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Buy Now and Price Later: Supply Contracts with Time-Consistent Mean-Variance Financial Hedging

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Highlights

- Flexible price contract linked to the future spot price of commodity is formulated.
- Manufacturer and retailer can be *win-win* under flexible price contract.
- Flexible price contract with financial hedging is formulated to mitigate the risk-averse retailer's exposure to commodity price risk.
- Closed-form time-consistent financial hedging policy is derived.

Buy Now and Price Later: Supply Contracts with Time-Consistent Mean-Variance Financial Hedging

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Abstract

We consider a two-stage supply chain comprising one risk-neutral manufacturer (he) and one risk-averse retailer (she), where the manufacturer procures consumption commodities in spot market as major inputs for production and sells the final products to the retailer. The retailer then sells the final products to the market at a stochastic clearance price. We investigate a *flexible price contract* that allows the manufacturer to determine the product wholesale price, and the retailer to determine the order quantity, based on the *future spot price* of consumption commodities. Compared with the simple wholesale price contract, a *win-win* situation can be achieved under the flexible price contract when the manufacturer's postponed processing cost is lower than a threshold. However, under this flexible price contract the retailer may suffer from the commodity price volatility, even if she does not procure the commodities directly. We further investigate how the risk-averse retailer conducts mean-variance financial hedging by purchasing consumption commodity futures contracts. We formulate the problem using a dynamic programming model and derive a closed-form time-consistent financial hedging policy. Through numerical experiments, we show that the commodity price risk from the manufacturer to the retailer is effectively mitigated with the hedging, and the benefits of the flexible price contract are maintained.

Keywords: Risk management, pricing, mean-variance, demand forecast update, contracts

1 Introduction

Consumption commodities, by definition, refer to those highly standardized materials that are held mainly for consumption (Hull, 2009). Metals, energies, and agricultural products are the most common consumption commodities that are used extensively in various industries (Wang et al. 2014). These consumption commodities are commonly traded in standardized spot markets, thanks to the standardization of trading contracts and the development of electronic trading platforms. However, the spot prices of these commodities have shown substantial fluctuation in the past decades (Li et al. 2017, Tong et al. 2017). Under today's volatile business environment, it is not uncommon that the price of various commodities can greatly vary, sometimes by more than 50 percent in a short period of time (Brown, 2008). For commodity-based supply chains, the growing price volatility has been one of the primary challenges for the supply chain members (Wang et al. 2017, Wang et al. 2016). Especially, consider the common phenomena that a lead-time exists between the signing of the supply contract and the physical production of the product, due to transportation, production set-up, *inter alia*. This lead-time can take as long as several months, during which the spot price could undergo significant changes. Therefore, the traditional wholesale price contract might not work well under the environment with stochastic commodity prices.

This paper explores how the supply chain members can benefit from incorporating the commodity markets (spot and futures markets) in the design of supply contracts based on the wholesale price contract, which probably is the most widely used contract in practice. Under the traditional wholesale price contract, the transfer payment between the supply chain parties is fixed before the production activity takes place. For instance, the wholesale price may be set based on the commodity spot price on the date when the contract is signed. However, the spot price could hike when the manufacturer procures the commodities for production and his profit will be greatly undermined. On the other hand, the retailer suffers if the current commodity price is significantly lower than the contract price committed earlier. To address this issue, this study attempts to investigate a form of supply contract signed between the manufacturer and the retailer, and that the wholesale price of the products is based on the spot price of the commodities on future date, e.g., when the physical transaction takes place.

Indeed, this form of supply contract has been adopted by practitioners, who are exposed to significant uncertainty of raw materials' cost, to actively manage the price fluctuation (Feng et al., 2013). For instance, it is reported that flexible price contract contingent on the input commodity price has been found useful in utility industry to deal with the uncertainty cost of fuel, such as oil, coal, and natural gas (Charles, 2005). As for the medical device manufacturing industry, precious metal could contribute up to 90% of the manufacturing cost (e.g., medical wire). Considering the significant volatility of the precious metal price, the finished product is usually invoiced to the small and medium-sized buyer depending on the market price of the metal on the delivery date rather than the price on the date when the contract is signed (Bestrom, 2009). From these examples, it can be seen that such a contract is signed early but the realized product wholesale price is based on the spot price of the commodities when physical transaction takes place. This type of supply contract is known as *flexible price contract*. Despite its proven value in practice, flexible price contract is still under-studied by researchers. Such contracts can be further explored to see how they could be applied to bring greater benefit to members in a supply chain.

Motivated by the practice mentioned above, this paper conducts a thorough research on flexible price contract under which the upstream and downstream supply chain parties agree on a transfer payment contingent on the future spot market price of the input commodities. In particular, we are interested in the following research questions. First, *is it a win-win solution for both supply chain parties to sign such a flexible price contract?* Second, *how the supply chain performance in terms of production output and competition penalty would be affected under this contract?*

To answer these questions, we will compare the effects of two different supply contracts on a supply chain as well as on its members individually. The first is a flexible price contract (referred to as F-contract) and the second is a traditional fixed wholesale price contract without flexibility (referred to as S-contract). This study will be conducted based on a stylized model where a risk-neutral manufacturer (he), acting as the leader, proposes a 'take-it-or-leave-it' offer (regarding the product wholesale price) to a risk-averse retailer (she). If the retailer accepts the offer, she will decide on how many units to order. Also, a mean-variance criterion is adopted to characterize the retailer's risk attitude. The proposed supply chain model is considered appropriate for industries involving consumption commodities (Zhao et al. 2015). In such industry, compared with the downstream retailers, the upstream manufacturers usually require huge amount of capital for the procurement of input commodities and investment on the production technology. As Chod and Lyandres (2011) and Zhao et al. (2015) discuss, it is reasonable to consider such kind of supply chain to be composed of a risk-neutral manufacturer and a risk-averse retailer. Also, such kind of assumption is popular in the operations management literature. For instance, Gan et al. (2004), Xiao and Yang (2009), Chiu et al. (2011), Li et al. (2014), Jiang et al. (2016), and Li et al.(2016) all consider a supply chain consisting of a risk-neural manufacturer (or supplier) and a risk-averse retailer.

In practice, many risk-averse firms have employed financial instruments such as commodity futures and options to reduce the risk exposure incurred by *financial random factors* such as fluctuating commodity price and uncertain foreign exchange rates. See, for example, a recent empirical study by Bartram et al. (2009). However, most of the current studies investigate the problem from the perspective of the direct commodity buyer while ignoring the potential impact on the downstream firms, which may suffer from the uncertainties transferred from the upstream commodity buyers. In practice, it is not uncommon that these uncertainties could propagate along the supply chain. For instance, under the aforementioned flexible supply contract, the ordering quantities and transfer payments are influenced by the commodity spot price fluctuation. To address this issue, we further raise the following question, (3) *how could the financial derivatives (such as futures or options) on the input commodity be effectively incorporated in the design of the flexible price contract?*

To answer the foregoing research question, this research further investigates a form of flexible price contract that involves financial hedging (referred to as H-contract). We first compute the equilibrium solutions and the corresponding profits/payoffs of the supply chain parties, and then compare the F-contract and S-contract both analytically and numerically. Our goal is to examine the effectiveness of flexible price contracts and identify the conditions under which the supply chain parties should select S-contract, F-contract or H-contract. Also, we will show how the time-consistent financial hedging strategy embedded in the H-contract can be derived analytically. Some managerial insights are obtained on supply chains that have consumption commodities as the main inputs. The main findings are summarized as follows:

- The flexible price contract contingent on the future spot price of the input commodity helps to achieve mutual benefits for the manufacturer and the retailer, when the manufacturer's postponed processing cost is lower than a threshold.
- The risk exposure of the retailer can be effectively reduced by adopting the time-consistent financial hedging policy.
- Financial hedging can be incorporated without affecting the manufacturer's operational decisions.

The remainder of the paper is as follows. The related literature is reviewed in Section 2. The model settings and notations are presented in Section 3. The equilibrium results under the F-contract and the comparison between the F-contract and the S-contract are given in Section 4. The equilibrium solutions under the H-contract are derived in Section 5. Numerical study is conducted in Section 6 to show the effectiveness of the H-contract. Concluding remarks and directions for further research are presented in Section 7. All the proofs are given in the online appendix.

2 Literature Review

The primary purpose of this paper is to investigate how the commodity markets can be fruitfully used in the design of supply contracts. Specifically, our study is related to three streams of literature: (i) supply chain management in the presence of spot market, (ii) procurement management with financial hedging, and (iii) ordering decision making with demand forecast update.

Early work in the first stream has been critically reviewed by Haksöz and Seshadri (2007). In recent years, several studies have investigated the benefits of incorporating spot market into supply chain operations and planning. For example, Chen et al. (2013) characterize the structural ordering policy for a firm procuring components from a spot market and a supplier with a minimum-order commitment. From the perspective of a commodity reseller, Xing et al. (2012) investigate how the buyer's procurement and pricing strategies are affected by a B2B spot market. Xing et al. (2014) further explore the strategic role of spot market in a two-stage supply chain, and find that spot market is not solely a second channel for procurement. Ma et al. (2015) study a firm's advance-booking decisions by assuming a firm can procure from the spot market as well as by using long-term contracts. To sum up, we find that most existing studies along this stream consider the spot market as a sourcing channel and/or a selling channel. Their focus is on the supply chain parties' performance when the spot market is introduced. Different from these studies, this paper investigates how spot market could be fruitfully used to design the flexible supply contract signed between a manufacturer and a retailer. We examine the efficacy of such a contract and derive the conditions under which both the manufacturer and the retailer can both benefit from such a contractual agreement.

We note that the flexible price contract we studied in this paper could also be called contingent contract in a general sense, which is extensively studied in the literature. For instance, Bazerman and Gillespie (1999) demonstrate the benefits of contingent contracts in various settings. We, however, identify the conditions under which such benefits can be realized and propose a feasible contract that could be employed in a supply chain in the presence of commodity markets. In addition, we recognize that the flexible price contract might also be referred to as pass-through contract in practice (Matthews 2011), which has rarely received attention in the literature (Wu et al. 2013). Zhang et al. (2013) and Bolandiar and Chen (2015) studied an index-linked/based contract, which is actually similar to the pass-through contract and hence similar to our flexible price contract. Specifically, Bolandiar and Chen (2015) consider a risk-neutral commodity-based supply chain with two competitive commodity processors and one common retailer. They show that the processors do not prefer to fully pass the commodity price risks to the downstream retailer. Differently, we consider a supply chain in which the retailer is risk-averse and we show that under certain conditions both the commodity processor and the retailer could be benefited by letting the downstream retailer fully bear the commodity price risks.

Our work also contributes to the growing literature on procurement management with financial hedging. In particular, our work is related to the studies on hedging under the mean-variance criterion. It has been observed that mean-variance framework is commonly adopted in the operations management literature. Chiu and Choi (2016) conduct a comprehensive review of literature applying a mean-variance evaluation framework to analyze supply chain risks. For studies involving financial hedging, commodity price risk mitigation using financial instrument has gained attention from the perspective of *commodity buyers*. See, e.g., Smith and Stulz (1985), Froot et al. (1993), Neuberger (1999), Gaur and Seshadri (2005), Ni et al. (2012, 2016), and Kouvelis et al. (2013). However, few researchers have considered the problem from the perspective of the other supply chain parties and/or the entire supply chain, with the possible exception of Caldentey and Haugh (2009) and Turcic et al. (2015). In particular, Caldentey and Haugh (2009) examine the efficacy of flexible and wholesale price contracts in a risk-neutral supply chain where the retailer is budget-constrained and the supply chain's profit is correlated to some general economic indices. Turcic et al. (2015) consider a decentralized risk-neutral supply chain in which financial hedging is employed to avoid possible default. We also notice that financial market could be simply used as a means for the physical acquisition of the commodity. For example, Ni et al. (2015) consider a two-echelon supply chain where the downstream manufacturer procures an intermediate product through a bilateral contract with the upstream supplier and a futures market. Different from their works, we model the retailer's risk-averse attitude and consider the impact of demand forecast update. In our research, the rationale for trading financial derivatives is to reduce the risk-averse retailer's exposures, while Caldentey and Haugh (2009) aim at relaxing the risk-neutral retailer's budget constraint.

