

## MODELLING AND ANALYSIS OF TWO PRODUCTION SOLUTIONS FOR PRECAST CONCRETE ELEMENTS: A PETRI-NETS APPROACH

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**Abstract.** *The adoption of precast concrete construction has been increasingly emphasised globally, due to its advancements in productivity, waste management, cost control, quality and safety. Meanwhile, the production solutions differ greatly in the off-site plants in terms of traditional stationary production and modern circulation production with different levels of automation. Practical cases show that improper selection or upgrade of production solution could lead to resource slack or even business failure, and there is limited knowledge to quantitatively support the decision making. Furthermore, existing works seldom dig deep into the fundamental setup of the production plant, and reckon without the stochasticity and variability of production operation. Therefore, this paper aims to develop an analytical model to facilitate the understanding of the production solutions of precast concrete elements, which should further support the decision making in factory planning or upgrade towards increased level of automation. Stochastic Petri-nets approach has been applied, with stochastic features embraced, to graphically modelled stationary and circulation solutions. Simulation and comparative performance analysis was conducted with a numerical case study. Results demonstrate that stationary solution can achieve even higher outputs during production period provided that enough resources are given, whilst circulation solution has a more flexible and stable production.*

**Keywords:** Precast concrete, Production, Stochastic Petri nets, Modelling, Prefabrication.

### 1 INTRODUCTION

Building with precast concrete (PC) is increasingly prevalent in the construction world, which has the great potential to provide productivity enhancement, waste reduction, cost effectiveness, labour saving, improved safety and quality control [1]. Despite the consentient benefits gained on-site, the production solutions differ widely in the off-site plants in terms of traditional stationary production and modern circulation production with different levels of automation [4]. In view of requirement of production capability, property size, products profiles, economic efficiency, resource and other constraints, plants may often require tailor-made solutions for the production lines [2, 3]. Practical cases also show that improper selection or upgrade of production solution could lead to substantial resource slack or even business failure, and there is limited knowledge to quantitatively support the decision making for an optimum solution with a full understanding of the process differences and resource utilization of different production methods, whilst the detailed design of the plant is always specialized to fulfil different requirements. Besides, existing works have focused extensively on precast production scheduling with regard to different resource constraints [5-7], but there is a lack of study to dig deep into the fundamental setup of the production plant with proper resource allocation, which is essential to underpin the decision-making in plant design and investment. Additionally, these studies [5-7] generally based on the hypothesis that the processing time of each production operation is a constant value, which reckon without the stochasticity and variability caused by operators and machinery.

Therefore, this paper aims to develop an analytical model to facilitate the understanding of the basic production solutions of PC elements in the construction industry, which should further support the

decision making for relevant construction companies in factory planning or upgrade towards increased level of automation. Stochastic Petri-nets (SPN) approach is applied for modelling and analysis, which can graphically model the production process with various constraints and stochastic parameters. Stationary and circulation solutions are modelled and comparative performance analysis was conducted with a numerical case study of four different sets of parameters. Stationary production is the traditional approach, in which elements are manufactured on stationary tables with exclusive moulds in one production hall. Circulation production, equipped with pallet carousel devices, is the modern way to produce elements in a circulation system to improve production capacity, which is apposite to laminary panels. These two basic systems are the cornerstone for the setup of the plant and configuration of machinery.

## 2 METHODS

Stochastic Petri nets (SPNs) approach has been applied to model and analyze the production solutions, which is a prevailing modelling technique to graphically and mathematically describe and evaluate discrete event dynamic systems [8]. It is isomorphic with a Markov process and the underlying Markov chain can be generated to solve the complexity and time-related features of the system. The extensions of SPN have been proposed to enable the modelling of time probability and enhance the capability to describe the structural behavior [8].

