

# Source Depletion Analogy for Reactive Plume Dispersion over **Schematic** Urban Areas

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## Abstract

Gaussian plume models have been used to estimate pollutant distribution for decades. In view of the empirically determined dispersion coefficients (largely based on atmospheric stability), their application in urban setting needs to be interpreted cautiously. It is even more complicated if chemically reactive pollutants are considered. In this technical note, we examine the reactive plume dispersion over schematic urban areas in attempt to excel the functionality of the conventional Gaussian plume models. Open-channel flows over an array of identical ribs in crossflows serve the theoretical platforms of atmospheric surface layer (ASL) over buildings. The irreversible ozone  $O_3$  titration oxidizes nitric oxide NO to nitrogen dioxide  $NO_2$ , representing the typical anthropogenic air pollution chemistry. Large-eddy simulation (LES) is employed to calculate the flows and pollution physics/chemistry coupling around/over the explicitly resolved roughness elements. The LES results show that, unlike the (larger) mesoscale ones, the conventional approach of modifying dispersion coefficients in terms of the timescales of pollution physics/chemistry is inapplicable due to inhomogeneous vertical mixing. We thus switch to the source depletion analogy which, however, estimates well the NO concentrations only above the plume rise mean height. A noticeable discrepancy is caused by the dominated NO oxidation in the near-wall region. Finally, the regression of LES output shows that the vertical dimensionless NO concentrations exhibit the Gamma  $\gamma$ -distribution for a range of background  $O_3$  concentrations, unveiling a new, primitive parameterization of reactive plume dispersion over urban areas.

*Keywords:* Dispersion coefficient  $\sigma_z$ , Gaussian plume models, large-eddy simulation (LES), ozone  $O_3$  titration, reactive nitric oxide NO plume transport and schematic urban areas.

## 1 Introduction

Air pollution poses major threat to premature mortality (Lelieveld et al. 2015) but its levels over 80% of the cities in the world are unhealthy (WHO 2016). Most air pollutants

4 are chemically reactive that evolve to their secondary counterparts in the atmospheric  
5 boundary layer (ABL). The conventional [Gaussian](#) plume model (Roberts 1923) has been  
6 widely employed in practical problems (Moreira et al. 2006), regulatory enactment (Briant  
7 et al. 2013), air toxic assessment (Scheffe et al. 2016) as well as continental pollutant trans-  
8 port (Tsuang et al. 2003). Its results, however, must be interpreted cautiously because  
9 of the inert-pollutant assumption (Harrison and McCartney 1980) and the complicated  
10 near-wall turbulent transport processes in the atmospheric surface layer (ASL; Britter  
11 and Hanna 2003).

12 In view of dense buildings, dispersion schemes have been developed to handle the  
13 rapid mixing in urban [ASLs](#) (Briggs 1973). The widening plume coverage is attributed  
14 to the elevated turbulence kinetic energy (TKE) in response to ground-level aerodynamic  
15 resistance (Walcek 2002). Early Gaussian plume models adopted the power-law wind  
16 profile (Sharma and Myrup 1975) together with the empirically determined dispersion  
17 coefficients (Skupniewicz and Schacher 1986) to handle the enhanced pollutant transport  
18 over urban areas. Whereas, the solution approach was basically site specific (Venkatram  
19 et al. 2005) that hindered from the understanding of fundamental mechanism. Extensive  
20 field measurements (Mavroidis and Griffiths 2001), laboratory experiments (Chung et al.  
21 2015) and mathematical modeling (Inagaki et al. 2012) have been conducted to elucidate  
22 the influence of rough surfaces on ASL transport processes. However, the [functional form](#)  
23 [of plume dispersion](#) is not yet developed likely because of complicated urban morphology.

24 ABL pollutants are seldom mixed uniformly with ambient air in view of [the](#) inho-  
25 mogeneity in both flows and sources (Georgopoulos and Seinfeld 1986). Chemical kinetics  
26 and dynamics are therefore coupled with each other to modify the pollutant compo-  
27 sitions which, however, are often ignored in reactive plume dispersion models (Chiogna  
28 et al. 2010). Mathematical modeling has been adopted for air pollution [physics/chemistry](#)  
29 coupling, such as the chemical evolution of nitrogen oxides  $\text{NO}_x$  ( $= \text{NO} + \text{NO}_2$ ; where  
30  $\text{NO}$  and  $\text{NO}_2$  are nitric oxide and nitrogen dioxide, respectively), for [decades](#) (Lamb and  
31 Seinfeld 1973). In the engineering community, modeling turbulent transport by computa-  
32 tional fluid dynamics (CFD) is generally grouped into stochastic (Bullin and Dukler 1974),

33 deterministic (Wang and Zhang 2009) and large-eddy simulation (LES; Tseng et al. 2006).

34 Simple, non-CFD models, such as AERMOD (USEPA 2016) and CALINE4 (Benson  
35 1984), possess the benefit of quick solution together with the ability performing multiple  
36 simulations concurrently. To the authors' best knowledge, perhaps due to the challenging  
37 pollution physics/chemistry coupling in inhomogeneous flows, there is no non-CFD model  
38 developed so far for reactive plume dispersion over urban areas. It therefore motivates  
39 our interest in rapid, non-CFD modeling techniques for pollutant-concentration estimates.

