Hierarchical-environment-assisted non-Markovian speedup dynamics control

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We investigate the qubit in the hierarchical environment where the first layer is just one lossy cavity while the second layer is the N-coupled lossy cavities. In the weak-coupling regime between the qubit and the first-layer environment, the dynamics crossovers from Markovian to non-Markovian and from no-speedup to speedup can be realized by controlling the hierarchical environment. We find that the coupling strength between two nearest-neighbor cavities and the number of cavities in the second-layer environment have the opposite effect on the non-Markovian dynamics and speedup evolution of the qubit. However, in the case of strong coupling between the qubit and the first-layer environment, we can be surprised to find that, compared with the original non-Markovian dynamics, the added second-layer environment cannot play a beneficial role on the speedup of the dynamics of the system. In addition, a phenomenon can be noticed that the complete opposite role of the coupling strength between two nearest-neighbor cavities (the number of cavities) in the second-layer environment in the above weak- and strong-coupling regimes.

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I. INTRODUCTION

With the rapid development of quantum information technology [1,2], the dynamics of open systems has drawn more and more interest. Any open system inevitably takes environmental considerations into account [3]. The role of an environment that interacts with the open system is divided into memory effects and memoryless effects. In a memoryless environment, the information flows only from the system to the environment, so Markovian dynamics appears. When the information flows in both directions (i.e., the information can backflow from the environment to the system), it leads to non-Markovian dynamics in the memory environment. Recently, the non-Markovian dynamics has been studied in theory [4–13] and in experiments [14–16]. It plays a leading role in many real physical processes such as quantum state engineering, quantum control [17–19], and the quantum information processing [20–24]. For example, the non-Markovianity has recently been shown experimentally to increase the success probability of the Deutsch–Jozsa algorithm in diamond [25]. So far, the non-Markovian dynamics have proven to be motivated by the strong system-environment coupling, structured reservoirs, low temperatures, initial system-environment correlations, and so on. Furthermore, to quantify non-Markovianity, several measures based on memory effects [8,26–44] have been proposed. There exists a vast literature on different definitions of the non-Markovianity [45–47], but there is no general consensus on one definition. So at present, the non-Markovian evolution process cannot be detected in some cases, such as the dynamical maps whose non-Markovianity with respect to violation of completely-positive-divisibility cannot be detected with non-Markovianity measures based on memory effects [48,49].

More importantly, some efforts have been recently devoted to investigating the role played by the non-Markovianity on the speed of evolution of a quantum system [50–62]. For the damped Jaynes–Cummings model, in which a qubit resonantly interacts with a leaky single-mode cavity [51], the non-Markovian effect could lead to speedup dynamics in the strong-system-environment-coupling regime. In addition, this theoretical result has been confirmed by increasing the system-environment coupling strength and the controllable number of atoms in the environment [57]. So far, some works usually consider the quantum system coupled to a single-layer environment. However, in realistic scenarios, the quantum systems often inevitably couple to the multilayer environments [63–67]. For example, the neighbor nitrogen impurities constitute the principle bath for a nitrogen-vacancy center, while the carbon-13 nuclear spins may also couple to them [63]. In a quantum dot the electron spin may be strongly affected by the surrounding nuclei [64,66]. A similar situation also occurs for a single-donor electron spin in silicon [65,67].

Recently, motivated by these facts, many efforts have been devoted to studying the effects of the multilayer environments on the non-Markovian dynamics of an open system. The dynamics of a two-level atom has been studied in the presence of an overall environment composed of two layers [38]. In their model, the first layer is just a single lossy cavity while the second layer consists of a number of non-coupled lossy cavities. And the non-Markovian dynamics of the atom is affected by the coupling strength between the two layers and

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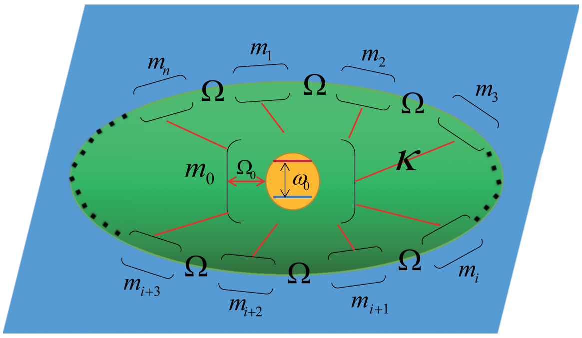

FIG. 1. Schematic representation of the model. The stratified lossy cavities are selected as the controllable environment.

the number of cavities in the second layer. In the laboratory, some actual physical systems (such as the coupled cavity array, the coupled spin chain, and the coupled superconducting resonators) usually have been chosen to implement quantum computing and quantum simulation [68–70]. So the influence of the coupling between two nearest-neighbor cavities in the second-layer environment on the dynamics of the system cannot be ignored.

So here we mainly study the dynamics of a qubit coupled with the two-layer environments. By using the quantum speed limit (QSL) time [51,71,72] to define the speedup evolutional process, we discuss the influence of the coupling strength between two nearest-neighbor cavities and the number of cavities in the second-layer environment on the quantum evolutional speed of the qubit. In the weak-coupling regime between the qubit and the first-layer environment, the Markovian and no-speedup dynamics of the qubit would follow when the second-layer environment has not been added [51]. In this paper, we elaborate how the non-Markovian speedup dynamics of the system can be obtained by controlling the coupling strength between two nearest-neighbor cavities and the number of cavities in the second-layer environment. Furthermore, we also analyze the influence of the parameters of the added second-layer environment on the non-Markovian speedup evolution of the qubit in the strong-coupling regime between the qubit and the first-layer environment.

II. THEORETICAL MODEL

We consider that the total system consists of a qubit and the hierarchical environment where the cavity m_0 and its corresponding memoryless reservoir serve as the first-layer environment for the two-level atom and the other coupled cavities m_n $(n = 1,\ 2,\ ...,\ N)$ and memoryless reservoirs involved act as the second-layer environment, as depicted in Fig. 1.