Basically, a SPN =  $\{P, T; F, W, M_0, \lambda\}$ , is a bipartite graph, with places  $P(p_1, p_2, \dots)$  and transitions  $T(t_1, t_2, \dots)$  connected by arcs  $F$  [9]. Places are depicted by circles with tokens inside to represent the state of event or the quantity of resources. Transitions are drawn as rectangle boxes, with black ones are immediate transitions representing time-independent actions and white ones are timed transitions representing time-dependent actions associated with firing rate  $\lambda = \{\lambda_1, \lambda_2, \dots\}$ . Priority can be preset by integers for conflicting transitions. Arcs, drawn as arrowed lines, indicate the relationship between places and transitions.  $W$  defines the weight of arcs that can transfer the tokens, with  $w_{ij}(t_i, p_j)$  is the weight of arc from  $t_i$  to  $p_j$  and  $w_{ji}(p_i, t_j)$  is the weight of arc from  $p_i$  to  $t_j$ . The default values of weights are unlimited. Inhibitor arcs, as the extension, are a special type of arcs depicted as a line end with a small circle, define that the output transitions can only be enabled when there is no token in the input places. A marking  $M$  is a state, denoted by tokens in places, whilst  $M_0$  is the starting state. The system can be dynamically described as the marking process, with the positions of tokens changed by firing the corresponding transitions. Tangible states are stable states when no immediate transition is enabled, whilst vanishing states are those with enabled immediate transitions.

The developed SPN models allow the performance analysis to be conducted. First, all the reachable markings can be generated from the model and the embedded Markov chain can be acquired. The stable state probability distribution can thereby be derived [8]. Suppose that  $P[M]$  is the probability of the stable state  $M$ .  $P[M(p_j)=j]$  is the probability that the number of tokens in place  $p_j$  is  $j$ .

The average number of tokens  $N(p_i)$  in place  $p_i$  as an essential measure can be computed as:

$$N(p_i) = \sum j \times P[M(p_i) = j] \tag{1}$$

The utilization of place  $p_i$  can be utilized to calculate the station or machine utilization, which can be obtained as:

$$U(p_i) = \sum P[M(p_i) > 0] \tag{2}$$

According to the Little's Law [10], the response time, or the processing time, of the system can be measured with the average number of tokens of system  $N(p)$  and the time rate of the start transition  $\mu$  in the system.

$$t = N(p) / \mu \tag{3}$$

In PC production, workstation idle time (WIT), throughput and makespan (MS) are commonly used to evaluate the production performance [5, 11]. WIT is the waiting time of the jobs to be processed in one station, which is treated as the waste time during production. Throughput represents the production rate. MS, or the cycle time represent the time duration of the one production order. This study considers MS as the performance measure based on the detailed SPN models.

### 3 MODELLING

#### 3.1 Model description

The typical production procedures of PC panels can be summarized into six major steps: preparation, mould arrangement, reinforcement and fixing setting, concrete casting, curing and demoulding [4]. Stationary solution (Figure 1) allows flexible production of customizable products with moulds manufactured in advance. The number of moulds required is often determined by the time requirement of on-site construction. During the production, the pallets with moulds are stationarily mounted on the shop floor. In circulation production (Figure 2), by contrast, the production pallets are transported by the carousel devices. Generally, circulation solution applies in the production of uncomplicated, laminary elements, supported by a spectrum of standardized shuttering profiles to form the various shapes for different products. Both production solutions are compatible with tailor-made moulds or adaptable shuttering profiles, but the investment on the circulation production system often involves the use of standardized shuttering profiles.

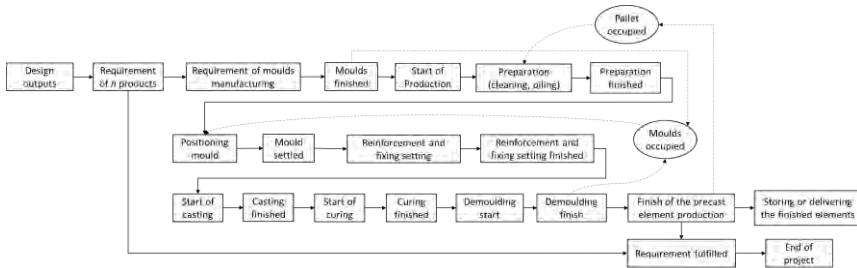
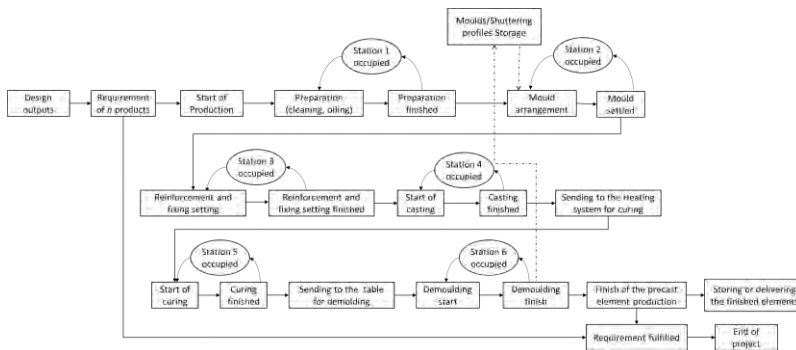


Figure 1. Stationary production solution for precast concrete elements.