40 This section introduces the problem background and reviews the existing literature. The  
41 theory of reactive plume dispersion is detailed in Section 2. The solution approach, com-  
42 putation configuration and numerical methods are described in Section 3. The findings,  
43 especially the turbulent transport processes and the newly proposed non-CFD, reactive  
44 plume parameterization (Gamma  $\gamma$ -distribution), are reported in Section 4. Conclusions  
45 are finally drawn in Section 5.

## 46 2 Theoretical Background

47 The irreversible ozone  $O_3$  titration



48 is considered where  $O_2$  is oxygen molecule and  $k_3$  ( $= 0.411 \text{ ppm}^{-1} \text{ sec}^{-1}$ ) the chemical  
49 reaction rate constant at 293.15 K. The consumption (production) rates of nitric oxide  
50 and ozone (nitrogen dioxide) by chemistry Equation (1) are equal to

$$\frac{d\bar{c}_{NO}}{dt} = \frac{d\bar{c}_{O_3}}{dt} = -\frac{d\bar{c}_{NO_2}}{dt} = -k_3 \times \bar{c}_{NO} \times \bar{c}_{O_3} \quad (2)$$

51 where  $c_\phi$  is the concentration of chemical species  $\phi$  and overbar  $\bar{\psi}$  the resolved-scale com-  
52 ponent in the LES. Hence, the chemistry timescales of nitric oxide  $\tau_{NO} = 1/k_3\bar{c}_{O_3}$  and  
53 ozone  $\tau_{O_3} = 1/k_3\bar{c}_{NO}$ . Given the model configuration of uniform inflows doped with a  
54 constant background ozone concentration  $[O_3]_0$ , our preliminary LES results show that  
55 the ozone consumption  $\Delta [O_3]$  ( $= [O_3]_0 - \langle \bar{c}_{O_3} \rangle$  where angle brackets  $\langle \psi \rangle$  denote the en-  
56 semble averaged properties) is less than 10%. We therefore essentially assume a constant  
57 chemistry timescale of nitric oxide  $\tau_{NO} = 1/k_3 [O_3]_0$  in the following analyses.

## 3 Methodology

### 3.1 Mathematical model

LES of the open-source CFD code Open-FOAM 2.3.0 (OpenFOAM 2015) is used in this technical note. The flows are assumed to be isothermal and incompressible that are calculated by the continuity and the Navier-Stokes equations in filtered variables. Source terms  $\mathcal{S}_\phi$ , which handle the chemical kinetics in the irreversible ozone titration Equation (1), are integrated into the filtered convection-diffusion equation

$$\frac{D}{Dt}\phi = \mathcal{D}(\phi) + \mathcal{S}_\phi \quad (3)$$

for the transport of chemical species  $\phi$  where  $D/Dt$  is the material derivative and  $\mathcal{D}(\phi)$  the diffusion term. Here, the source terms in the transport equations of NO, NO<sub>2</sub> and O<sub>3</sub> are  $\mathcal{S}_{\text{NO}}$  ( $= -k_3\bar{c}_{\text{NO}}\bar{c}_{\text{O}_3}$ ),  $\mathcal{S}_{\text{NO}_2}$  ( $= k_3\bar{c}_{\text{NO}}\bar{c}_{\text{O}_3}$ ) and  $\mathcal{S}_{\text{O}_3}$  ( $= -k_3\bar{c}_{\text{NO}}\bar{c}_{\text{O}_3}$ ), respectively. Hence, the source terms for NO and O<sub>3</sub> are consumptions while NO<sub>2</sub> production. The subgrid-scale (SGS) motions are modeled by the Smagorinsky model (Smagorinsky 1963). The one-equation SGS model is employed to enforce SGS TKE conservation (Schumann 1975). Only resolved scales are included in the source terms  $\mathcal{S}_\phi$ . The timescales are compared by the dimensionless Damköhler number  $Da$  ( $= \tau_p/\tau_\phi$ ; where  $\tau_p$  and  $\tau_\phi$  are the timescales of physical and chemical processes, respectively; Janssen et al. 1990). We focus on the mixing processes so the diffusion timescale  $\tau_d$  is used to measure the physical processes. We look into the reactive plume dispersion of nitric oxide so its chemical timescale  $\tau_{\text{NO}}$  is used to measure the chemical processes.

