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Research Article

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The Shapiro-Stiglitz Model with Non-constant Marginal Utility

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Abstract: The Shapiro-Stiglitz model plays an important role in the employment theory. Woodford pointed out the theoretic limitation of the linear worker's utility function in that model. He questioned the model's implication of the secular decline in the unemployment rate when such rate was in fact trendless. He proposed to resolve this by allowing diminishing marginal utility of income. In this paper, the Shapiro-Stiglitz model is generalized using a nonlinear utility function implicit in the Stiglitz Efficiency-wage paper, thus linking these two well-known models. The nonlinear utility function in this generalized model not only allows for diminishing marginal utility of income but also allows for the analysis of parameters representing various factors affecting the secular unemployment rate. In particular, we can specify the condition under which the diminishing marginal utility can cause such rate to be trendless.

JEL Classifications: E24; J31; J64
Keywords: Macroeconomics, Efficiency-Wage, S-shaped Effort Supply, Shirking model.

1 Introduction

Efficiency-wage theory asserts that for various reasons, firms pay above market-clearing wages and thus cause unemployment. The static textbook model in this field is based on Stiglitz [1973, 1976] Efficiency Wage (SEW) papers, wherein labor supply and demand are replaced by effort supply and demand. The effort demand curve is the argmax of the profit function of a representative firm. The effort supply function is assumed to be given and S-shaped to guarantee the existence of an equilibrium. This supply function should be the argmax of a worker's utility function, yet this underlying utility function was almost never made explicit in the literature.[1]

Shapiro & Stiglitz [1984] (S-S) introduced the notion that it is costly for a firm to monitor its workers. The firm will therefore pay an efficiency wage to prevent shirking. The S-S model is in a dynamic optimization setting and has also become standard in macroeconomics textbooks. However, instead of a nonlinear worker's utility function implicit in SEW, Shapiro & Stiglitz employed a more tractable utility function in the form of $U^*(w, \lambda) = w - \lambda$ with w as wage and λ as effort. This $U^*(w, \lambda)$ has constant marginal utility (MU) with respect to income. Woodford [1994] noted that such a functional form would preclude the effect of household wealth in wage determination and constitute an important theoretic limitation. In addition, Woodford maintained that if the S-S model were placed in a growth setting, increases in labor productivity over time would shift labor demand upwards and result in a secular decline in the unemployment rate.

1 See Wu and Ho (2012).

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However, such secular rate had been trendless. Woodford proposed using diminishing MU of income to explain why workers' reservation wages needed to rise with wealth over time. This would cause a rise in the no-shirking efficiency wage and increase unemployment, thus explaining the trendless rate. We are curious to see if the SEW model utility function can be used to provide an analytical platform for Woodford's conjecture.

2 Literature Review

In unemployment models, incomplete and asymmetric information plays a major role.

From the incomplete information perspective, Stigler (1961) introduced the notion of an economic agent searching for the most favorable price. Different versions of labor market search models were introduced by Stigler (1962)[2], McCall (1970)[3], Mortensen (1970)[4] and Pissarides (1976)[5]. This search feature was integrated into a dynamic framework to explain both the efficiency-wage and unemployment over time. For example, Salop's (1979) labor turnover model asserted that firms set higher wages to avoid turnover costs.

From the asymmetric information perspective, Shapiro & Stiglitz (1984) recognized that it is costly for the firm to determine and monitor workers' effort. This motivates the notion of using an efficiency-wage to avoid shirking. As we mentioned, Shapiro & Stiglitz's used the utility function $U^*(w, \lambda) = w - \lambda$. Subsequent studies in the literature have generally adopted this approach. Based on this utility function, Strand (1987) examined the effects of heterogeneous workers on the labor market; Oi (1990) modeled unemployment in a dual labor market; Strand (1991) investigated how firm-specific shocks and moral hazard affect unemployment and wages; Meier (2002), Holzner *et al.*(2010) examined the effect of work requirements for welfare recipients on the wage rate; Boeria and Jimeno (2005) analyzed employment protections in the labor market; Ose (2005) explored the relationship among working conditions, compensation and absenteeism. Skott (2006) examined over-education and wage inequality; Sparks (1986) highlighted the role of imperfect monitoring and dismissal threats in underpinning real wage rigidity; and Zenou & Smith (1995) and Zenou (2006) studied the worker's decision to live in central business district or urban-fringe; Phelps (1994) and Brecher *et al.* (2010) generalized this model by incorporating asset accumulation and modifying the no-shirking condition.

