Title:

A Charging-Scheme Decision Model for Electric Vehicle Battery Swapping Station Using Varied Population Evolutionary Algorithms

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Abstract:

This paper proposes a new battery swapping station (BSS) model to determine the optimized charging scheme for each incoming Electric Vehicle (EV) battery. The objective is to maximize the BSS's battery stock level and minimize the average charging damage with the use of different types of chargers. An integrated objective function is defined for the multi-objective optimization problem. The genetic algorithm (GA), differential evolution (DE) algorithm and three versions of particle swarm optimization (PSO) algorithms have been implemented to solve the problem, and the results show that GA and DE perform better than the PSO algorithms, but the computational time of GA and DE are longer than using PSO. Hence, the varied population genetic algorithm (VPGA) and varied population differential evolution (VPDE) algorithm are proposed to determine the optimal solution and reduce the computational time of typical evolutionary algorithms. The simulation results show that the performances of the proposed algorithms are comparable with the typical GA and DE, but the computational times of the VPGA and VPDE are significantly shorter. A 24-hour simulation study is carried out to examine the feasibility of the model.

Keywords:

Battery swapping stations, electric vehicles, evolutionary algorithms, varied population.

1. Introduction

The business of EVs is growing and blooming. People are buying them not just due to environmental concerns but also the modern status symbol. Yet, EVs are not perfect substitutes of traditional fossil fuel vehicles [1]- [3]. They would post problems especially when one has to travel long distance in a day. One solution to this limited travel distance problem is battery swapping. A driver can drive to a nearby battery swapping station (BSS) for a battery swap. The entire process takes around three minutes and then the BSS would have to recharge the swapped battery for future swapping service.

Electric vehicle technology has been applied in many countries all over the world, and many different kinds of vehicles, such as private cars, taxicabs, buses and trucks have gone electric. During 2015, the new EV registration was 550,000, which was up 68% from the year of 2014. Globally, the number of EVs has reached 1.5 million in May 2016 [4].

However, the battery limitation remains the chief drawback to the development of EV technology. Firstly, the battery is deemed to be an expensive component of an electric vehicle, considering its initial purchase cost related to the life of the battery. Secondly, the travel range per charge mainly depends on the chemistry of the battery. The most commonly used battery on EVs is the lithium-ion battery, which can provide a travel range from 320 to 480 km per charge. For example, Model S from Tesla Motors is equipped with an 85 kWh battery containing 7,104 lithium-ion battery cells with a rated distance of 510 km [5]. Yet, the range is still too short for some travelers who need to travel long distance within a day. Thirdly, charging time varies depending on different types of charging technology and equipment. For a Model S Tesla charging from a 120V/15A household outlet, the travel-range only increases by 6 km per hour of charge. If charged from a 240V/50A power outlet, the charging rate increases to 46 km per hour charging. When charged from a Tesla charging station using fast charging technology, the rate may reach 92 km per hour of charge [6].

Some research on battery swapping station model has focused on maximizing the BSS's revenue by applying renewable energy resource [7][8], selling electricity back to grid [9][10] and building centralized charging stations [11]. Some researchers also aimed at determining the location distribution of BSSs to maximize the revenue [12][13]. Others proposed a battery swapping station model for electric buses [14]. To summarize, previous work on BSS aimed at maximizing its revenue with the use of optimization algorithms.

The BSS operation model of this paper is described next. Firstly, this decision model is only designed for a single BSS, and the swapped batteries are charged at the same location. Secondly, the BSS would buy electricity from the power grid, and the selling of electricity from battery to grid (B2G) is not considered in this model. Thirdly, the proposed model would not forecast the battery swapping demands, but the BSS would encourage the EV drivers to make advanced notice, so that the BSS would make the charging decision with the known orders. Lastly, when a battery is swapped and being charged, its method cannot be changed during the charging period.

Compared with previous work, the main contribution of this paper is on the development of a new battery swapping station model for determining the near-optimal scheme for recharging batteries at a battery swapping station, aimed at minimizing the running cost. The typical optimization algorithms including genetic algorithm (GA), differential evolution (DE) algorithm and particle swarm optimization (PSO) algorithms are implemented to solve the problem. The results with 100 EV case show that GA and DE can achieve an optimal objective value, but the PSO algorithms fail to obtain the acceptable primary objective even though the computational times of PSO algorithms are shorter than GA and DE. As the GA and DE can obtain a desired objective but the computational times are relatively longer, we have proposed the varied population genetic algorithm (VPGA) and varied population differential evolution (VPDE) algorithms to reduce the computational time and improve the performance of the original version of GA and DE. Hence, the model and algorithms developed in this paper are inherently different from previous work.

This paper is organized as follows. Section 2 describes literature review on the battery swapping station model and battery charging technologies. Section 3 introduces the new operation model. The optimization model is defined in Section 4. Section 5 introduces the proposed algorithms in the paper. A case study, with two examples and a 24-hour simulation, is presented and discussed in Section 6. Finally, Section 7 gives the conclusions and describes the contribution of the paper.

2. Literature Review

2.1. Battery Swapping Station Model

Some studies have proposed EV battery swapping with the aim of reducing carbon emissions by applying renewable energy sources (RES). A new economic dispatch model considering wind power for an EV battery swapping station using the Particle Swarm Optimization (PSO) method is proposed in [7]. The results showed that a battery swapping station operated on wind power can be profitable. In [8], battery swapping station distribution and power distribution models were established based on the energy exchanges in the battery swapping system. The "feed-in shift" method was used to realize the optimal configuration of wind, solar and hydro power.

Other battery swapping station models have been proposed as mediators between the power system and EV owners by an optimized business and operating model in [9] and [10]. In the business model, the BSS aimed to meet battery demand and maximize its profits. Taking the time dependencies of electricity prices into account, the BSS would make a profit by buying electricity during low-price periods and selling electricity during high-price periods. An intelligent battery information management system [11] for swapping batteries was designed to eliminate the limitations of the long charging process and huge infrastructure cost. The idea was to charge swapped batteries in a management hub and then deliver them to a switching station via an optimized route with minimal supply chain costs. Nevertheless, the model assumes that all battery charging takes place only in the hub, which would result in many shipments of batteries.

A distribution model of the BSSs in a particular area can be used to optimize the cost-benefit and enhance safety [12]. The life cycle cost (LCC) criterion was used to specify the objective of their model, which qualified the cost of investment, operation, maintenance, failure, and disposal. In addition, considering the fluctuation of electricity prices during the day, a BSS would adjust the charging strategies based on the electricity price in order to generate more revenue. The proposed model was a multistage, nonlinear, constrained mixed-integer optimization problem, which is difficult to solve by classic mathematical techniques. A differential evolution (DE) algorithm, which is a heuristic optimization technique, was used to solve the problem. The optimization model would give a candidate solution by determining the location, size, and strategy of the charging station based on the fixed demand of electric vehicles.

A robust optimization model was used for battery swapping infrastructure in [10]. The decision model would determine the location of a battery swapping station serving EVs on freeways. The operation policy of the charging service provider was determined with the aim of preparing sufficient stock batteries for incoming swapping demand. Two particular policies, FIFO (first-in first-out) and Highest State-of-Charge (SOC) first, were compared in order to operate the swapping station with different numbers of batteries in stock. Considering the fixed costs of operating the swapping stations plus the battery holding costs, a cost-concerned model was proposed to minimize the total cost. In addition, the authors also proposed a goal-driven model that aimed at obtaining a target return-on-investment (ROI). Simulations were carried out based on a realistic data set of the San Francisco Bay Area freeway network. The results showed that a fast charging speed was critical to the profitability of the BSS. However, when the charging speed was improved to an acceptable level, the battery cycle life becomes very important.

A battery swapping station model was designed for the bus terminal at the Hong Kong International Airport in [14], which serves the electric bus routes in the airport. The BSS model aimed to optimize the battery charging methods to maximize the number of batteries in stock. However, there are many limitations discussed in the paper. Firstly, the target of the model is only the airport bus terminal serving electric buses. Secondly, the mathematical formulation was over simplified, and did not sufficiently consider the conditions of the charging stations and the batteries. Thirdly, only a basic version of the genetic algorithm was used to obtain the optimal solution.

In this paper, based on the review of previous work, we assume that the battery swapping station carries out the charging at the same location, which would result in a more economical and efficient system for battery swapping and management. The BSS does not aim to make a profit by selling electricity back to the grid, as each discharging process would reduce the battery's lifecycle and increase the potential expense. Some typical optimization algorithms are studied, and then new extension has been made to obtain improved solution for the optimization model.

2.2. Different Types of Battery Charging

Many factors, such as power and energy density, charging cycle life, calendar life, weight, and environmental friendliness, affect the development of battery technologies. Compared with lead acid, nickel metal hydride and sodium batteries, the lithium-ion battery is regarded as the best type of battery for use in EVs [15] - [18].

It has been widely reported that the battery's charging strategy is related to its charging rate, cycle life, temperature, and safety [19] - [21]. The constant current (CC), constant voltage (CV) algorithm [22] is the common standard for charging lithium-ion batteries. As a typical charging operation would take much longer than refilling a vehicle with gasoline, fast- and ultra-fast charging technologies were proposed in [23] to improve the charging efficiency. However, these fast-charging schemes require certain conditions. First, the battery must be designed to be charged with a high current. Secondly, the fast charging only applies to the first stage of charging, which is typically the constant current stage. Thirdly, the high current should be reduced after the battery is around 70 percent charged, in order to protect the circuit and prolong the battery life. Lastly, the fast charging can only be used in an environment in which a certain temperature can be maintained.

Cycle life is considered as one of the most important characteristics of EV batteries. There have been many experiments and studies that have focused on factors related to battery cycle life, such as material type, environmental temperatures, and charging depth [24] - [28]. After a battery has reached a long cycle life, the available electricity capacity is usually lower. However, recent research has shown that the new lithium-ion battery with $Li_4Ti_5O_{12}$ (LTO) cells has almost no capacity loss, even after many charging cycles [29].

Suggestions for good battery charging practice are given in [23]. First of all, a moderate rate should be used for normal charging even if the battery can be charged by high current. Also, fast and ultra-fast charging schemes cannot fully charge the battery. Lastly, the fast-charging process must be carefully monitored to avoid battery overheating. A table of charging parameters is listed in Table 1. The battery charger topologies, charging power levels, and charging infrastructure for electric vehicles are also reviewed in [30].