The LES model for schematic urban area consists of a number of idealized urban street canyons fabricated by identical square ribs of size  $h$  (Figure 1). The spatial domain sizes  $72h$  (length)  $\times$   $12h$  (width)  $\times$   $12h$  (height) that is composed of 36 idealized street canyons of the same geometry. The street width  $b$  is the same as the building height  $h$  so the building-height-to-street-width (aspect) ratio is equal to unity. The flows thus fall into the skimming flow regime (Oke 1988). The prevailing flows in the urban ASL are driven by the (background) pressure gradient perpendicular to the street axes, representing the worst scenario of pollutant removal from street canyons. Ensemble average of the LES-calculated

85 pollutant concentrations  $\langle \bar{c}_\phi \rangle$  is applied in the homogeneous spanwise  $y$  direction in  
 86 the data analyses. The infinitely long streamwise  $x$  domain for flows is constructed by  
 87 periodic boundary conditions (BCs). Wall BCs are applied on all the solid boundaries and  
 88 shear-free BCs ( $\partial \bar{u} / \partial z = \partial \bar{v} / \partial z = \bar{w} = 0$ ) along the domain top  $z = H$ . The prevailing  
 89 wind enters the computational domain from the upstream inflow doped with a constant  
 90 background ozone concentration  $[\text{O}_3]_0$ . The sensitivity to NO chemistry timescales  $\tau_{\text{NO}}$   
 91 is tested by controlling  $[\text{O}_3]_0$  (Table 1). An area source of nitric oxide with constant  
 92 concentration  $[\text{NO}]_0$  is placed on the ground surface of the first street canyon that serves as  
 93 a reactive pollutant being continuously emitted into the computational domain, simulating  
 94 the vehicular exhaust in urban areas. Neumann BCs of pollutants ( $\partial \bar{c}_\phi / \partial z = 0$ ) are  
 95 applied on the remaining solid and shear-free boundaries. An open BC of pollutants  
 96 ( $\partial \bar{c}_\phi / \partial t + \bar{u} \partial \bar{c}_\phi / \partial x = 0$ ) is prescribed at the downstream outflow so all the chemical species  
 97 are removed from the computational domain without reflection. The entire computational  
 98 domain is discretized into  $4.6 \times 10^6$  (hexahedral) cells approximately.

### 99 3.2 Modified Dispersion Coefficient

100 Analytical solution, following Sutton (1953) and Seinfeld (1986), to the advection-  
 101 diffusion equation of a first-order chemically reactive pollutant at a consumption rate  
 102 proportional to its local concentration  $d\bar{c}_\phi/dt = -\mathcal{L}\bar{c}_\phi = -k_3 \times [\text{O}_3]_0 \times \bar{c}_{\text{NO}}$  in reactive  
 103 nitric oxide plume exhibits the Gaussian form in which the modified pollutant dispersion  
 104 coefficient is given by (Wu and Liu 2016a)

$$\hat{\sigma}_z(x) = \frac{\sigma_z(x)}{(1 + 2\mathcal{K}\mathcal{L}/U^2)^{1/2}} = \frac{\sigma_z(x)}{[1 + 2(\tau_a/\tau_d \times \tau_a/\tau_{\text{NO}})]^{1/2}} \quad (4)$$

105 where  $\sigma_z(x)$  is the dispersion coefficient of inert pollutants in the streamwise direction  
 106  $x$  and  $\mathcal{K}$  the (constant) eddy diffusivity. Uniform flows with a constant wind speed  $U$  is  
 107 assumed so  $\tau_a (= h/U$ ; where  $h$  is the size of roughness elements) and  $\tau_d (= h^2/\mathcal{K})$  are the  
 108 timescales of advection and diffusion, respectively. Equation (4) is in line with that has  
 109 been reported in McKenna (1997) and Dore et al. (2015) in which the timescales of the  
 110 dynamics and the chemistry are tightly coupled with each other, modifying the pollutant  
 111 concentrations collectively. Hence, the classic assumption of diluent reactive gases with

negligible pollution physics/chemistry coupling (i.e. well-mixed conditions with prolonged  $\tau_{\text{NO}}$ ) should be applied cautiously in the ASL over urban areas. The discrepancy is more noticeable when the variation in the ratio of residence time scales, which is defined as the ratio of advection-time-scale-to-diffusion-time-scale  $\tau_a/\tau_d$ , is large in the vertical direction. The modified dispersion coefficient  $\hat{\sigma}_z$  is no longer a function of streamwise distance  $x$  only that violates the prime assumption of Gaussian plume models (dispersion coefficient is a function of streamwise distance  $x$  after the source that is independent from the vertical height  $z$ ; Wu and Liu 2016b). This finding arouses our interest to improve the current parameterization of reactive plume dispersion in the inhomogeneous ASL over urban areas.

### 3.3 Source Depletion Analogy

In view of the variation in physical timescales  $\tau_a$  and  $\tau_d$  across the urban ASL, the conventional approach of modified pollutant dispersion coefficient Equation (4) is no longer applicable to reactive plume. Because the chemistry is dominated in the near-wall region, we modify the Gaussian plume model by source depletion analogy (Arya 1998) in attempt to handle ozone titration Equation (1) by a non-CFD model. As a preliminary study, the depleted source is calculated by integrating the mean streamwise pollutant flux across the turbulent boundary layer

$$Q_{dp}(x) = \int_0^H \langle \bar{u}(x, z) \rangle \times \langle \bar{c}_{\text{NO}}(x, z) \rangle dz. \quad (5)$$

## 4 Results and Discussion

### 4.1 Flow and Turbulence Structure

Figure 2 depicts the flow and turbulence properties in the urban ASL over the rib-type schematic urban surface described previously. The mean-wind speed profile (Figure 2a) agrees well with our previous wind tunnel measurement (Ho and Liu 2017) but over-predicts slightly than that available in literature (Wood and Antonia 1975). The mild dissimilarity is attributed to the different roughness Reynolds number  $\text{Re}_*$  ( $= u_* h/\nu$ ; where  $u_*$  is the friction velocity and  $\nu$  the kinematic viscosity) that was approximately 60

138 in Wood and Antonia (1975), was over 600 in Ho and Liu (2017) and is 470 in this paper.