Lastly, our research is related to the studies on ordering decisions with demand forecast update. Choi (2007) investigates the pre-season inventory decisions and product pricing decisions with multiple information updates. Choi and Sethi (2010) review literature on quick response supply chain systems in which various technologies are employed to improve the demand forecast and production/distribution schedules. Ma et al. (2012) consider a loss-averse newsvendor ordering problem with demand information update. Shang et al. (2015) investigate the incentive of a retailer to share the updated demand information in a supply chain with two competing manufacturers that sell substitutive products through a common retailer. Zhao et al. (2015) investigate demand information updating, demand-spot-price correlation, and their impacts on the profits of the manufacturer and the retailer. Some other typical studies on information update include Iyer and Bergen (1997), Gurnani, and Tang (1999), Donohue (2000), and Yang et al. (2011).

3 Model Formulation

3.1 Model settings and notations

Table 1 Summary of mathematical notations

Symbols	Parameters
S_τ	Commodity spot market price at time τ
w_τ	Wholesale price at time τ
q_τ	Ordering quantity at time τ
c_τ	Commodity processing cost at time τ
m	Fixed markup charged by the manufacturer
D	The random market size
D_t	Market size forecast in period t
ϵ_i	Demand forecast adjustments during period $i = 2, \cdots, n$
I_t	Cumulative demand forecast adjustment up to period t
ξ	Demand elasticity
γ	The degree of risk aversion
$\mathbb{F} = \{\mathcal{F}_t\}_{0 \le t \le T}$	Filtration that is generated by $\{S_t\}_{0 \le t \le T}$ and $\{D_t\}_{0 \le t \le T}$ on a given probability space $(\Omega, \mathcal{F}, Q)$
F_t	Price of the futures contract at time t maturing at time τ
θ_t	Quantity of futures contracts held at time t
W_t	Retailer's wealth level at time t
χ_t	Short-term deviation in prices
ω_t	Long-term equilibrium price level
κ	Short-term mean-reversion rate

σ_χ	Short-term volatility
λ_χ	Short-term risk premium
σ_ω	Long-term volatility
$\mu_\omega{}^*$	Equilibrium risk-neutral drift rate
ρ	Correlation in increments

In the proposed model, the contract terms are determined at time 0 while the market clearing price and the resulting cash flow are realized at a future time $T > 0$. Between time 0 and T, the manufacturer processes the commodities into products at a time τ, i.e., $0 \le \tau \le T$. As we have mentioned, the risk-neutral manufacturer acts as a Stackelberg leader and proposes a unit wholesale price w_τ for the products; and this price depends on the commodity spot market price at time τ (we denote this price as S_τ). Also, the risk-averse retailer determines the ordering quantity q_τ. The manufacturer's unit cost is composed of a commodity processing cost (denoted as c_τ) and a commodity procurement cost. As the products should be delivered to the retailer on or before time T, it is clear that the production response time decreases in the value of τ. As it is generally costly to delay production, without loss of generality, we assume that c_τ is strictly positive when τ takes a positive value and c_τ is zero when $\tau = 0$. The commodity procurement cost denotes the cost to acquire the input commodity from the spot market.

Manufacturer's problem. We assume that the wholesale price is a linear function of the spot price of the input commodity, i.e., $w_\tau = m + S_\tau$, where m denotes the fixed markup pre-determined by the manufacturer. This kind of transfer payment is widely used in existing literature, e.g., Li and Kouvelis (1999). Then the manufacturer's expected payoff is given by

$$\Pi_M = E[w_\tau q_\tau - C(q_\tau)] \tag{1}$$

where $C(q_\tau)$ denotes the cost to provide q_τ units of products to the retailer.

Retailer's problem. Following Caldentey and Haugh (2009), we adopt the stochastic linear clearance price model to compute the retailer's revenue. Given the retailer's ordering quantity, q_τ, the amount of revenue the retailer could collect at time T is $(D - \xi q_\tau)q_\tau$.

$D - \xi q_\tau$ is the price at which the retailer sells (clears) the products she have. The random variable D represents the market size and the fixed parameter ξ captures demand elasticity. The retailer's payoff under the mean-variance criterion then takes the following form

$$\Pi_R = E[\pi_R] - \frac{\gamma}{2} Var(\pi_R) \,, \tag{2}$$

where γ is a positive constant representing risk aversion, and π_R is the retailer's random profit, i.e., $\pi_R = (D - \xi q_\tau) q_\tau - w_\tau q_\tau$.

We assume that the demand information can be updated over the planning horizon $[0, T]$. For example, the supply chain parties can perform a market research to collect information about demand. The forecast of the market size D is based on the assumption that the market evolves following the additive martingale model of forecast evolution (MMFE), see Wang et al. (2012). The time interval $[0, \tau]$ is divided into n periods and the interval from τ to T is regarded as the $(n + 1)$-th period. The market size forecast in period $t = 2, \cdots, n$ is given by $D_t = d_1 + \epsilon_2 + \cdots + \epsilon_t$, where d_1 is the expected market size in period 1 and ϵ_i represents forecast adjustments during period $i = 2, \cdots, n$. The adjustments are assumed to be independent with each other and normally distributed with mean zero and variance σ_i^2. $I_t = \epsilon_2 + \cdots + \epsilon_t$ is used to represent the cumulative forecast adjustment up to period t. Then, the estimate of D after observing I_t is normally distributed with parameters $(d_1 + I_t, \sum_{i=t+1}^{n+1} \sigma_i^2)$. Throughout this paper, we assume that there is no information asymmetry between the retailer and the manufacturer.

It is clear that w_τ and q_τ depend on the spot price of the underlying commodity at time τ. Also, Π_M and Π_R depend on the spot price of the commodity at time τ because both of them are functions of w_τ and q_τ. The risk-averse retailer may then have incentive to trade in the financial market during the time interval $[0, \tau]$ to mitigate the revenue volatility. In contrast, the risk-neutral manufacturer has no incentive to trade in the financial market, given the perfect capital market assumption (Modigliani and Miller, 1958).

3.2 Financial market

We assume that there exists a futures contract written on the spot price of the same commodity used for production. In particular, we assume that the contract perfectly matches the planning horizon i.e., $[0, \tau]$, which is a standard assumption in the hedging literature (see e.g., Ni et al. (2012), Kouvelis et al. (2013), Turcic et al. (2015), Goel and Tanrisever (2017), and Ni et al. (2017))[1]. F_t is used to denote the price of the futures contract at time t maturing at time τ. Filtration $\mathbb{F} = \{\mathcal{F}_t\}_{0 \leq t \leq T}$ is generated by $\{S_t\}_{0 \leq t \leq T}$ and $\{D_t\}_{0 \leq t \leq T}$ on a given probability space $(\Omega, \mathcal{F}, Q)$. In this model, Q is assumed to be a combination of the real-world probability measure on demands and the risk-neutral measure on spot prices. Following the literature, we assume that $\{S_t\}_{0 \leq t \leq T}$ and $\{D_t\}_{0 \leq t \leq T}$ are independent. This implies that the real-world and risk-neutral measures of the demand distribution are equivalent because the demand risk is completely diversifiable (Berling and Rosling, 2005; Goel and Tanrisever, 2011). In this setting, the discounted futures contract prices are then Q-martingales. We also assume that there is a risk-free cash account available for depositing cash. Without loss of generality, the risk-free interest rate is assumed to be zero. Therefore, F_t is a Q-martingale. The use of risk-neutral probability measure ensures that trading in the futures market is for hedging only and any other speculation for financial gains is ruled out. This is consistent with how the financial markets are used by nonfinancial firms in practice.

Let θ_t denote the quantity of futures contracts held at time t. $G_\tau(\theta)$ represents the gain (or loss) at time τ that results from following a $\mathcal{F}_\tau$-predictable self-financing trading strategy, $\boldsymbol{\theta}_{\boldsymbol{\tau}} = \{\theta_t\}_{0 \leq t \leq \tau}$. See Shreve (2004) for details about the property of self-financing. Because the risk-free interest rate is assumed to be zero, the gain at time τ takes the following form in the discrete time trading setting:

$$G_\tau(\theta) = \sum_{t=0}^{\tau-1} \theta_t \cdot (F_{t+1} - F_t) \tag{3}$$

[1] We would like to thank the anonymous referee for comment on this. Specifically, if the maturity date of the futures contract mismatches with the planning horizon, basis risk would arise. Readers may refer to Castelino (1992) for the effect of basis risk on the hedging effectiveness. Following the literature on the interface of operations management and finance, research on the effect of basis risk on hedging is out of the scope of this paper. Also, according to the World Bank report (Varangis and Larson (1996)), basis risk is relatively small.

In addition, a complete financial market is assumed for simplifying the analysis (Smith and Nau, 1995).

3.3 Three supply contracts

We investigate three contractual agreements between the manufacturer and the retailer. As stated earlier, the contract is signed at time 0 whereas the actual physical transaction takes place at time $\tau \geq 0$.

1) Supply contract without flexibility (S-contract)

Under this simple wholesale price contract, $\tau = 0$, i.e., the product wholesale price and ordering quantity are both based on the commodity price at the beginning of the planning horizon. The model is then reduced to the case where the commodity spot market is used as the source to purchase raw materials.

2) Flexible price contract (F-contract)

Under the flexible price contract, the physical transaction takes place at time $\tau \in [0, T]$ while the contract is signed at time 0. It is obvious that S-contract is a special case of F-contract with $\tau = 0$. Under the F-contract, the manufacturer offers an $\mathcal{F}_\tau$-measurable wholesale price, $w_\tau = m + S_\tau$, to the retailer. Given this, the retailer determines an $\mathcal{F}_\tau$-measurable ordering quantity, $q_\tau = q(w_\tau)$, which should be interpreted as the retailer's response to w_τ rather than a function of w_τ. In this case, besides serving as a sourcing channel to purchase raw materials, the commodity spot market provides an index that is used to determine the wholesale price.

3) Flexible price contract with financial hedging (H-contract)

This contract is similar to F-contract except that the risk-averse retailer now has access to the financial market to hedge her uncertain payoff. In particular, the retailer can trade in the futures market and dynamically rebalance the positions of the futures contracts over time horizon $[0, \tau]$.

To simplify the exposition, in the ensuing analysis the superscripts S, F, and H are used to index the results obtained under the S-contract, F-contract, and H-contract, respectively. As the simple contract is a special case of the flexible supply contract, we study F-contract in the following section.

4 Flexible Price Contract

4.1 Supply chain parties' performance

In response to the wholesale price, w_τ, the risk-averse retailer determines the ordering quantity, $q_\tau = q(w_\tau)$, by solving the following optimization problem:

$$\Pi^{\mathrm{F}}_{\mathrm{R}|\tau}(w_\tau) = \max_{q_\tau \geq 0}\{\mathrm{E}^{\mathrm{Q}}_{\tau}[\pi_R] - \frac{\gamma}{2}\mathrm{Var}^{\mathrm{Q}}_{\tau}(\pi_R)\}\,, \tag{4}$$

where $\pi_R = (D - \xi q_\tau)q_\tau - w_\tau q_\tau$. Note that $\mathrm{E}^{\mathrm{Q}}_{\tau}[\cdot]$ and $\mathrm{Var}^{\mathrm{Q}}_{\tau}(\cdot)$ represent the expectation and variance taken under the probability measure Q conditional on $\mathcal{F}_\tau$, respectively. For notational convenience, let us define $\overline{D}_\tau = E^{Q}_{\tau}[D]$. Specially, we let $\overline{D} = \mathrm{E}^{\mathrm{Q}}_{0}[D]$. Notice that the choice of the risk neutral probability measure with respect to the uncertain demand is not unique because the market is not complete (the uncertain demand is not a random factor that can be fully hedged in the financial market in the proposed model), though the financial market itself is complete; see Ni et al. (2016) for a technical discussion. Thus, without loss of generality, one can choose the real-world probability measure as the Q - measure with respect to the demand uncertainty.