Other articles based on constant marginal utility include Altenburg & Brenken (2008), who studied equilibrium unemployment monopolistic competition where workers are paid efficiency wages. Aloi & Lloyd-Braga (2010) incorporated constant marginal labor disutility in a two-country general equilibrium model to study the relationship between international factor mobility and macroeconomic instability. Similar utility functions were also employed in Moen & Rosén (2006), Fuchs (2007) and Yang (2008) when studying optimal wage contracts, and in Stähler (2008) when studying judicial mistakes in the labor market.

This worker's utility function was slightly modified in some other papers. Hendricks & Kahn (1991) used the form $U^{**} = w - \lambda^2$ to consider wage determination in an economy with unionized sectors. Fehr *et al.* (1996) employed the utility function $U^{**} = w - g(\lambda)$, where g is a convex function, to test the wage effect on involuntary unemployment in an experimental market. Walsh (1999) used the same utility function U^{***} to study the pattern of effort and wages in a multi-sector model; Allgulin & Ellingsen (2002) also used this utility function in their paper to determine whether monitoring and pay are substitute instruments for motivating workers.

A few papers employed slightly more complex utility function forms. These included Kiley (1997), who used the form $U^{\#\#} = wh - \frac{(\lambda h)^\theta}{\theta}$ to study the cyclical behavior of real wages and marginal cost; Hoon (2001) developed a dynamic Ricardian model of the world economy with the utility function $U^{\#\#\#} = aw - \lambda$.

2 Stigler proposed that one way to reduce a firm's search cost is to pay higher wage rates.

3 McCall derived the optimal search for an unemployed worker using the distribution of wages in the labor market and search costs.

4 Mortensen used a model with heterogeneous workers to calculate the natural rate of unemployment.

5 Pissarides derived a worker's optimal job acceptance policy by assuming that workers can only have a finite number of searches in their lifetime.

The above literature review shows that most of the studies in labor market unemployment adopt a constant marginal utility function. For the few that venture out to nonlinear utility functions, the implication of non-constant MU is not analyzed. Woodford (1994) is the exception in proposing to use diminishing MU to explain the trendless secular unemployment rate.

We add parenthetically that a parallel strand of work was initiated by Diamond (1982), Mortensen & Pissarides (1994) who added the matching function to the employment process. This allows the firm and the worker to interact in a search-matching model. Bertola & Caballero (1994) and Davis (2001) further extended the model to include dynamically heterogeneous productivity and search costs across firms. Jellal & Zenou (1999) included the quality of job matching in the workers' effort function to determine the efficiency wage. Within a general equilibrium framework, Trigari (2004) combined search and matching with a monetary model and used nominal price rigidities to explain economic fluctuations.[6] Gertler *et al.* (2008) extended the Trigari (2004) paper to include a government spending shock. Pries (2004), Hall (2005) and Shimer (2005) conducted empirical studies to test whether the calibrated search model matches the cyclical behavior of the labor market. Lentz (2009) determined the optimal unemployment insurance benefit in a search model with savings. Boone & Bovenberg (2004) explored the optimal interaction between the tax system and unemployment insurance. Lehmann *et al.* (2011) determined the optimal redistributive taxation policy for heterogeneous workers when search frictions are present.

3 S-S Model with Non-Constant Marginal Utility

We introduce into the S-S Model a SEW type worker's utility function in the form of

$$U(w_t, \lambda_t(w_t)) = M + w_t^{\beta} \lambda_t(w_t) - \gamma w_t^{\beta} \lambda_t(w_t)^2 - \alpha \lambda_t(w_t)^2 \quad (1)$$

where $w_t > 0$ is the wage and $\lambda_t(w_t) \geq 0$ is the effort at time t. The constant $M > 0$ is the utility derived from unemployment benefits when the worker is unemployed and $M = 0$ when he is employed. Constants $\alpha > ½$, $\beta > 1$ and $y > 0$.[7] The two models are thus linked by the same worker's utility function U. Furthermore, this nonlinear function can be used to address Woodford's concern regarding the limitation of constant MU.