139 The sharp near-wall **wind-speed** gradient signifies the locally elevated TKE also. The  
 140 streamwise  $\langle u''u'' \rangle^{1/2}$  (Figure 2b) and the vertical  $\langle w''w'' \rangle^{1/2}$  (Figure 2c) fluctuating ve-  
 141 locities are normalized by the friction velocity  $u_*$  instead of the prevailing **wind** speed  
 142  $u_\infty$  similar to most studies of turbulent shear flows. **Here, the angle bracket  $\langle \psi \rangle$  denotes**  
 143 **spatio-temporally (ensemble) averaged properties (horizontal domain for flows and span-**  
 144 **wise domain for pollutants) and the double prime denotes the deviation of the property**  
 145 **from its ensemble average  $\psi'' = \bar{\psi} - \langle \bar{\psi} \rangle$ .** A good agreement of fluctuating velocities,  
 146 especially in the lower urban ASL, between the current LES and the wind tunnel mea-  
 147 surements (Burattini et al. 2008, Ho and Liu 2017) is observed. Minor differences, such  
 148 as a higher LES-calculated streamwise fluctuating velocity  $\langle u''u'' \rangle^{1/2}$  compared with that  
 149 of the direct numerical simulation (DNS) of Coceal et al. (2007), are found. The current  
 150 LES-calculated streamwise fluctuating velocity (Figure 2b) agrees well with that of the  
 151 wind tunnel measurements of Krogstad and Antonia (1999) but underpredicts the vertical  
 152 component (Figure 2c). Besides, **the LES-calculated** fluctuating velocities in both stream-  
 153 wise and vertical direction **are smaller** than the wind tunnel measurements of Djenidi  
 154 et al. (1999). Apart from the different Reynolds number, the aforementioned discrepancy  
 155 in various aspects is likely attributed to the dissimilar turbulence generation mechanism  
 156 in mathematical models and laboratory experiments. Horizontally homogeneous flows  
 157 are assumed in the current LES so the turbulence is generated only by the interaction  
 158 between momentum flux  $\langle u''w'' \rangle$  and **gradient of mean-wind speed**  $\partial \langle \bar{u} \rangle / \partial z$ , resulting in  
 159 a TKE intensity smaller than that of laboratory measurements.

160 As shown in Figure 2d, both our previous wind-tunnel measurements (Ho and Liu  
 161 2017) and the current LES results exhibit the conventional characteristics of ensemble  
 162 averaged vertical momentum flux  $\langle u''w'' \rangle$  **in turbulent shear flows** that decreases with  
 163 increasing height. The wind-tunnel measurements deviate slightly from linearity because  
 164 of **the** background turbulence intensity ( $\approx 10\%$ ). On the other hand, the LES-calculated  
 165 total momentum flux ( $= \langle u''w'' \rangle - \tau_{xz} - \nu \partial \langle \bar{u} \rangle / \partial z$ ; **where  $\tau_{xz}$  is the SGS momentum flux**)  
 166 is almost linearly proportional to the wall-normal distance  $z$  that agrees well with the



167 theoretical solution. The small SGS and viscous momentum fluxes support that the LES  
 168 spatial resolution is sufficient. Whereas, the layer of constant momentum flux in the  
 169 near-wall region is shallow compared with that of the wind tunnel measurements.

## 170 4.2 Inert Plume Dispersion

171 Nitrogen conserves in the sensitivity tests regardless of the background ozone concen-  
 172 trations  $[O_3]_0$  so the nitrogen oxides  $NO_x$  plume can be taken as inert-pollutant disper-  
 173 sion. Figure 3a depicts the dimensionless profiles of nitrogen oxides concentration  $\langle \bar{c}_{NO_x} \rangle$   
 174 as functions of wall-normal distance  $z$  at different streamwise locations  $x$ . The current  
 175 LES-calculated concentrations agree well with the theoretical Gaussian-form solutions.  
 176 Scaled concentration profiles of nitrogen oxides are independent from the streamwise lo-  
 177 cation  $x$  in which the dispersion coefficient is calculated by

$$\sigma_z(x) = \left[ \frac{\int_{z=0}^{z=H} \langle \bar{c}_{NO_x}(x, z) \rangle z^2 dz}{\int_{z=0}^{z=H} \langle \bar{c}_{NO_x}(x, z) \rangle dz} \right]^{1/2}. \quad (6)$$

178 In view of the rapid change of LES-calculated mean-wind speed  $\langle \bar{u} \rangle$  in the near-wall region,  
 179 the average mean-wind speed across 95% of plume coverage

$$\hat{u} = \frac{\int_{z=0}^{z=2\sigma_z} \langle \bar{u}(z) \rangle dz}{\int_{z=0}^{z=2\sigma_z} dz} \quad (7)$$

180 is introduced in the analysis to facilitate the comparison between the LES and analytical  
 181 solutions.