Let q^F_τ denote the optimal response of the retailer under the F-contract given a wholesale price w_τ. Given the retailer's optimal response, the manufacturer then obtains the optimal wholesale price by solving:

$$\Pi^{\mathrm{F}}_{\mathrm{M}} = \mathrm{E}^{\mathrm{Q}}_{0}\left[\max_{w_\tau \geq c_\tau}\{w_\tau q^F_\tau - C(q^F_\tau)\}\right] \tag{5}$$

In this paper, the objective of the retailer is to maximize the cash flow under a mean-variance criterion while the manufacturer's objective is to maximize the expected cash flow. Note that the expectation and variance involved throughout this paper are all taken under the Q measure. Under this assumption, all cash flows can be valued using an appropriate equivalent martingale measure (EMM) and the expected payoff of any financial hedge would be zero. Following Caldentey and Haugh (2009), we simply assume that there exists such an EMM without concerning how it should be identified. Although this is a standard assumption in the literature (e.g., Chod et al. 2010, Goel and Gutierrez 2011, Chod and Zhou 2014), we notice that the financial risk exposed to the firms is actually discounted under the risk-neutral measure. However, we note that our model considers both financial risk (stochastic commodity price) and nonfinancial risk (uncertain demand), which means that the market is incomplete and the discounting for risk cannot be fully addressed with the risk-neutral measure. Thus, in an incomplete market, it is still necessary to use a utility function to model the retailer's risk-averse behavior (Ni et al. 2016). Alternatively, it would also be interesting to investigate the case where real-world probability measure is employed on both the demand uncertainty and spot prices (Kouvelis et al. 2013). We leave it for future study due to the technical complexity.

Notice that the condition of production postponement written in the F-contract allows the manufacturer to procure in the forward market rather than relying solely on the spot market. However, the risk-neutral manufacturer cannot gain any benefit through procuring in the forward market under the perfect capital market condition (Modigliani and Miller, 1958). Therefore, all the input commodities will be procured in the spot market only. In other words, the input commodity is procured at the prevailing price S_τ. As stated earlier, compared with the S-contract case, an additional commodity processing cost, c_τ, is incurred due to the reduction in the production lead time. Therefore, we have $C(q_\tau) = (S_\tau + c_\tau)q_\tau$. The following proposition shows the equilibrium results for the manufacturer and the retailer.

Proposition 1: The equilibrium wholesale price and ordering quantity for the F-contract are:

$$w_\tau^F = \frac{\bar{D}_\tau + S_\tau + c_\tau}{2} \text{ and } q_\tau^F = \frac{\bar{D}_\tau - S_\tau - c_\tau}{4\xi + 2\gamma\sigma_{n+1}^2} \tag{6}$$

The corresponding expected payoffs of the manufacturer and of the retailer are:

$$\Pi_{\mathrm{M}|\tau}^{\mathrm{F}} = \frac{(\bar{D}_\tau - S_\tau - c_\tau)^2}{8\xi + 4\gamma\sigma_{n+1}^2} \tag{7}$$

$$\Pi_{\mathrm{R}|\tau}^{\mathrm{F}} = \frac{(\bar{D}_\tau - S_\tau - c_\tau)^2}{16\xi + 8\gamma\sigma_{n+1}^2} \tag{8}$$

From the above proposition, we observe that the retailer's payoff under the mean-variance criterion is *proportional* to the manufacturer's expected profit. This makes their profits *aligned* and *predictable*. We further conduct sensitive analysis with respect to the risk-aversion parameter γ and find that the supply chain parties' profits are reduced if the retailer's degree of risk aversion is high. The main factor is found to be the ordering quantity, which is decreasing in γ.

We now compare the equilibrium outcomes for the F-contract when $\tau > 0$ and the results for the S-contract when $\tau = 0$. As the following proposition shows, it is not a straightforward comparison because the production cost changes in a complicated way, especially when the transaction is postponed by the retailer to wait for more updated demand information.

Proposition 2:

1. For the equilibrium wholesale price, we have $\mathrm{E}_0^{\mathrm{Q}}[\mathrm{w}_\tau^{\mathrm{F}}] = \mathrm{w}^{\mathrm{S}} + \frac{\mathrm{c}_\tau}{2}$.
2. As for the equilibrium ordering quantity and the manufacturer's expected payoff, we have the following comparison results depending on the relationship between c_1 and $\bar{D} - S_0 - \sqrt{c_2}$, where $c_1 = \frac{\gamma\sum_{i=2}^{n}\sigma_i^2}{2\xi+\gamma\sum_{i=2}^{n+1}\sigma_i^2}(\bar{D} - S_0)$ and $c_2 = \frac{2\xi+\gamma\sigma_{n+1}^2}{2\xi+\gamma\sum_{i=2}^{n+1}\sigma_i^2}(\bar{D} - S_0)^2 - \sum_{i=2}^{n}\sigma_i^2 - Var_0^Q(S_\tau)$.

2.1) If $c_2 < 0$, we have $E_0^Q[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] \geq \Pi_{\mathrm{M}}^{\mathrm{S}}$. As for the ordering quantity, if $c_\tau \leq c_1$, then $E_0^Q[q_\tau^F] \geq q^S$; otherwise, $E_0^Q[q_\tau^F] < q^S$.

2.2) If $\mathrm{c}_2 \geq 0$, the comparison results are as follows.

Case 1: $c_1 \geq \bar{D} - S_0 - \sqrt{c_2}$.

1) If $c_\tau \leq \bar{D} - S_0 - \sqrt{c_2}$, then $\mathrm{E}_0^{\mathrm{Q}}[q_\tau^F] \geq q^S$ and $\mathrm{E}_0^{\mathrm{Q}}[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] \geq \Pi_{\mathrm{M}}^{\mathrm{S}}$.

2) If $\bar{D} - S_0 - \sqrt{c_2} \leq c_\tau \leq c_1$, then $\mathrm{E}_0^{\mathrm{Q}}[q_\tau^F] \geq q^S$ but $\mathrm{E}_0^{\mathrm{Q}}[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] \leq \Pi_{\mathrm{M}}^{\mathrm{S}}$.

3) If $c_\tau \geq c_1$, then $\mathrm{E}_0^{\mathrm{Q}}[q_\tau^F] \leq q^S$ and $\mathrm{E}_0^{\mathrm{Q}}[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] \leq \Pi_{\mathrm{M}}^{\mathrm{S}}$.

Case 2: $c_1 \leq \bar{D} - S_0 - \sqrt{c_2}$.

1) If $c_\tau \leq c_1$, then $\mathrm{E}_0^{\mathrm{Q}}[q_\tau^F] \geq q^S$ and $\mathrm{E}_0^{\mathrm{Q}}[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] \geq \Pi_{\mathrm{M}}^{\mathrm{S}}$.
2) If $c_1 \leq c_\tau \leq \bar{D} - S_0 - \sqrt{c_2}$, then $\mathrm{E}_0^{\mathrm{Q}}[q_\tau^F] \leq q^S$ but $\mathrm{E}_0^{\mathrm{Q}}[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] \geq \Pi_{\mathrm{M}}^{\mathrm{S}}$.
3) If $c_\tau \geq \bar{D} - S_0 - \sqrt{c_2}$, then $\mathrm{E}_0^{\mathrm{Q}}[q_\tau^F] \leq q^S$ and $\mathrm{E}_0^{\mathrm{Q}}\left[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}\right] \leq \Pi_{\mathrm{M}}^{\mathrm{S}}$.

From Proposition 2, three major observations can be made. Firstly, the expected optimal ordering quantity under F-contract is larger than that under S-contract, when the manufacturer's postponed processing cost is lower than a threshold ($c_\tau \leq c_1$). Notice that c_1 is increasing in the reduced variability of the market size. Therefore, given c_τ, the retailer has more opportunities to order more under F-contract when the information updating procedure is effective, i.e., the demand forecast is more accurate. Otherwise, the retailer is expected to order less under F-contract.

Secondly, we compare the manufacturer's expected payoff (the retailer's expected payoff is proportional to the manufacturer's) and find that, if the reduced demand variability is large, i.e. when c_2 is negative, then the manufacturer will earn more under F-contract. When the reduced demand variability is moderate such that $c_2 \geq 0$ and $c_\tau \leq \bar{D} - S_0 - \sqrt{c_2}$, the manufacturer and the retailer are also better off under F-contract. However, if the reduced demand variability is low such that $c_2 \geq 0$ and $c_\tau \geq \bar{D} - S_0 - \sqrt{c_2}$, then the manufacturer and the retailer will be worse off under F-contract. In short, a *win-win* situation can be achieved under F-contract only if the manufacturer's postponed processing cost is lower than a threshold, $\bar{D} - S_0 - \sqrt{c_2}$.

Thirdly, the expected profit of the manufacturer and the expected ordering quantity placed by the retailer under F-contract both decrease in the commodity processing cost. When the cost is sufficiently high, F-contract will be dominated by S-contract. However, the conditions under which these two values fall below that under S-contract are different. This result suggests that the ordering quantity does not play a critical role in the supply chain parties' profit gains. Instead, the profit margin, which is defined as the wholesale price subtracting the commodity processing cost, is the major factor affecting their payoffs. In Case 1, the condition $c_1 \geq \bar{D} - S_0 - \sqrt{c_2}$ is satisfied while in Case 2 the condition $c_1 \leq \bar{D} - S_0 - \sqrt{c_2}$ is satisfied. Specifically, the value of the major parameters in Case 1 and Case 2 is given as follows. For Case 1, $d_1 = 2.5, \sigma = 0.1, \xi = 1, \tau = 9/12$; for Case 2, $d_1 = 1, \sigma = 0.05, \xi = 1, \tau = 9/12$. The remaining parameters are the same as those in the base case in Section 6. As we can see from Figure 1, the profit margin decreases more sharply than the ordering quantity does in both cases.

