to master some new positional rules concerning the symbolic number system, it makes sense that children's understanding of place-value structure of whole numbers plays a role in the understanding of both fractions and decimals. On the other hand, the structural difference between fractions and decimals may explain the differential prediction from the nonsymbolic RPS. Fractions, owing to a bipartite structure, facilitate the processing of relations between quantities (DeWolf et al., 2015a; H. S. Lee et al., 2016). In one-dimensional decimals, however, the focus is on magnitudes instead of the relations between magnitudes (DeWolf et al., 2014). As the focus of the nonsymbolic RPS seems to lie in the ratio between quantities, it is reasonable to expect a specific linkage between this construct and the understanding of fractions.

Limitations

Readers may want to note the following limitations when they interpret the current findings. The first one concerns about the correlational nature of the current study, and causality could not be claimed based on the current findings. The second one is about the relatively low reliability of the nonsymbolic ratio comparison task. This could be due to the dichotomous nature of the task (Sun et al., 2007), resulting in a relatively large effect of guessing (Grosse, 1985). While the dichotomous nature is hard to change, future studies may include more items in this task in order to yield a more satisfactory reliability. The third one concerns about the time point at which the measures were conducted. Due to time constraints, the working memory measure was conducted when the participants were in kindergarten. Yet, given that working memory was shown to be a highly stable construct (K. Lee & Bull, 2015) and that the correlation between working memory and mathematics achievement in the current study (i.e., .31) fell within the 95% confidence interval [.30, .37] found in a recent meta-analysis on the same topic (Peng, Namkung, Barnes, & Sun, 2016), the difference in measurement period was not expected to affect the current findings in any significant ways. Future studies may further examine this issue by using a concurrent working memory measure.

Educational implications

Given the difficulties that children, adolescents, and adults have in processing rational numbers (Kloosterman, 2010; Siegler & Lortie-Forgues, 2015), the identification of the correlates of different kinds of rational number magnitude understanding is helpful to educators in different ways. For instance, educators may now have more cues on the reasons behind children's difficulty in understanding rational numbers. Some of the students may have such difficulty because they are confused about the positional rules behind the symbolic number system, while others may have difficulty in processing fractions because of an imprecise nonsymbolic RPS. Knowing the potential issues in processing rational numbers enables teachers to put more focused attention to the important aspects that affect students' understanding of rational number.

More importantly, the current study has shed light on potential interventions that may help children to overcome the difficulties in mastering rational numbers. With children's place-value understanding being a significant predictor of both kinds of rational number magnitude understanding and mathematics achievement, future studies may investigate whether the effects of the interventions on place-value understanding could be generalized to rational number understanding. If that is the case, educators should devote more attention to ensure that children fully understand these basic number rules. The potential benefits of place-value interventions have been demonstrated in previous studies. For example, by drawing the equivalence of grouping straws into bundles (10s) and glasses (100) and grouping 1s into 10s and 100s, Ho and Cheng (1997) successfully improved children's understanding of place-value as well as their addition skills. Mix, Smith, Stockton, Cheng, and Barterian (2016) further demonstrated that different kinds of place-value training may help children with different abilities to master the place-value concept. The training that involves base-10 blocks seems to be particularly helpful to those who start with lower level of place-value understanding, while the training that involves only symbols are more beneficial to those higher achievers. These interventions may help prevent children from having difficulties in the acquisition of rational numbers at a later stage of development. Interventions that target at the processing of nonsymbolic ratios, on the other hand, may benefit students who have more specific difficulties in processing fractions. Although such intervention is not yet available for the time being, the successful attempts to train our ANS (Hyde, Khanum, & Spelke, 2014; Park & Brannon, 2016) encourage researchers to develop similar interventions for the nonsymbolic RPS. These interventions, together with those targeting at rational number processing itself (Fuchs et al., 2016, 2013), may be considered as means to minimize children's difficulty in learning rational numbers in the future.

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The roles of place-value understanding and nonsymbolic ratio processing system in symbolic rational number processing

Table 1 – Descriptive statistics and correlations among variables.