Notes: Station 1 - Preparation station; Station 2 - Moulding station works;  
Station 3 - Reinforcement and fixing station; Station 4 - Casting station; Station 5 - Curing station; Station 6 - Demoulding station

Figure 2. Circulation production solution for precast concrete elements.

We define that there are  $n$  types of products to be produced, while the required number for type  $k$  product ( $k=1, 2, \dots, n$ ) is  $n_k$ , then the total number of products to be produced can be calculated as:

$$N = \sum_{k=1}^n k \cdot n_k \quad (4)$$

Considering that the number of moulds for type  $k$  product is  $m_k$ , then the resource sharing issue occurs to produce the  $n_k$  elements. The processing time for each job can be modelled by timed transitions. We assume that each job is processed independently and the processing time is exponentially distributed. The firing delay of the corresponding transition  $t$  can thus be defined with the following exponential probability distribution function, where  $\lambda_t$  denote the associated firing rate.

$$\forall t \in T : F_t = 1 - e^{-\lambda_t x} (x \geq 0) \quad (5)$$

Then, the average processing time for operation is  $1/\lambda_t$ . To develop the SPN models for the above production solutions, there are some further assumptions:

- Machine sharing issue in stationary production is only considered in casting process and not considered in other processes that may be done manually or mechanically. There is only one concrete spreader for casting, and it can only process one job at a time.
- In circulation production, each station except curing can only process one job at a time. Curing is a parallel process and can handle more than one job simultaneously with capacity of  $Z$ , and the idle state of curing is not considered.
- There are two widely used methods of crew organization, comprehensive method that the same crew performs all the operation works for producing one element, and specialized method that the whole process has been broken into specific activities with each performed by a different crew [6]. Since the workers can be arranged differently to affect the production performance, worker resource is not considered in the model to simplify the analysis.
- The models consider the production of two-dimensional PC elements, while assuming that the traditional stationary production is based on certain number of tailor-made premanufactured moulds and circulation production is based on adaptable shuttering profiles that the number will not limit the production.
- The storage capacity is sufficient for moulds or shuttering profiles.
- The plant is operated all day with three shifts.
- The operation is non-preemptive and interruption is not allowed during the processing period.

### 3.2 SPN models

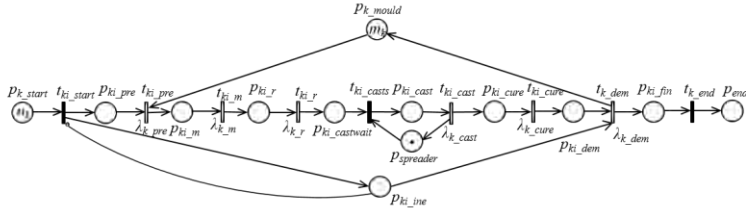
Based on the model description and assumptions, the SPN models of two production solutions can be established as seen in Figure 3 and Figure 4. The models can be further extended to include the manpower resource limitation by adding the place representing workers  $p_{man}$  and linking to each manual-based job operation  $p_x$  as Figure 5.

The MS of the stationary (S) production and circulation (C) production can be calculated based on the following equations.

$$MS_S = \max \{ \max [N_{ki}(p)] / \lambda_{k\_pre} \} \quad (6)$$

$$MS_C = \max [N_{ki}(p)] / \lambda_{k\_pre} \quad (7)$$

where  $N_{ki}(p)$  is the sum of the number of average tokens in places that represent either job operation or waiting state. MS can also be obtained from the simulation results that the time duration for place  $p_{end}$  to have  $N$  tokens.