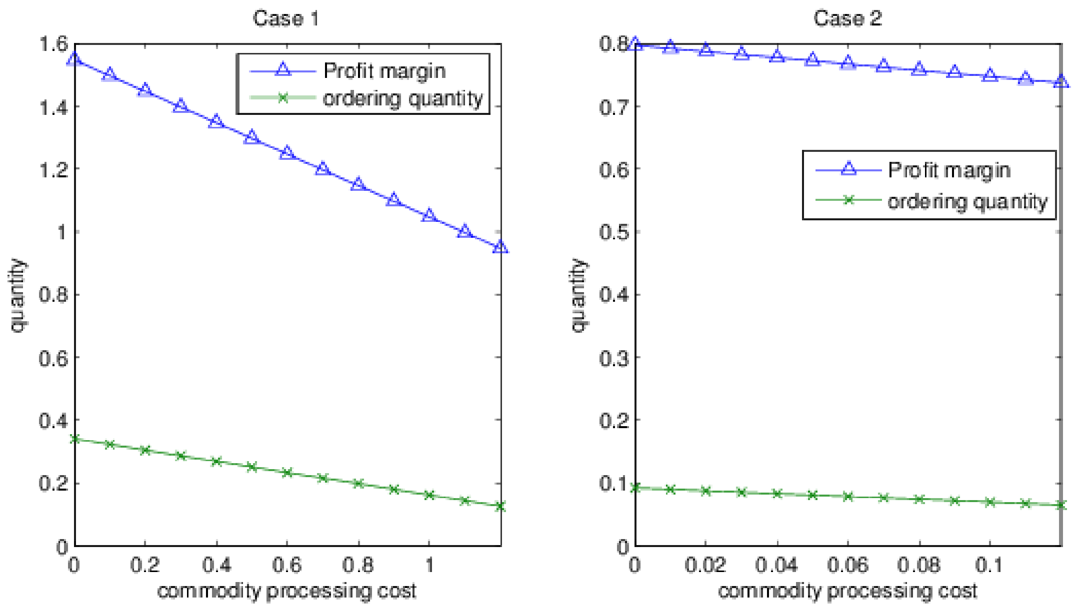

Figure 1: Profit margin and ordering quantity under F-contract

4.2 Supply chain's performance

We next investigate the supply chain's performance. The optimal ordering quantity and the corresponding profit of the centralized supply chain can be obtained by solving the following optimization problem

$$\Pi^{F}_{C|\tau} = \max_{q_\tau \geq 0} E^{Q}_{\tau}[(D - \xi q_\tau - S_\tau - c_\tau) q_\tau]. \quad (9)$$

The optimal ordering quantity and the corresponding system profit are, respectively

$$q^{F}_{C|\tau} = \frac{\overline{D}_\tau - S_\tau - c_\tau}{2\xi}, \qquad \Pi^{F}_{C|\tau} = \frac{(\overline{D}_\tau - S_\tau - c_\tau)^2}{4\xi}. \quad (10)$$

Immediately, we have $\Pi^{\mathrm{F}}_{\mathrm{C}|\tau} \geq \Pi^{\mathrm{S}}_{\mathrm{C}}$ if **a)** $c_2 < 0$, or **b)** $c_2 \geq 0$ and $c_\tau \leq \overline{\mathrm{D}} - \mathrm{S}_0 - \sqrt{c_2}$. The proof of the above result is very similar to the proof of Proposition 2 and is therefore omitted. Two ratios are introduced for the ensuing analysis:

$$\mathcal{Q}^{\mathrm{F}}_\tau = \frac{q^{\mathrm{F}}_\tau}{q^{\mathrm{F}}_{\mathrm{C}|\tau}}, \tag{11}$$

$$\mathcal{P}^{\mathrm{F}}_\tau = 1 - \frac{\Pi^{\mathrm{F}}_{\mathrm{M}|\tau} + \Pi^{\mathrm{F}}_{\mathrm{R}|\tau}}{\Pi^{\mathrm{F}}_{\mathrm{C}|\tau}}. \tag{12}$$

The first ratio is used to measure the ineffectiveness of the decentralized solution from the aspect of production output (Caldentey and Haugh, 2009). The second ratio is similar to the competition penalty in Cachon and Zipkin (1999). However, here the expected utility of the retailer is adopted due to her risk-aversion. Similarly, the overall effectiveness of F-contract is high when $\mathcal{P}^{\mathrm{F}}_\tau$ is small. The following proposition characterizes the two ratios in terms of the risk-aversion parameter γ.

Proposition 3:

(1) $\mathcal{Q}^F_\tau = \frac{\xi}{2\xi+\gamma\sigma^2_{n+1}}$, $\mathcal{Q}^S = \frac{\xi}{2\xi+\gamma\sum_{i=1}^{n+1}\sigma_i^2}$; $\mathcal{P}^F_\tau = 1 - \frac{3\xi}{4\xi+2\gamma\sigma^2_{n+1}}$, $\mathcal{P}^S = 1 - \frac{3\xi}{4\xi+2\gamma\sum_{i=1}^{n+1}\sigma_i^2}$.

(2) $\mathcal{Q}^F_\tau > \mathcal{Q}^S$; $\mathcal{P}^F_\tau < \mathcal{P}^S$.

Proposition 3 is immediate from the definition of the ratios and the optimal solutions we have obtained. From Proposition 3, two observations can be made. Firstly, the efficiency of the F-contract and S-contract decreases as the retailer becomes more risk averse. Secondly, F-contract is more effective than S-contract with respect to both production output and competition penalty. The result holds irrespective of the degree of risk aversion of the retailer.

Although F-contract will lead to higher payoff for both the manufacturer and the retailer under certain conditions, it will also increase the variability of the retailer's profit due to the latter's exposure to the commodity price fluctuation. Fortunately, such kind of financial risk can be controlled using financial instruments such as the commodity futures contract written on the spot price of the input commodity (see e.g., Kouvelis et al., 2013). In practice, it has been observed that it is not uncommon for nonfinancial firms to reduce their risk exposures by using financial derivatives (Bartram et al., 2009). Intuitively, the volatility in the retailer's profit incurred by the volatile commodity price could be partially mitigated by engaging in commodity futures markets. Therefore, the next section will focus on how a commodity futures contract could interplay with the design of flexible supply contract.

5. Flexible Price Contract with Financial Hedging

For simplicity, we assume that there exists a futures contract written on the spot price of input commodity maturing exactly on time τ. F_t is used to denote the price of futures contracts at time t, which matures at time τ. θ_t represents the position of the futures contracts held by the retailer at time t (a long position is represented as $\theta_t < 0$). The time horizon $[0, \tau]$ is divided into n stages, depending on the frequency of rebalancing desired by the retailer (Ni et al., 2012). Note that in this model, $F_n = F_\tau = S_\tau$. The uncertain payoff of the retailer at time τ including the profit earned (or loss incurred) by dynamically trading in the futures market takes the form of

$$\pi_{R|\tau}^{H} = \pi_R + \sum_{t=0}^{n-1} \theta_t (F_{t+1} - F_t), \tag{13}$$

where $\pi_R = (D - \xi q_\tau) q_\tau - w_\tau q_\tau$.

In response to the wholesale price, w_τ, the risk-averse retailer determines the ordering quantity, $q_\tau = q(w_\tau)$, by solving the following optimization problem:

$$\Pi_{\mathrm{R}|\tau}^{\mathrm{F}}(w_\tau) = \max_{q_\tau \geq 0} \{ \mathrm{E}_\tau^{\mathrm{Q}}[\pi_{R|\tau}^{H}] - \frac{\gamma}{2} \mathrm{Var}_\tau^{\mathrm{Q}}(\pi_{R|\tau}^{H}) \}. \tag{14}$$

Given the initial wealth level W_0, the retailer's wealth level at time t can be formulated as:

$$W_t = W_{t-1} + \theta_{t-1}(F_t - F_{t-1}), 0 < t < n = \tau \tag{15}$$

$$W_\tau = W_n = W_{n-1} + \theta_{n-1}(S_\tau - F_{n-1}) + \pi_R \tag{16}$$

The optimal financial hedging policy can be obtained by solving the following problem

$$\max_{\{\theta_i\}_{i=0}^{n-1}} \{\mathrm{E}_0^{\mathrm{Q}}[W_\tau] - \frac{\gamma}{2}\mathrm{Var}_0^{\mathrm{Q}}(W_\tau)\} \tag{17}$$

Following the approach developed in Basak and Chabakauri (2010, 2012), the following proposition provides a time-consistent policy. The proof in the Appendix illustrates how such a policy can be derived.

Proposition 4: Given the spot price S_τ, the retailer's optimal ordering quantity is $q_\tau^H = q_\tau^F = \frac{\bar{D}_\tau - S_\tau - c_\tau}{4\xi + 2\gamma\sigma_{n+1}^2}$ and the manufacturer's optimal wholesale price is $w_\tau^H = w_\tau^F = \frac{\bar{D}_\tau + S_\tau + c_\tau}{2}$. At the last stage, the optimal position in futures contracts held for hedging is given as follows:

$$\theta_{n-1}^* = -\frac{Cov_{n-1}^Q(S_\tau, \pi_R^*)}{Var_{n-1}^Q(S_\tau)} \tag{18}$$

where $\pi_R^* = (D - \xi q_\tau^H)q_\tau^H - w_\tau^H q_\tau^H$

For $0 \leq t < n-1$, the optimal position in the futures contracts can be represented as:

$$\theta_t^* = -\frac{Cov_t^Q\left(F_{t+1}, E_{t+1}^Q[\pi_R^*]\right)}{Var_t^Q(F_{t+1})}. \tag{19}$$

From Proposition 4, three observations can be made. Firstly, it is observed that the optimal wholesale price and ordering quantity in this case is the same as those in the F-contract case. This implies that the manufacturer is not affected by the retailer's financial hedging behavior. In other words, financial hedging will not incur additional complexity to the manufacturer's operational decision making, which is desirable from practical interests. Secondly, the retailer should conduct long hedge, i.e., buy futures contracts, as the value of the optimal positions is negative. This is because the hedgeable risk for the retailer is the payment that she is going to transfer to the manufacturer. Also, these results are largely due to the assumption that the manufacturer is risk neutral and that financial trading is employed to hedge the uncertain payoff of the retailer without any speculative purpose. Thirdly, although it is well-known that mean-variance criterion is not time consistent, we can see that the derived optimal hedging policy is time-consistent in the sense that the optimal decisions in the future are also optimal now. In other words, the derived policy is not static or myopic and considers the optimal actions that are going to be taken in the future. Readers can refer to Basak and Chabakauri (2010, 2012) for a more detailed discussion about time-consistent policy under the mean-variance framework.

5.1 Price modeling of commodity futures

In order to implement the financial hedging policy proposed in Proposition 4, the short-term/long-term model of Schwartz and Smith (2000) is employed to price the futures contracts. However, it is worth noting that the financial hedging policy as well as the other results previously obtained do not depend on any specific commodity price model. Following Schwartz and Smith (2000), the natural logarithm of the spot price at time t is defined as $ln(S_t) = \chi_t + \omega_t$, where χ_t denotes the short-term factor and ω_t denotes the long-term factor. Under the risk-neutral valuation framework, the processes of the two-factor model are of the form:

$$d\chi_t = (-\kappa\chi_t - \lambda_\chi)dt + \sigma_\chi dW_\chi{}^* \tag{20}$$

$$d\omega_t = (\mu_\omega - \lambda_\omega)dt + \sigma_\omega dW_\omega{}^* \tag{21}$$

where $dW_\chi{}^*$ and $dW_\omega{}^*$ are increments of standard Brownian motion under the risk-neutral probability measure with $dW_\chi{}^* dW_\omega{}^* = \rho dt$. The price of the futures contracts written on the spot price of the commodity is:

$$ln(F_t) = ln\left(E_t^Q[S_T]\right) = \chi_t e^{-\kappa(T-t)} + \omega_t + K(T-t) \tag{22}$$

where

$$K(t) = \mu_\omega{}^* t - \frac{\lambda_\chi}{\kappa}(1 - e^{-\kappa t}) + \frac{1}{2}\left(\frac{\sigma_\chi{}^2}{2\kappa}(1 - e^{-2\kappa t}) + \sigma_\omega{}^2 t + 2(1 - e^{-\kappa t})\frac{\rho\sigma_\chi\sigma_\omega}{\kappa}\right).$$

That is, at the beginning of the planning horizon, the possible trend of the futures price can be estimated based on the initial price information. As time progresses, new observations on the prices of the commodities are obtained. Based on the updated information on commodity prices, the estimation of possible futures price trend can be improved. The following lemma gives the information process of stochastic price. These results are required for deriving the exact expression of the optimal position of futures contract at each stage.

Lemma 1:

$E_t^Q[F_{t+1}] = F_t$, $E_t^Q[(F_{t+1})^n] = (F_t)^n(A(t))^{n(n-1)/2}$, $E_t^Q[(S_T)^n] = (F_t)^n B(t)^{n(n-1)/2}$, $Var_t^Q(F_{t+1}) = (F_t)^2(A(t)-1)$, and $Var_t^Q(S_T) = (F_t)^2(B(t)-1)$.