The cycle life of a battery is also related to the charging rate. When the discharge capacity drops to 500 mAh, we assume this is the retirement condition of the battery, and the cycle life of a battery charged using 1C, 2C and 3C is around 500, 300, and 100 respectively [23]. Here, the charge and discharge rates are governed by C-rates. The charging rate 1C means that a fully charged battery rated at 1Ah should provide 1A for one hour, or an empty battery can be fully recharged using a 1A charger for one hour [31]. Hence, the damage to a battery charged using an ultra-fast charger is considerably increased. Therefore, the cost of using fast chargers would be higher than that of using normal or slow chargers.

Table 1
List of different charging methods [22]

Type	Chemistry	Charging Rate	Time	Charge termination
Slow Charger	NiCd Lead acid	0.1C	14h	Continuous low charge or fixed timer
Normal Charger	NiCd, NiMH, Li-ion	0.3-0.5C	3-6h	Senses battery by voltage, current, temperature and time-out timer
Fast Charger	NiCd, NiMH, Li-ion	1C	1h+	Same as a rapid charger with faster service
Ultra-fast Charger	Li-ion, NiCd, NiMH	1-10C	10-60 minutes	Applies ultra-fast charge to 70% SoC; limited to specialty batteries

3. Operation Model

There are different strategies for a battery swapping station to maximize its profits while satisfying the swapping demand. Firstly, considering the initial purchase cost of EV batteries, a BSS should minimize the battery stock level. This is because the price of an EV battery is about US$200 per kWh, and an 85kWh battery pack would cost around $21,000 [32]. Secondly, the BSS should determine an optimal battery charging scheme to recharge the swapped batteries, knowing that a fast charging scheme using high current/voltage would degrade the battery life cycle and hence an inherent cost to the BSS. Therefore, the BSS would need to find a balance between meeting demand and the potential cost of batteries using fast chargers.

When the battery is running low, an EV driver can drive directly to the BSS for battery swapping. Alternatively, the driver can send a swapping notice with an estimated arrival time to the BSS. Based on all the estimated arrival times of vehicles, the BSS can determine a charging scheme for each incoming battery. If any vehicle suddenly arrives for battery swapping, or a new swapping notice is received, the BSS will plan for a new charging scheme for the incoming batteries. In Section 6.3, the simulation results show that the BSS can maintain a higher battery stock level after receiving more advanced-notice orders. Hence, the BSS can offer a price discount to entice the EV drivers to send advanced swapping notice.

From the EV driver's perspective, in Fig. 1, the BSS works like a traditional gasoline station. The only difference is that if they send an advanced notice to a BSS and arrive at the appointment time, the price will be lower than that offered without advanced notice. On one hand, the advanced notice approach is to minimize any sudden or unexpected orders to the BSS. On the other hand, if drivers can give early notice of a swapping plan, it will save them money.

From the BSS's perspective as shown in Fig. 2, the model helps the station to forecast the swapping demand and to prepare the batteries needed for swapping. The BSS can also try to optimize its operation and maximize its profits in the process. When an EV arrives, the BSS installs a fully-charged battery into the vehicle and determines the optimal charging scheme for the depleted battery.

It is important to note that the proposed model is designed for one single BSS to serve for the incoming EVs, and the BSS would recharge the swapped batteries at the same location. The same location criterion would result in a more economical and efficient system considering the potential cost on delivery and management. The BSS does not aim to make a profit by selling electricity back to the grid because the discharging process would reduce the battery's lifecycle and increase the potential expense. When a battery is swapped and in the charging process, the method cannot be changed during the charging period.

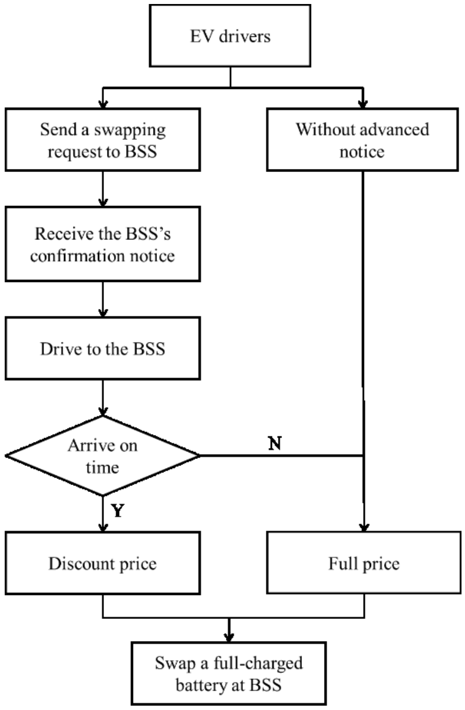


Fig. 1. BSS operation model from EV drivers' perspective.

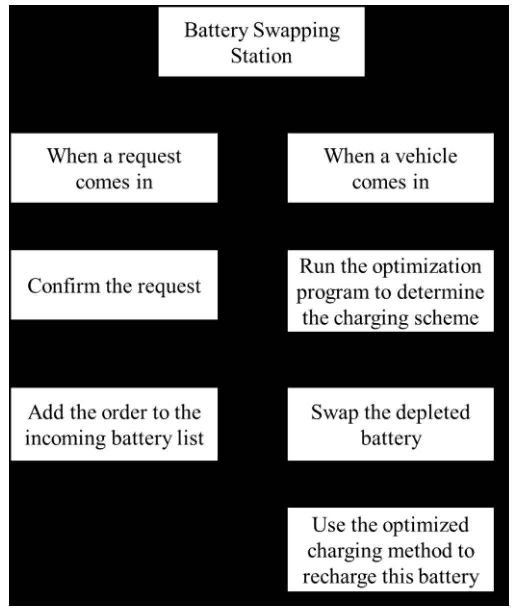


Fig. 2. BSS operation model from BSS' perspective.

4. Optimization Model

4.1. Assumptions

In the proposed model, only one type of battery is considered for swapping and charging in the BSS. Yet, the model can be easily extended for different type of batteries. The BSS would determine the charging scheme for the incoming batteries. Advanced notice is received and updated in real time. Four charging methods, M ={1, 2, 3, 4}, are proposed in the model, corresponding to ultra-fast, fast, normal and slow charging methods. The damage value to batteries using the four types of charger is normalized from 0 to 1. Specifically, the values corresponding to ultra-fast charger, fast charger, normal charger and slow charger are 1, 2/3, 1/3 and 0 respectively.

4.2. Notation

The notation of variables used in this model is shown in Table 2.

Table 2
Notation of Variables

Sets:	
T	Set of time with index t.
O	Set of swapping order with index o.
B	Set of battery with index b.
M	Set of charging method with index m.

Parameters:

$NumO$	Number of orders.
T_{max}	Maximum value of the time T
C_{rated}	Rated capacity of battery (kWh).
R_m	Charging rate of method m.
C_{init}	Initial number of batteries in stock
$Ch(m)$	Scaled charging damage using charging method m.
η_m	charging/ discharging efficiency of method m
α_m	Normalized charging damage to battery using charging method m (%).
β_i	Cycle aging after the i-th charging.
R_m	Rate charging power using method m.
P_{max}	Maximum load for individual battery swapping station (kW).
φ	The percentage of orders comes with advanced notice (%)
τ	The maximum arrival time of orders that the BSS take into computation (minutes)

Variables:

Obj_1	Primary objective function.
Obj_2	Secondary objective function.
Obj	Integrated objective function.
C_t	Quantity of charged batteries at time t.
D_t	Swapping demand of battery at time t.
M_o	The charging method of order o.
T_o^S	Start charging time of order o's battery.
T_o^F	Finish charging time of order o's battery.
C_o^R	Remaining capacity of order o's battery (%)
$C_{b,i}^F$	The full capacity of battery b at i-th charging (kWh).
SOC_b	State of charge of battery b (%).
$SOH_{b,i}$	State of health of battery b after $(i\text{-}1)$ charging cycles (%).
n_t^m	Number of orders being charged using method m at time t.
N_m	Number of chargers using method m in the BSS.
E	The daily charging energy/volume of a battery swapping station (kWh).

4.3. Criterion for BSS

The battery stock level and battery charging damage using different chargers are crucial considerations in minimizing a BSS's cost. Due to the expensive purchasing and maintenance cost, the BSS should determine a strategy to use only a limited number of batteries serving for the swapping demand. Fast chargers can help a BSS to recharge the battery in a shorter time than slower chargers. Then, the fully-charged battery would be ready for the next swapping. In such a case, the number of batteries that the BSS needs to prepare in stock is fewer. However, the fast charging method causes greater damage to a battery than the normal charging method, which means an inherent expense for the BSS.

In this model, the price of an EV battery is considered as the primary issue for a BSS. Thus, the BSS would aim to prepare fewer batteries in stock by using the fast chargers as far as possible. However, slower charging methods are also used to reduce the potential battery damage. Hence, we propose the primary objective value to indicate the infimum value of stock for a BSS, and propose the secondary objective value to indicate the average charging damage using different chargers.

The criterion of this model is defined as follows:

- The battery stock level is defined as the primary objective value. When a higher stock level is obtained, the solution is regarded as the desired solution in this iteration.
- The average charging damage is defined as the secondary objective value. When the primary objective is no longer changed, the model would search for an optimal solution with lower charging damage.

4.4. Objective Function

In this model, the criterion is intended to maintain a higher battery stock level, which is noted as the primary objective. If the primary objective is obtained, the secondary objective is used to reduce the charging damage to the batteries.

The primary objective Obj_1 is formulated as:

$$Obj_1 = \inf_{t \in T} (C_t - D_t) \quad . \tag{1}$$

C_t denotes the quantity of charged batteries and D_t denotes the swapping demand of battery at time t. The difference between them indicates the stock level, and hence Obj_1 would indicate the infimum number of the stock level. The definition of C_t and D_t is given in (4) and (5) below. Obj_1 is an integer ranging from 0 to the number of initial stock batteries.

The secondary objective Obj_2 is defined as:

$$Obj_2 = \frac{1}{NumO} \sum_{o \in O} Ch(M_o) \, , \tag{2}$$

which represents the average charging damage. Here, M_o is the assigned charging method to order o. $Ch(M_o)$ indicate the scaled charging damage using M_o for recharge the battery, which is in the range of 0 and 1. Obj_2 is a decimal number ranging from 0 to 1. This value will be close to zero if slow chargers are mostly used.

In order to compare the performance of the optimization algorithms, the primary and secondary objectives are combined to an objective value Obj, and the integrated-objective function is defined as:

$$\text{maximize} \quad Obj = Obj_1 + (1 - Obj_2) \tag{3}$$

As mentioned, Obj_1 is a natural number ($Obj_1 \in N$), and Obj_2 is a number ranging from 0 to 1 ($Obj_2 \in [0, 1]$). The aim of this criterion is to maximize *Obj*. When a greater Obj_1 occurs, *Obj* would increase by the same variation as Obj_1. In this case, Obj_2 would not affect *Obj* because the variation of Obj_2 is smaller than the variation on Obj_1. When Obj_1 is steady, the Obj_2 would be used to get an optimal solution with lower damage to the batteries. So, *Obj* would increase by the variation of Obj_2. In summary, *Obj* is combined with the two objective values. The integer part of *Obj* is the Obj_1, while the decimal part of *Obj is* calculated by $(1 - Obj_2)$.