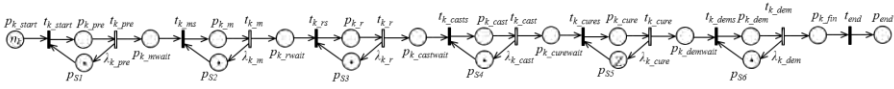
Places ( $k=1,2,\dots,n, i=1,2,\dots,m_k$ ):

- $P_{k\_start}$ : the requirement of producing  $n_k$  number of type  $k$  PC element.
- $P_{k\_pre}$ : pallet preparation on production line  $i$  of type  $k$  product.
- $P_{k\_m}$ : mould arrangement on production line  $i$  of type  $k$  product.
- $P_{k\_r}$ : reinforcement and fixing setting on production line  $i$  of type  $k$  product.
- $P_{k\_castwait}$ : waiting for concrete casting on production line  $i$  of type  $k$  product.
- $P_{k\_cast}$ : concrete casting on production line  $i$  of type  $k$  product.
- $P_{spreader}$ : the concrete spreader
- $P_{k\_cure}$ : curing on production line  $i$  of type  $k$  product.
- $P_{k\_dem}$ : demoulding on production line  $i$  of type  $k$  product.
- $P_{k\_fin}$ : completion of the production on line  $i$  of type  $k$  product.
- $P_{end}$ : completion of the production requirement.

Transitions ( $k=1,2,\dots,n, i=1,2,\dots,m_k$ ):

- $t_{k\_start}$ : the immediate transition, representing start time of the production of type  $k$  PC element.
- $t_{k\_pre}$ : the timed transition, representing time for processing pallet preparation works on production line  $i$  of type  $k$  product with processing rates  $\lambda_{k\_pre}$ .
- $t_{k\_m}$ : representing time for mould arrangement on production line  $i$  of type  $k$  product with processing rates  $\lambda_{k\_m}$ .
- $t_{k\_r}$ : representing time for reinforcement and fixing setting on production line  $i$  of type  $k$  product with processing rates  $\lambda_{k\_r}$ .
- $t_{k\_cast}$ : representing start time of concrete casting on production line  $i$  of type  $k$  product.
- $t_{k\_cast}$ : representing time for concrete casting on production line  $i$  of type  $k$  product with processing rates  $\lambda_{k\_cast}$ .
- $t_{k\_cure}$ : representing time for curing with on production line  $i$  of type  $k$  product processing rates  $\lambda_{k\_cure}$ .
- $t_{k\_dem}$ : representing time for demoulding on production line  $i$  of type  $k$  product with processing rates  $\lambda_{k\_dem}$ .
- $t_{end}$ : the end time of completion of the production requirement.

Figure 3. Stationary production solution for precast concrete elements.



Places ( $k=1,2,\dots,n$ ):

- $P_{k\_start}$ : the requirement of producing  $n_k$  number of type  $k$  PC element.
- $P_{k\_pre}$ : pallet preparation of type  $k$  product.
- $P_{sj}$ : availability of preparation station.
- $P_{k\_mwait}$ : waiting for mould arrangement of type  $k$  product.
- $P_{k\_m}$ : mould arrangement of type  $k$  product.
- $P_{sj}$ : availability of moulding arrangement station.
- $P_{k\_rwait}$ : waiting for reinforcement and fixing setting of type  $k$  product.
- $P_{k\_r}$ : reinforcement and fixing setting of type  $k$  product.
- $P_{sj}$ : availability of reinforcement and fixing station.
- $P_{k\_castwait}$ : waiting for concrete casting of type  $k$  product.
- $P_{k\_cast}$ : concrete casting of type  $k$  product.
- $P_{sj}$ : availability of casting station.
- $P_{k\_curewait}$ : waiting for curing of type  $k$  product.
- $P_{k\_cure}$ : curing of type  $k$  product.
- $P_{sj}$ : availability of curing station, with capacity of  $Z$ .
- $P_{k\_demwait}$ : waiting for demoulding of type  $k$  product.
- $P_{k\_dem}$ : demoulding of type  $k$  product.
- $P_{sj}$ : availability of demoulding and finishing station.
- $P_{k\_fin}$ : completion of the precast elements production of type  $k$  product.
- $P_{end}$ : completion of the production requirement.