182 The far-field behavior  $\sigma_z \propto x^{1/2}$  might not be fully developed in the near-field plume  
 183 dispersion so a noticeable discrepancy between the theoretical Gaussian plume model and  
 184 the current LES is observed at  $x = 15.5h$ . In the near-wall region  $z \leq 0.25 \times 2^{1/2}\sigma_z$   
 185 right over the roughness elements, the current LES shows a higher level of inert-pollutant  
 186 concentrations than does the analytical Gaussian solution by 10%. In addition to the  
 187 elevated near-wall wind-speed gradient, the difference could be attributed to the pollutant  
 188 reflection from the ground surface, which, however, cannot be simply represented by the

189 method of reflection and superposition for plume dispersion. Moreover, the slower flows  
 190 suppress the pollutant dilution over rough surfaces.

### 191 4.3 Reactive Plume Dispersion

192 Including chemistry in the LES results in an obvious plume rise mean height

$$z_r(x) = \frac{\int_{z=0}^{z=H} \langle \bar{c}_{\text{NO}}(x, z) \rangle z dz}{\int_{z=0}^{z=H} \langle \bar{c}_{\text{NO}}(x, z) \rangle dz} \quad (8)$$

193 in terms of the streamwise location  $x$ . The consumption of nitric oxide by ozone titration  
 194 is much more significant in the near-wall region so the peaked concentrations of nitric  
 195 oxides are elevated (Figure 3). The plume rise mean height  $z_r$  increases with increasing  
 196 background ozone concentration  $[\text{O}_3]_0$  that varies over five times from  $z_r = 0.1 \times 2^{1/2} \sigma_z$   
 197 for  $[\text{O}_3]_0 = 1$  ppb up to  $z_r = 0.6 \times 2^{1/2} \sigma_z$  for  $[\text{O}_3]_0 = 500$  ppb. The deviation from  
 198 Gaussian solution can be explained by the change in Damköhler number so the relative  
 199 extent of physical/chemical processes is nonuniform in the wall-normal direction. Hence,  
 200 applying the conventional Gaussian plume model to reactive plume dispersion must be  
 201 careful because the fully well-mixed conditions are seldom complied with. In particular,  
 202 the primary-pollutant concentrations would be under-predicted because of secondary-  
 203 pollutant production.

204 Figure 3 compares the dimensionless profiles of nitric oxide concentrations  $\langle \bar{c}_{\text{NO}} \rangle$  in dif-  
 205 ferent background ozone concentrations  $[\text{O}_3]_0$ . The source of nitric oxide in the theoretical  
 206 Gaussian plume model is depleted in order to handle the ozone titration (Equation 5). The  
 207 LES-calculated dimensionless nitric oxide concentration at different streamwise locations  
 208 clearly exhibit self-similarity, supporting the validity of Gaussian-shape functional form  
 209 of the source depletion models for reactive plume dispersion. Alike their inert-pollutant  
 210 counterpart, early plume-dispersion characteristics are clearly observed in the near field  
 211 at  $x = 15.5h$  where the plume dispersion is not yet fully developed, deviating from the  
 212 self-similarity. It is caused by the implicit limitation of Gaussian plume model. Ozone  
 213 titration is nonuniform that is dominated in the near-wall region where a sharp drop  
 214 in local nitric concentrations is observed, unveiling the drawback of Gaussian solution

215 to reactive plume dispersion. The nitric oxide concentrations decrease with increasing  
 216 background ozone concentrations. Almost all the nitric oxide in the near-wall region is  
 217 consumed in the case of high-level background ozone concentration  $[O_3]_0 = 500$  ppb so  
 218 the functional form of concentration distribution does not fully follow Gaussian shape.  
 219 This is another major limitation of the source depletion model, calling for an alternative  
 220 solution to reactive plume dispersion.

221 For the far-field nitric oxide concentrations  $\langle \bar{c}_{NO} \rangle$ , the analytical Gaussian-form source  
 222 depletion plume model and the current LES-calculated results exhibit various degrees  
 223 of disagreement in response to the background ozone concentrations  $[O_3]_0$ , i.e. different  
 224 chemistry timescales  $\tau_{NO}$ . An obvious discrepancy in nitric oxide concentrations, which  
 225 increases with increasing background ozone concentration, is observed in the near-wall  
 226 region. Ozone governs the life time of nitric oxide in the plume dispersion. Higher  
 227 background ozone concentrations shorten the life time of nitric oxide  $\tau_{NO}$ . The effect is  
 228 more promising for a longer residence time such as in the near-wall region where turbulent  
 229 diffusion dominates the transport processes, i.e. residence is determined by the diffusion  
 230 timescale  $\tau_d$ . The discrepancy diminishes with increasing wall-normal distance because of  
 231 the reducing residence time in advection-dominated transport processes. It is interesting  
 232 that the error is noticeable mainly below the plume rise mean height  $z_r$ . Over the plume  
 233 rise mean height the prediction of the newly developed source depletion analogy is good,  
 234 suggesting a handy parameterization of reactive plume dispersion.