4.5. Constraints

The constraints of this model are as follows:

$$C_t = C_{init} + \sum_{o \in O} \sum_{t=T_o^F}^{T_{\max}} (C_t + 1) \tag{4}$$

$$C_t \geq 0 \tag{5}$$

$$D_t = \sum_{o \in O} \sum_{t=T_o^S}^{T_{\max}} (D_t + 1) \tag{6}$$

$$D_t \geq 0 \tag{7}$$

$$T_o^F = T_o^S + (1 - C_o^R) \frac{C_{b,i}^F}{\eta_m R_m} \tag{8}$$

$$0 \leq T_o^S < T_o^F \tag{9}$$

$$0 \leq SOC_b \leq 100\% \tag{10}$$

$$0 \leq SOH_{b,i} \leq 100\% \tag{11}$$

$$C_O^R = SOC_{b,i} \tag{12}$$

$$C_{b,i}^F = C_{rated} \times SOH_{b,i} \tag{13}$$

$$0 < C_{b,i}^F \leq C_{rated} \tag{14}$$

$$SOH_{b,i} = SOH_{b,i-1} - f(\alpha_m, \beta_i) \tag{15}$$

$$0 \leq n_t^m \leq N_m \qquad \forall t \in T, \forall m \in M \tag{16}$$

$$\sum_{m \in M} n_t^m \cdot R_m \leq P_{\max} \qquad \forall t \in T, \forall m \in M \tag{17}$$

$$E = \sum_{t \in T} t \cdot (\sum_{m \in M} n_t^m \cdot R_m) \qquad \forall t \in T, \forall m \in M \tag{18}$$

$$E \geq 0 \tag{19}$$

In (4), C_t is denoted as the quantity of charged batteries at time *t*. At the beginning of the time, the BSS need to prepare C_{init} batteries in stock to serve for the incoming EVs. When the battery swapped from order *o* is fully recharged at time T_o^F, C_t will increase within the time space from T_o^F to T_{max}. In (6), D_t is denoted as the swapped demand of battery at time *t*. When the order *o* comes at time T_o^S, D_t will increase once from the time T_o^S to time T_{max}. Hence, the difference between C_t and D_t indicates the stock level of the BSS at time t, and Obj_1 in (1) is the infimum value of the stock level during the time period T.

In (8), the finish charging time of order o is T_o^F, which depends on the start charging time T_o^S, the battery's remaining capacity C_o^R, the fully capacity $C_{b,i}^F$ of the battery b at the i-th charging, the charging efficency η_m and the charging rate R_m.

Each battery has a profile to record its charging method in the past, state-of-health (SOH) and state-of-charge (SOC) for each charging cycle. The battery's SOC and SOH have the basic constraints (10) and (11). In the practical application [33], the SOC of a Li-ion battery cannot completely drop to 0% due to fast degradation issues. Also, it cannot be recharged fully due to safety considerations. Similarly, the SOH of a battery cannot degrade to a very low level.

In (12), the battery b's state-of-charge is taken as its remaining capacity C_o^R of order o. In (13), $C_{b,i}^F$ denotes the fully capacity after the i-th cycle of recharging, and it is defined as the product of the rated capacity C_{rated} and the state of health $SOH_{b,i}$.

SOH is a metric to evaluate the condition of a battery. Equation (15) gives the state of health $SOH_{b,i}$ of the battery b after the i-th charging. The decrease of the SOH from the previous cycle is a function of the normalized charging damage α_m due to charging method m and the cycle aging β_i. Specifically, the charging damage α_m using fast charging is larger than that using slow charging. The cycle aging β_i corresponds to the capacity change relation in [27], which shows that the battery's capacity decreases steadily in the early phase (around 500 cycles) and then the capacity degradation is aggravated near the end of the battery's life.

The constraint (16) ensures that the number of chargers using method m would not exceed the number of available chargers at any time. Constraint (17) disallows the power load to exceed the limitation of the electricity grid. In (18), the overall energy charged to the batteries is calculated.

The charging method for each incoming battery can be decided according to the advanced notice received from the EV drivers and the estimated completion time of the batteries being charged. If the foreseeable demand rises and there are not enough batteries to meet the swapping demand, the BSS will decide to charge the incoming battery using a faster charger. On the other hand, if the demand drops, the BSS may use the normal or slow charging method, which can prolong a battery's life and hence lower the cost.

5. Methodology

The objective of this paper is to determine an optimal charging scheme for the incoming batteries with maximum profit. Assuming that there are N incoming batteries and M types of charging methods, the solution is an array with N elements. The value of each element is chosen from the M types of chargers. Specifically, S is defined as one solution of the model, $S = [\ O_1, O_2, \ldots, O_i, \ldots, O_{N-1}, O_N\ ]$. O_i is denoted as the charging method of order i, which is selected from the list of charging methods. In this model, the total number of possible solutions is M^N.

Accordingly, the solution of the model is a discrete array with exponential dependence of the number of orders and charging methods. As a result, the defined model is a non-deterministic polynomial-time hard (NP-hard) problem with high computational complexity [34]. Optimization algorithms are regarded as reasonable methods to solve the problem. Three well-known computational algorithms, genetic algorithm (GA), differential evolution (DE) algorithm, and particle swarm optimization (PSO) algorithm are adapted to obtain a solution of the model.

Some research on adaptive genetic algorithm has been proposed in [35] – [37]. Gupta et.al. [35] presented an adaptive genetic algorithm by using a heuristic to generate initial population to solve the reconfiguration of radial distribution systems. Jiang et. al. [36] proposed a cloud-theory-based adaptive genetic algorithm to solve a fuzzy multi-objective model for serving restoration in the shipboard power system. The cloud model was designed to generate the probabilities of crossover and mutation in each generation by X-condition cloud generator. Wei et al. [37] also proposed an improved self-adaptive genetic algorithm with quantum scheme to solve electromagnetic optimization problems. They used a constant crossover probability and mutation rate in the early iterations, and then self-adaptive scheme would change the probability of crossover and mutation operations in the later iterations. However, these adaptive genetic algorithms use a constant number of populations during each generation.

5.1. Algorithm analysis and comparison

Even though the GA, DE and PSO algorithms are typical optimization algorithms for solving complicated problems, their principles and characteristics are different from each other.

Firstly, the origins of these algorithms are different. GA was inspired by the process of natural selection such as mutation and crossover, so that it is commonly applied on solving the optimization and search problems. DE was also motivated by real-world problems like most of the evolutionary algorithms, but DE is much simpler and straightforward to improve on a candidate solution. PSO was first designed for simulating social behavior such as a bird flock or fish school, and the movement of the optimal solution is influenced by its local best known position and the best known position in the whole search space. In this paper, the complexity of the problem is extremely high, which is influenced by the EVs' arrival pattern and a series of constraints. Hence, the mentioned algorithms are suitable for solving the optimization problem.

Secondly, the methods for updating the population are different. In GA, DE and PSO, a set of candidate solutions is generated randomly through the search-space at the beginning. In the optimization process, GA selects a portion of the existing solutions with better fitness values, and then the crossover and mutation operations are applied to generate some new solutions. In DE, a difference vector between two solutions is determined and then the difference vector is added to the original solution to generate a new candidate solution. In PSO, the particles/solutions are moved around in the search-space, and the movements are guided by the local best position and global position. In the proposed model, the solution is a set of charging methods assigned to each EV orders and the aim is to find an optimal solution with the maximum objective value. Here, the three optimization algorithms are chosen as viable methods to determine an optimal charging methods, and it is essential to compare the performances of the algorithms considering the use of different updating methods.

Thirdly, the level of changes with the use of the algorithms are different. In GA, the crossover operation can obtain a child solution by combining parts from two parents, so that the child solution only inherits from its parents and the change is small. Also, the mutation operation of GA only changes a small portion of the initial state. Hence, the difference between the new and original solutions is on small scale using GA. In DE, the new solutions are adjusted by a difference vector, so that only a small part of the solutions are changed. In PSO, all the candidate solutions/particles are influenced by the local and global best solutions, and then almost most of the solutions are changed in each optimization iteration. In this case, the difference between the new solution and the original solution can be wide with the use of PSO algorithm. In this model, PSO can achieve its objective value in a shorter time because the change in each iteration is greater than the GA and DE. However, the PSO algorithm may not outperform on searching for a better solution when it reaches a near-optimal situation.

After studying on the original algorithms and simulations in Section 6.2.3, we found that GA and DE can achieve an optimal objective value, but the PSO algorithms fail to obtain the acceptable primary objective even though the computational times of PSO algorithms are shorter than GA and DE. As the GA and DE can obtain a desired objective but the computational times are relatively longer, new algorithms called varied population genetic algorithm (VPGA) and varied population differential evolution (VPDE) algorithm are proposed to reduce the computational time and improve the performance of the original version of GA and DE in this paper.

5.2. Varied Population Genetic Algorithm

In order to obtain an optimized solution for the BSS based on the use of different charging methods, the genetic algorithm (GA) is considered as an efficient and powerful method for NP-hard problems with such high computational complexity. GA is a powerful metaheuristic algorithm proposed by Holland in 1970s [38] and has been applied in many areas for solving optimization problems.

Algorithm 1 presents the implementation of the varied population genetic algorithm, and the steps of the proposed VPGA are as follow:

1) Set the initial parent population of solutions as $\boldsymbol{P}$, where *NumIP* is the population size.

2) For each parent, crossover and mutation operations are used to obtain some solutions as the children $\boldsymbol{C}$ in Line 6 – 16.

3) Combine the parents and children as set $\boldsymbol{T}$, and rank the solution in $\boldsymbol{T}$ with objective function. Choose the solution with the best score in $\boldsymbol{T}$ as $\boldsymbol{P_{best}}$.

4) Use $\boldsymbol{P_{best}}$ as reference, sum up the absolute differences between $\boldsymbol{P_{best}}$ and each solution in $\boldsymbol{T}$ as *diff(ig)*.

5) In the first *NumGen/4* generations, use the same number of parent *NumIP*.

6) When the number of generation is greater than *NumGen/4* generations, execute the varied population process in Line 24 – 40.

7) If the difference *diff(ig)* is less than the difference in the previous generation, reduce the number of population by a ratio α defined in Line 25. If the difference *diff(ig)* is greater than the difference in the previous generation, increase the number of population by a ratio β defined in Line 26.

