Editors' Suggestion

Test One to Test Many: A Unified Approach to Quantum Benchmarks

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Quantum benchmarks are routinely used to validate the experimental demonstration of quantum information protocols. Many relevant protocols, however, involve an infinite set of input states, of which only a finite subset can be used to test the quality of the implementation. This is a problem, because the benchmark for the finitely many states used in the test can be higher than the original benchmark calculated for infinitely many states. This situation arises in the teleportation and storage of coherent states, for which the benchmark of 50% fidelity is commonly used in experiments, although finite sets of coherent states normally lead to higher benchmarks. Here, we show that the average fidelity over all coherent states can be indirectly probed with a single setup, requiring only two-mode squeezing, a 50-50 beam splitter, and homodyne detection. Our setup enables a rigorous experimental validation of quantum teleportation, storage, amplification, attenuation, and purification of noisy coherent states. More generally, we prove that every quantum benchmark can be tested by preparing a single entangled state and measuring a single observable.

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Introduction.—Quantum information processing offers compelling advantages over its classical counterpart. However, realistic implementations suffer from unavoidable noise and imperfections. To demonstrate a quantum advantage, one needs to ensure that, despite the imperfections, such implementations achieve performances that could not be achieved classically.

For every given task, such as the transmission of information or its storage in a quantum memory, the limit that has to be surpassed in order to demonstrate a quantum advantage is called the quantum benchmark [1]. Quantum benchmarks are routinely used in experiments of quantum teleportation [2–6] and in the realization of quantum memories [7–10]. The theoretical values of the benchmarks have been determined in a variety of scenarios, including the teleportation and storage of finite-dimensional quantum systems [11,12], coherent states [1,13], and squeezed states [14–16]. Benchmarks for the amplification of coherent states are important for assessing the realization of deterministic [17] as well as probabilistic [18–21] amplifiers, and have been theoretically studied in Refs. [22,23]. Many benchmarks are fidelity based, meaning that they use the fidelity [24,25] as the figure of merit. Other benchmarks are entanglement based, meaning that the figure of merit is (a measure of) the ability to preserve entanglement [26–29].

In theory, quantum benchmarks provide rigorous criteria of quantumness. In practice, the application of these criteria can be problematic. The benchmarks often rank quantum devices based on their average performance on an infinite set of input states, such as the set of all coherent states [1,2,4,5,7–10,13,17–23]. In a real experiment, however, only a finite subset of inputs can be tested. The evaluation of the performance on each input requires many sessions of data collection, often amounting to a full tomography of the state [19]. Now, the problem is that the value of the benchmark for the finite subset of states used in the experiment can be much larger than the theoretical benchmark. For example, the fidelity benchmark for the teleportation of uniformly distributed coherent states is 50% [1,13], while the benchmark for just two coherent states is at least 93.3%, the minimum value over all pairs of coherent states [30]. Comparing the experimental fidelity with the theoretical benchmark requires additional assumptions on the device—e.g., assumptions on how it would have worked if it had been tested on other inputs. But making such assumptions is in contradiction to the purpose of quantum benchmarks, i.e., to certify quantum advantages without having to trust the devices. An alternative approach would be to perform a full tomography of the device [31–36], but this would require a large number of measurement settings (or even an infinite number in the case of continuous variable systems).

In this Letter, we show that every quantum benchmark can be tested by preparing a single entangled state and performing a single measurement on the output. More broadly, we develop a unified framework for quantum benchmarks, including fidelity-based and entanglement-based benchmarks as special cases. We observe that the same benchmark can be tested in multiple equivalent ways, among which one can choose the most experimentally friendly one. Using the idea of equivalent tests, we propose a benchmark setup for the demonstration of continuous-variable quantum memories [7–10] and for the demonstration of quantum-enhanced amplification [17–21]. Our proposal allows one to measure the average fidelity over all possible coherent states, using only two-mode squeezing, a 50-50 beam splitter, and homodyne detection. The same approach can be applied to benchmarks for quantum attenuation [22,37–40] and cloning [41–43] of coherent states, as well as the purification of displaced thermal states [39,44,45].

General benchmark framework.—The scenario of quantum benchmarks can be conveniently viewed as a game between an experimenter and a verifier [46]. The experimenter builds a device performing a quantum task, such as teleportation or cloning. The verifier sets up a test in order to determine whether the device offers a quantum advantage. The test consists in sending inputs to the device and performing measurements on the outputs.