#### 235 4.4 Reactive Plume Parameterization

236 In view of the discrepancy in the calculation of nitric oxide concentrations  $\langle \bar{c}_{NO} \rangle$   
 237 observed below the plume rise mean height, additional effort is sought to improve the  
 238 reactive plume dispersion parameterization. We focus on the near-wall region below the  
 239 plume rise mean height  $z \leq z_r$  where the source depletion analogy shows a major over-  
 240 prediction in ground-level nitric oxide concentration  $\langle \bar{c}_{NO} \rangle$ . Gamma  $\gamma$ -distribution in  
 241 terms of  $\gamma$  function  $\Gamma(\alpha)$

$$\sqrt{\frac{\pi}{2}} \times \frac{\hat{u}\sigma_z}{Q_{dp}} \times \langle \bar{c}_{NO} \rangle = \frac{(z/\sqrt{2}\sigma_z)^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} \exp\left(-\frac{z/\sqrt{2}\sigma_z}{\beta}\right) \quad (9)$$

242 is tested by regression where  $\alpha$  and  $\beta$  are parameters which can be determined by the  
 243 mean  $\mu$  ( $= \alpha\beta$ ) and the variance  $\sigma^2$  ( $= \alpha\beta^2$ ) of the pollutant distribution. The reduction  
 244 in nitric oxide concentrations below the plume rise mean height  $z_r$  in the near-wall region  
 245 is predicted well by the Gamma  $\gamma$ -distribution (Figure 3). Over the plume rise mean  
 246 height, both the Gaussian-form source depletion model and the Gamma  $\gamma$ -distribution  
 247 show similar dimensionless profiles which are close to the current LES-calculated nitric  
 248 oxide concentration. The root-mean-square (RMS) error between the nitric oxide concen-  
 249 trations  $\langle \bar{c}_{\text{NO}} \rangle$  in various background ozone concentrations  $[\text{O}_3]_0$  calculated by the source  
 250 depletion model and the Gamma  $\gamma$ -distribution is tabulated in Table 1 as well. Except  
 251 the one with  $[\text{O}_3]_0 = 10$  ppb, the Gamma  $\gamma$ -distribution outperforms the source deple-  
 252 tion analogy in which the accuracy is improved substantially such that the RMS error is  
 253 reduced by an order of magnitude. The improvement is more prominent at higher back-  
 254 ground ozone concentrations because the Gamma  $\gamma$ -distribution is able to handle well the  
 255 rapid near-wall ozone titration.

256 Coincidentally, the Gamma  $\gamma$ -distribution was used to handle reactive plume disper-  
 257 sion over smooth surfaces in stochastic approaches (Ferrero et al. 2013). The large-scale  
 258 plume meandering was calculated by stochastic equations and the internal mixing was  
 259 modeled using bivariate Gamma function. No roughness element was considered. On  
 260 the other hand, the Gamma  $\gamma$ -distribution was used to model the concentration proba-  
 261 bility density function (PDF) of plume dispersion over a large array of obstacles (Gailis  
 262 et al. 2007). Although chemistry was not considered, the favorable agreement of using  
 263 Gamma  $\gamma$ -distribution could be attributed to the strong mixing and rapid dilution of  
 264 pollutants in the near-wall region. In this study, the LES results show that the plume  
 265 shape is modified by ozone titration instead of surface roughness.

## 266 5 Conclusions

267 Gaussian plume model is well received in industry for the quick estimate of pollutant  
 268 distribution in open-terrain configurations. Whereas, its functionality for reactive plume  
 269 dispersion over urban areas is rather limited because most of the dispersion coefficients

270 were determined empirically as functions of atmospheric stability only yet overlooked  
271 surface roughness. The drawback on dynamics is thus often biased toward temperature  
272 stratification. The interaction between pollution physics and chemistry is more compli-  
273 cated because of the coupling between transport ( $\tau_a$  and  $\tau_d$ ) and reaction ( $\tau_\phi$ ) timescales.  
274 Hence, the conventional Gaussian plume model must be applied with caution to urban  
275 setting. In this technical note, we report a preliminary study of reactive plume dispersion  
276 over schematic urban areas in the form of identical ribs in crossflows (idealized urban  
277 street canyons). The LES flow properties over rough surfaces are similar to those of open-  
278 channel flows that agree well with other results available in literature. The irreversible  
279 ozone titration is adopted to demonstrate the effect of pollution chemistry on plume dis-  
280 persion. Because ozone titration mainly takes place in the near-wall region but not evenly  
281 throughout the urban ASL, the LES results show that most nitric oxide is oxidized to  
282 nitrogen dioxide in the near-wall region so the dimensionless profiles of reactive plume dis-  
283 persion is no longer Gaussian form. As such, we first use the analogous pollutant source  
284 depletion to handle the near-wall chemistry dominance. The source depletion analogy  
285 calculates well the reactive plume dispersion in the region over plume rise mean height  
286  $z_r$ . However, the large difference in timescales in urban ASL, especially below the plume  
287 rise mean height, cannot be rectified. In view of the major discrepancy below the plume  
288 rise mean height, the Gamma  $\gamma$ -distribution is proposed as the regression to the pollutant  
289 distribution in which an appreciable improvement is clearly demonstrated. The current  
290 solution framework could in turn formulate a new, primitive parameterization of reactive  
291 plume dispersion over urban areas. Additional effort is undertaken to advance our under-  
292 standing of the physics/chemistry mechanism as well as to refine the parameterization.