To be concrete, the qubit is coupled with strength Ω_0 to a mode m_0 which decays to a memoryless reservoir with a lossy rate Γ_0 and then the mode m_0 is further coupled simultaneously with strengths κ to modes m_n which also decay to their respective memoryless reservoirs with rates $\Gamma_n = \Gamma$. Besides, the coupling strength between two nearest-neighbor cavities is Ω in the second-layer environment. The total Hamiltonian can be given by $H = H_0 + H_I$ and reads

$$H_0 = \frac{\omega_0}{2}\sigma_z + \omega_c a^\dagger a + \sum_{n=1}^{N} \omega_n b_n^\dagger b_n,$$

$$H_I = \Omega_0(\sigma_+ a + \sigma_- a^\dagger) + \sum_{n=1}^{N} \kappa(a b_n^\dagger + a^\dagger b_n) + \sum_{\langle ij\rangle} \Omega(b_i^\dagger b_j + b_j^\dagger b_i). \tag{1}$$

In the above expressions, $\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$ is a Pauli operator for the qubit with transition frequency ω_0, $\sigma_\pm$ represent the raising and lowering operators of the qubit, and a $(a^\dagger)$ is the annihilation (creation) operator of mode m_0 with frequency ω_c. Besides, $b_n^\dagger$ (b_n) is the creation (annihilation) operator of mode m_n $(n = 1,\ 2,\ ...,\ N)$ with frequency ω_n. $\langle ij\rangle$ means the nearest-neighbor cavities in the second-layer environment. Then the density operator $\rho(t)$ of the total system obeys the following master equation:

$$\frac{d\rho}{dt} = -i[H,\rho] - \frac{\Gamma_0}{2}(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a) - \sum_{n=1}^{N} \frac{\Gamma}{2}(b_n^\dagger b_n \rho - 2b_n \rho b_n^\dagger + \rho b_n^\dagger b_n). \tag{2}$$

For simplicity, we suppose that the atom is initially in its excited state $|1\rangle_s$, while all the cavities are in their ground states $|0\ldots 0\rangle_{m_0\ldots m_n}$, i.e., the initial state of the total system is $\rho(0) = |10\ldots 0\rangle\langle 10\ldots 0|$. Since there exists at most one excitation in the total system at any time, then at time t the evolutional state of the total system can be written as $\rho(t) = [1 - d(t)]|\psi(t)\rangle\langle\psi(t)| + d(t)|00\ldots 0\rangle\langle 00\ldots 0|$, where $0 \leqslant d(t) \leqslant 1$, and $|\psi(t)\rangle = g(t)|10\ldots 0\rangle + c_0(t)|01\ldots 0\rangle + c_1(t)|001\ldots 0\rangle + \ldots + c_n(t)|000\ldots 1\rangle$, where $g(t), c_0(t), c_1(t), ..., c_n(t)$ correspond to probability amplitudes of the excited state for the atom or the modes m_0, m_1, ..., m_n, respectively. And when $t = 0$, $d(0) = 0$, $g(0) = 1$, $c_0(0) = c_1(0) = \cdots = c_n(0) = 0$. In addition, the probability amplitudes $G(t) = \sqrt{1 - d(t)}g(t)$, $C_0(t) = \sqrt{1 - d(t)}c_0(t)$, $C_1(t) = \sqrt{1 - d(t)}c_1(t)$, ..., $C_n(t) = \sqrt{1 - d(t)}c_n(t)$ of the unnormalized state vector $\sqrt{1 - d(t)}|\psi(t)\rangle$ are related to the master equation in Eq. (2). To give the dynamics of the system, we use the Schrödinger equation to acquire the approximate solutions of the above master equation. The non-Hermitian Hamiltonian, which includes the additional terms $-\frac{i\Gamma_0}{2}a^\dagger a - \sum_{n=1}^{N}\frac{i\Gamma}{2}b_n^\dagger b_n$, is considered to approximate the dissipative effect of the lossy cavities. Then a set of differential equations can be written:

$$i\dot{G}(t) = \Omega_0 C_0(t),$$

$$i\dot{C}_0(t) = \left(-\frac{i}{2}\Gamma_0\right)C_0(t) + \Omega_0 G(t) + \sum_{n=1}^{N}\kappa C_n(t), \tag{3}$$

$$i\sum_{n=1}^{N}\dot{C}_n(t) = \sum_{n=1}^{N}\left(2\Omega - \frac{i}{2}\Gamma\right)C_n(t) + \kappa N C_0(t).$$

The solutions of the above equations can be obtained by Laplace transformation and Laplace inverse transformation.

And the probability amplitude $G(t)$ can be acquired by numerical simulations. Then the reduced density matrix of the qubit in the atomic basis $\{|1\rangle_s, |0\rangle_s\}$ can be expressed as

$$\rho^s(t) = \begin{pmatrix} \rho_{11}(0)|G(t)|^2 & \rho_{01}(0)G(t)^* \\ \rho_{10}(0)G(t) & \rho_{00}(0) + \rho_{11}(0)[1-|G(t)|^2] \end{pmatrix}, \quad (4)$$

where $\rho_{11}(0) = 1$ and $\rho_{00}(0) = \rho_{01}(0) = \rho_{10}(0) = 0$. From now on, we mainly focus on how the dynamical behavior of the atomic system can be modified by the second-layer controllable environment in the weak qubit-m_0 coupling regime ($\Omega_0 < \Gamma_0/4$) and the strong qubit-m_0 couple regime ($\Omega_0 > \Gamma_0/4$).