8) As shown in Line 31 – 34, the boundary of the varied population is set to $0.1\times NumIP$ and $2\times NumIP$.

9) If the number of varied population is smaller than the $NumT$, the $\boldsymbol{P_{best}}$ and other ($NumP$ - 1) solutions are chosen as the parents of the next generation. If the number of varied population is greater than the $NumT$, ($NumP - NumT$) solutions are generated randomly as ***tmpP***. Then, ***tmpP*** and ***T*** would become the parents of the next generation.

10) After the *NumGen-th* generation, the solution with the best objective score is identified as the optimal solution.

Some detailed explanation on the varied population algorithm and related parameters are discussed as follows.

The procedure is divided into two stages in VPGA. In the first stage (from the first iteration to the *NumGen/4-th* generation), due to the large differences among the candidate solutions at the beginning, the population size is set as the number of initial parent population $NumIP$ in order to search in a larger scale of search-space. In the second stage (after the *NumGen/4-th* generation), the candidate solutions converge to some similar points, and then the varied population algorithm would calculate the difference between the best solution and the other solutions denoted as $diff(ig)$. If the difference is smaller than the previous generation, which means more candidate solutions are similar to the best solution, then the population size can be reduced by a reduction ratio α. If the difference is larger, it indicates better solutions may be obtained, then the population size should be enlarged by an increase ratio β to widen the search-space. Here, the reduction ratio α is set as

$$\alpha = \max[1 - 0.01\times(ig - NumGen), 0.8] \tag{20}$$

and the increase ratio β is set as

$$\beta = \min[1/\alpha, 1.2]. \tag{21}$$

In this algorithm, α varies from 1 to 0.8 linearly to reduce the population size and β is obtained by an inverse ratio to α. In the optimization process, the similarity of the candidate solutions would increase so that the ratio α would reduce to remove the repeated/similar solutions. However, once the average difference rises in a generation, the increase ratio β would increase so that the population size is enlarged to search in a wider space. The experimental results on the average difference and the varied population size are illustrated in Section 6.2.2.

Two boundary values are defined to limit variation of the population size. Here, the upper and lower limits of the proposed algorithm are set to $0.1\times NumIP$ and $2\times NumIP$ respectively. With the use of the reduction and increase ratios, the number of populations would decrease or increase exponentially. In this case, the boundary values are defined in case that the population size is close to zero or reach an extremely high level.

As the results shown in Section 6.2.2, the number of initial parent population $NumIP$ is set to 100, and the lower and upper boundaries are set to 10 and 200 respectively. Fig. 6 shows that the tendency of the population size drops from 100 to 10, and the number of populations increases when the average difference increases occasionally in Fig. 5. In this case, the upper boundary ($2\times NumIP$) is not achieved because the average difference drops in most iterations. When it reaches the 42nd iteration, the average difference is stable at zero, which indicates that the algorithms cannot find a better solution. Hence, the number of populations would drop to the lower boundary (*0.1×NumIP*) to keep searching for a better solution with the minimum cost of population.

Compared with the fixed population size algorithms, the proposed varied population algorithm can use the minimum population size to improve on the computational time when the differences within the solutions are narrow, but can increase the number of populations to expand the search-space when a better candidate solution is found. It is essential to emphasize that the mentioned parameters, such that reduction ratio α, increase ratio β, upper boundary and lower boundary, have been studied with the use of control variates method.

5.3. Varied Population Differential Evolution Algorithm

Differential evolution (DE), proposed by Price and Storn in 1995 [39], is also a typical evolutionary computation algorithm to obtain an optimized solution for non-linear optimization problems. The basic idea of DE is to maintain a population of candidates, and then use the DE formulas to create new candidates by combining the existing solutions. After generating new solutions, the candidate with the best objective value would be stored.

Algorithm 1 Varied Population Genetic Algorithm

parameters

NumO: number of incoming orders
NumIP: number of initial parents
NumP: number of parents in the GA process
NumGen: number of generations in the GA process

Input: a set of order data with arrival time and remaining capacity
Output: charging schemes corresponding to each incoming orders as vector P_{best}

```
generate an initial parent population P←{X^1, X^2, …, X^NumP}
NumP ← NumIP
for each generation g less than NumGen do
  C←0
  T←0
  for each parent p less than NumP do
    randomly choose two parents from P as P_1 and P_2
    randomly choose a breakpoint from P_1 as b
    combine P_1(1:b) and P_2((b+1):NumO) as a child C_1
    combine P_2(1:b) and P_1((b+1):NumO) as a child C_2
    Add C_1 and C_2 to C
  for each parent p less than NumP do
    randomly choose a number i from 0 to NumO
    assign the charger of order i randomly as C_3
    Add C_3 to C
  combine P and C as T
  NumT ← size of T
  calculate the scores of each solution from T
  rank the scores
  P_best ← solution with the best score in T
  diff(ig) ← 0
  for each solution p in T do
    Diff(ig) ← diff(ig) + absolute difference between P_best and p
  if ig > NumGen/4 then
    α ← max [1 - (ig - NumGen/4)×0.01, 0.8]
    β ← min (1/α, 1.2)
    if diff(ig) <= diff(ig-1) then
      NumP ← round( α × NumP )
    else
      NumP ← round( β × NumP )
    if NumP < 0.1 × NumIP then
      NumP ← round( 0.1 × NumIP )
    if NumP > 2 × NumIP then
      NumP ← 2 × NumIP
    if NumP <= NumT then
      tmpP ← randomly choose (NumP - 1) solutions from T
      P ← add P_best and tmpP
    else
      tmpP ← randomly generate (NumP - NumT) solutions
      P ← add T and tmpP
return P_best
```

Algorithm 2 Varied Population Differential Evolution Algorithm

parameters

NumO: number of incoming orders
NumIP: number of initial parents
NumP: number of parents in the GA process
NumGen: number of generations in the GA process
λ: a decimal value between 0 and 1
m, n: two decimal values between 0 and 1

Input: a set of order data with arrival time and remaining capacity
Output: charging schemes corresponding to each incoming orders as vector P_{best}

```
generate an initial parent population P←{X^1, X^2, …, X^NumP}
NumP ← NumIP
for each generation g less than NumGen do
  calculate the scores of each solution from P
  choose the solution with highest score as X_best
  for each parent p less than NumP do
    C ← 0
    randomly choose two parents from P as P_1 and P_2
    for each order i less than NumO do
      obtain a random decimal between 0 and 1 as r
      if r < λ then
        X_i^p = X_i^p + m × (X_Best − X_i^p) + n × (P_1(i) − P_2(i))
        if X_i^p < 1 then
          X_i^p = 1
        if X_i^p > 4 then
          X_i^p = 4
        add the integer part of X_i^p to the vector C
      else
        add X_i^p to the vector C
    if the objective score of C is larger than X^p then
      change the p-th parent in P to C
  NumP' ← size of P
  P_best ← solution with the best score in P
  diff(ig) ← 0
  for each solution p in P do
    Diff(ig) ← diff(ig) + absolute difference between P_best and p
  if ig > NumGen/4 then
    α ← max [1 - (ig - NumGen/4)×0.01, 0.8]
    β ← min (1/α, 1.2)
    if diff(ig) <= diff(ig-1) then
      NumP ← round( α × NumP )
    else
      NumP ← round( β × NumP )
    if NumP < 0.1 × NumIP then
      NumP ← round( 0.1 × NumIP )
    if NumP > 2 × NumIP then
      NumP ← 2 × NumIP
    if NumP <= NumP' then
      tmpP ← randomly choose (NumP - 1) solutions from P
      P ← add P_best and tmpP
    else
      tmpP ← randomly generate (NumP – NumP') solutions
      P ← add P and tmpP
return P_best
```

Algorithm 3 Particle Swarm Optimization Algorithm

parameters

NumO: number of incoming orders
NumP: number of particles in the PSO process
NumGen: number of generations in the PSO process
V_{max}: the maximum value of velocity
c_1,c_2: two values set for adjustment parameter
r_1,r_2: two random decimal value between 0 and 1

Input: a set of order data with arrival time and remaining capacity
Output: charging schemes corresponding to each incoming orders as vector $\boldsymbol{P_{best}}$

generate an initial population of particles $\boldsymbol{P \leftarrow \{X^1, X^2, \ldots, X^{NumP}\}}$
generate an initial velocity of each particle $\boldsymbol{V \leftarrow \{V^1, V^2, \ldots, V^{NumP}\}}$
calculate the scores of all particles from $\boldsymbol{P}$
set the particle with highest score as $\boldsymbol{X_p}$ and $\boldsymbol{X_g}$
for each generation g less than *NumGen* **do**
  calculate the scores of each solution from $\boldsymbol{P}$
  choose the solution with highest score as $\boldsymbol{X_{Best}}$
  for each particle p less than *NumP* **do**
    if PSO is original **then**
      $V_i^p = V_i^p + c_1 r_1 (X_{p,i} - X_i^p) + c_2 r_2 (X_g - X_i^p)$
    if PSO is PSO-In **then**
      $$w = (0.8 - 0.4) \times \frac{NumGen - g}{NumGen} + 0.4$$
      $V_i^p = wV_i^p + c_1 r_1 (X_{p,i} - X_i^p) + c_2 r_2 (X_g - X_i^p)$
    if PSO is PSO-Co **then**
      $$\varphi = c_1 + c_2$$
      $$\chi = \frac{2}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|}$$
      $V_i^p = \chi V_i^p + c_1 r_1 (X_{p,i} - X_i^p) + c_2 r_2 (X_g - X_i^p)$
    if $V_i^p > V_{max}$ **then**
      $$V_i^p = V_{max}$$
    if $V_i^p < -V_{max}$ **then**
      $$V_i^p = -V_{max}$$
      $$X_i^p = X_i^p + V_i^p$$
    if the objective score of $\boldsymbol{X_i^p}$ is larger than that of $\boldsymbol{X_p}$ **then**
      $$X_p = X_i^p$$
    if the objective score of X_i^p is larger than $\boldsymbol{X_{Best}}$ **then**
      $$X_g = X_i^p$$
  $\boldsymbol{X_{Best}} \leftarrow$ objective value of $\boldsymbol{X_g}$
return $\boldsymbol{X_{Best}}$

Algorithm 2 presents the pseudo-code of the varied population differential evolution. Assuming that a candidate solution is a permutation in the vector $\boldsymbol{X}$, each individual in the vector is indexed by i, and each generation is indexed by g. For the initial generation, the elements in parent $\boldsymbol{P}$ are generated randomly. The next populations will be created from the previous generation

$$X_i^p = X_i^p + m(X_{best} - X_i^p) + n(P_1(i) - P_2(i)) \tag{22}$$

, where $X_i^{\,p}$ is the solution of the order i of parent p, X_{best} is the best solution in this generation, P_1 and P_2 are two other solution selected randomly, m and n are two parameters defined less than 1. Similar to VPGA, after the *NumGen/4-th* generation, the algorithm would use the varied population process to change the number of populations according to the difference between each solution to the best solution in Line 24 – 43.