As in the S-S model we assume there are a large number of identical firms, N, and a large number of identical risk-neutral workers, n_F. A firm maximizes its instantaneous real profits Π at each moment t: (Such instantaneous maximization applies to each t so we do not need to analyze the present value of profits. We simplify the notations by omitting the subscript t for w_t and $\lambda_t(w_t)$)

$$\max \pi = \max(AF(\lambda(w)L) - wL),\ F'(\bullet) > 0,\ F''(\bullet) < 0. \quad (2)$$

L is the number of workers in the firm and F is its production function with λL as the multiplicative input. We include a traditional shifter A to illustrate an unusual characteristic of this production function which is peculiar to the efficiency-wage theory. Due to the multiplicative term, there will be two conditions for maximization, namely

$$\lambda(w)\, AF'(\lambda(w)L) = w \quad (3)$$

and

$$AF'(\lambda(w)L)\, \lambda' L - L. \quad (4)^8$$

From these two conditions, we can derive a downward sloping labor demand

$$w = \frac{Z}{L}, \text{ where } Z \text{ is some constant.} \quad (5)$$

6 See Yashiv (2007) for an excellent survey of search and matching models. Pissarides (2000) provides an overview of macroeconomic equilibrium unemployment theory.

7 Under static optimization, this U implies an equilibrium similar to that of the SEW model. (See Appendix A.)

8 (3) and (4) imply the Solow condition $\frac{w\lambda'(w)}{\lambda(w)} = 1$ and $\rightarrow \lambda'(w) > 0$, which is the crucial assumption of efficiency wage models and is implicit in the Stiglitz's S-shaped curve.

Since A is not in the demand function, it cannot be used to shift the function when there is an increase in the productivity. Such an increase is now reflected by Z which acts as the shifter. (See Appendix B)

As in the S-S model the worker has a binary effort choice: $\lambda_t(w_t)= 0$ or $\lambda_t(w_t)=\bar{\lambda}>0$ (Each worker exerting $\bar{\lambda}$ contributes 1 unit of effective labor). Let n_e be the number of workers who are exerting effort in the firm. The firm's objective is to set w sufficiently high to prevent shirking and find the effective labor n_e at each moment to maximize its real profits. This is accomplished by hiring workers up to the point where the marginal product of effective labor equals the wage $(\bar{\lambda}\ AF'(\bar{\lambda}n_e)=w)$. By the identical firm assumption, these firm-level labor demand conditions can be extended to the aggregate level for the whole economy. Instead of the S-S model assumption of $\bar{\lambda}AF'[\frac{\bar{\lambda}n_F}{N}]>\bar{\lambda}$, we need to assume

$$\left[\bar{\lambda}AF'(\frac{\bar{\lambda}n_F}{N})\right]^{\beta}\bar{\lambda}-\gamma\left[\bar{\lambda}AF'(\frac{\bar{\lambda}n_F}{N})\right]^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2> M$$ (See Appendix C) to ensure there will be full employment if there is perfect monitoring.

The other S-S model assumptions are: (following rates are per unit time)

(A-1) an exogenous job breakup rate $\chi>0$;

(A-2) an exogenous shirking detection rate $\rho>0$;

(A-3) the unemployed worker finds employment at the rate θ, which is endogenously determined.

When the worker maximizes his expected lifetime utility, the value function in state i will be

$$V_i=\max\quad E[\int_0^{\infty}e^{-rt}(M+w^{\beta}\lambda-\gamma w^{\beta}\lambda^2-\alpha\lambda^2)dt]. \tag{6}$$

where $r \geq 0$ is the worker's discount rate.

There are three possible states for i, namely E, S, and X, where E denotes the state of the worker being employed and exerting effort $\bar{\lambda}$; S the state when the worker is employed and shirking ($\lambda = 0$) and X the state when the worker is unemployed.