III. NON-MARKOVIAN DYNAMICS CONTROL

To further illustrate the roles of the parameters in the considered second-layer environment, i.e., the number N of cavities and the coupling strength Ω between two nearest-neighbor cavities, in what follows we describe how to tune the controllable second-layer environment from Markovian to non-Markovian by manipulating N and Ω. A general measure $\mathbf{N}(\Phi)$ for non-Markovianity, which can be used to distinguish the Markovian dynamics and the non-Markovian dynamics of the quantum system, has been defined by Breuer *et al.* [8]. For a quantum process $\Phi(t)$, $\rho^s(t) = \Phi(t)\rho^s(0)$, where $\rho^s(0)$ and $\rho^s(t)$ denote the density operators at time $t = 0$ and at any time $t > 0$ of the quantum system, respectively, the non-Markovianity $\mathbf{N}(\Phi)$ is quantified by $\mathbf{N}(\Phi) = \max_{\rho^s_{1,2}(0)} \int_{\sigma>0} d\tau\sigma[t, \rho^s_{1,2}(0)]$, with $\sigma[t, \rho^s_{1,2}(0)] = \frac{d}{dt}\mathcal{D}(\rho^s_1(t), \rho^s_2(t))$ is the rate of change of the trace distance. The trace distance $\mathcal{D}$ describing the distinguishability between the two states is defined as [1] $\mathcal{D}(\rho^s_1, \rho^s_2) = \frac{1}{2}\|\rho^s_1 - \rho^s_2\|$, where $\|M\| = Tr(\sqrt{M^\dagger M})$ and $0 \leqslant \mathcal{D} \leqslant 1$. And $\sigma[t, \rho^s_{1,2}(0)] \leqslant 0$ [$\forall\ \rho^s_1(0), \rho^s_2(0)$] corresponds to all dynamical semigroups and all time-dependent Markovian processes. A process is non-Markovian if there exists a pair of initial states such that, at some time t, $\sigma[t, \rho^s_{1,2}(0)] > 0$. We should take the maximum over all initial states $\rho^s_{1,2}(0)$ to calculate the non-Markovianity. The states of these optimal pairs must be orthogonal and lie on the boundary of the space of physical states [34]. According to the dynamical maps of the form Eq. (4), it is proven that the optimal state pair of the initial states can be chosen as $\rho^s_1(0) = (|0\rangle_s + |1\rangle_s)/\sqrt{2}$ and $\rho^s_2(0) = (|0\rangle_s - |1\rangle_s)/\sqrt{2}$ [8,35]. Here, for this optimal state pair, the rate of change of the trace distance can be acquired $\sigma[t, \rho^s_1(0), \rho^s_2(0)] = \partial_t|G(t)|$. Then the non-Markovianity of the quantum system dynamics from $\rho^s(0)$ to $\rho^s(t)$ can be calculated by $\mathbf{N}(\Phi) = \int_0^t [\partial_t|G(t)|]|_{>0} dt$.

If there is no other-layer environment, the system's dynamics mainly depends on the parameters Ω_0 and Γ_0 in such a way that $\Gamma_0 > 4\Omega_0$ ($\Gamma_0 < 4\Omega_0$), identified as the weak-coupling (strong-coupling) regime, leads to Markovian (non-Markovian) dynamics. In the case of adding the second-layer environment, the atomic dynamics from ρ^s_0 to ρ^s_τ would be considered in the weak qubit-m_0 coupling regime (here τ is the actual evolution time). In Fig. 2, the non-Markovianity $\mathbf{N}(\Phi)$ of the atomic dynamics as function of the controllable second-layer environment parameters (κ, Ω, N) has been plotted. First, by fixing $\Omega = \Omega_0$, $\Gamma_0 = 5\Omega_0$ in Fig. 2(a), in

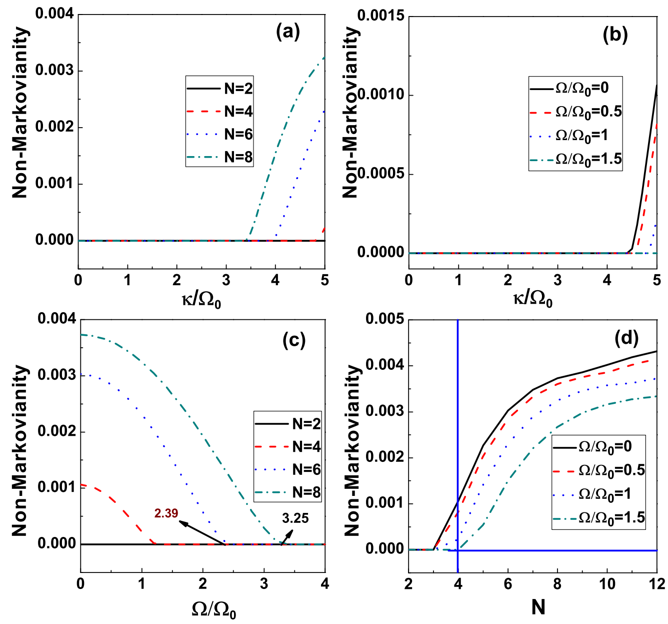

FIG. 2. (a), (b) The non-Markovianity $\mathbf{N}(\Phi)$ of the atomic system dynamics as a function of the coupling strength κ between the first-layer environment and the second-layer environment in the weak qubit-m_0 coupling regime $\Gamma_0 = 5\Omega_0$. (c), (d) The non-Markovianity $\mathbf{N}(\Phi)$ of the atomic system dynamics as a function of the coupling strength Ω between two nearest-neighbor cavities and the number of cavities N in the weak qubit-m_0 coupling regime $\Gamma_0 = 5\Omega_0$. The other parameters are (a) $\Omega = \Omega_0$, $\Gamma = 5\Omega_0$, $\tau = 3$; (b) $N = 4$, $\Gamma = 5\Omega_0$, $\tau = 3$; (c) and (d) $\kappa = 5\Omega_0$, $\Gamma = 5\Omega_0$, $\tau = 3$. Here, the optimal state pair of the atomic initial states $\rho^s_1(0) = (|0\rangle_s + |1\rangle_s)/\sqrt{2}$ and $\rho^s_2(0) = (|0\rangle_s - |1\rangle_s)/\sqrt{2}$ is considered to calculate the $\mathbf{N}(\Phi)$ numerically.