Let us start from the case of a deterministic device, which generates an output whenever it receives an input. Such a device can be described by a quantum channel (completely positive trace-preserving linear map), transforming states of the input system into states of the output system. Let us denote by A (A') the input (output) system, and by $\mathcal{C}$ a generic channel with input A and output A'.

In order to rate the performance of the channel $\mathcal{C}$, the verifier could use the setup described in Fig. 1. First, the verifier prepares system A in an input state ρ_x, randomly drawn from some set $\{\rho_x\}$ with probability p_x. Then, the verifier submits the input to the experimenter, who returns the output $\mathcal{C}(\rho_x)$. Finally, the verifier performs a measurement, described by a positive operator-valued measure (POVM) $\{P_y^{(x)}\}$ where x labels the measurement setting

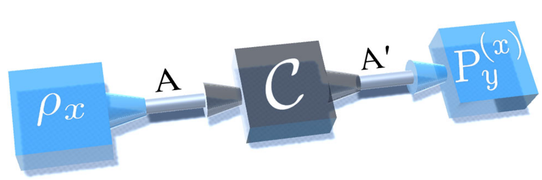

FIG. 1. Input-output test of a quantum device. To test the device $\mathcal{C}$, the verifier prepares an input state, randomly drawn from the set $\{\rho_x\}$. Upon receiving the input, the device generates an output, which is then measured by the verifier with the POVM $\{P_y^{(x)}\}$. The outcome is assigned a score and the average score is used as a measure of performance.

and y labels the measurement outcome. For every setting x, the outcome y is assigned a score $\omega(x,y)$. The average score

$$S^{(\mathrm{det})} = \sum_x \sum_y \omega(x,y) p_x \mathrm{Tr}[P_y^{(x)} \mathcal{C}(\rho_x)], \qquad (1)$$

is then used as a figure of merit. The typical example of Eq. (1) is that of the fidelity-based benchmarks [1–3, 11–16,19,22,23,47], where the goal is to transform an unknown input state ρ_x into a pure target state $|\phi_x\rangle$. Fidelity benchmarks are expressed in terms of the average fidelity

$$F^{(\mathrm{det})} \coloneqq \sum_x p_x \langle \phi_x | \mathcal{C}(\rho_x) | \phi_x \rangle, \qquad (2)$$

which can be viewed as the special case of Eq. (1) where each POVM $\{P_y^{(x)}\}$ has an outcome y_x associated to the projector $P_{y_x}^{(x)} = |\phi_x\rangle\langle\phi_x|$ and the score $\omega(x,y)$ is either 1 or 0, depending on whether or not y is equal to y_x.

The benchmark for a genuine quantum implementation has the form $F^{\mathrm{det}} > F_C^{\mathrm{det}}$, where F_C^{det} is the classical fidelity threshold, namely, the maximum fidelity achievable by measure-and-prepare channels [1]. The direct way to evaluate the score (1)—or the average fidelity (2)—is to test the action of the channel $\mathcal{C}$ on all the input states $\{\rho_x\}$ and to use the experimental data to compute the average. However, this approach is not viable when the set of input states is infinite. Now, we show that many indirect ways to experimentally measure the average score (1) or the average fidelity (2) exist. Among these indirect measurements, some can be dramatically simpler than the direct approach of Fig. 1.

First of all, we note that every test with random input states can be reformulated as a test with a single, mixed, input state σ_{AR}. This is because one can regard the preparation of the state ρ_x with probability p_x as the preparation of a single quantum-classical state $\sigma = \sum_x p_x \rho_x \otimes |x\rangle\langle x|_R$, where R is an auxiliary system keeping track of the index x. Likewise, one can formally write down a single quantum observable $O = \sum_{x,y} \omega(x,y) P_y^{(x)} \otimes |x\rangle\langle x|_R$, so that the average score (1) takes the form

$$S^{(\mathrm{det})} \coloneqq \mathrm{Tr}[O(\mathcal{C} \otimes \mathcal{I}_R)(\sigma_{AR})]. \qquad (3)$$

Per se, this reformulation does not make the problem easier. The merit of Eq. (3) is that it reveals a general structure, suggesting new ways to measure the average score. This reformulation also offers a unified approach, which can be adopted not only for fidelity-based benchmarks, but also for other types of quantum benchmarks, such as the entanglement-based benchmarks [26–29].