The corresponding value functions are

$$V_E=\frac{(w^{\beta}\bar{\lambda}-\gamma w^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2)+\chi V_X}{r+\chi} \qquad \text{(See Appendix D)} \tag{7}$$

$$V_S=\frac{w^{\beta}\bar{\lambda}+(\chi+\rho)V_X}{r+\chi+\rho} \qquad \text{(See Appendix E)} \tag{8}$$

$$V_X=\frac{M}{r+\theta}+\frac{\theta}{r+\theta}V_E \qquad \text{(See Appendix F).} \tag{9}$$

Each firm will pay wage that is just sufficient to induce effort. This is done by equating V_E with V_S resulting in

$$w=[\frac{r\rho V_X}{\bar{\lambda}\rho-(r+\chi+\rho)\gamma\bar{\lambda}^2}+\frac{\alpha(r+\chi+\rho)\bar{\lambda}^2}{\bar{\lambda}\rho-(r+\chi+\rho)\gamma\bar{\lambda}^2}]^{\frac{1}{\beta}} \tag{10}$$

We follow the S-S-model in assuming a steady-state in which the movement in and out of unemployment is balanced:

$$\theta(n_F-n)=\chi n, \qquad \text{(Note } \Theta \text{ then is a function of } n\text{)} \tag{11}$$

where $n = NL$ is the employment level of all firms.

From equations (7), (9), and (10), we derive the following "No-Shirking Condition" curve (NSC) which is analogous to that of the S-S model: (See Appendix G)

$$\hat{w}(n)=[\frac{\rho M+(r+\chi+\rho+\theta)\alpha\bar{\lambda}^2}{\rho\bar{\lambda}-(r+\chi+\rho+\theta)\gamma\bar{\lambda}^2}]^{\frac{1}{\beta}}\text{, with} \tag{12}$$

$$\frac{d\hat{w}(n)}{dn} = \frac{n_F \rho \chi \overline{\lambda}^2 w^{1-\beta}(\alpha\overline{\lambda} + \gamma M)}{\beta\{[\rho\overline{\lambda} - (r+\chi+\rho)\gamma\overline{\lambda}^2]n_F - [\rho\overline{\lambda} - (r+\rho)\gamma\overline{\lambda}^2]n\}^2} > 0, \tag{13}$$

$\rightarrow \hat{w}(n)$ is upward sloping.

We note that since w and λ cannot move in opposite directions ($\because \lambda'(w) > 0$, see footnote 9) and $\beta > 1$, the term $w^{\beta}\lambda$ moves in the same direction as $w\lambda$. This enables us to use it as a proxy for the worker's income. Our utility function U exhibits diminishing MU characteristics with respect to the income proxy $w^{\beta}\lambda$.[9]

The comparative statics are straightforward and consistent with the shirking models. When the utility from unemployment benefits M in the numerator of $\hat{w}(n)$ is increased, ceteris paribus, the no-shirking wage will be raised to induce workers not to shirk.[10] If the worker's disutility from exerting effort α increases, the second numerator term $(r+\chi+\rho+\theta)\alpha\overline{\lambda}^2$ becomes larger and will increase $\hat{w}(n)$. This means a higher no-shirking wage is necessary when there is more disutility. Since equation (12) can be rewritten as $\hat{w}(n) = \left[\dfrac{M + (\frac{r+\chi+\theta}{\rho}+1)\alpha\overline{\lambda}^2}{\overline{\lambda} - (\frac{r+\chi+\theta}{\rho}+1)\gamma\overline{\lambda}^2}\right]^{\frac{1}{\beta}}$, an increase in the detection rate ρ will reduce the no-shirking wage. Higher degree of diminishing MU (as represented by an increase in the positive value of γ) will increase the negative denominator term $(r+\chi+\rho+\theta)\gamma\overline{\lambda}^2$ in (12) to raise $\hat{w}(n)$. This shows a higher no-shirking wage is needed to compensate for the higher degree of diminishing MU when income rises over time.[11]

Equilibrium occurs at the intersection of the NSC and labor demand L^d curves. There will be unemployment ($n_F - n^*_{E0}$) as shown in Figure 1.