293

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Table 1: Root mean square (RMS) error comparing source depletion model and Gamma  $\gamma$ -distribution.

| Background ozone<br>concentration $[\text{O}_3]_0$ (ppb) | RMS error              |                              |
|--|------------------------|------------------------------|
|  | Source depletion model | Gamma $\gamma$ -distribution |
| 1  | 0.009745               | 0.001761                     |
| 10   | 0.0006537              | 0.002166                     |
| 50   | 0.008048               | 0.002672                     |
| 100  | 0.01582                | 0.002972                     |
| 200  | 0.02984                | 0.003605                     |
| 500  | 0.05867                | 0.004318                     |

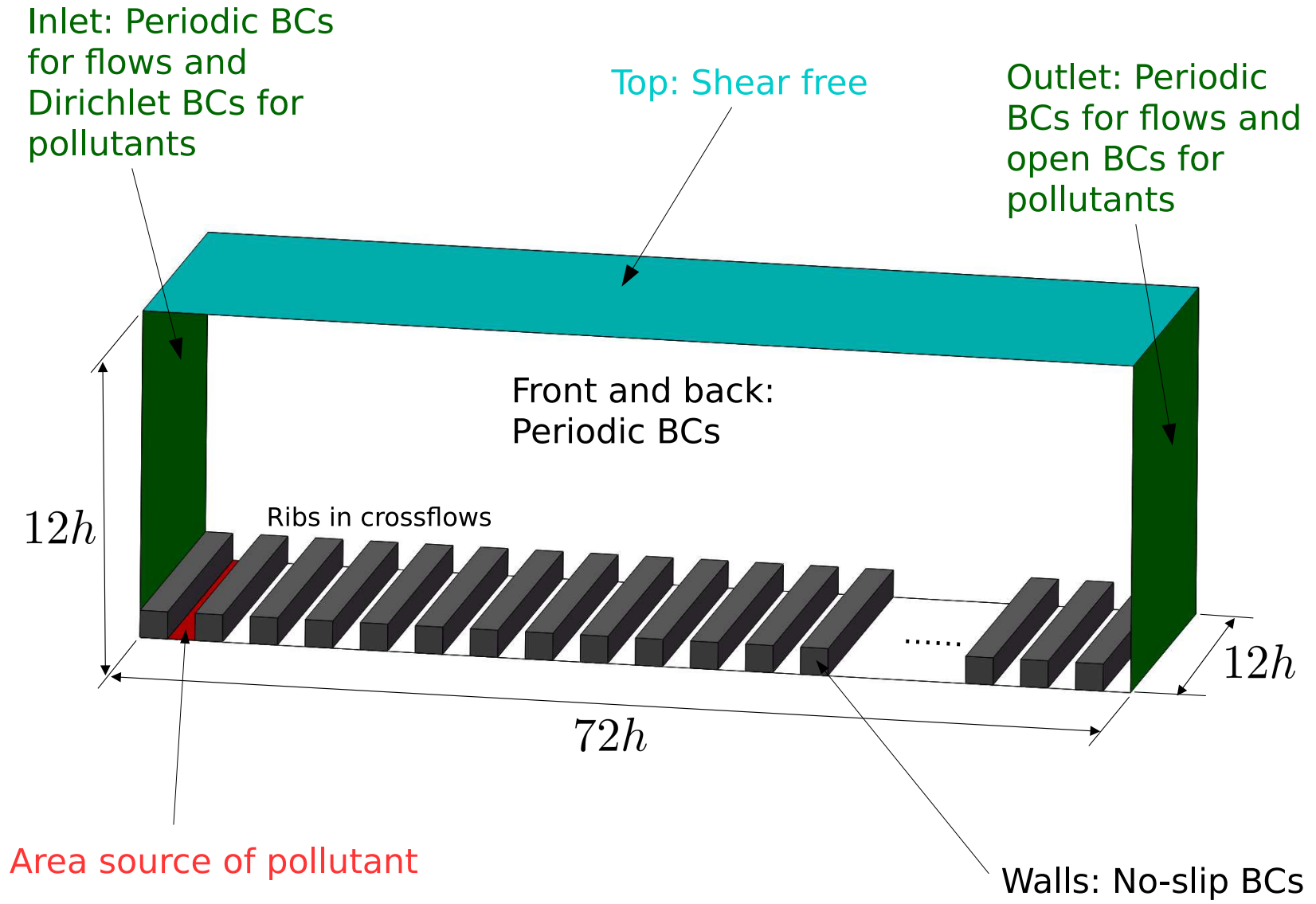


Figure 1: Spatial domain of the LES.