The single-input setup for testing quantum channels is depicted in Fig. 2. Now, the key observation is that many different tests are equivalent, meaning that they assign the

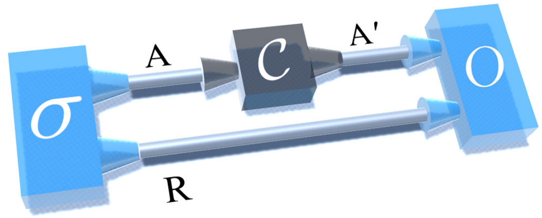

FIG. 2. Test with a single input and a single observable. A composite system AR in a joint state σ. Then, system A is sent to the device $\mathcal{C}$, which transforms it into the output system A'. Finally, systems A' and R undergo a joint measurement, described by the observable O. The expectation value of O is then used as the figure of merit.

same average score to all possible channels. This observation is important because, among the many equivalent tests, one can choose the easiest to realize experimentally. Now, we develop a framework that captures the equivalence of tests and facilitates the search for the most convenient realization. The framework is based on the Jamiołkowski operator [48], defined as

$$C := \sum_{ij} \mathcal{C}(|i\rangle\langle j|) \otimes |j\rangle\langle i|, \tag{4}$$

where $\{|i\rangle\}$ is a fixed orthonormal basis for system A. In terms of the Jamiołkowski operator, the average score can be written as [49]

$$S^{(\mathrm{det})} = \mathrm{Tr}[\Omega C], \tag{5}$$

where Ω is the operator on $A'A$ defined by

$$\Omega := \mathrm{Tr}_R[(O_{A'R} \otimes I_A)(I_{A'} \otimes \sigma_{AR})]. \tag{6}$$

Here, it is understood that the Hilbert spaces are rearranged in the appropriate order, so that the operators in the right hand side can be multiplied.

We call Ω the performance operator of the test. For fidelity-based benchmarks, the performance operator is simply the average input-output state

$$\Omega = \sum_x p_x |\phi_x\rangle\langle\phi_x| \otimes \rho_x. \tag{7}$$

where ρ_x is the input and $|\phi_x\rangle$ is the target output.

Canonical tests for deterministic devices.—Clearly, two tests with the same performance operator are equivalent, even if they correspond to totally different testing procedures. Now, we exploit the equivalence to realize every test through the preparation of a single pure state and the measurement of a single observable.

Theorem 1 [49].—Every test for deterministic devices is equivalent to a canonical test of the following form.

Step 1: Choose a mixed state τ_A, with the property that the operator $I_{A'} \otimes \tau_A$ is invertible on the support of the operator Ω^{T_A}, where T_A denotes the partial transpose on system A.

Step 2: Prepare a purification of τ_A, denoted by $|\Psi\rangle_{AR}$.

Step 3: Apply the channel $\mathcal{C}$ on system A.

Step 4: Measure systems A' and R with the observable

$$O = (I_{A'} \otimes \tau_R^{-1/2} T_{AR}^\dagger)\Omega^{T_A}(I_{A'} \otimes T_{AR}\tau_R^{-1/2}). \tag{8}$$

where $\tau_R = \mathrm{Tr}_A[|\Psi\rangle\langle\Psi|_{AR}]$ is the marginal of the state $|\Psi\rangle_{AR}$ on system R, and T_{AR} is the partial isometry such that $T_{AR}^\dagger \tau_A T_{AR} = \tau_R$.

The best way to understand Theorem 1 is to use it in a concrete example. Consider the problem of amplifying coherent states [17,22,23]. Here, the task is to transform a generic coherent state $|\alpha\rangle \propto \sum_n \alpha^n |n\rangle/\sqrt{n!}$ into the amplified coherent state $|g\alpha\rangle$, where $g \geq 1$ is the gain of the amplifier. For $g = 1$, the problem is to teleport coherent states [2,4,5] or to store them in a quantum memory [7–10]. Assuming that the inputs are Gaussian-distributed, the average fidelity is

$$F^{\mathrm{det}} = \int \frac{d^2\alpha}{\pi} \lambda e^{-\lambda|\alpha|^2} \langle g\alpha|\mathcal{C}(|\alpha\rangle\langle\alpha|)|g\alpha\rangle, \tag{9}$$

where $\lambda \geq 0$ is the inverse of the variance. In practice, the average cannot be evaluated directly, because this would require sampling over an infinite set of input states. Moreover, in the actual experiments [19], the fidelity is evaluated through a full tomography of the output state, meaning that each value of α requires a large (ideally infinite) number of experimental settings, making the evaluation of the average fidelity prohibitively expensive. Luckily, Theorem 1 offers a way out. Instead of sampling over all coherent states, it is enough to prepare a two-mode squeezed vacuum state

$$|\Psi\rangle_{AR} = \sqrt{1-x} \sum_n x^{n/2} |n\rangle_A \otimes |n\rangle_R, \tag{10}$$

where the squeezing parameter x can be any number in the interval (0,1). Instead of evaluating the fidelity on each coherent state, it is enough to measure a single observable, given by Eq. (8) with the performance operator

$$\Omega = \int \frac{d^2\alpha}{\pi} \lambda e^{-\lambda|\alpha|^2} |g\alpha\rangle\langle g\alpha| \otimes |\alpha\rangle\langle\alpha|. \tag{11}$$