The procedure for varying the population of VPDE is similar to the VPGA. Hence, the detailed explanation on the varied population principle and how the variants are decided can be referred to Section 5.2.

5.4. Particle Swarm Optimization Algorithm

Particle swarm optimization (PSO) is a computational algorithm inspired by the social behaviors in the artificial life [40]. PSO is a powerful method of solving the optimization problem by exploring the particles. In each iteration, the particles would be updated to a better position according to the particles' position and velocity.

Algorithm 3 shows the detailed algorithm of the PSO algorithm, which is initialized with a group of random particles. Each particle has a corresponding fitness value which is evaluated by the observation model, and has a relevant velocity which directs the movement of the particle. In each iteration, the particle moves with the adaptable velocity, which is a function of the best state found by that particle (for individual best), and of the best state found so far among all particles (for global best). In this paper, three versions of PSO algorithms, the original PSO [41], the PSO-In [42] and the PSO-Co [43], are implemented in lines 9 – 17.

In the next section, the simulation results obtained by GA, DE, PSO, PSO-In, PSO-Co, VPGA and VPDE are shown and compared.

6. Case Study

Based on the proposed model and assuming that EV batteries are served by the BSS, this case study simulates the swapping and charging operations for a swapping station. Referring to the Society of Automotive Engineers (SAE) charging configurations and rating terminology [44], the J1772 standard defines the EV battery charging specifications. In this paper, the charging standard used to recharge the swapped battery is DC Level 2, which would reach a peak power of 90kW. Four charging schemes, ultra-fast, fast, normal and slow are assigned by the rated power of 90kW, 60kW, 45kW and 30kW respectively. So, the charging rate R_m is specified as $\{90\;\;60\;\;45\;\;30\}$.

Considering the electricity power load, the charging load of the battery swapping station would be limited to a safe range. According to the charging statistics suggested by [45], the maximum load of a charging station should be less than 2400 kW and the total daily load should not exceed 10,080 kWh. In this paper, P_{max} is set to 2400, and E_{max} is set to 10080.

The battery's cyclic age accelerates with the charging current rate. As explained in [26], more than 80 cells were tested with different aging protocols to understand the degradation mechanisms. The battery's cyclic age linearly accelerates with charging current rate. The result shows that when the battery's capacity fades 20%, the battery could be recharged to around 520, 560, 600 and 640 cycles using the charging rates of 2C, C/1, C/2 and C/5 respectively [31].

In this study, there is only one BSS serving the incoming orders. We assume that the capacity of the EV battery is 85 kWh, and the times for full-charging for the four charging methods are 90, 150, 210 and 270 minutes. Two examples and many 24-hour simulation cases are described below to demonstrate the effectiveness of the algorithms in finding solutions to our optimization problem.

6.1. Simple Case with 15 orders

6.1.1. Initialization

This simple case would illustrate a typical operation of the BSS with the use of an optimization scheme. Suppose fifteen advanced-notice orders are received as shown in Table 3. The first three columns show their characteristics. The second column gives the arrival time at the BSS. The third column indicates the estimated remaining capacity of the batteries before recharging. The arrival time and the remaining capacity are randomly generated within 150 minutes and 40% respectively. For example, assuming that the current time is 06:00am, the first order would arrive at the BSS at 06:17am (in 17 minutes), and the remaining capacity of this battery would be 31.32%.

Table 3
Result of Orders

Order index	Arrival time (in minutes)[a]	Remaining capacity (in %)	**Charging method (1, 2, 3, 4)[b]**	Battery fully charged time (in minutes)[a]
1	17	31.32	**2**	120
2	18	24.12	**2**	132
3	34	13.55	**1**	112
4	65	14.16	**1**	143
5	81	12.64	**3**	265
6	90	22.25	**4**	300
7	96	13.09	**4**	331
8	112	16.75	**4**	336
9	113	15.21	**4**	342
10	119	12.05	**4**	357
11	130	33.69	**4**	309
12	130	26.00	**4**	330
13	140	31.84	**4**	324
14	140	20.42	**4**	355
15	147	15.14	**4**	376

a. The time is counted from 6:00am.
b. Charging method: 1-ultra-fast charging; 2-fast charging; 3-normal charging; 4-slow charging

6.1.2. Optimized scheme

Column 4 shows the assigned charging scheme for each order. Two out of the fifteen batteries are charged using the ultra-fast charging method. In this case, these ultra-fast charged batteries may be swapped for the incoming orders later. For example, the third battery is assigned to a ultra-fast charger and will be recharged at 07:52am (112 minutes), so that this battery will be available for swapping by a subsequent order. Some can observe that the optimal charging scheme would assign fast charging method to early arrived orders, then these recharged batteries can be swapped to other incoming orders. On the other hand, the scheme would assign slow charging methods to the latter orders to reduce the charging cost of BSS.

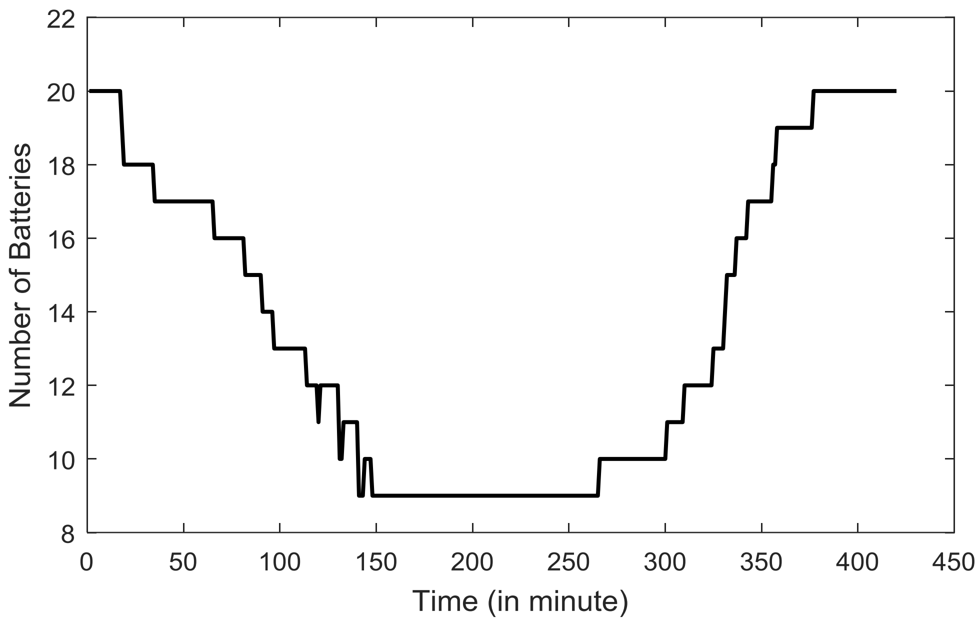


Fig. 3. Graph of stock

Fig. 3 shows the variation of the number of batteries in stock from 06:00 to 13:30 hours (the next 450 minutes). The figure indicates that the first 4 orders would swap the batteries in stock, and then the first swapped battery is fully recharged at 112 minute. In this case, the infimum of stock level is 9, which means that only 11 batteries in stock are used for the 15 incoming orders.

6.1.3. Result

In this study, we use a typical version of genetic algorithm (GA) to obtain an optimal solution, and the solution is listed in column 4 of Table 3. As shown in Fig. 4, we have used 50 iterations in the optimization process, and set the initial number of batteries as 20. It is clear that the infimum of stock level, which is defined as primary objective (Obj_1), increase from 7 to 9 during the iterations, while the secondary objective value (Obj_2) fluctuates with the change of Obj_2. In the first stage (iteration 1 to 15), the Obj_1 is stable at 7 and the Obj_2 drops from 0.33 to 0.24. At the 16th iteration, a higher Obj_1 is obtained and the Obj_2 also increases. In the second stage (iteration 16 to 27), the Obj_1 is stable at 8 and the charging cost Obj_2 continues to drop from 0.31 to 0.24. After the 28th iteration, the Obj_1 increases to 9 and no longer change in the later iterations. In the third stage (iteration 28 - 50), the optimal Obj_1 is obtained as 9 and then the Obj_2 drops from 0.29 to 0.24.

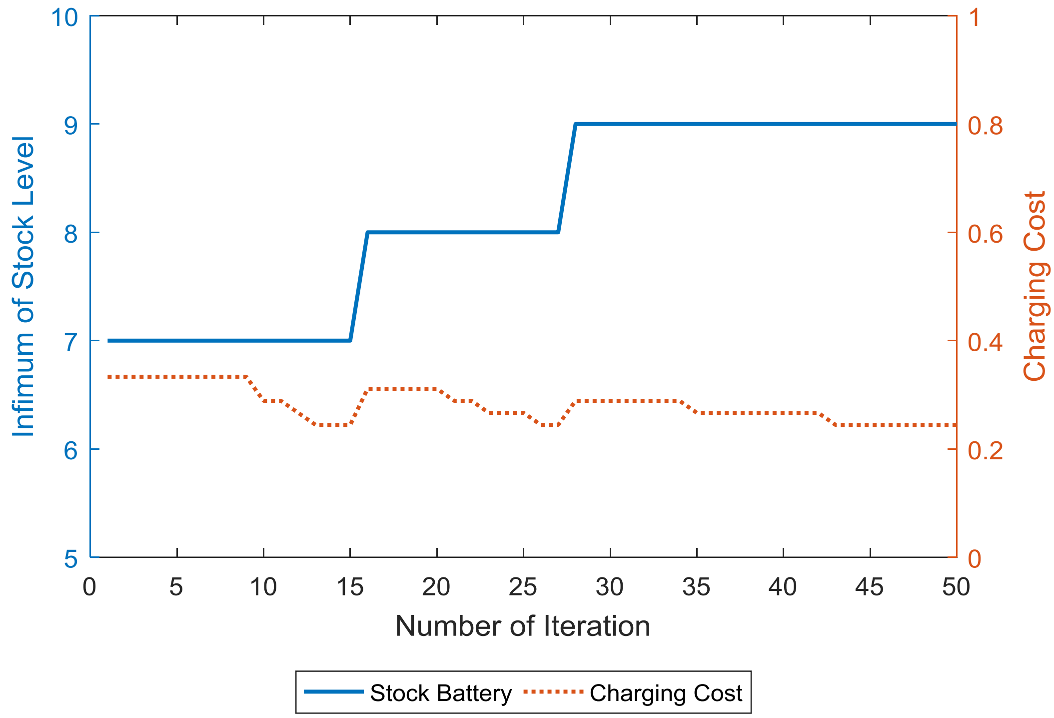


Fig. 4. Optimization curve

6.2. Case with 100 orders

6.2.1. Initialization

We simulate 100 EVs coming to the BSS and their arrival times are randomly distributed within the next three hours. The remaining capacity of each incoming battery is randomly assigned between 10% and 35%.