the case $N = 2$, the non-Markovian dynamics of the atomic system cannot be obtained by tuning the $m_0 - m_n$ coupling strength κ. However, if the lossy cavity m_0 simultaneously interacts with the second-layer environment constituted by more than two cavities, then a remarkable dynamical crossover from Markovian behavior to non-Markovian behavior can occur at a certain critical coupling strength κ_{c1}. When $\kappa < \kappa_{c1}$, the dynamics abides Markovian behavior, and then the non-Markovianity increases monotonically with increasing κ. Similarly, in the case $\Omega = 1.5\Omega_0$ in Fig. 2(b), no matter how we adjust the parameter κ, the dynamics is Markovian. However, the non-Markovian dynamics can be triggered at a threshold κ_{c2} by decreasing the coupling strength Ω between two nearest-neighbor cavities, such as $\Omega = 0, 0.5\Omega_0, \Omega_0$. And we can clearly point out that, when $\kappa < \kappa_{c2}$, the dynamics is Markovian, and the non-Markovianity increases with increasing κ in the case $\kappa > \kappa_{c2}$. Finally, in the weak qubit-m_0 coupling regime, it needs to be emphasized that, both the larger the value κ (N) and the smaller value Ω can be requested to trigger the stronger non-Markovianity of the system.

In the following, in order to more intuitively explain the effect of parameters Ω and N on the non-Markovianity of the system in the weak qubit-m_0 coupling regime, we first fix $\kappa = 5\Omega_0$ larger than the critical value κ_{c1} or κ_{c2} in the Figs. 2(c) and 2(d). It is worth noting that the dynamical crossover from Markovian behavior to non-Markovian behavior for the

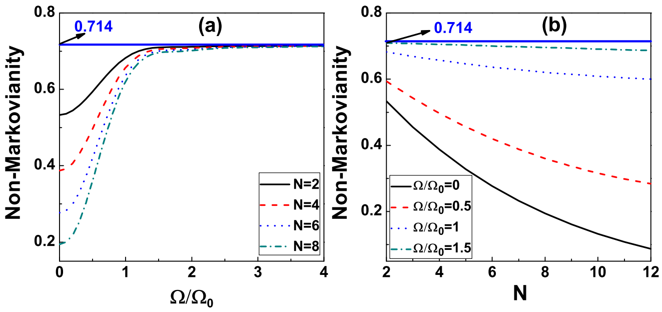

FIG. 3. (a), (b) The non-Markovianity $\mathbf{N}(\Phi)$ of the atomic system dynamics as a function of the coupling strength Ω between two nearest-neighbor cavities and the number of cavities N in the strong qubit-m_0 coupling regime $\Gamma_0 = 0.2\Omega_0$. The parameters are $\Gamma = 0.2\Omega_0$, $\kappa = 0.2\Omega_0$, $\tau = 3$. Here, the optimal state pair of the atomic initial states $\rho_1^s(0) = (|0\rangle_s + |1\rangle_s)/\sqrt{2}$ and $\rho_2^s(0) = (|0\rangle_s - |1\rangle_s)/\sqrt{2}$ is considered to calculate $\mathbf{N}(\Phi)$ numerically.

atomic dynamics (ρ_0^s to ρ_τ^s) could appear by tuning Ω and N. When the value N is confirmed in Fig. 2(c), in the case $\Omega > \Omega_c$ (Ω_c means the critical value of Ω), the dynamics always follows Markovian behavior, but the non-Markovianity increases monotonically with reducing Ω when $\Omega < \Omega_c$. This means that increasing of the coupling strength Ω in the second-layer environment could suppress the non-Markovian dynamics of the atomic system. Moreover, the larger the value N, the larger the critical value Ω_c should be acquired. Take the cases in Fig. 2(c): when $N = 6$, we find the critical value $\Omega_{c1} = 2.39\Omega_0$. While in the case $N = 8$, $\Omega_{c2} = 3.25\Omega_0$ should be given. Furthermore, it is interesting to find that, by fixing $\Omega = 1.5\Omega_0$ in the Fig. 2(d), the dynamics abides by Markovian up to $N = 3$, but becomes non-Markovian starting from $N = 4$. However, the non-Markovian dynamics is induced from $N = 3$ by decreasing the coupling strength Ω, say, $\Omega = 0, 0.5\Omega_0, \Omega_0$. In general, it is worth noting that a larger coupling strength Ω (N) could lead to a larger critical N (Ω). Finally, in the weak qubit-m_0 coupling regime, we find that the non-Markovian dynamics can be triggered by manipulating the added second-layer environment parameters.

It is well known that in the absence of the second-layer environment, the dynamics of the system is non-Markovian in the strong qubit-m_0 coupling regime. Next, Figs. 3(a) and 3(b) show how the non-Markovianity $\mathbf{N}(\Phi)$ of the atomic dynamics from ρ_0^s to ρ_τ^s is affected by the parameters Ω or N with the actual evolution time $\tau = 3$ when the strong qubit-m_0 coupling satisfied $\Gamma_0 = 0.2\Omega_0$. The blue solid line $\mathbf{N}(\Phi) = 0.714$ in Fig. 3 means the value of non-Markovianity without the second-layer environment. By fixing N in Fig. 3(a), the non-Markovianity increases and tends to the original non-Markovianity $\mathbf{N}(\Phi) = 0.714$ of the atomic system with increasing coupling strength Ω between two nearest-neighbor cavities, implying the coupling strength Ω between the two nearest neighbors in the second-level environment can promote the non-Markovian dynamics of the system. Furthermore, by fixing Ω in the Fig. 3(b), the non-Markovianity of the system increases with decreasing the number of cavities N in the second level environment. And it is clear that, irrespective of Ω, N, the non-Markovianity always is smaller than the original non-Markovianity (0.714) of the system. That is to say, compared with the previous non-Markovian dynamics in the strong qubit-m_0 coupling regime, the added second-level environment cannot contribute to the non-Markovian dynamics of the system. Besides, in stark contrast, when the parameters N and Ω make it possible to trigger the non-Markovianity of the system under the weak qubit-m_0 coupling regime, the non-Markovianity of the system is weakened under the strong qubit-m_0 coupling regime at this time. That is to say, in the case of the weak qubit-m_0 coupling regime and the strong qubit-m_0 coupling regime, the second-layer environment parameters Ω and N have opposite effects on the non-Markovian dynamics of the system.