Now, we take advantage of the fact that every value of the squeezing parameter x is allowed, and therefore, one can choose the most convenient x. Specifically, we notice that the observable (8) takes a simple form when $x = 1/(1+\lambda)$. For $g^2 \leq \lambda + 1$, we find [49]

$$O = S_\theta^\dagger (I \otimes G_\theta) S_\theta, \tag{12}$$

where $S_\theta = \exp[\theta(ab - a^\dagger b^\dagger)]$ is a two-mode squeezer with $\tanh\theta = g/\sqrt{\lambda+1}$, and G_θ is the Gaussian observable $G_\theta = \sum_n (\tanh\theta)^{2n}|n\rangle\langle n|$. In practice, this means that the observable O can be measured by sending the two output modes A' and R through a two mode squeezer and by measuring the observable G_θ on the second port. In turn, the observable G_θ can be measured by sending the mode through a 50-50 beam splitter, measuring the quadratures $X = (a + a^\dagger)/2$ and $P = (b - b^\dagger)/(2i)$ on the two output modes, respectively, and, finally, averaging the outcomes with a Gaussian weight (see [49] for the exact expression). The setup for $g^2 < \lambda + 1$ is identical, except that one has to set $\tanh\theta = \sqrt{\lambda+1}/g$ and the observable G_θ is measured on the first output port [49].

Our method makes the average fidelity (9) experimentally accessible, thus, enabling a rigorous experimental test of the quantum advantage. The same method can be used to test the fidelity of attenuation [22,37,38,40], cloning [41–43], purification of displaced thermal states [39,44,45], and phase conjugation [57], as shown in the Supplemental Material [49]. A limitation of the present approach is that the verifier should be able to preserve the reference mode from noise. In the case of quantum memories, this means that the verifier should possess a good quantum memory for the reference mode. Basically, the test of Fig. 3 compares the untrusted quantum memory implemented by the experimenter with a trusted quantum memory in the verifier's lab.

Canonical tests for nondeterministic devices.—Now, let us consider the case of devices that return an output with some nonunit probability. Examples of such devices are the noiseless probabilistic amplifier [58], experimentally realized in Refs. [18–21], and the noiseless probabilistic attenuator of Refs. [37,38,40]. In general, a probabilistic device can be described by a quantum operation $\mathcal{C}$ (completely positive trace-nonincreasing linear map). To test the device, one can prepare a single input state σ and measure an observable O on the output, as in Fig. 2. Sometimes, the device will report failure instead of producing an output. The probability that an output is produced is

$$p_{\mathrm{succ}} = \mathrm{Tr}[(\mathcal{C}\otimes\mathcal{I}_R)(\sigma_{AR})] = \mathrm{Tr}[\mathcal{C}(\sigma_A)], \qquad (13)$$

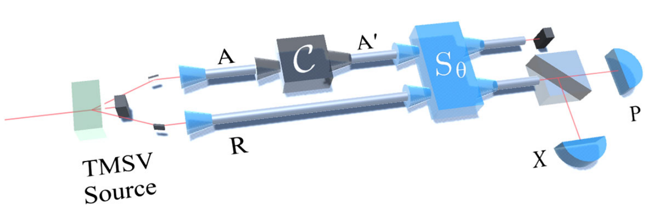

FIG. 3. Canonical test for coherent state amplifiers. The input mode and a reference are prepared in the two-mode squeezed vacuum (TMSV). After the action of the amplifier, the output mode and the reference are sent through a two mode squeezer S_θ, followed by a 50-50 beam splitter and two quadrature measurements on the output modes.

where $\sigma_A = \mathrm{Tr}_R[\sigma_{AR}]$ is the marginal of σ_{AR} on system A. The average score is then

$$S^{(\mathrm{prob})} := \frac{\mathrm{Tr}[O(\mathcal{C}\otimes\mathcal{I}_R)(\sigma_{AR})]}{\mathrm{Tr}[\mathcal{C}(\sigma_A)]}, \qquad (14)$$

and can be expressed as

$$S^{(\mathrm{prob})} = \frac{\mathrm{Tr}[C\Omega]}{\mathrm{Tr}[C(I_{A'}\otimes\sigma_A)]}, \qquad (15)$$

where Ω is the performance operator (6) and C is the Jamiołkowski operator. Note that, now, the score depends both on the performance operator Ω and on the marginal input state σ_A, which determines the probability of success via Eq. (13).