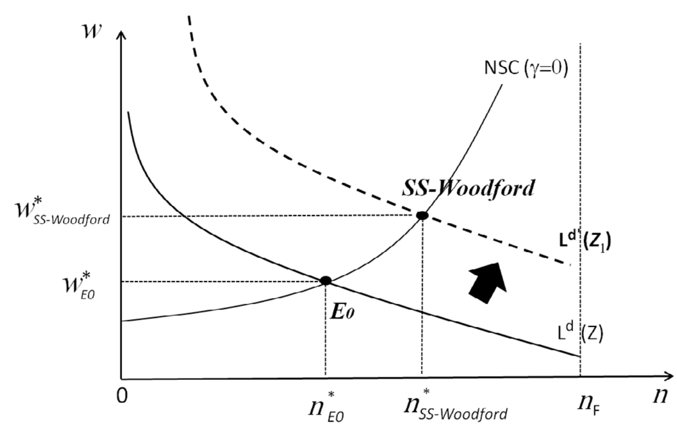

Figure 1: Labor market equilibrium (γ = 0 case)

From the labor equilibrium graph, we can analyze the following Woodford's (1994) statements:

(A) secular growth in labor productivity would reduce unemployment rate; and

(B) such result can be avoided by abandoning the Shapiro-Stiglitz assumption of constant marginal utility of income.

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9 U is concave with respect to $w^{\beta}\lambda$ (See Appendix H).

10 This is in line with the Shapiro-Stiglitz assertion that "the existence of unemployment benefits reduces the "penalty" associated with being fired. Therefore, to induce workers not to shirk, firms must pay higher wages." (Shapiro and Stiglitz 1984, p. 434)

11 An increase in γ raises $\hat{w}(n)$ in two ways. (See Appendix G).

To reflect the secular growth in labor productivity, Z is increased to Z_1 in the labor demand function (equation (5)). This will shift the labor demand curve L^d to $L^{d'}$ in Figure 1. The equilibrium will move from E_0 to *SS-Woodford*, where there is a rise in the no-shirking wage and a decline in unemployment. This is the statement (A) described by Woodford.

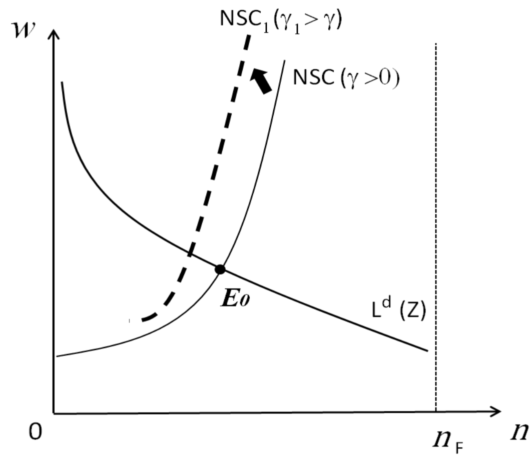

Figure 2: Effect of an increase in the degree of diminishing MU.

For statement (B), when y is increased to y_1 in equation (12), the NSC curve in Figure 2 will be shifted upwards; and from equation (13), its slope will steepen. Hence, NSC will be shifted to NSC$_1$. This will reduce the employment level. If, over time, this reduction offsets the increase in employment caused by the upward shift in labor demand (i.e. Z to Z_1), then the secular unemployment rate will be trendless. The condition necessary for this to happen is for (Z_1, y_1) to conform to

$$\frac{NZ_1}{n^*_{E0}} = \left[\frac{\rho M + (r + \chi + \rho + \frac{\chi n^*_{E0}}{n_F - n^*_{E0}})\alpha\bar{\lambda}^2}{\rho\bar{\lambda} - (r + \chi + \rho + \frac{\chi n^*_{E0}}{n_F - n^*_{E0}})\gamma_1\bar{\lambda}^2}\right]^{\frac{1}{\beta}} \tag{14}$$

(See Figure 3 and Appendix I)

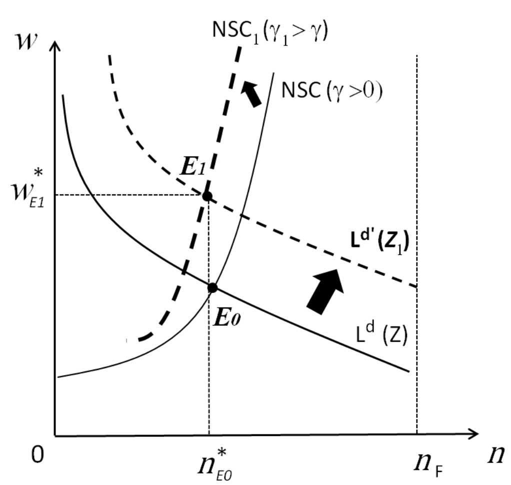

Figure 3: Trendless secular unemployment rate

If the trendless secular unemployment rate persists, it might suggest that the intertemporal movements of (Z, y) to (Z_1, y_1) follow some related patterns so that the condition (14) will be maintained over time and the shifted labor demand and NSC curves will keep intersecting near . This of course can be the subject of further research.