IV. QUANTUM SPEEDUP OF THE ATOMIC DYNAMICS

To characterize how fast the quantum system evolves, here we use the definition of the QSL time for an open quantum system, which can be helpful to analyze the maximal speed of evolution of an open system. The QSL time between an initial state $\rho^s(0) = |\phi_0\rangle\langle\phi_0|$ and its target state $\rho^s(\tau)$ (the evolutional state of the quantum system at the actual evolution time τ) for open system is defined by [51] $\tau_{\text{QSL}} = \sin^2\{\mathbf{B}[\rho^s(0),\ \rho^s(\tau)]\}/\Lambda_\tau^\infty$, where $\mathbf{B}[\rho^s(0),\ \rho^s(\tau)] = \arccos\sqrt{\langle\phi_0|\rho^s(\tau)|\phi_0\rangle}$ denotes the Bures angle between the initial and target states of the system, and $\Lambda_\tau^\infty = \tau^{-1}\int_0^\tau \|\dot{\rho}^s(t)\|_\infty dt$ with the operator norm $\|\dot{\rho}^s(t)\|_\infty$ equaling the largest singular value of $\dot{\rho}^s(t)$. When the ratio between the QSL time and the actual evolution time equals unity, i.e., $\tau_{\text{QSL}}/\tau = 1$, the quantum system evolution is already along the fastest path and possesses no potential capacity for further quantum speedup. When $\tau_{\text{QSL}}/\tau < 1$, the speedup evolution of the quantum system may occur, and the smaller τ_{QSL}/τ, the greater the capacity for potential speedup will be.

Recently, according to the dynamical maps of the form Eq. (4),the relationship between non-Markovianity and the QSL times has been given by [56,59]

$$\frac{\tau_{\text{QSL}}}{\tau} = \frac{1 - |G(\tau)|^2}{2\mathbf{N}(\Phi) + 1 - |G(\tau)|^2}$$

So the same critical values (Ω, N) can be obtained for the transition from Markovian to non-Markovian process and from no-speedup of quantum evolution to speedup. The above equation implies that larger non-Markovianity would lead to lower QSL times (that is to say, the greater the capacity for potential speedup could be). In our model, it is easy to find that accelerating the evolution of quantum system can also be achieved by the controllable non-Markovianity discussed above. Obviously, in this section, by choosing the atomic excited state $\rho^s(0) = |1\rangle_s\langle 1|$ as the initial state, we focus on how the coupling strength Ω and the number of cavities N to accelerate the evolution of quantum system.

For the weak qubit-m_0 coupling regime, the variations of the QSL time τ_{QSL}/τ with respect to Ω and N are plotted in Fig. 4. It is clear that, in the Fig. 4(a), the quantum system has no potential capacity for the further quantum speedup for $N \leqslant 2$. In Fig. 4(a), in the cases $N = 2$, 4, 6, 8, the no-speedup evolution ($\tau_{\text{QSL}}/\tau = 1$) could be followed, and the speedup evolution ($\tau_{\text{QSL}}/\tau < 1$) would occur when the coupling strength Ω is less than a certain critical coupling strength Ω_c. In addition, it is worth noting that the larger the

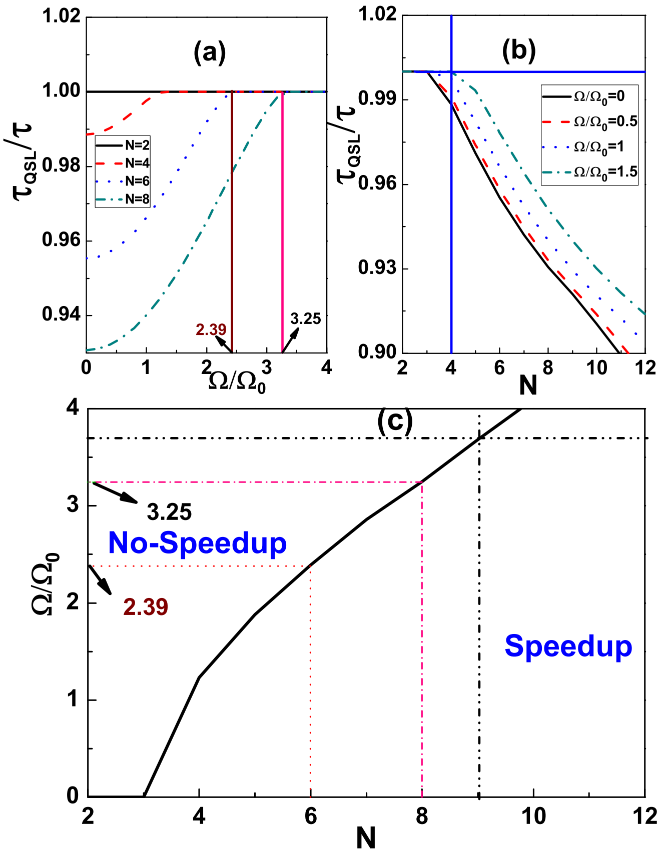

FIG. 4. (a), (b) The QSL time for the atomic system dynamic with $\Gamma_0 = 5\Omega_0$ in the weak qubit-m_0 coupling regime as a function of the coupling strength Ω and the number of cavities N in the second-layer environment. (c) Phase diagram of QSL time in the Ω-N plane with $\Gamma_0 = 5\Omega_0$ in the weak qubit-m_0 coupling regime. The parameters are $\Gamma = 5\Omega_0$, $\kappa = 5\Omega_0$.

value of N, the larger the critical value Ω_c may be requested. As shown in the Fig. 4(a), when $N = 6$, $\Omega_{c1} = 2.39\Omega_0$. While $N = 8$, $\Omega_{c2} = 3.25\Omega_0$. That is the same as the above discussion for the non-Markovianity. As for Fig. 4(b), by fixing $\Omega = 1.5\Omega_0$, the speedup evolution may occur when $N > 4$ in the second-layer environment. However, considering $\Omega = 0$, $0.5\Omega_0$, Ω, the speedup evolution can be induced by $N > 3$. So we get an interesting conclusion that the coupling strength Ω and the number of cavities N in the second-layer environment have the opposite effect on the speedup quantum evolution of the atomic system; namely, the quantum evolution would speedup by decreasing Ω or increasing N. So in the weak qubit-m_0 coupling regime, the purpose of accelerating evolution can be achieved by controlling the hierarchical environment.