Transitions ( $k=1,2,\dots,n$ ):

- $t_{k\_start}$ : the immediate transition, representing start time of the production of type  $k$  PC element, which can be designed with scheduling strategies.
- $t_{k\_pre}$ : the timed transition, representing time for processing pallet preparation works of type  $k$  product with processing rates  $\lambda_{k\_pre}$ .
- $t_{k\_m}$ : representing start time of mould arrangement of type  $k$  product.
- $t_{k\_m}$ : representing time for mould arrangement of type  $k$  product with processing rates  $\lambda_{k\_m}$ .
- $t_{k\_r}$ : representing start time of reinforcement and fixing setting of type  $k$  product.
- $t_{k\_r}$ : representing time for reinforcement and fixing setting of type  $k$  product with processing rates  $\lambda_{k\_r}$ .
- $t_{k\_cast}$ : representing start time of concrete casting of type  $k$  product.
- $t_{k\_cast}$ : representing time for concrete casting of type  $k$  product with processing rates  $\lambda_{k\_cast}$ .
- $t_{k\_cure}$ : representing start time of curing of type  $k$  product.
- $t_{k\_cure}$ : representing time for curing with processing rates  $\lambda_{k\_cure}$ .
- $t_{k\_dem}$ : representing start time of demoulding of type  $k$  product.
- $t_{k\_dem}$ : representing time for demoulding of type  $k$  product with processing rates  $\lambda_{k\_dem}$ .
- $t_{end}$ : the end time of completion of the production requirement.

Figure 4. Circulation production solution for precast concrete elements.

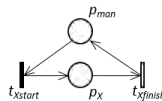


Figure 5. Extended module for modelling the manpower resource.

#### 4 NUMERICAL CASE STUDY

To enable the quantitative comparative analysis, a numerical case study has been conducted based on Snoopy [12], a software tool with continuous development for Petri nets modelling and simulation.

The example study was carried out with a simple model considering  $n=2, Z=8$ . Manpower resource and scheduling issues are not considered. The time parameters of each operation job are assumed as given in Table 1 based on previous studies [5, 6, 11].

Table 1. Parameter of the study.

Operation Job	PC element type 1		PC element type 2	
	Duration (hour)	$\lambda_1$	Duration (hour)	$\lambda_2$
Preparation	0.2	5	0.5	2
Mould arrangement	0.4	2.5	1.0	1
Reinforcement and fixing setting	0.5	2	0.8	1.25
Concrete casting	0.6	1.67	1.2	0.83
Curing	10	0.1	10	0.1
Demoulding	0.25	4	1.5	0.67

We set 10000 simulation runs from Gillespie’s algorithm with 0.1 split time. The influence of different production requirement  $n_k$  and number of moulds  $m_k$  on the production performance is then studied. Table 2 presents the calculated results of the  $MS$  of two production solutions with four different sets of  $n_k$  and  $m_k$ . Results demonstrate that stationary production solution has a higher  $MS$  value in case 2 to 4, and circulation one has a higher  $MS$  value in case 1. The better  $MS$  performance of stationary solution can be explained by its parallel production capability, while case 1 and 2 have three production lines in parallel and case 3 and 4 have five. However, the distinction in  $MS$  between two solutions is not significant considering the differences in production line. The most significant one is the case 4 where  $MS_S$  is 33.6% less than  $MS_C$ , while stationary production should provide multiple times of manpower resources and land space input. Taking into consideration of the efforts for tailor-made premanufactured moulds, stationary production may have little or no advantage in the time performance of the whole project. Additionally, the capability of circulation production can be restricted by the adaptive shuttering profiles which may not suit for non-standardized complicated PC elements,

Table 2. Results of the  $MS$  of two production solutions under four studied cases.

Parameters	Case 1	Case 2	Case 3	Case 4
$n_1$	10	20	20	40
$m_1$	2	2	3	3
$n_2$	10	10	20	20
$m_2$	1	1	2	2
$MS_S$ (time unit)	194.0	196.4	333.9	339.4
$MS_C$ (time unit)	161.1	265.7	379.0	511.0

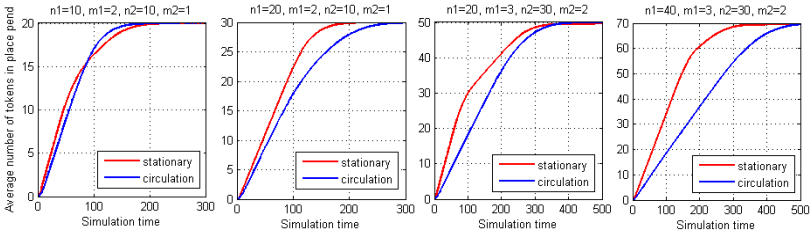


Figure 5. Plots of simulation results of average number of tokens in place  $p_{end}$ .