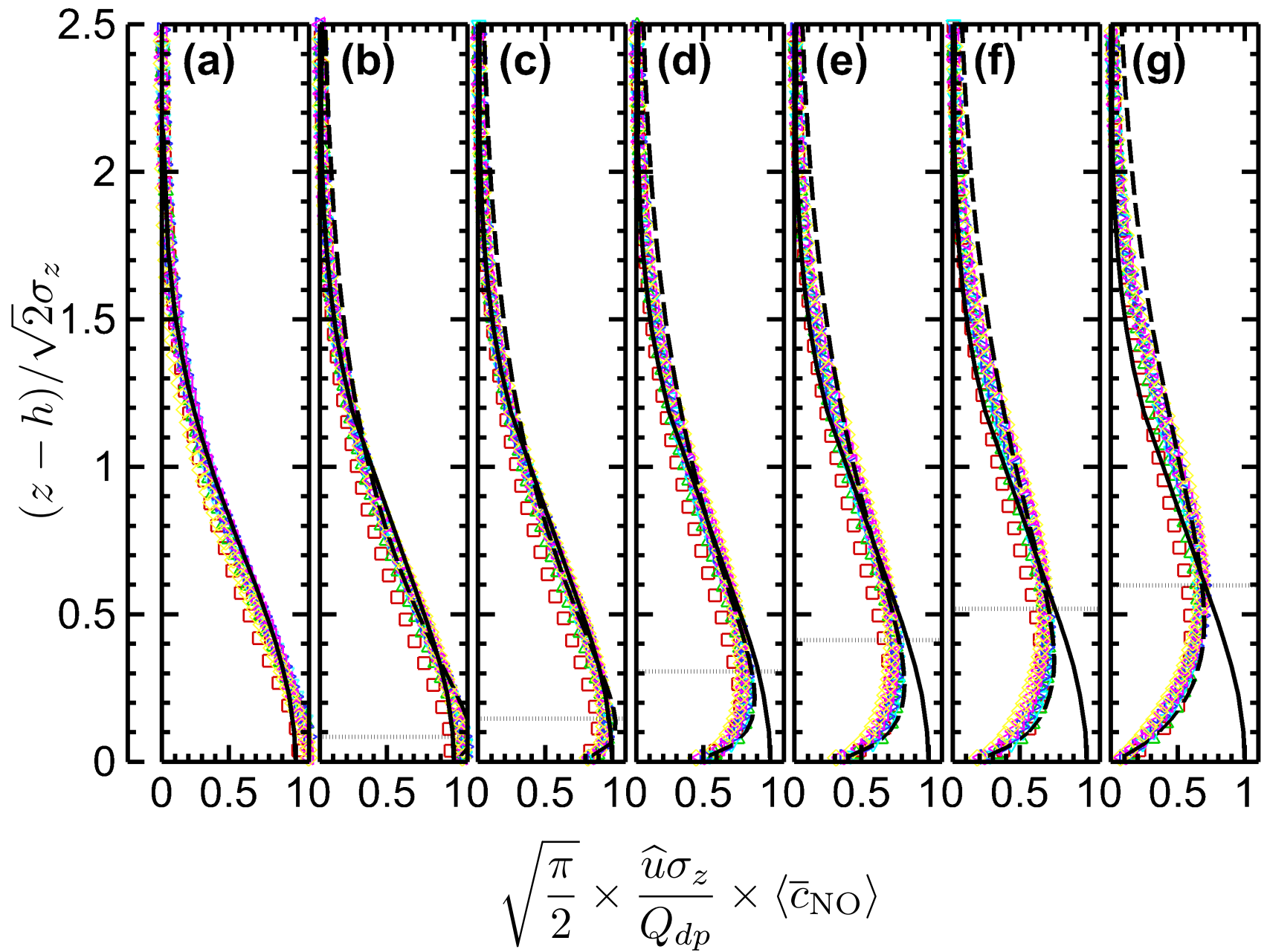


Figure 3: Dimensionless profiles of reactive pollutant nitric oxide concentration  $(\pi/2)^{1/2} \times \hat{u}\sigma_z/Q_{dp} \times \langle \bar{c}_{\text{NO}} \rangle$  in the streamwise direction at  $x/h = 15.5$ :  $\square$ ,  $25.5$ :  $\triangle$ ,  $35.5$ :  $\nabla$ ,  $45.5$ :  $\triangleright$ ,  $55.5$ :  $\triangleleft$  and  $65.5$ :  $\diamond$  expressed in dimensionless wall-normal coordinate  $z/2^{1/2}\sigma_z$  for background ozone concentration  $[\text{O}_3]_0$  of (a) 0 ppb (insert pollutant nitrogen oxides  $\text{NO}_x$ ); (b) 1 ppb; (c) 10 ppb; (d) 50 ppb; (e) 100 ppb; (f) 200 ppb and (f) 500 ppb. Also shown are the profiles of the theoretical Gaussian plume model: — and the Gamma  $\gamma$ -distribution: - - - - - Equation (9). The plume-rise level  $z_r$  is indicated by  $\cdots\cdots$ .