To clear the region of parameters in which the speedup dynamics of the system can be eventuated in the weak qubit-m_0 coupling regime, Fig. 4(c) describes the Ω-N phase diagrams. The transition points from no-speedup to speedup regime could be acquired. Take the cases in Fig. 4(c): when the value $N = 6$ or 8 is fixed, the corresponding critical values of Ω are also given respectively by $\Omega_{c1} = 2.39\Omega_0$ and $\Omega_{c2} = 3.25\Omega_0$. By fixing N, when $\Omega < \Omega_c$, the quantum speedup evolution would occur and the transition to no-speedup would happen when increasing Ω. It is worth noting that the number of the cavities in the second-layer environment must be greater than two. Decreasing Ω and increasing N can drive the purpose of speedup evolution of the quantum system in the weak qubit-m_0 coupling regime.

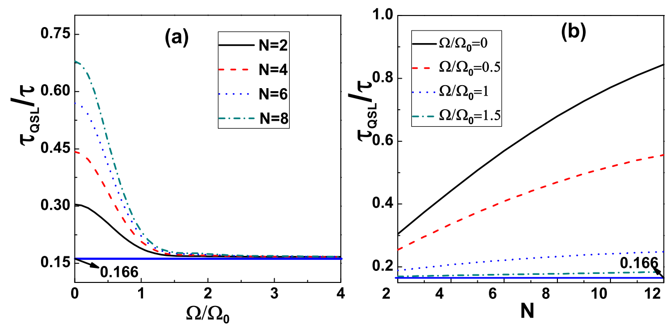

FIG. 5. (a), (b) The QSL time for the atomic system dynamic with $\Gamma_0 = 0.2\Omega_0$ in the strong qubit-m_0 coupling regime as a function of the coupling-strength Ω and the number of cavities N. The parameters are $\Gamma = 0.2\Omega_0$, $\kappa = 0.2\Omega_0$.

For the strong qubit-m_0 coupling regime, by fixing $\Gamma_0 = 0.2\Omega_0$ and $\kappa = 0.2\Omega_0$, we plot the variations of the QSL time τ_{QSL}/τ with respect to Ω and N in Fig. 5. The blue solid line indicates that when there is no second-layer environment, the value of τ_{QSL}/τ is equal to 0.166. Clearly shown in Fig. 5(a), the value of τ_{QSL}/τ decreases and tends to 0.166 as Ω increases. That is to say, the increasing of Ω can accelerate the evolution of the system in our two-layer environments. Besides, by fixing Ω in the Fig. 5(b), the QSL time always increase as N increases. However, regardless of how to tune parameters N or Ω in Fig. 5, the QSL time is always larger than 0.166. That is to say, although N or Ω can affect the QSL time of the dynamics of the system in the strong qubit-m_0 coupling regime, the added second-level environment cannot play a beneficial role on the speedup of the dynamics of the system.

V. CONCLUSION

In conclusion, we investigated the dynamics of the qubit in a controllable hierarchical environment where the first-layer environment is a lossy cavity m_0 and the second-layer environment is the other coupled lossy cavities. Some interesting phenomena are observed. By controlling the number of cavities N and the coupling strength Ω between two nearest-neighbor cavities in the second layer, two dynamical crossovers of the quantum system, from Markovian to non-Markovian dynamics and from no-speedup evolution to speedup evolution, have been achieved in the weak qubit-m_0 coupling regime. And it is worth noting that the coupling strength Ω and the number of cavities N have the opposite effect on the non-Markovian dynamics and speedup evolution of the atomic system in the weak qubit-m_0 coupling regime. The transitions from no-speedup phase to speedup phase and from Markovian to non-Markovian effect for the system, have been demonstrated in our work. To further illustrate the physics behind the above results, we try to give a discussion for this problem based on the effective coupling strength between the first layer (the single lossy cavity) and the second layer (mutually interacting lossy cavities). The effective coupling strength κ_{eff} between the first layer and the second layer would increases with the number of the second-layer cavities increasing, as shown in Eq. (3). However, the larger intercavity strength Ω in the second-layer environment would weaken the effective coupling strength. The stronger effective coupling strength $\kappa_{\rm eff}$ could characterize the impact of the added second-layer environment on the dynamics of the atomic system.

On the other hand, by considering the strong qubit-m_0 coupling regime, compared with the non-Markovian dynamics without the second-layer environment, the added second-layer environment cannot promote the speedup evolution of the system. And for the weak qubit-m_0 coupling regime and strong qubit-m_0 coupling regime, the second-layer environment parameters Ω and N have opposite effects on the non-Markovian speedup dynamics of the system.

In addition, a recent experiment allowed one to drive the open system from no-speedup to speedup by altering the number of independent atoms (corresponding to the cavities in the second-layer environment), and a slow atomic beam of ^{85}Rb atoms should be produced in the magneto-optical trap [57]. So the number of the cavities could be experimentally controlled. The second-layer environment is also hopefully realizable in the coupled cavity array [73] or the capacitively coupled transmon qubits [74]. The nearest-neighbor coupling strength in the second layer not only could be controlled by all-optical modulation of intercavity couplings via ac Stark or the stimulated Raman adiabatic passage [68], but also could be manipulated by the value of the capacitor in the coupled superconducting resonators [74]. Besides, recent experiments have demonstrated that the scheme with a metal nanoparticle surrounded by a collection of quantum emitters is used to implement the reversible exchange energy with the quantum emitters [75]. Therefore, according to these potential candidates for the controllable environment, our proposed scheme is experimentally feasible. This hierarchical environment-assisted non-Markovian speedup dynamics control is essential to most purposes of quantum optimal control.