Where

$$A(t) = \exp\left\{\frac{\sigma_\chi{}^2}{2\kappa}\left(e^{-2\kappa(T-t-1)} - e^{-2\kappa(T-t)}\right) + \sigma_\omega{}^2 + 2\left(e^{-2\kappa(T-t-1)} - e^{-2\kappa(T-t)}\right)\frac{\rho\sigma_\chi\sigma_\omega}{\kappa}\right\}$$

$$B(t) = \exp\left\{\frac{\sigma_\chi{}^2}{2\kappa}\left(1 - e^{-2\kappa(T-t)}\right) + \sigma_\omega{}^2(T-t) + 2\left(1 - e^{-\kappa(T-t)}\right)\frac{\rho\sigma_\chi\sigma_\omega}{\kappa}\right\}$$

Based on Lemma 1, the expression of the optimal hedging position can be written as follows.

Proposition 5: *At the beginning of stage t, the optimal amount of futures contract to hold during this stage can be represented as:*

$$\theta_t^* = \frac{\xi + 2\gamma\sigma_{n+1}^2}{(4\xi + 2\gamma\sigma_{n+1}^2)^2}\left(2(d_1 + I_t) - F_t B(t+1)\left(A(t)^2 + A(t)\right)\right), t < n-1 \quad (23)$$

$$\theta_{n-1}^* = \frac{\xi + 2\gamma\sigma_{n+1}^2}{(4\xi + 2\gamma\sigma_{n+1}^2)^2}\left(2(d_1 + I_{n-1}) - F_t\left(B(n-1)^2 + B(n-1)\right)\right). \quad (24)$$

Proposition 5 provides the optimal hedge positions in the futures contract in closed-form expressions. From the expressions, we can see that the hedge quantity can be decomposed into two components which corresponds to the demand uncertainty and the commodity price fluctuation respectively.

6. Numerical Studies

In this section, numerical experiments are conducted to illustrate the benefits of H-contract and compare the performance of different supply contracts. We consider a planning horizon of three months period (or 12 weeks). Let $T = 1$ denote the end of the planning horizon.

The planning horizon is divided into five periods. The five points are time $\frac{\tau}{5}, \frac{2\tau}{5}, \frac{3\tau}{5}, \frac{4\tau}{5}$, and time τ.

The uncertainty of the market size is assumed to be reduced linearly over time, i.e., $\sigma_n^2 = (t_n - t_{n-1})\sigma^2$, where $t_0 = 0, t_n = \frac{n\tau}{5}, n = 1,2,\cdots,5$, and σ^2 is the initial variance of the market size. The residual uncertainty is given by $\sigma_6^2 = (1 - t_5)\sigma^2$. Under this additive MMFE model, the forecast essentially evolves following a Brownian motion.

In the base model, we assume that the initial estimated demand, d_1, is 2.5 and the uncertainty, σ, is 0.1. Without loss of generality, we divide the prices of the commodity and the corresponding futures contract by 100000. By doing so, we can make sure that the ordering quantity under the clearance price model is non-negative with the value of demand and cost being a small number. Another interpretation for this is that we could consider the units of demand and the processing cost should be multiplied by 100000. In addition, without loss of generality, c_τ is set to zero to make sure that the comparison between F-contract and H-contract is conducted under the situation that F-contract dominates S-contract.

The parameters of the stochastic commodity process are estimated following the approach introduced in Schwartz and Smith (2000). The dataset consists of weekly observations of prices for the Shanghai Futures Exchange (SHFE) copper futures contracts maturing in the next month and in approximately 3, 5, 7, and 10 months. The data is collected from 02/26/2010 to 04/12/2013, with 163 weekly observations in total. Table 2 shows the maximum likelihood parameter estimates of the commodity price process based on this dataset.

Table 2 Summary of stochastic price process parameters (copper)

Parameters	**Symbols**	**Value (standard deviation)**
Short-term mean-reversion rate	κ	0.2410 (0.0541)
Short-term volatility	σ_χ	0.2392 (0.0558)
Short-term risk premium	λ_χ	-0.2750 (0.1501)
Long-term volatility	σ_ω	0.3901 (0.0268)
Equilibrium risk-neutral drift rate	μ_ω^*	-0.0515 (0.1060)
Correlation in increments	ρ	-0.8861 (0.0374)

Initial condition		
Short-term factor	χ_0	0.9555
Long-term factor	ω_0	10.0363
Spot price	S_0	59382

We first compare the performance of F-contract and S-contract. The following figure shows the performance with respective to c_τ, the unit commodity processing cost at time τ. Note that Case 1 and Case 2 corresponds to the two cases in Proposition 2.

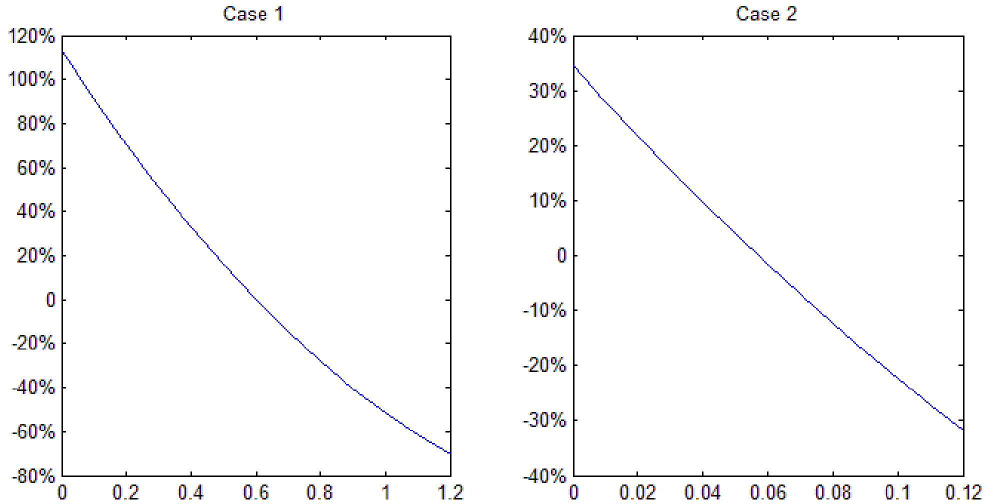

Figure 2: Performance comparison between F-contract and S-contract

In Figure 2, the value of X Axis denotes the commodity processing cost and the value of Y Axis denotes the performance improvement of F-contract compared with S-contract, which is defined as $\frac{\Pi^F_{M|\tau}-\Pi_M}{\Pi_M}$. Therefore, positive value means that F-contract dominates S-contract while negative value means that S-contract performs better than the F-contract. Note that we use the manufacturer's profit only because the results of Proposition 1 show that the retailer's payoff is proportional to the manufacturer's expected profit. From Figure 2, we can see that F-contract dominates S-contract when c_τ, the commodity processing cost, is no larger than a threshold value. Specially, the threshold value in Case 1 is 0.6 while the threshold value in Case 2 is 0.05, which are calculated according to the expression given in Proposition 2. In addition, we can see that in both cases the advantage of the flexibility embedded in F-contract increases significantly when the commodity processing cost approaches to zero. However, the advantage of F-contract will disappear and S-contract will outperform as the commodity processing cost increases. Therefore, the supply chain members should evaluate the operational parameters carefully before deciding whether F-contract should be adopted.

Next, we compare the performance of H-contract and F-contract. To do so, three different measures are employed with respect to the retailer's profits obtained by adopting these two contracts. The results are summarized in Table 3. Before we explain the results in details, we would like to illustrate a bit more how the results are calculated so that better understanding on the implementation of H-contract and F-contract could be achieved. As we generate 5000 samples using the Monte Carlo simulation, we will use the data in the first sample for illustration. At time 0, the manufacturer first announces the wholesale price and then the retailer decides the ordering quantity. Both of them depend on the future spot price and the demand information at the end of the 9th week (the planning horizon is 12 weeks in total), which are given by Equation (6). At the end of the 9th week, the spot price and demand information are updated, according to Equation (6) the realized wholesale price and ordering quantity are 1.6309 and 0.3419, respectively. These are the decisions the supply chain members need to make under F-contract. For the H-contract, the decisions for the wholesale price and ordering quantity are the same as those under F-contract. But the retailer will engage in financial hedging at time 0. Using the data generated in the first sample, the initial quantity of futures contract that the retailer should buy is 0.2113, which is calculated using Equation (24). According to the setting, the retailer will review her hedging position every 9 days (We assume that there are 5 trading days per week and the hedging horizon (9 weeks) is divided into 5 stages). According to Proposition 5, the quantities of futures contract that the retailer holds in the remaining 4 stages are 0.2099, 0.2007, 0.2035, and 0.2187, respectively. At the end of the hedging horizon, i.e., the end of the 9th week, the retailer will close the hedging position with cash transaction and the realized profit from hedging is 0.0164. At the same time, the retailer orders 0.3419 from the manufacturer at the wholesale price 1.6309. The manufacturer's profit is 0.3545 and the retailer's hedged payoff under mean-variance criterion is 0.2268.

Table 3 Performance comparison of H-contract and F-contract

	SD	VaR (5%)	CVaR (5%)
H-Contract	0.0137	0.1869	0.1819
F-Contract	0.0181	0.1796	0.1725

Table 3 shows that H-contract outperforms F-contract with respect to all three measures: the standard deviation (SD), the value-at-risk (VaR) and the conditional value-at-risk (CVaR). As we can see, the SD of the retailer's profit is reduced from 0.0181 to 0.0137 by employing the proposed financial hedging policy. Also, the downside risk measured by VaR and CVaR also increase by 4.1% and 5.4%, respectively. This means that the worst case is improved by applying H-contract. As CVaR can measure the risk beyond VaR, in the ensuing analysis CVaR will be applied as the major measure of the downside risk.

6.1 Sensitivity analysis of the changing demand variance

Following Ni et al. (2012), this analysis is conducted by increasing or decreasing the demand variance by using a changing ratio, i.e.,

$$\sigma_{changed} = \sigma_{initial} \cdot (1 + changing\ ratio).$$

In particular, the changing ratio takes value from the ranges from -80% to 80%. The performance of the H-contract is shown in the following figure.

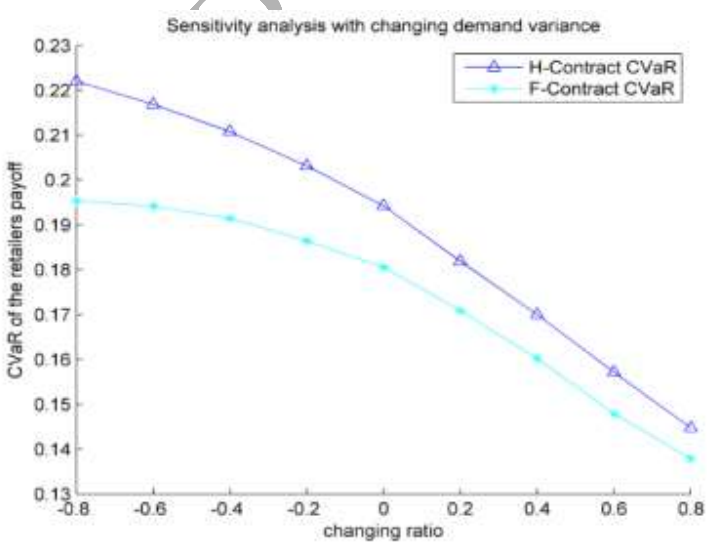

Figure 3 Performance of F-contract and H-contract with changing demand variance

It is observed from Figure 3 that the performance of the contracts is affected by the demand variance. H-contract will be more powerful when the demand variance decreases as the gap between the CVaR under H-contract and F-contract widens slightly. However, the financial hedging policy is still effective when the demand becomes more volatile since the CVaR under H-contract is strictly larger than that under F-contract.