In this case, we compare the performance of the seven algorithms, GA, DE, PSO, PSO-In, PSO-Co, VPGA and VPDE, with the same set of EV orders. In order to compare the performances of the algorithms, the initial number of parents of each algorithm is set to 100, and the number of iterations (generations) is set to 100. In DE, the two adjustment parameters λ and μ are set to 0.9 and 0.8 respectively. The acceleration coefficients c_1 and c_2 are both set to 2. To the PSO with the inertia weight (PSO-In) algorithm, the weight w is varied linearly from 0.9 to 0.4 depending on the generation variable.

6.2.2. Optimal Solution

We have used the proposed VPDE algorithm to obtain an optimal solution with 100 orders, and one typical example of the solutions is discussed in this part. The best objective value of the solution is 38.6233, which mean that the stock level Obj_1 is 38 and the average charging cost Obj_2 to batteries is 0.3767 (1 - 0.6233). The time for obtaining the results is 12.54s.

As explained in Algorithm 2, a fixed number of populations was used at the first 25 iterations, and then the number of populations was varied depending on the average difference between the optimal solution to other solutions. One typical example of this average difference is shown in Fig. 5, and the corresponding number of varied generation is shown in Fig. 6. It is clear that the number of populations is stable at 100 in the first 25 iterations as shown in Fig. 6. After the 25th iteration, the number of populations would fluctuate according to the average difference with the previous iteration. As shown in Fig. 5, the average difference rises at the 26th, 33rd, and 37th iteration, and the number of populations also increases at the corresponding iterations. The result shows that the proposed VPDE algorithm is viable to solve the BSS optimization problem.

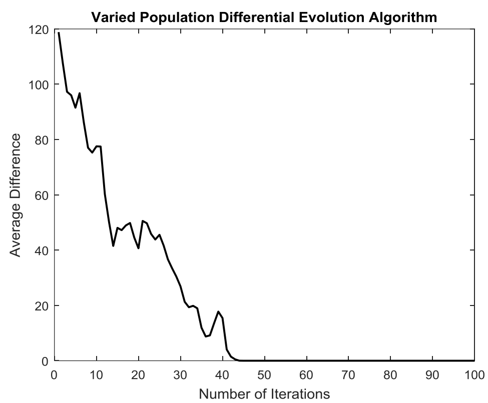


Fig. 5. Average Difference with the VPDE Algorithm

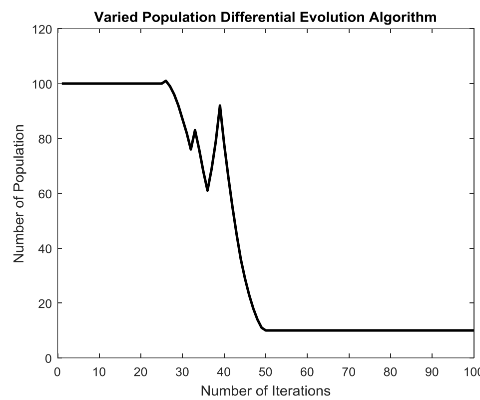


Fig. 6. Number of Populations with the VPDE Algorithm

6.2.3. Algorithms Comparison

In this part, we use the proposed algorithms to obtain solutions for the same order set with 100 EVs. Each of the seven algorithms are repeated for 100 times.

The performances with the use of different algorithms are shown in Fig.7, and the fitness value in each iteration is calculated by taking the average value of the 100 repeats. It is clear that the PSO, PSO-In and PSO-Co algorithms fail to obtain the maximum primary objective (Obj_1 = 38), which means that the PSO algorithm is not suitable for solving the proposed optimization problem. Hence, the PSO algorithm is not adapted to varied population PSO algorithm in this paper.

Comparing the original versions of GA and DE, both algorithms can obtain the maximum primary objective. However, the DE algorithm can achieve a better secondary objective for decreasing the charging damages to the batteries. It is clear that the performances by VPGA and VPDE are slightly better than the original GA and DE, while the computational times are significantly shorter as shown in Table 4.

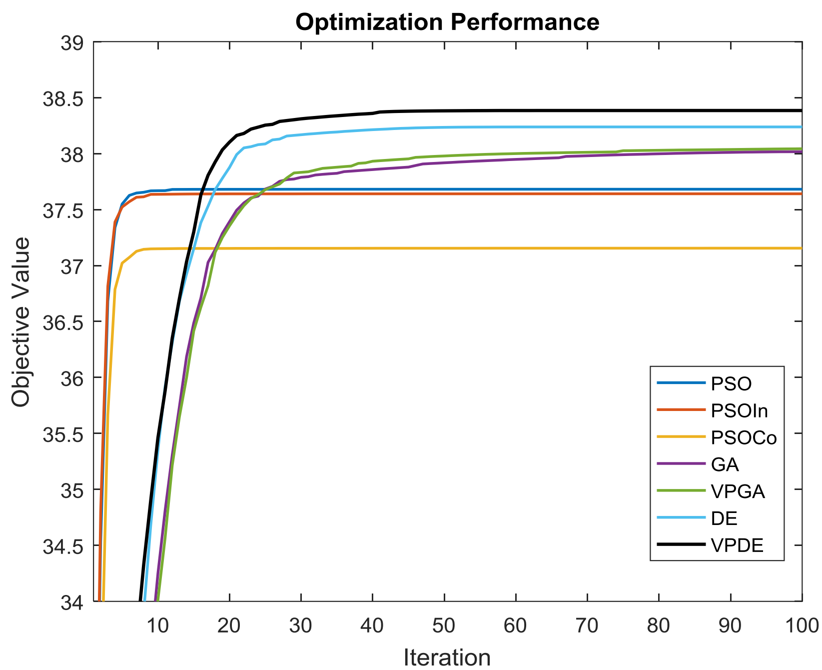


Fig. 7. Optimization Performances by Different Algorithms

Table 4
Comparison Between Algorithms

	Integrated Objective Value					Average Time	Average Number of Fitness Evaluations
	Average	Best	Worst	Median	Standard Deviation		
GA	38.0169	**38.6233**	35.6600	**38.6117**	0.7267	18.62s	35000
DE	38.2380	38.6133	36.6000	38.5750	0.4974	26.74s	50000
PSO	37.6815	38.5400	35.4500	37.9700	0.9355	5.52s	**10101**
PSO-In	37.6408	38.5500	34.4500	37.5200	0.8858	**5.51s**	**10101**
PSO-Co	37.1549	38.5100	34.4900	37.4650	1.0453	5.52s	**10101**
VPGA	38.0433	38.6133	36.5867	38.5367	0.5988	9.81s	18859
VPDE	**38.3844**	**38.6233**	**37.5400**	38.5700	**0.3982**	13.20s	23311

The results show that the VPDE algorithm outperforms all other algorithms in terms of the integrated objective value. As shown in Table 4, the average, best and worst of objective value obtained by VPDE is the highest within the algorithms, and the median value is comparable with the GA. The worst value means the minimum objective value obtained in the 100 repeats corresponding to the algorithm. The worst objective determined by VPDE is the highest, which means VPDE is better than other algorithms even in the worst case. The standard deviation of VPDE is smaller than other algorithms, which indicates that the proposed VPDE algorithms is more desirable. In terms of the primary objective, the GA, DE, VPGA and VPDE algorithms can obtain the same stock level of 38, while the stock level by the PSO, PSO-In and PSO-Co algorithms is 37. Hence, even though the computational time of the PSO algorithms is shorter than other algorithms, the performance of PSO algorithms is not acceptable in this case.

The proposed varied population algorithms improve the original versions of GA and DE. The average objective value obtained by typical GA and VPGA are 38.0169 and 38.0433 respectively, while the computational time of VPGA is 47% less than the typical GA. Also, the objective value obtained by the VPDE is greater than the typical DE, and the computational time of VPDE and typical DE are 13.20s and 26.74s respectively.

The average numbers for calling the fitness evaluations of the algorithms are shown in Table 4. When the program calls the fitness function to calculate the objective value, the number is counted once. The PSO algorithms call the fitness evaluation for 10101 times in this case, which is also the reason why the computational time of PSO algorithms is shorter than the GA and DE. However, because of the poor solution quality, the PSO algorithms are not considered as the acceptable method for solving the problem. The original version of GA calls the fitness evaluation for 35000 times because the 100 parents would generate 200 children with crossover and 50 children with mutation in each iteration, which is presented in Section 5.2. Refer to the original version of DE, the program would call the fitness evaluation for 500 times in each iteration. With the help of varied population algorithm, the average number of fitness evaluation of the VPGA and VPDE are 18859 and 23311 respectively, which only account half of the original version the algorithms. The reduction of the number of fitness evaluation using varied population algorithms is because of the decline in the number of population in the optimization process.

In summary, the proposed varied population genetic algorithm and varied population differential evolution algorithm are more efficient and performs better than the original version the evolutionary algorithms. As the PSO, PSO-In and PSO-Co algorithms fail to obtain the maximum primary objective, the PSO algorithms are not adapted to varied population PSO algorithm in this paper.

6.3. 24-hour simulation

6.3.1. Initialization

In this example, we simulate the actual operation of a BSS in a 24-hour period, and use the proposed optimization method to decide on the charging scheme for each incoming battery. Fig. 8 shows the swapping demand characteristics during a 24 hours of operation. This curve gives the swapping demand every ten minutes for one day. This figure is based on the statistics for a typical gasoline refueling station in [46].

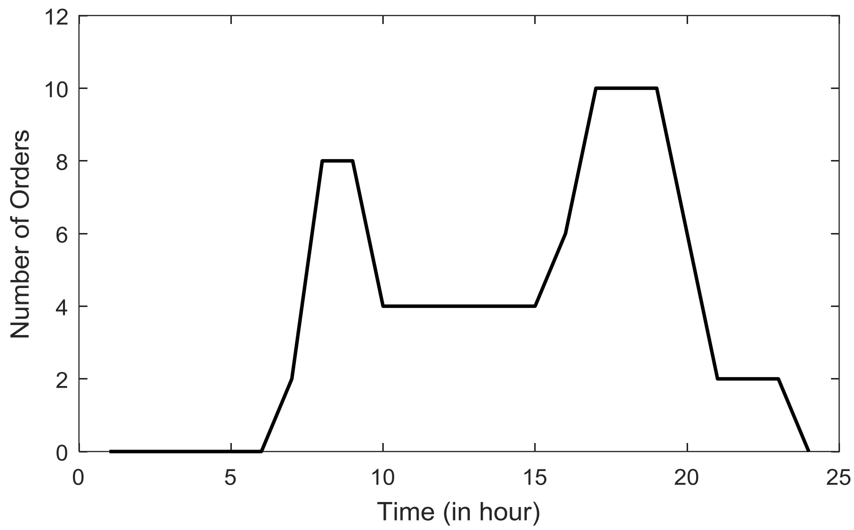


Fig. 8. Swapping demand distribution for every ten-minute interval.