It is easy to see that two tests are equivalent in terms of score and success probability if and only if they have the same pair of operators (Ω, σ_A). Leveraging on the equivalence, we can construct a canonical realization.

Theorem 2.—Every test of probabilistic devices is equivalent to a canonical test of the following form.

Step 1: Prepare a purification of the marginal input state σ_A, denoted by $|\Phi\rangle_{AR}$.

Step 2: Apply the quantum operation $\mathcal{C}$ on system A.

Step 3: Measure systems A' and R with the observable

$$O = (I_{A'}\otimes\tilde{\sigma}_R^{-1/2}T_{AR}^\dagger)\Omega^{T_A}(I_{A'}\otimes T_{AR}\tilde{\sigma}_R^{-1/2}). \qquad (16)$$

where $\tilde{\sigma}_R$ is the marginal of the state $|\Phi\rangle_{AR}$ on system R and T_{AR} is the partial isometry such that $T_{AR}^\dagger\sigma_A T_{AR} = \tilde{\sigma}_R$.

Theorem 2 offers the first rigorous way of testing the fidelity benchmark for noiseless nondeterministic amplifiers [18–21]. In this case, the marginal state σ_A is

$$\sigma_A = \int \frac{d^2\alpha}{\pi}\lambda e^{-\lambda|\alpha|^2}|\alpha\rangle\langle\alpha|. \qquad (17)$$

Its purification is a two-mode squeezed vacuum, given by Eq. (10) with $x = 1/(1+\lambda)$. Then, one can obtain the observable O from Eqs. (16) and (11). Again, the observable has a simple experimental realization. In fact, this is the same realization described in the deterministic case. Using this realization, it is now possible to set up a conclusive demonstration of quantum advantage for noiseless amplifiers. The same holds for nondeterministic attenuation [37,38,40].

The fully black box test.—We analyzed, separately, the tests of deterministic devices and the tests of probabilistic devices. In practice, however, we may not know the success probability of the tested device. This would be a problem, because the benchmark generally depends on the success probability [46]: in general, the smaller the success probability, the higher the benchmark. A solution to the problem would be to use the highest benchmark, calculated in the limit of vanishing success probability. However, this could set an unreasonably high bar for the experiment. Now, we show that the verifier can devise a fully black box test, where the value of the benchmark is independent of the probability of success.

Theorem 3 [49].—Given a test $\mathcal{T}$ for deterministic devices, one can construct a new test $\mathcal{T}'$ for probabilistic devices, with the following properties.

Property 1:$\mathcal{T}'$ has the same performance operator as the original test $\mathcal{T}$. Therefore, $\mathcal{T}'$ assigns the same score as $\mathcal{T}$ to all deterministic devices.

Property 2: For probabilistic devices, the benchmark for $\mathcal{T}'$ is independent of the success probability.

The new test $\mathcal{T}'$ is described by a pair of operators (Ω, σ_A), with the following properties: the performance operator Ω is chosen to be the same as the performance operator of the old test $\mathcal{T}$. This choice guarantees that the test $\mathcal{T}'$ assigns the same score as $\mathcal{T}$ when applied to deterministic devices. The marginal state σ_A is chosen to be the state that reduces the probabilistic benchmark to its minimum: this means that σ_A minimizes the best score (15) over all measure-and-prepare channels. The test for amplification or attenuation shown earlier in the Letter is an example of a fully black box test: the same experimental test and the same benchmark value can be used for both deterministic and probabilistic devices. More examples of this situation are shown in Sec. VI of [49], which focusses on the scenario where the test $\mathcal{T}$ enjoys a symmetry with respect to a group of physical transformations.

Conclusions.—In this Letter, we showed that a verifier can experimentally evaluate the performance of a quantum device on an infinite set of inputs by preparing a single entangled input and measuring a single joint observable. As an application, we constructed a test for the realization of quantum memories, amplifiers, and attenuators of coherent states, and purifiers of displaced thermal states. The test can be realized using two-mode squeezers, beam splitters, and homodyne detection. Using these ingredients, one can experimentally assess the average fidelity over all possible coherent states (or all possible displaced thermal states), thus, providing a fully rigorous demonstration of genuine quantum advantage.

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