6.2 Sensitivity analysis of changing transaction time

In order to assess the effect of the physical transaction time, we assume that τ takes the values of $\frac{6}{12}, \frac{7}{12}, \frac{8}{12}, \frac{9}{12}$, and $\frac{11}{12}$. The performance of the H-contract is shown in the following figure.

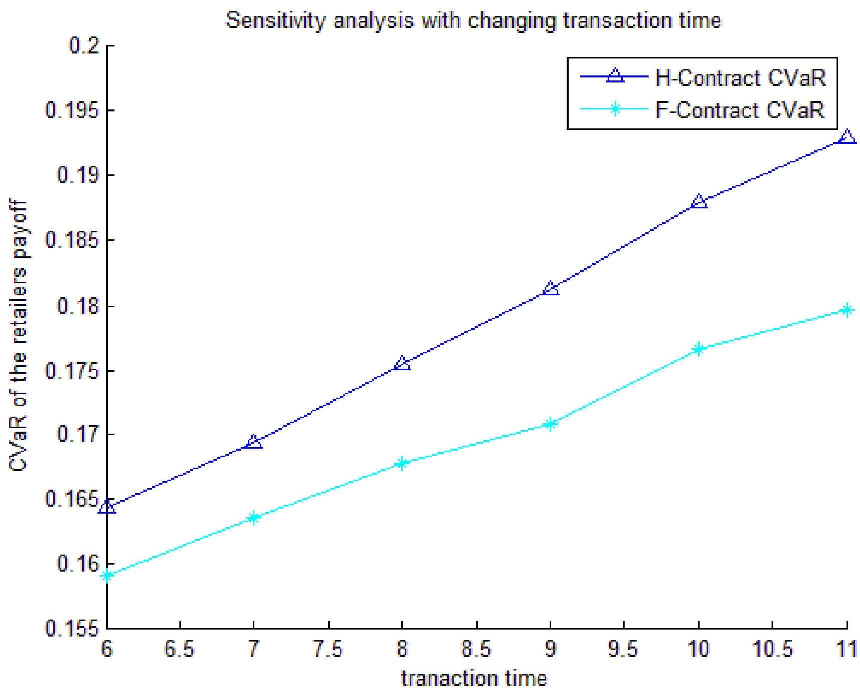

Figure 4 Performance of F-contract and H-contract with changing transaction time

It is observed from Figure 4 that the performance of the contracts will be affected by the transaction time. H-contract will be more powerful when the transaction time delays even more as the gap between the CVaR under H-contract and F-contract widens slightly. However, the financial hedging policy is still effective with relatively short postponement of the physical transaction.

6.3 Comparison between multi-stage hedging policy and one-stage hedging policy

In the implementation of H-contract, a multi-stage hedging policy is employed. However, one-stage hedge could also be adopted to mitigate the commodity price risks. In contrast to multi-stage hedge, one-stage hedge does not require rebalancing of the futures position over the planning horizon. The performances of the two hedging policies are evaluated in this section.

H-contract with one-stage hedge also aims to optimize the same utility function as the multi-stage hedge case. In such a case, the futures position is not rebalanced during the planning horizon. The calculation is similar to the procedure of deriving the multi-stage hedging policy in H-contract. Therefore, the process is omitted here. Under this framework, the optimal futures position is given by:

$$\theta_{os}^{*} = -\frac{\mathrm{Cov}_0^Q(S_\tau, \pi_R^*)}{\mathrm{Var}_0^Q(S_\tau)}. \tag{25}$$

In the following, Monte Carlo simulation is conducted to compare the performance of the multi-stage policy and one-stage policy with respect to CVaR, which is the major measure of downside risk. Specifically, 5000 samples of the stochastic spot and futures prices and the demand information are generated based on the parameters we set at the beginning of this section. The results are shown in the following three figures.

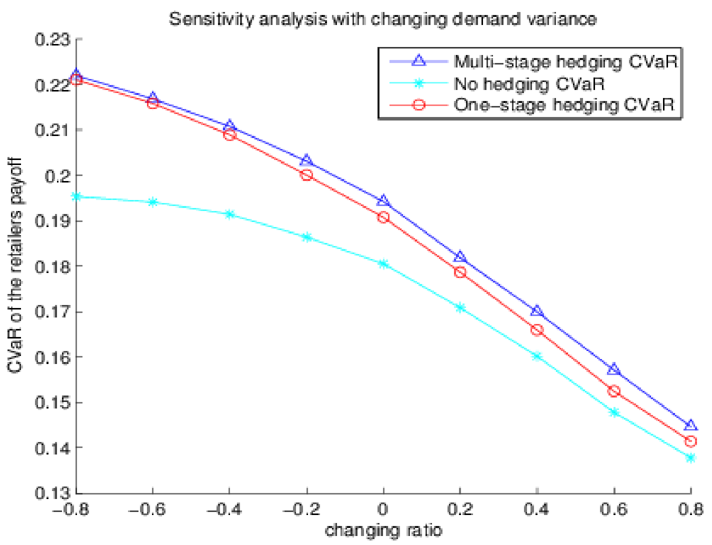

Figure 5 Performance of multi-stage and one-stage hedging with changing demand variance

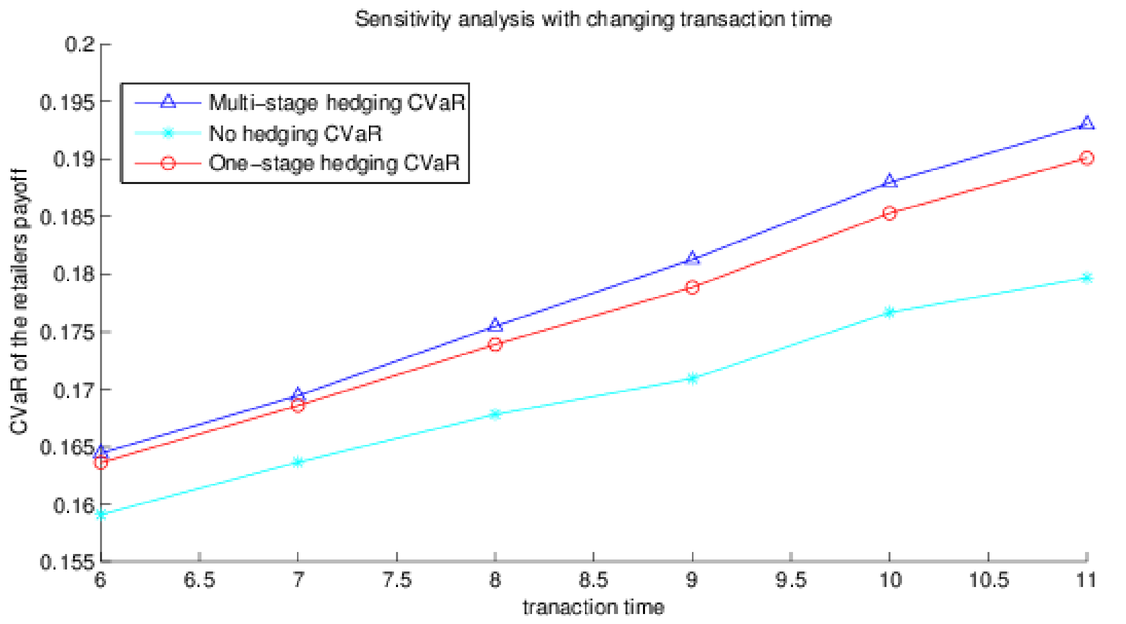

Figure 6 Performance of multi-stage and one-stage hedging with changing transaction time

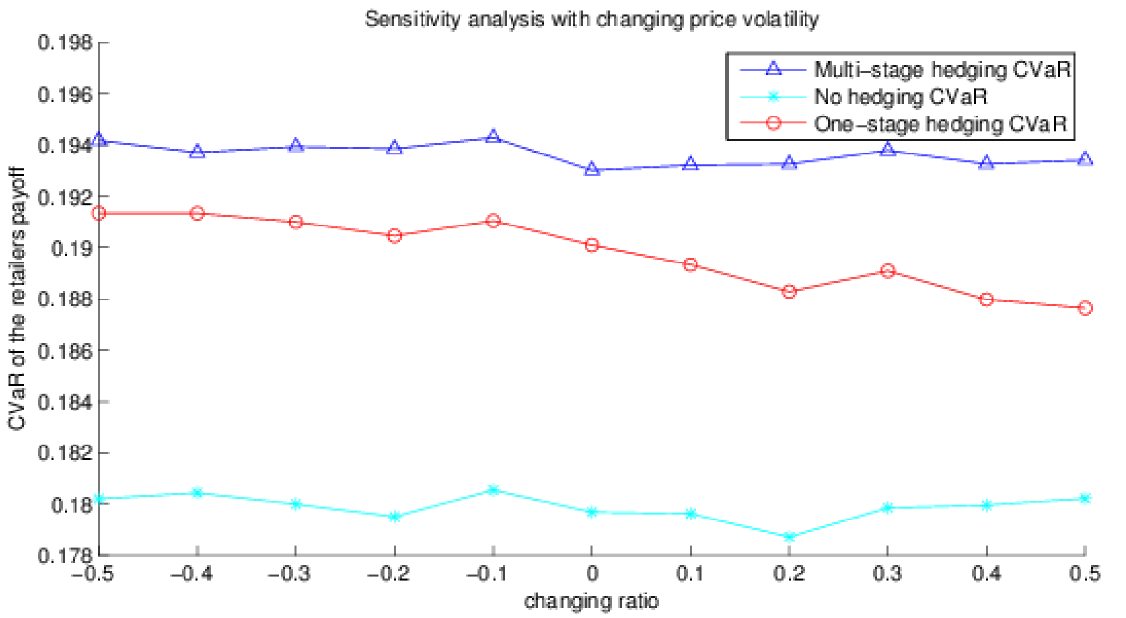

Figure 7 Performance of multi-stage and one-stage hedging with changing price volatility

As can be seen in Figures 5 to 7, the CVaR under H-contract with multi-stage hedge is strictly larger than that under H-contract with one-stage hedge indicates that the former is more powerful in commodity price risk hedging than the latter. Also, H-contract with one-stage hedge significantly outperforms F-contract, in which no hedge is employed, as the gap between the CVaR under H-contract with one-stage hedge and F-contract is quite wide. Figure 5 shows that H-contract with multi-stage hedge outperforms the H-contract with one-stage hedge when the demand becomes more uncertain. Meanwhile, financial hedging becomes less powerful as the gap between the hedged and unhedged CVaR decreases. Figure 6 shows that the H-contract with multi-stage hedge is more powerful in risk hedging than the H-contract with one-stage hedge when the transaction time has a larger delay. Meanwhile, financial hedging becomes more powerful as the gap between the hedged and unhedged CVaR increases. Figure 7 shows that the performance of the H-contract with multi-stage hedge is not affected significantly when the price volatility varies within the range of -50% to 50%. However, the H-contract with one-stage hedge becomes less effective when the price becomes more volatile.

7. Conclusion Remarks

This paper considers a form of "flexible supply contract", which is widely used in supply chains with consumption commodities as the main inputs. Specifically, we consider the management of input commodity price fluctuation in a supply chain context in which the upstream manufacturer procures commodities from the spot market. We also investigate the value of demand forecast update during the lead time between contract signing and the manufacturer's at-once production of the products. Our analysis shows that, for both the manufacturer and the retailer, flexible supply contract (F-contract) outperforms the traditional simple wholesale price contract (S-contract) when the manufacturer's postponed processing cost is lower than a threshold. That is, a win-win situation can be achieved under F-contract.