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[1] M. Nielsen and I. Chuang, *Quantum Optics* (Cambridge University Press, Cambridge, 2000).

[2] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O'Brien, Nature (London) **464**, 45 (2010).

[3] H. P. Breuer and F. Petruccione, *Theory of Open Quantum Systems* (Oxford University Press, New York, 2002).

[4] J. H. An and W. M. Zhang, Phys. Rev. A **76**, 042127 (2007).

[5] J. P. Paz and A. J. Roncaglia, Phys. Rev. Lett. **100**, 220401 (2008); Phys. Rev. A **79**, 032102 (2009).

[6] M. M. Wolf, J. Eisert, T. S. Cubitt, and J. I. Cirac, Phys. Rev. Lett. **101**, 150402 (2008).

[7] M. W. Y. Tu and W. M. Zhang, Phys. Rev. B **78**, 235311 (2008).

[8] H. P. Breuer, E. M. Laine, and J. Piilo, Phys. Rev. Lett. **103**, 210401 (2009); E. M. Laine, J. Piilo, and H. P. Breuer, Phys. Rev. A **81**, 062115 (2010).

[9] D. Chruscinski and A. Kossakowski, Phys. Rev. Lett. **104**, 070406 (2010); D. Chruscinski, A. Kossakowski, and A. Rivas, Phys. Rev. A **83**, 052128 (2011).

[10] H. N. Xiong, W. M. Zhang, X. Wang, and M. H. Wu, Phys. Rev. A **82**, 012105 (2010); C. U. Lei and W. M. Zhang, *ibid.* **84**, 052116 (2011).

[11] A. Rivas, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. **105**, 050403 (2010).

[12] M. Znidaric, C. Pineda, and I. Garcia-Mata, Phys. Rev. Lett. **107**, 080404 (2011).

[13] W. M. Zhang, P. Y. Lo, H. N. Xiong, M. Wei-Yuan Tu, and F. Nori, Phys. Rev. Lett. **109**, 170402 (2012).

[14] B. H. Liu, L. Li, Y. F. Huang, C. F. Li, G. C. Guo, E. M. Laine, H. P. Breuer, and J. Piilo, Nat. Phys. **7**, 931 (2011).

[15] K. H. Madsen, S. Ates, T. Lund-Hansen, A. Loffler, S. Reitzenstein, A. Forchel, and P. Lodahl, Phys. Rev. Lett. **106**, 233601 (2011).

[16] J. S. Tang, C. F. Li, Y. L. Li, X. B. Zou, G. C. Guo, H. P. Breuer, E. M. Laine, and J. Piilo, Europhys. Lett. **97**, 10002 (2012).

[17] S. B. Xue, R. B. Wu, W. M. Zhang, J. Zhang, C. W. Li, and T. J. Tarn, Phys. Rev. A **86**, 052304 (2012).

[18] A. D'Arrigo, R. Lo. Franco, G. Benenti, E. Paladino, and G. Falci, Ann. Phys. (NY) **350**, 211 (2014).

[19] B. Bylicka, D. Chruscinski, and S. Maniscalco, Sci. Rep. **4**, 5720 (2014).

[20] Z. L. Xiang, S. Ashhab, J. You, and F. Nori, Rev. Mod. Phys. **85**, 623 (2013).

[21] C. H. Bennett and D. P. Divincenzo, Nature (London) **404**, 247 (2000).

[22] J. S. Xu, K. Sun, C. F. Li, X. Y. Xu, G. C. Guo, E. Andersson, R. Lo Franco, and G. Compagno, Nat. Commun. **4**, 2851 (2013).

[23] B. Aaronson, R. Lo Franco, and G. Adesso, Phys. Rev. A **88**, 012120 (2013).

[24] L. M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature (London) **414**, 413 (2001).

[25] Y. Dong, Y. Zheng, S. Li, C. C. Li, X. D. Chen, G. C. Guo, and F. W. Sun, npj Quantum Inf. **4**, 3 (2018).

[26] S. Lorenzo, F. Plastina, and M. Paternostro, Phys. Rev. A **88**, 020102(R) (2013).

[27] F. Caruso, V. Giovannetti, C. Lupo, and S. Mancini, Rev. Mod. Phys. **86**, 1203 (2014).

[28] E. M. Laine, J. Piilo, and H. P. Breuer, Europhys. Lett. **92**, 60010 (2010).

[29] J. Dajka and J. Łuczka, Phys. Rev. A **82**, 012341 (2010).

[30] A. Smirne, H. P. Breuer, J. Piilo, and B. Vacchini, Phys. Rev. A **82**, 062114 (2010).

[31] S. Luo, S. Fu, and H. Song, Phys. Rev. A **86**, 044101 (2012).

[32] X. M. Lu, X. Wang, and C. P. Sun, Phys. Rev. A **82**, 042103 (2010).

[33] D. Chruscinski and S. Maniscalco, Phys. Rev. Lett. **112**, 120404 (2014); C. Addis, B. Bylicka, D. Chruscinski, and S. Maniscalco, Phys. Rev. A **90**, 052103 (2014).