	Mean	SD	Range	α	Correlations													
					1	2	3	4	5	6	7	8	9	10	11	12	13	14
1. FC (4b)	30.55	4.78	15 - 40	.752	--													
2. FNL (4b)	12.94	9.28	3.29 - 57.43	.912	**-.334**	--												
3. DC (4b)	7.91	2.30	2 - 10	.713	**.352**	**-.487**	--											
4. DNL (4b)	17.37	11.13	2.68 - 42.86	.892	**-.268**	**.455**	**-.519**	--										
5. NRC (4b)	32.94	3.13	19 - 38	.489	**.240**	**-.270**	.142	-.176	--									
6. NRNL (4b)	17.22	7.90	6.4 - 47.16	.808	**-.352**	**.423**	**-.233**	**.218**	**-.313**	--								
7. PV (1b)	7.89	2.38	1 - 10	.822	**.186**	**-.519**	**.456**	**-.412**	.118	**-.224**	--							
8. AR (1b)	24.24	7.39	10 - 48	.908	**.281**	**-.413**	**.364**	**-.413**	**.272**	**-.276**	**.454**	--						
9. NC (4b)	37.26	3.66	27 - 45	.616	**.309**	**-.228**	**.245**	**-.179**	**.231**	**-.227**	.051	**.206**	--					
10. WNL (1b)	7.64	3.13	2.73 - 16.91	.829	-.169	**.373**	**-.256**	**.353**	**-.253**	.165	**-.433**	**-.458**	-.076	--				
11. WNC (1b)	1.08	.34	-.06 - 2.07	.960	**-.281**	.175	**-.292**	**.211**	**-.322**	**.252**	**-.212**	**-.395**	-.141	.077	--			
12. IQ (1a)	111.65	12.70	84 - 135	.841	**.263**	**-.362**	**.374**	**-.364**	.143	**-.329**	**.401**	**.479**	**.229**	**-.354**	**-.194**	--		
13. WM (Kb)	5.36	1.37	2 - 11	.503	.056	-.063	.137	-.165	.036	-.056	**.309**	**.345**	-.066	**-.279**	-.008	**.243**	--	
14. WR (1b)	72.08	27.27	6 - 131	.980	**.239**	**-.274**	**.424**	**-.309**	.100	-.104	**.394**	**.412**	.155	**-.355**	**-.224**	**.252**	**.234**	--
15. Inatt. (4b)	4.94	2.21	0 -10	.686	-.113	.157	-.165	.131	-.067	.154	**-.352**	-.147	-.100	.053	.120	-.123	-.122	-.142
16. LAMK (4b)	40.02	8.33	8 - 50	.897	**.443**	**-.510**	**.580**	**-.453**	**.213**	**-.282**	**.605**	**.580**	**.240**	**-.378**	**-.287**	**.462**	**.307**	**.490**

FC = fraction comparison, FNL = fraction number line, DC = decimal comparison, DNL = decimal number line, NRC = nonsymbolic ratio comparison, NRNL = nonsymbolic ratio number line, PV = place-value, AR = arithmetic, NC = nonsymbolic comparison, WNL = whole number line, WNC = whole number comparison, IQ = nonverbal intelligence, WM = working memory, WR = word reading, Inatt. = inattention rating, LAMK = mathematics achievement test, the numbers and the letter in the bracket indicated the grade and semester (a = semester 1, b = semester 2) at which the measure was conducted.

p <.05 when $|r|$>.177; p<.001 when $|r|$>.309, correlations that are significant at p = .05 are bolded

Table 2 –Fit indices of the CFA models

Model	df	χ^2	AIC	BIC	CFI	RMSEA	SRMR
1. One factor	9	16.838	1952.343	2002.962	.928	.084	.055
2. Two factors (symbolic vs. nonsymbolic)	8	13.863	1949.898	2003.329	.946	.077	.047
3. Three factors (purely symbolic, symbolic-nonsymbolic mapping, purely nonsymbolic)	6	13.995	1953.348	2012.404	.927	.104	.049
4. Three factors (fractions, decimals, nonsymbolic ratio)	6	1.601	1941.350	2000.406	1.000	.000	.016
5. Two factors (comparison vs. number line)	8	16.784	1954.157	2007.589	.920	.094	.054

Based on Hu and Bentler (1999), the model fitness was evaluated based on the following criteria: insignificant Chi-squared (χ2) test, Comparative Fit Index (CFI) > .95, Root Mean Square Error of Approximation (RMSRA) <.006, and Standardized Root Mean Square Residual (SRMR) <.08. Models that satisfied the above criteria were suggested to fit the data well.

Table 3 – Factor loadings of the rational number variables

	Fraction factor	Decimal factor	Nonsymbolic ratio factor
Fraction comparison	.495		
Fraction number line	-.665		
Decimal comparison		.772	
Decimal number line		-.672	
Nonsymbolic ratio comparison			.451
Nonsymbolic ratio number line			-.685

Table 4 – Multiple regression predicting fraction magnitude, decimal magnitude, and math achievement

Step	Predictor	Fraction magnitude			Decimal magnitude			Math achievement			Math achievement (without rational number items)		
		ΔR^2	β	sr^2	ΔR^2	β	sr^2	ΔR^2	β	sr^2	ΔR^2	β	sr^2
1	Nonverbal intelligence	.209***	.088	.005	.288***	.160	.018	.399***	.053	.002	.359***	.039	.001
	Working memory		-.100	.008		-.031	.001		.083	.005		.073	.004
	Word reading		.092	.006		.182	.024*		.142	.014*		.133	.013
	Inattention rating		.011	<.001		.012	<.001		-.101	.008		-.088	.007
2	Nonsymbolic comparison	.096**	.164	.023*	.050*	.116	.011	.045*	.044	.002	.037	.052	.002
	Whole number line		-.049	.002		-.046	.001		.020	<.001		.020	<.001
	Whole number comparison		-.030	.001		-.089	.006		-.043	.001		-.022	<.001
3	Arithmetic	.017	.101	.006	.011	.094	.005	.046**	.200	.022*	.051**	.221	.027*
4	Place-value	.101***	.244	.034*	.048*	.281	.047**	.050**	.182	.018*	.056**	.209	.023*
	Nonsymbolic ratio		.299	.067***		.049	.002		-.060	.002		-.090	.005
5	Fraction magnitude							.068***	.256	.035**	.057**	.256	.035**
	Decimal magnitude								.159	.014†		.118	.008
	Total R^2	.424			.398			.608			.560		

† *p* < .06, * *p* < .05, ** *p* < .01, *** *p* < .001

All the variance inflation factors (VIF) in these regression models were smaller than 2, suggesting that multicollinearity was not an issue in these regression models.