In this simulation, the 540 orders are distributed from 6:00 to 24:00. Two demand peaks are reached at 8:00 and 16:00. Two parameters on the incoming orders are considered. Let φ be the percentage of orders with advanced notice. Suppose the BSS knows the advanced-notice orders within a period of τ minutes and this parameter is called the ahead predicting time. The percentage φ is examined for values at 80%, 90% and 100%, and the ahead predicting time is set to 120 and 240 minutes.

To evaluate the model, the results are compared with the charging methods, which are all fast (use of all ultra-fast chargers), all slow (use of all slow chargers), and random assignment of chargers. If a solution is assigned using all fast chargers, the maximum primary objective would always be achieved by this solution. However, the charging damage using the all fast charging solution would be much higher than that using normal charger. If a solution is assigned using all slow chargers, the charging damage is the lowest, but the BSS has to keep many batteries in stock to serve for the swapping demand. The random assignment simulates the operation of a BSS that assign a random charger to each incoming battery.

In this case, the proposed VPDE algorithm is used to obtain the optimal charging scheme for the incoming batteries, and the number of initial population is set to 100.

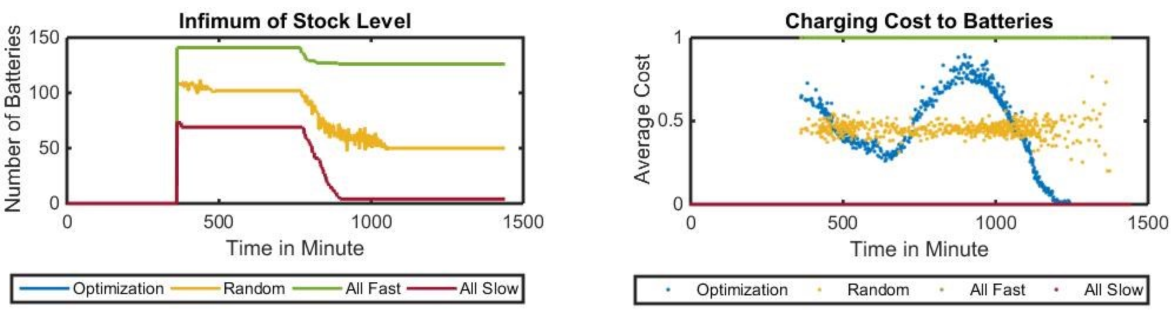
6.3.2. Evaluation of the model (φ = 100%, τ = 120 or 240)

In the initial study, the BSS is assumed to have advanced notice from all the EVs (φ = 100%), and the BSS knows the orders within the following 120 and 240 minutes (τ). The initial number of batteries in stock is set to 200.

When an EV comes in, the BSS considers this battery along with the incoming orders within τ minutes in order to carry out the optimization. The charging method of the current battery is then obtained, and the time to fully-charge would be estimated. This calculation is carried out for each incoming order.

The optimization curves of τ for 240 and 120 minutes are shown in Figs. 9 and 11 respectively. The abscissa axis is defined as the point in time from 0 to 1440 minutes (24 hours). Figs. 8 (a) and 10 (a) show the lowest stock levels in the 24-hour period for four different charging strategies. It should be mentioned that the optimization result is identical and overlaps with the results of the all-fast charging strategy. Both results are higher than the results of the random and all-slow strategies.

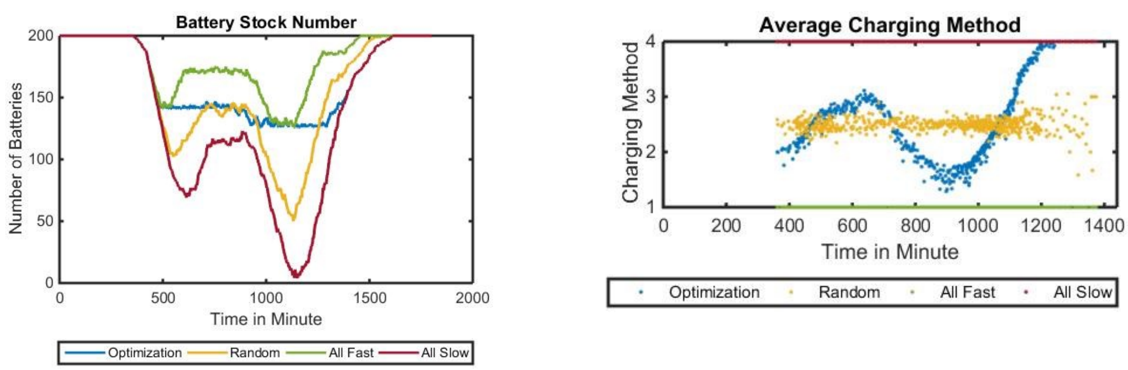
Figs. 9 (b) and 11 (b) show the charging cost/damage for the four charging schemes. The all-fast and all-slow charging schemes stand for the highest and lowest cost for the batteries respectively. The random assignment result is always distributed near the middle range. However, only the optimization result can adjust the charging scheme to balance the charging cost and the battery stock level.



(a) Primary objective: number of stock batteries (the "Optimization" is overlapped with the "All Fast")

(b) Secondary objective: charging damage

Fig. 9. Optimization curve of 24-hour simulation (φ = 100%, τ = 240)



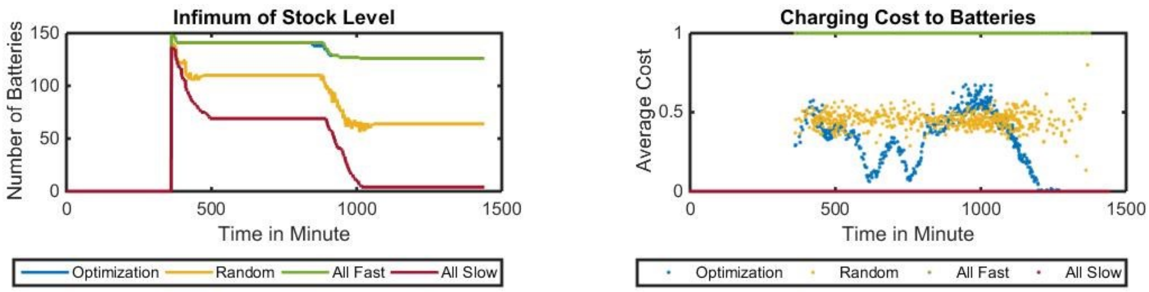
(a) Stock battery graph

(b) Charging method assigned result

Fig. 10. Results of 24-hour simulation (φ = 100%, τ = 240)

Fig. 10 (a) shows the battery stock level during the day. Due to the variations in daily demand, the lines vary considerably during the peak times, except in the optimization case. This is because the optimization model determines the charging scheme with the use of the advanced notice orders. Note that the lowest stock level for the optimization and the all-fast results are both 126, but the charging cost shown in Fig. 9 (b) of the optimization scheme is much less than the all-fast strategy. A similar result is obtained in Fig. 12 (a), where the lowest stock levels for the all-fast and optimization strategy are 126.

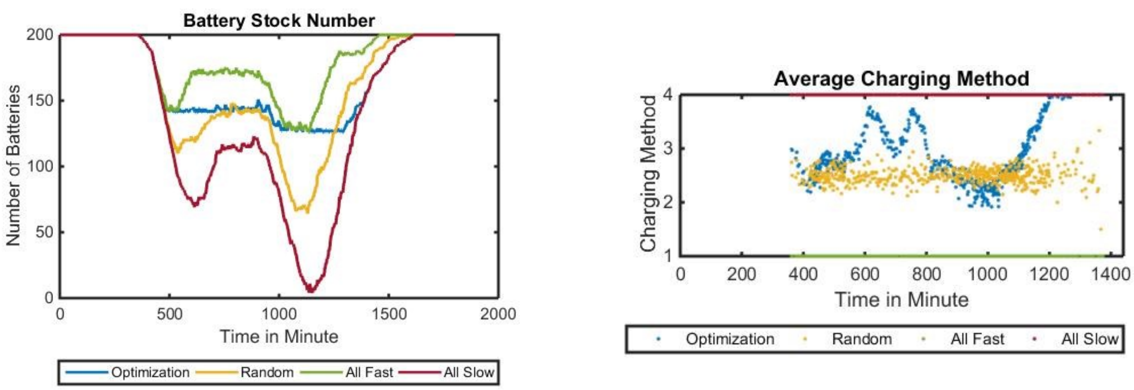
The charging methods are denoted by the integers 1 to 4, which stand for ultra-fast charger, fast charger, normal charger and slow charger respectively. When an order comes in, the model considers the advanced-notice orders within τ minutes, obtaining an optimized charging method for each battery. The average values of the charging methods over the 24-hour period are shown in Figs. 10 (b) and 12 (b).



(a) Primary objective: number of stock batteries (the "Optimization" is overlapped with the "All Fast")

(b) Secondary objective: charging damage

Fig. 11. Optimization curve of 24-hour simulation (φ = 100%, τ = 120)



(b) Charging method assigned result

(a) Stock battery graph

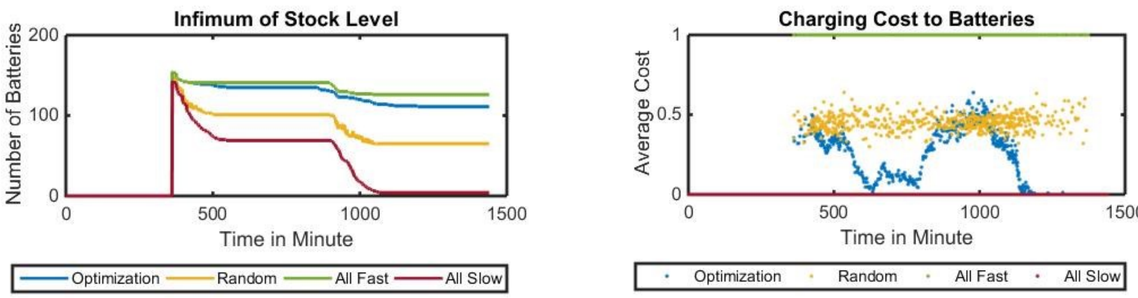
Fig. 12. Results of 24-hour simulation (φ = 100%, τ = 120)

To summarize, the results from the 24-hour simulation show that the proposed optimization model can achieve the same stock levels as the case of only using "ultra-fast chargers". Yet, the optimized scheme uses various chargers so as to lower the damage to the batteries.