Figure 5 further illustrates the simulation results of average number of tokens in place  $p_{end}$ , which show that circulation production solution is more stable in production than stationary one. Results in Table 2 and Figure 5 also reveal that stationary production suffers the production line idle problem in both case 1 and 3, using same number of production lines for producing fewer elements as case 2 and 4, with a decreased production rate during later simulation period.

#### 4 DISCUSSIONS

The developed SPN models should provide an elementary framework to support the understanding of different production solutions for PC elements, which should further support the decision making in factory planning or upgrade towards an increased level of automation. The numerical case study is an example to showcase and verify how the models can be applied. Although simulation results from the numerical case study is highly theoretical, based on a series of assumptions, there are still informative findings with practical value for PC production in the construction industry. Stationary and circulation solution suit for different production conditions and subject to different constraints. Several arguments can be drawn as follows.

- Results from the case study show that stationary solution can achieve even higher outputs than circulation one during production period provided that enough moulds, pallets, manpower and land resources are given. In fact, stationary solution has a strong competitive edge in areas where labour and land costs are low compared with machinery cost for carousel and other devices, and for companies have limited or moderate customer demand [4]. Therefore, although circulation solution equipped with advanced machinery and increased automation potential has proved to achieve productivity gains in PC production [13], there is a market for the traditional stationary approach to obtain satisfied output.
- Results from the case study also demonstrate that stationary solution is vulnerable to resource allocation, and inappropriate upfront planning could lead to severe production line idle problems and low productivity. In contrast, circulation solution is more flexible to resources, with a stable output and high utilization of production line. In many construction projects, design changes or variations might occur whilst construction works are in progress, as one leading factor for time and cost overruns [15]. In this respect, circulation solution is more resilience to the sporadic or unforeseeable changes in the construction world.
- There is an increasing trend for the standardization of PC components in the construction industry, which is propitious to further reduction of construction and maintenance costs, alleviation of interface and tolerance problems, and fulfillment of recycling and reuse potentials [14]. From the production perspective, standardized design of PC elements that adaptable to shuttering profile also plays an essential role in circulation production to achieve a high output per production line and capacity improvement by increasing the level of automation.
- Overall the decision making for PC plant design and investment for upgrade in terms of production solution should synthesise a number of factors particularly including technical and qualitative requirements of the PC product in the targeted market, product range, manpower and land cost.

Besides, factors regarding safety and well-being of workers, accuracy of products should also be incorporated. A mixed production solution, which inherits the merits of both, may also preferable.

## 5 CONCLUSIONS

This paper applied SPN approach to develop the analytical model to facilitate the understanding of the basic production solutions of PC elements in the construction industry, embracing the stochastic features of the processes. Traditional stationary and modern circulation solutions are graphically modelled and comparative performance analysis was conducted with a numerical case study with four different sets of parameters. Results suggest that stationary and circulation production methods suit for different production conditions regarding technical and qualitative demand of PC products, output requirement during production, manpower and facility resources, land spaces, etc. Although stationary solution can achieve even higher outputs than circulation one during production period provided that enough resources are given, circulation solution has a more flexible and stable production and high utilization of production line.

This paper should provide a twofold contribution. First, the proposed SPN approach enables a graphical and mathematical modelling of the production solutions of PC elements in a series of discrete events, incorporating stochasticity and resources sharing issues. Second, production performance can be quantitatively assessed based on the developed SPN models by jointly considering production pattern and product requirements, which provide an innovative perspective for the comparative study of different production solutions and could further support the decision-making in selection or upgrade of production solution in plant. The work can be extended to study the scheduling strategies in the PC production by defining the enabling predicates and the random switches of transitions in the SPN models. More performance measures should be taken into account and further validation is suggested with more sophisticated numerical studies in the future.

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