[34] S. Wißmann, A. Karlsson, E. M. Laine, J. Piilo, and H. P. Breuer, Phys. Rev. A **86**, 062108 (2012).
[35] J. G. Li, J. Zou, and B. Shao, Phys. Rev. A **81**, 062124 (2010).
[36] Md. M. Ali, P.-Y. Lo, M. Wei-Yuan Tu, and W.-M. Zhang, Phys. Rev. A **92**, 062306 (2015).
[37] T. T. Ma, Y. S. Chen, T. Chen, S. R. Hedemann, and T. Yu, Phys. Rev. A **90**, 042108 (2014).
[38] Z. X. Man, N. B. An, and Y. J. Xia, Opt. Express **23**, 5763 (2015).
[39] A. Smirne, D. Brivio, S. Cialdi, B. Vacchini, and M. G. A. Paris, Phys. Rev. A **84**, 032112 (2011).
[40] R. Lo Franco, B. Bellomo, E. Andersson, and G. Compagno, Phys. Rev. A **85**, 032318 (2012).
[41] E. M. Laine, H. P. Breuer, J. Piilo, C. F. Li, and G. C. Guo, Phys. Rev. Lett. **108**, 210402 (2012).
[42] B. H. Liu, D. Y. Cao, Y. F. Huang, C. F. Li, G. C. Guo, E. M. Laine, H. P. Breuer, and J. Piilo, Sci. Rep. **3**, 1781 (2013).
[43] Z. X. Man, Y. J. Xia, and R. Lo Franco, Phys. Rev. A **92**, 012315 (2015).
[44] Z. X. Man, N. B. An, and Y. J. Xia, Phys. Rev. A **90**, 062104 (2014).
[45] H. P. Breuer, E. M. Laine, J. Piilo, and B. Vacchini, Rev. Mod. Phys. **88**, 021002 (2016).
[46] I. de Vega and D. Alonso, Rev. Mod. Phys. **89**, 015001 (2017).
[47] A. Rivas, S. F. Huelga, and M. B. Plenio, Rep. Prog. Phys. **77**, 094001 (2014).
[48] M. J. W. Hall, J. D. Cresser, L. Li, and E. Andersson, Phys. Rev. A **89**, 042120 (2014).
[49] J. Teittinen, H. Lyyra, B. Sokolov, and S. Maniscalco, arXiv:1803.06844.
[50] Y. J. Zhang, Y. J. Xia, and H. Fan, Europhys. Lett. **116**, 30001 (2016).
[51] S. Deffner and E. Lutz, Phys. Rev. Lett. **111**, 010402 (2013).
[52] Z. Sun, J. Liu, J. Ma, and X. Wang, Sci. Rep. **5**, 8444 (2015).
[53] X. Meng, C. Wu, and H. Guo, Sci. Rep. **5**, 16357 (2015).
[54] Y. J. Zhang, W. Han, Y. J. Xia, J. P. Cao, and H. Fan, Sci. Rep. **4**, 4890 (2014).
[55] Y. J. Zhang, W. Han, Y. J. Xia, Y. M. Yu, and H. Fan, Sci. Rep. **5**, 13359 (2015).
[56] Z. Y. Xu, S. Luo, W. L. Yang, C. Liu, and S. Q. Zhu, Phys. Rev. A **89**, 012307 (2014).
[57] A. D. Cimmarusti, Z. Yan, B. D. Patterson, L. P. Corcos, L. A. Orozco, and S. Deffner, Phys. Rev. Lett. **114**, 233602 (2015).
[58] H. B. Liu, W. L. Yang, J. H. An, and Z. Y. Xu, Phys. Rev. A **93**, 020105(R) (2016).
[59] Y. J. Zhang, W. Han, Y. J. Xia, J. P. Cao, and H. Fan, Phys. Rev. A **91**, 032112 (2015).
[60] K. Xu, W. Han, Y. J. Zhang, Y. J. Xia, and H. Fan, Ann. Phys. (NY) **388**, 1 (2018).
[61] Z. Y. Xu and S. Q. Zhu, Chin. Phys. Lett. **31**, 020301 (2014).
[62] M. Cianciaruso, S. Maniscalco, and G. Adesso, Phys. Rev. A **96**, 012105 (2017).
[63] R. Hanson, V. V. Dobrovitski, A. E. Feiguin, O. Gywat, and D. D. Awschalom, Science **320**, 352 (2008).
[64] R. Hanson, L. P. Kouwehoven, J. R. Petta, S. Tarucha, and L. M. K. Vandersypen, Rev. Mod. Phys. **79**, 1217 (2007).
[65] J. J. Pla, K. Y. Tan, J. P. Dehollain, W. H. Lim, J. J. L. Morton, D. N. Jamieson, A. S. Dzurak, and A. Moreello, Nature (London) **489**, 541 (2012).
[66] E. A. Chekhovich, M. N. Makhonin, A. I. Tartakovskii, A. Yacoby, H. Bluhm, K. C. Nowack, and L. M. K. Vandersypen, Nat. Mater. **12**, 494 (2013).
[67] A. M. Tyryshkin, S. Tojo, J. J. L. Morton, H. Riemann, N. V. Abrosimov, P. Becker, H. J. Pohl, T. Schenkel, M. L. W. Thewalt, K. M. Itoh, and S. A. Lyon, Nat. Mater. **11**, 143 (2012).
[68] K. Y. Xia and J. Twamley, Phys. Rev. X **3**, 031013 (2013).
[69] J. Q. You and F. Nori, Nature (London) **474**, 589 (2011).
[70] C. L. Degen, F. Reinhard, and P. Cappellaro, Rev. Mod. Phys. **89**, 035002 (2017).
[71] M. M. Taddei, B. M. Escher, L. Davidovich, and R. L. de Matos Filho, Phys. Rev. Lett. **110**, 050402 (2013).
[72] A. del Campo, I. L. Egusquiza, M. B. Plenio, and S. F. Huelga, Phys. Rev. Lett. **110**, 050403 (2013).
[73] F. Nissen, S. Schmidt, M. Biondi, G. Blatter, H. E. Tureci, and J. Keeling, Phys. Rev. Lett. **108**, 233603 (2012).
[74] S. Hacohen-Gourgy, V. V. Ramasesh, C. De Grandi, I. Siddiqi, and S. M. Girvin, Phys. Rev. Lett. **115**, 240501 (2015).
[75] A. Delga, J. Feist, J. Bravo-Abad, and F. J. Garcia-Vidal, Phys. Rev. Lett. **112**, 253601 (2014).