It is worth noting that, although F-contract can lead to higher payoff for both players, the commodity price volatility is also transmitted to the downstream risk-averse retailer who does not face such variability under the traditional wholesale price contract. In order to mitigate the risk arising from commodity price fluctuation, H-contract is proposed under which commodity futures contracts are traded dynamically. The numerical study shows that the risk-averse retailer's performance can be improved with hedging, with three major risk measures serving as metrics. Lastly, a comparative study is conducted between H-contract with multi-stage hedge and that with one-stage hedge. We observe that H-contract with multi-stage hedge is more powerful in commodity price risk hedging.

There are three possible directions along which this research can be extended. In this paper, the commodity price risks are born either by the upstream manufacturer through S-contract or the downstream retailer through F-contract or H-contract. Thus, a potentially useful extension is to consider a risk-sharing mechanism between the manufacturer and the retailer. However, it might be challenging to derive the strategy as the optimal strategy might then be a combination of both risk-sharing and risk-hedging. The second extension is to consider a supply chain where both the manufacturer and the retailer are risk averse. It is expected that model solvability would be a big problem. The last possible extension is to consider a "closed" input commodity spot market, in which powerful supplier(s) can determine the spot price by controlling the supply of the commodity in the spot market. In this case, it would be more appropriate to consider a three-stage supply chain in which the commodity supplier is explicitly considered. We shall leave these issues for future research.

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Appendix A:

Proof of Proposition 1:

Given the wholesale price w_τ, the retailer's optimal response is

$$q_\tau^F = \frac{\bar{D}_\tau - w_\tau}{2\xi + \gamma \mathrm{Var}_\tau^{\mathrm{Q}}(\mathrm{D})} \tag{A1}$$

Given the retailer's optimal response, the best wholesale price, w_τ^F, can be obtained by solving the manufacturer's problem. Substituting w_τ^F and q_τ^F into the payoff function of the manufacturer and the retailer, the equilibrium expected payoffs can be obtained. □

Proof of Proposition 2:

Following the procedure in the F-contract case, the equilibrium optimal decisions under the S-contract can be readily obtained:

$$w^S = \frac{\bar{D}+S_0}{2}, q^S = \frac{\bar{D}-S_0}{4\xi+2\gamma\mathrm{Var}_0^{\mathrm{Q}}(D)}, \Pi_{\mathrm{M}}^{\mathrm{S}} = \frac{(\bar{D}-S_0)^2}{8\xi+4\gamma\mathrm{Var}_0^{\mathrm{Q}}(D)}, \Pi_{\mathrm{R}}^{\mathrm{S}} = \frac{(\bar{D}-S_0)^2}{16\xi+8\gamma\mathrm{Var}_0^{\mathrm{Q}}(D)}.$$

Recall that the following conditional variances are known: $\mathrm{Var}_0^{\mathrm{Q}}(D) = \sum_{i=1}^{n+1} \sigma_i^2$ and $\mathrm{Var}_\tau^{\mathrm{Q}}(D) = \sigma_{n+1}^2$.

a) As for the wholesale price, the comparison can be obtained immediately as follows:

$$\mathrm{E}_0^{\mathrm{Q}}[w_\tau^F] = \mathrm{E}_0^{\mathrm{Q}}\left[\frac{\bar{D}_\tau + S_\tau + c_\tau}{2}\right] = w^S + \frac{c_\tau}{2}. \tag{A2}$$

b) We now compare the expected ordering quantity under the F-contract and the ordering quantity under the S-contract. Following the results of Proposition 1, we have

$$\mathrm{E}_0^{\mathrm{Q}}[q_\tau^F] - q^S = \mathrm{E}_0^{\mathrm{Q}}\left[\frac{\bar{D}_\tau - S_\tau - c_\tau}{4\xi + 2\gamma\sigma_{n+1}^2}\right] - \frac{\bar{D} - S_0}{4\xi + 2\gamma\sum_{i=2}^{n+1}\sigma_i^2}$$

$$= \frac{\bar{D} - S_0 - c_\tau}{4\xi + 2\gamma\sigma_{n+1}^2} - \frac{\bar{D} - S_0}{4\xi + 2\gamma\sum_{i=2}^{n+1}\sigma_i^2}$$

$$= \frac{2\gamma\sum_{i=2}^{n}\sigma_i^2(\bar{D} - S_0) - (4\xi + 2\gamma\sum_{i=2}^{n+1}\sigma_i^2)c_\tau}{(4\xi + 2\gamma\sigma_{n+1}^2)(4\xi + 2\gamma\sum_{i=2}^{n+1}\sigma_i^2)} \qquad (A3)$$

From (A3), we can see that $\mathrm{E}_0^{\mathrm{Q}}[q_\tau^F] \geq q^S$ if $c_\tau \leq c_1$ where $c_1 = \frac{\gamma\sum_{i=2}^{n}\sigma_i^2}{2\xi+\gamma\sum_{i=2}^{n+1}\sigma_i^2}(\bar{D} - S_0)$. Otherwise, we have $\mathrm{E}_0^{\mathrm{Q}}[q_\tau^F] < q^S$.

c) As the retailer's equilibrium expected payoff is proportional to the manufacturer's equilibrium expected payoff, the comparison will be conducted with respect to the manufacturer only. Again from Proposition 1, we have

$$\mathrm{E}_0^{\mathrm{Q}}[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] - \Pi_{\mathrm{M}}^{\mathrm{S}} = \mathrm{E}_0^{\mathrm{Q}}\left[\frac{(\bar{D}_\tau - S_\tau - c_\tau)^2}{8\xi + 4\gamma\sigma_{n+1}^2}\right] - \frac{(\bar{D} - S_0)^2}{8\xi + 4\gamma\sum_{i=2}^{n+1}\sigma_i^2}$$

$$= \frac{\mathrm{E}_0^{\mathrm{Q}}[(\bar{D}_\tau - S_\tau - c_\tau)^2]}{8\xi + 4\gamma\sigma_{n+1}^2} - \frac{(\bar{D} - S_0)^2}{8\xi + 4\gamma\sum_{i=2}^{n+1}\sigma_i^2} \qquad (A4)$$

Before conducting the comparison, the following result is introduced for the proof.

$$\mathrm{E}_0^{\mathrm{Q}}[(\bar{D}_\tau - S_\tau - c_\tau)^2] = \mathrm{E}_0^{\mathrm{Q}}[(\bar{D}_\tau - S_\tau)^2 - 2c_\tau(\bar{D}_\tau - S_\tau) + c_\tau^2]$$

$$= \mathrm{E}_0^{\mathrm{Q}}[(\bar{D}_\tau - S_\tau)^2] - 2c_\tau(\bar{D} - S_0) + c_\tau^2$$

$$= (\bar{D} - S_0)^2 - 2c_\tau(\bar{D} - S_0) + c_\tau^2 + \sum_{i=2}^{n}\sigma_i^2 + \mathrm{Var}_0^{\mathrm{Q}}(S_\tau) \qquad (A5)$$

The sign of expression (A4) is thus the same as the following expression

$$\Delta_{\Pi_M} = \mathrm{E}_0^{\mathrm{Q}}[(\bar{D}_\tau - S_\tau - c_\tau)^2] - \frac{2\xi + \gamma\sigma_{n+1}^2}{2\xi + \gamma\sum_{i=2}^{n+1}\sigma_i^2}(\bar{D} - S_0)^2$$

$$= \frac{\gamma \sum_{i=2}^{n} \sigma_i^2}{2\xi + \gamma \sum_{i=2}^{n+1} \sigma_i^2} \cdot (\bar{D} - S_0)^2 - \left[c_\tau^2 - 2c_\tau(\bar{D} - S_0) + \sum_{i=2}^{n} \sigma_i^2 + \mathrm{Var}_0^{\mathrm{Q}}(S_\tau) \right]$$

$$= \frac{2\xi + \gamma \sigma_{n+1}^2}{2\xi + \gamma \sum_{i=2}^{n+1} \sigma_i^2} \cdot (\bar{D} - S_0)^2 - \sum_{i=2}^{n} \sigma_i^2 - \mathrm{Var}_0^{\mathrm{Q}}(S_\tau) - (\bar{D} - S_0 - c_\tau)^2 \qquad (A6)$$

Let $c_2 = \frac{8\xi + 4\gamma \sigma_{n+1}^2}{8\xi + 4\gamma \sum_{i=2}^{n+1} \sigma_i^2} \cdot (\bar{D} - S_0)^2 - \sum_{i=2}^{n} \sigma_i^2 - \mathrm{Var}_0^{\mathrm{Q}}(S_\tau)$. If $c_2 < 0$ then immediately we have $\mathrm{E}_0^{\mathrm{Q}}[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] < \Pi_{\mathrm{M}}^{\mathrm{S}}$ as $\Delta_{\Pi_M} < 0$. Otherwise, the relationship depends on the value of c_τ. From expression (A6), the following results are straightforward. If $c_\tau \leq \bar{D} - S_0 - \sqrt{c_2}$, then $\mathrm{E}_0^{\mathrm{Q}}[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] \geq \Pi_{\mathrm{M}}^{\mathrm{S}}$; if $c_\tau > \bar{D} - S_0 - \sqrt{c_2}$, then $\mathrm{E}_0^{\mathrm{Q}}[\Pi_{\mathrm{M}|\tau}^{\mathrm{F}}] < \Pi_{\mathrm{M}}^{\mathrm{S}}$.

The proof can be finished by comparing the $\bar{D} - S_0 - \sqrt{c_2}$ with c_1. The procedure is quite straightforward and thus is omitted. □

Proof of Proposition 5:

First, let us define an auxiliary function $\mathrm{U}_t = \mathrm{E}_t^{\mathrm{Q}}[W_\tau] - \frac{\gamma}{2} \mathrm{Var}_t^{\mathrm{Q}}(W_\tau)$. Following the law of total variance and the law of total expectation, U_t can be rewritten as

$$\mathrm{U}_t = \mathrm{E}_t^{\mathrm{Q}}[W_\tau] - \frac{\gamma}{2} \{ \mathrm{E}_t^{\mathrm{Q}}[\mathrm{Var}_{t+1}^{\mathrm{Q}}(W_\tau)] + \mathrm{Var}_t^{\mathrm{Q}}(\mathrm{E}_{t+1}^{\mathrm{Q}}[W_\tau]) \}$$

$$= \mathrm{E}_t^{\mathrm{Q}} \left[\mathrm{E}_{t+1}^{\mathrm{Q}}[W_\tau] - \frac{\gamma}{2} \mathrm{Var}_{t+1}^{\mathrm{Q}}(W_\tau) \right] - \frac{\gamma}{2} \mathrm{Var}_t^{\mathrm{Q}}(\mathrm{E}_{t+1}^{\mathrm{Q}}[W_\tau])$$

$$= \mathrm{E}_t^{\mathrm{Q}}[\mathrm{U}_{t+1}] - \frac{\gamma}{2} \mathrm{Var}_t^{\mathrm{Q}}(\mathrm{E}_{t+1}^{\mathrm{Q}}[W_\tau]) \qquad (A6)$$

Given the optimal time-consistent hedging policy $\{\theta_i^*\}_{i=t}^{n-1}$, the value function of the retailer can be defined as:

$$\mathrm{J}_t = \mathrm{E}_t^{\mathrm{Q}}[W_\tau^*] - \frac{\gamma}{2} \mathrm{Var}_t^{\mathrm{Q}}(W_\tau^*) \qquad (A7)$$

where the terminal wealth W_τ^* is calculated under the optimal trading policy $\{\theta_i^*\}_{i=t}^{n-1}$ and the ordering quantity q_τ^H. From the recursive definition of U_t and the definition of J_{t+1}, the optimal trading policy, θ_t^*, can be obtained by solving the following problem:

$$\max_{\theta_t}\left\{\mathrm{E}_t^{\mathrm{Q}}[\mathrm{J}_{t+1}]-\frac{\gamma}{2}\mathrm{Var}_t^{\mathrm{Q}}\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[W_\tau]\right)\right\} \tag{A8}$$