6.3.3. Simulation of the situation without all advanced-notice orders (φ = 90%, 80%, τ = 120)

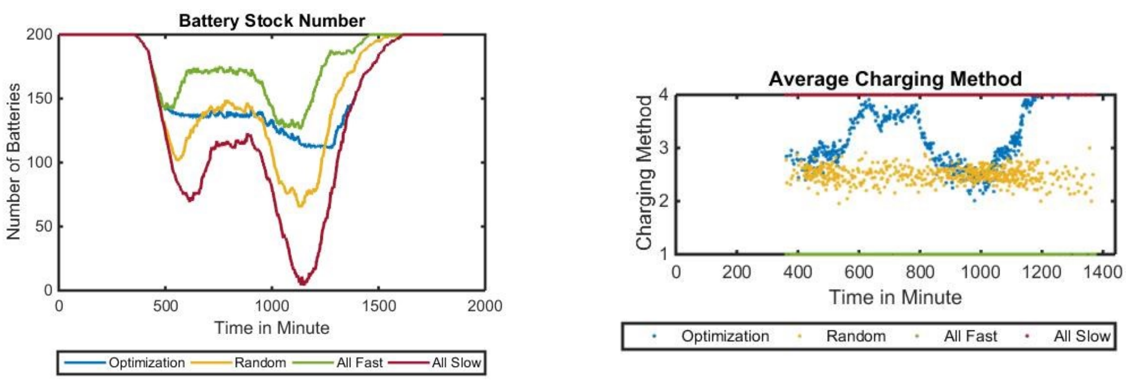
In an actual situation, not all EV drivers give advanced notice before arrival. The following experiments considers two cases (φ = 90%, 80%) of orders with advance notice. The order distribution is the same as in previous section. When a vehicle comes in without advanced notice, the BSS would add this order in the model, and determine the optimized charging method for it.

Figs. 13 (a) and 15 (a) indicate the battery stock level for the orders. As the red line shows, the stock level of the all-fast case is the theoretical highest stock level. The optimization cases are a little lower than the maximum level. However, the optimization result is always higher than the random and all-slow strategies. Comparing the optimization results, the stock level with more advanced-notice orders is higher.



(a) Primary objective: number of stock batteries (b) Secondary objective: charging damage

Fig. 13. Optimization curve of 24-hour simulation (φ= 90%, τ = 120)



(a) Stock battery graph (b) Charging method assigned result

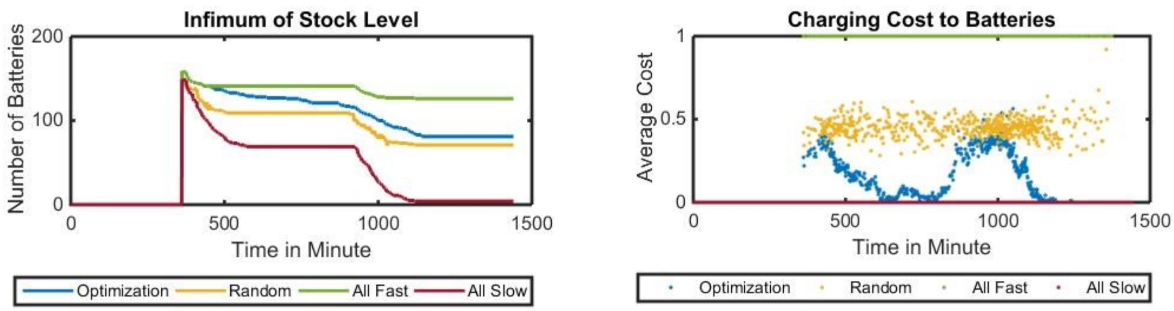
Fig. 14. Results of 24-hour simulation (φ= 90%, τ = 120)

The cost value, shown in Figs. 13(b) and 15(b), is normalized in this model between 0 and 1, where 0 means low cost while 1 stands for high cost. The charger costs of the all-fast and all-slow

charging methods are therefore 1 and 0 respectively. The random scheme result is distributed around 0.5. The optimization result is always lower than that of the random scheme, and the cost is also very low in some periods. The variation of charging costs is caused by the order fluctuations. For example, during the period between 900 and 1080 minutes (15:00–18:00), the optimized charging cost increases because the peak demand is at 120 minutes (17:00–20:00) and the BSS will then use faster charging to meet the incoming orders.

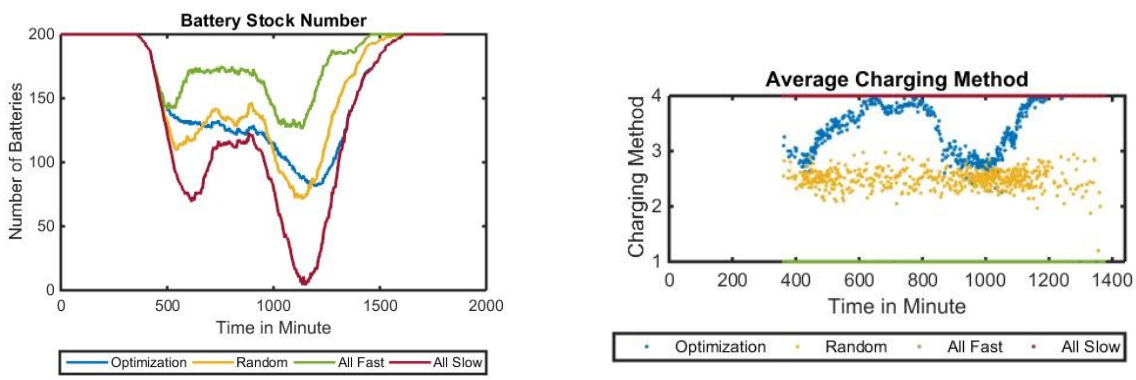
The battery stock levels are shown in Figs. 14 (a) and 16 (a). The battery stock level optimization curves vary considerably in the two cases. In Fig. 14 (a), the number of stock batteries drops from 130 to 111 in the period from 1000 to 1200 minutes. However, in Fig. 16 (a), the number drops from 120 to 81 in the same period. For these two cases, the differences between the all-fast and optimized schemes are 15 and 45 respectively. This shows that the BSS would use 30 fewer stock batteries with 90% advanced-notice orders compared to 80%.

The average values of the charging methods are shown in Figs. 14 (b) and 16 (b). In the optimization model, the charging method curves for the two cases (φ = 90% and 80%) are similar and correspond to the variation of the demand distribution.



(a) Primary objective: number of stock batteries (b) Secondary objective: charging damage

Fig. 15. Optimization curve of 24-hour simulation (φ= 80%, τ = 120)



(a) Stock battery graph (b) Charging method assigned result

Fig. 16. Results of 24-hour simulation (φ= 80%, τ = 120)

In summary, the results show that the number of stock batteries will be higher if there are more orders with advanced notice. The charging costs are comparable. This means that the BSS can obtain an optimized result if the charging orders are more deterministic. Hence, drivers should be encouraged to give advanced notice and can be motivated by a price discount.

7. Conclusion

In this paper, an operation and optimization model has been proposed for EV battery charging at a battery swapping station by determining the optimal charging scheme for each incoming batteries. The objective of this model is to minimize the BSS's running cost by maximizing the battery stock level and minimizing the average charging damage with the use of different types of chargers. The GA, DE and PSO algorithms have been implemented and compared in the paper, and the results show that the PSO algorithms fail to achieve an acceptable objective value even though the computational times of PSO algorithms are shorter than GA and DE. As the GA and DE can obtain a desired objective but the computational times are longer, we have proposed varied population algorithms (VPGA and VPDE) to reduce the computational time and improve the performance of the original version of GA and DE. The simulation results show that the performances of the proposed algorithms are comparable with the typical GA and DE, but the computational times are much shorter. A 24-hour simulation study is carried out to examine the feasibility of the model.

The contributions are summarized as follows:

-- This paper has proposed a new operation model for the operation of a viable battery swapping station. In this model, the BSS would determine an optimized charging scheme for the batteries to minimize the running cost to the BSS by maximizing the battery stock level and minimizing the battery charging damage. EV drivers are encouraged to give advanced notification to the BSS as to arrival by discount offers.

-- A varied population strategy is proposed to improve the performance of the typical optimization algorithms. Two new algorithms, VPGA and VPDE algorithm, are proposed to solve the optimization problem. As the results shown in Section 6.2, the objective values obtained by the PSO algorithms are inferior to GA, DE, VPGA and VPDE. Considering that the principal object of the proposed model is to minimize the running cost of the BSS, the performance of the PSO algorithms is not acceptable in this problem. Even though the performances determined by GA, DE, VPGA and VPDE are comparable, the computational times of the proposed VPGA and VPDE are almost half of the original version of GA and DE. Once an EV arrives at the station, a schedule would be obtained with the use of all the known orders and current stock status. In this 100 order case, the VPGA can obtain the solution in seconds. It must be mentioned that the computational time is an important factor in the performance of the algorithms considering the objective values are comparable.

-- The BSS would know in advance the time of the battery swap of an EV and the remaining capacity of the swapped battery. With the optimization algorithm, it can determine the best scheme to recharge the battery. From the simulation study, it is shown that the optimized scheme can achieve a stock level which is the same as the case of only using "ultra-fast chargers". Yet, the optimized scheme would use various chargers so as to lower the damage to the batteries.

-- When some orders arrive without advanced notice, the stock level would be lower than for the case of using "ultra-fast chargers", which is as expected. The stock level is always higher than the case of always using "slow chargers" or random assignment, while the average charging cost is always low. This study has confirmed the advantage of encouraging drivers to make advanced notice of arrival as it would help the BSS to maintain a higher stock level and provide better service for the drivers. The provision of a price discount is considered as a reasonable way to entice drivers. However, the quantification of the price discount is out of the scope of this study.

Currently, this research has designed the operation model for a battery swapping station and the mathematic model has been formulated to represent the problem. With the use of three typical optimization algorithms, a varied population algorithm has been proposed to solve the problem. In the future, we aim to extend this model to more than one battery swapping stations and use more than one type of batteries. Also, except using the integrated objective function, we will study the multi-objective algorithms or other mathematical methods to solve the proposed problem and then compare the performances with this paper. Lastly, we hope to obtain some real EV data from the public transportation system for the evaluation of our model.

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