Define $f_t=\mathrm{E}_t^{\mathrm{Q}}[W_\tau^*]-W_t=\mathrm{E}_t^{\mathrm{Q}}[\pi_R^*]$. The following recursive equation for J can be obtained

$$\mathrm{J}_t=\max_{\theta_t}\{\mathrm{E}_t^{\mathrm{Q}}[\mathrm{J}_{t+1}]-\frac{\gamma}{2}\mathrm{Var}_t^{\mathrm{Q}}(W_{t+1}+f_{t+1})\}$$

$$=\max_{\theta_t}\{\mathrm{E}_t^{\mathrm{Q}}[\mathrm{J}_{t+1}]-\frac{\gamma}{2}\mathrm{Var}_t^{\mathrm{Q}}(W_{t+1}-W_t+f_{t+1})\}$$

$$=\max_{\theta_t}\{\mathrm{E}_t^{\mathrm{Q}}[\mathrm{J}_{t+1}]-\frac{\gamma}{2}\mathrm{Var}_t^{\mathrm{Q}}\left(\theta_t(F_{t+1}-F_t)+\mathrm{E}_{t+1}^{\mathrm{Q}}[\pi_R^*]\right)\}$$

$$=\max_{\theta_t}\{\mathrm{E}_t^{\mathrm{Q}}[\mathrm{J}_{t+1}]-\frac{\gamma}{2}\{{\theta_t}^2\mathrm{Var}_t^{\mathrm{Q}}(F_{t+1})+\mathrm{Var}_t^{\mathrm{Q}}\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[\pi_R^*]\right)+2\theta_t\mathrm{Cov}_t^{\mathrm{Q}}\left(F_{t+1},\mathrm{E}_{t+1}^{\mathrm{Q}}[\pi_R^*]\right)\}$$

(A9)

It is clear that there exists a unique solution solving the above function. The solution is:

$$\theta_t^*=-\frac{\mathrm{Cov}_t^{Q}\left(F_{t+1},\mathrm{E}_{t+1}^{\mathrm{Q}}[\pi_R^*]\right)}{\mathrm{Var}_t^{Q}(F_{t+1})}. \tag{A10}$$

Specifically, when $t=n-1$, $F_{t+1}=F_n=S_\tau$ and $\mathrm{E}_{t+1}^{\mathrm{Q}}[\pi_R^*]=(\overline{D}_\tau-\xi q_\tau^H)q_\tau^H-w_\tau^H q_\tau^H$.

□

Proof of Lemma 1:

$$\mathrm{E}_t^{\mathrm{Q}}[F_{t+1}]=\mathrm{E}_t^{\mathrm{Q}}\left[\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right]=\mathrm{E}_t^{\mathrm{Q}}[\exp\{\ln(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T])\}]$$

$$=\exp\left\{\mathrm{E}_t^{\mathrm{Q}}\left[\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right]+\frac{1}{2}\mathrm{Var}_t^{Q}\left(\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right)\right\} \tag{A11}$$

$$\mathrm{E}_t^{\mathrm{Q}}\left[\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right]=e^{-\kappa(\mathrm{T}-\mathrm{t}-1)}\mathrm{E}_t^{\mathrm{Q}}[\chi_{t+1}]+\mathrm{E}_t^{\mathrm{Q}}[\omega_{t+1}]+K(T-t-1)$$

$$= e^{-\kappa(T-t-1)}\left(\chi_t e^{-\kappa} - (1-e^{-\kappa})\frac{\lambda_\chi}{\kappa}\right) + \omega_t + \mu_\omega{}^* + K(T-t-1)$$

$$= \chi_t e^{-\kappa(T-t)} + \omega_t + \mu_\omega{}^*(T-t) - \frac{\lambda_\chi}{\kappa}\left(1-e^{-\kappa(T-t)}\right)$$

$$+\frac{1}{2}\left(\frac{\sigma_\chi{}^2}{2\kappa}\left(1-e^{-2\kappa(T-t-1)}\right)\right.$$

$$\left.+\sigma_\omega{}^2(T-t-1) + 2\left(1-e^{-\kappa(T-t-1)}\right)\frac{\rho\sigma_\chi\sigma_\omega}{\kappa}\right) \qquad (A12)$$

$$\mathrm{Var}_t^Q\left(\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right)$$

$$= e^{-2\kappa(T-t-1)}\mathrm{Var}_t^{\mathrm{Q}}[\chi_{t+1}] + \mathrm{Var}_t^{\mathrm{Q}}[\omega_{t+1}] + 2e^{-\kappa(T-t-1)}\mathrm{Cov}_t^{\mathrm{Q}}[\chi_{t+1},\omega_{t+1}]$$

$$= \frac{\sigma_\chi{}^2}{2\kappa}\left(e^{-2\kappa(T-t-1)} - e^{-2\kappa(T-t)}\right) + \sigma_\omega{}^2 + 2\left(e^{-2\kappa(T-t-1)} - e^{-2\kappa(T-t)}\right)\frac{\rho\sigma_\chi\sigma_\omega}{\kappa}$$

$$(A13)$$

$$\mathrm{E}_t^{\mathrm{Q}}[F_{t+1}] = \exp\left\{\mathrm{E}_t^{\mathrm{Q}}\left[\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right] + \frac{1}{2}\mathrm{Var}_t^Q\left(\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right)\right\}$$

$$= \exp\{\chi_t e^{-\kappa(T-t)} + \omega_t + \mu_\omega{}^*(T-t) - \frac{\lambda_\chi}{\kappa}\left(1-e^{-\kappa(T-t)}\right)$$

$$+\frac{1}{2}\left(\frac{\sigma_\chi{}^2}{2\kappa}\left(1-e^{-2\kappa(T-t)}\right)\right.$$

$$\left.+\sigma_\omega{}^2(T-t) + 2\left(1-e^{-\kappa(T-t)}\right)\frac{\rho\sigma_\chi\sigma_\omega}{\kappa}\right\}$$

$$= \mathrm{E}_t^{\mathrm{Q}}[S_T] = F_t \qquad (A14)$$

$$\mathrm{E}_t^{\mathrm{Q}}[(F_{t+1})^n] = \mathrm{E}_t^{\mathrm{Q}}\left[(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T])^n\right] = \mathrm{E}_t^{\mathrm{Q}}[\exp\{n\cdot\ln(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T])\}]$$

$$= \exp\left\{n\cdot\mathrm{E}_t^{\mathrm{Q}}\left[\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right] + \frac{n^2}{2}\mathrm{Var}_t^Q\left(\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right)\right\}$$

$$= \exp\left\{n\cdot\left(\mathrm{E}_t^{\mathrm{Q}}\left[\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right] + \frac{1}{2}\mathrm{Var}_t^Q\left(\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right)\right)\right.$$

$$\left.+\frac{n(n-1)}{2}\mathrm{Var}_t^Q\left(\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right)\right\}$$

$$= (F_t)^n \exp\left\{\frac{n(n-1)}{2} \mathrm{Var}_t^Q \left(\ln\left(\mathrm{E}_{t+1}^{\mathrm{Q}}[S_T]\right)\right)\right\}$$

$$= (F_t)^n (A(t))^{\frac{n(n-1)}{2}} \qquad (A15)$$

$$\mathrm{E}_t^{\mathrm{Q}}[(S_T)^n] = \mathrm{E}_t^{\mathrm{Q}}[\exp\{n \cdot \ln(S_T)\}]$$

$$= \exp\left\{n \cdot \mathrm{E}_t^{\mathrm{Q}}[\ln(S_T)] + \frac{n^2}{2} \mathrm{Var}_t^Q(\ln(S_T))\right\}$$

$$= \exp\left\{n \cdot (\mathrm{E}_t^{\mathrm{Q}}[\ln(S_T)] + \frac{1}{2}\mathrm{Var}_t^Q(\ln(S_T))) + \frac{n(n-1)}{2} \mathrm{Var}_t^Q(\ln(S_T))\right\}$$

$$= (F_t)^n B(t)^{\frac{n(n-1)}{2}} \qquad (A16)$$

Given the above results, the conditional variance of the spot price and futures price can be readily derived following the definition of variance. □

Proof of Proposition 6:

Following lemma 8, the following results can be obtained as follows:

$$\mathrm{E}_{t+1}^{\mathrm{Q}}[\pi_R^*] = (\overline{D}_\tau - \xi q_\tau^H) q_\tau^H - w_\tau^H q_\tau^H \qquad (A17)$$

$$\mathrm{E}_{t+1}^{\mathrm{Q}}[\pi_R^*] = \mathrm{E}_{t+1}^{\mathrm{Q}}\left[\left(\overline{D}_\tau - \xi \frac{\overline{D}_\tau - s_\tau}{4\xi + 2\gamma\sigma_{n+1}^2}\right)\frac{\overline{D}_\tau - s_\tau}{4\xi + 2\gamma\sigma_{n+1}^2} - \frac{\overline{D}_\tau + s_\tau}{2}\frac{\overline{D}_\tau - s_\tau}{4\xi + 2\gamma\sigma_{n+1}^2}\right]$$

$$= \frac{\xi + 2\gamma\sigma_{n+1}^2}{(4\xi + 2\gamma\sigma_{n+1}^2)^2} \mathrm{E}_{t+1}^{\mathrm{Q}}[(\overline{D}_\tau - s_\tau)^2]$$

$$= \frac{\xi + 2\gamma\sigma_{n+1}^2}{(4\xi + 2\gamma\sigma_{n+1}^2)^2} (\mathrm{E}_{t+1}^{\mathrm{Q}}[(S_T)^2] - 2\mathrm{E}_{t+1}^{\mathrm{Q}}[\overline{D}_\tau]F_{t+1} + \mathrm{E}_{t+1}^{\mathrm{Q}}[(\overline{D}_\tau)^2]) \qquad (A18)$$

$$\mathrm{Cov}_t^Q\left(F_{t+1}, \mathrm{E}_{t+1}^{\mathrm{Q}}[\pi_R^*]\right)$$

$$= \frac{\xi + 2\gamma\sigma_{n+1}^2}{(4\xi + 2\gamma\sigma_{n+1}^2)^2} \mathrm{Cov}_t^Q(F_{t+1}, \mathrm{E}_{t+1}^{\mathrm{Q}}\left[(S_T)^2\right] - 2\mathrm{E}_{t+1}^{\mathrm{Q}}[\overline{D}_\tau]F_{t+1})$$

$$= \frac{\xi + 2\gamma\sigma_{n+1}^2}{(4\xi + 2\gamma\sigma_{n+1}^2)^2} \left(-2\mathrm{E}_{t+1}^{\mathrm{Q}}[\overline{D}_\tau]\mathrm{Var}_t^Q(F_{t+1}) + \mathrm{Cov}_t^Q(F_{t+1}, \mathrm{E}_{t+1}^{\mathrm{Q}}[(S_T)^2])\right)$$

(A19)

$$\mathrm{Cov}_t^Q(F_{t+1}, \mathrm{E}_{t+1}^{\mathrm{Q}}[(s_\tau)^2]) = \mathrm{Cov}_t^Q(F_{t+1}, (F_{t+1})^2 B(t+1))$$

$$= B(t+1)\mathrm{Cov}_t^Q(F_{t+1}, (F_{t+1})^2)$$

$$= B(t+1)(\mathrm{E}_t^{\mathrm{Q}}[(F_{t+1})^3] - F_t \mathrm{E}_t^{\mathrm{Q}}[(F_{t+1})^2])$$

$$= B(t+1)\left(\left(A(t)\right)^2 + A(t)\right)(F_t)^3(A(t)-1) \qquad (A20)$$

Substituting the above results into Proposition 7, the proof can be completed. □