Moreover, it is obvious from our model that the diminishing MU of income is not the only explanation of a trendless secular employment rate. An increase in parameters M, r, χ, α, $\overline{\lambda}$ or a decrease in ρ can produce the trendless effect, as shown in Figure 3. The policy implication of varying the parameters is interesting. For example, policies aimed at increasing the utility of unemployment benefit M can have dampening effect on the secular increase in labor productivity. And better detection rate by firms (i.e. an increase in ρ) can reduce the no-shirking wage, leading to a further reduction in the unemployment rate.

4 Conclusion

We find that we can link the SEW and S-S model with a common nonlinear worker's utility function and thus generalize the S-S model by relaxing the constant MU assumption. The parameters in our model enable us to make explicit the various factors affecting the secular unemployment rate, as well as to analyze the worker's preference representation regarding income and incentive to work. We can specify the various conditions necessary for a trendless unemployment rate. In particular, we specify the condition under which Woodford's conjecture will occur. Besides workers' diminishing marginal utility of income, other factors like rising unemployment benefit or declining shirking detection rate can also help to explain the trendless secular unemployment rate.

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Appendix

A. The conditions to generate an S-shaped effort supply curve

(To simplify the notations, we omit the subscript t)

To maximize $U(w,\lambda(w)) = M + w^{\beta}\lambda(w) - \gamma w^{\beta}\lambda(w)^2 - \alpha\lambda(w)^2$. The necessary condition $\frac{dU}{d\lambda}=0$

yields $w^{\beta} - 2\gamma w^{\beta}\lambda - 2\alpha\lambda = 0$ ➔ the optimal $\lambda(w) = \frac{w^{\beta}}{2(\alpha+\gamma\ w^{\beta})}$. The sufficient condition is also met since

$\frac{\partial^2 U}{\partial\lambda^2} = -2\gamma\ w^{\beta} - 2\alpha < 0$ .The first and second order derivatives of $\lambda(w)$ with respect to wage are respectively:

$\frac{d\lambda}{dw} = \frac{\alpha\beta\ w^{\beta-1}}{2(\alpha+\gamma\ w^{\beta})^2}$ and $\frac{d^2\lambda}{dw^2} = \frac{\alpha\beta\ w^{\beta-2}}{2(\alpha+\gamma\ w^{\beta})^3}[\alpha(\beta-1)-\gamma(\beta+1)w^{\beta}]$. To ensure an upward sloping $\lambda(w)$, we rule out the case of $\beta \le 0$. For $0<\beta\le 1$, $\frac{d^2\lambda}{dw^2}$ is negative over the whole domain and $\lambda(w)$ cannot be S-shaped. Hence $\beta>1$, $\alpha>0$, and $\gamma>0$ are necessary to obtain an upward-sloping S-shaped effort supply curve.

$$\beta = \frac{\partial(U+\alpha\ \lambda^2)}{\partial w} \Bigg/ \frac{(U+\alpha\ \lambda^2)}{w}$$

and this is the wage elasticity with respect to the middle two terms of the utility function.

Hence β partially measures the effect of wage on factors contributing to the utility of the worker. α represents the weight which determines acceleration of disutility change due to exerting effort. γ is the disutility factor of exerting effort associated with earnings. The term $-\gamma\lambda(w)$ accounts for the decrease in marginal utility related to $w^{\beta}\lambda(w)$. (See Figure A1).

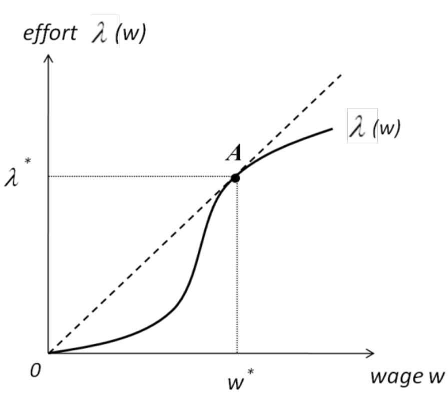

Figure A1. SEW static model equilibrium with an S-shaped effort function

B. Deriving labor demand curve (5):

The necessary and sufficient conditions for $\max \prod$ are:

$\frac{\partial\prod}{\partial L} = AF'[\lambda(w)L]\lambda(w) - w = 0$ ➔ equation (3); $\frac{\partial^2\prod}{\partial L^2} = AF''[\lambda(w)L][\lambda(w)]^2 < 0$ ($\because F''<0$)

$\frac{\partial\prod}{\partial w} = AF'[\lambda(w)L]\lambda'(w)L - L = 0$ ➔ equation (4);

$\frac{\partial^2\prod}{\partial w^2} = AF''[\lambda(w)L][\lambda'(w)L]^2 + AF'[\lambda(w)L]\lambda''(w)L < 0$ ($\because F''<0$, $\lambda''<0$).

From (3) and (4), we obtain the Solow condition $\frac{w\lambda'(w)}{\lambda(w)}=1$.

Differentiating equation (3) with respect to w yields

$$AF''[\lambda(w)L][\lambda'(w)L+\lambda(w)\frac{\partial L}{\partial w}]=\frac{1}{\lambda(w)}-\frac{1}{\lambda(w)}\cdot\frac{w\lambda'(w)}{\lambda(w)} \tag{B.1}$$

Substituting the Solow condition into the RHS of (B.1) we get $AF''[\lambda(w)L][\lambda'(w)L+\lambda(w)\frac{\partial L}{\partial w}]=0$, which implies $\frac{\partial L}{\partial w}=-\frac{L}{w}<0$.

➔ $\frac{dw}{dL}=-\frac{w}{L}$ ➔ $\int\frac{dw}{w}=-\int\frac{dL}{L}$ ➔ $\ln w=-\ln L+cons\tan t$

➔ $w=\frac{Z}{L}$ where $Z\equiv e^{constant}$ (B.2)

This is the labor demand curve (5).

C. Condition to ensure full employment if there is perfect monitoring

By the identical firm assumption, if each firm hires $\frac{1}{N}$ of the labor force (to reach full employment) at the profit-maximizing wage which is equal to the marginal product of labor $\left[\bar{\lambda}AF'(\frac{\bar{\lambda}n_F}{N})\right]$, and if we wish to have the worker exerting effort, then we need the following condition:

$$\left[\bar{\lambda}AF'(\frac{\bar{\lambda}n_F}{N})\right]^{\beta}\bar{\lambda}-\gamma\left[\bar{\lambda}AF'(\frac{\bar{\lambda}n_F}{N})\right]^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2>M.$$

This condition ensures that the worker's utility from working will exceed his utility derived from the unemployment benefits. In other words, the worker will be exerting effort as if there is absence of imperfect monitoring.

D. Value function for an employed and non-shirking worker (equation (7)):

$$V_E(\Delta t)=\int_{t=0}^{\Delta t}\left[e^{-\chi t}e^{-rt}(0+w^{\beta}\bar{\lambda}-\gamma w^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2)\right]dt+e^{-r\Delta t}[e^{-\chi\Delta t}V_E(\Delta t)+(1-e^{-\chi\Delta t})V_X(\Delta t)]$$

$$=(w^{\beta}\bar{\lambda}-\gamma w^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2)\int_{t=0}^{\Delta t}\left[e^{-(\chi+r)t}\right]dt+e^{-r\Delta t}[e^{-\chi\Delta t}V_E(\Delta t)+(1-e^{-\chi\Delta t})V_X(\Delta t)]$$

$$=(w^{\beta}\bar{\lambda}-\gamma w^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2)[\frac{1}{r+\chi}-\frac{e^{-(\chi+r)\Delta t}}{r+\chi}]+e^{-r\Delta t}[e^{-\chi\Delta t}V_E(\Delta t)+(1-e^{-\chi\Delta t})V_X(\Delta t)]$$

$$V_E(\Delta t)=\frac{(w^{\beta}\bar{\lambda}-\gamma w^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2)}{r+\chi}+\frac{e^{-r\Delta t}(1-e^{-\chi\Delta t})V_X(\Delta t)}{1-e^{-(\chi+r)\Delta t}}$$

➔ $$\frac{V_E(\Delta t)-\frac{(w^{\beta}\bar{\lambda}-\gamma w^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2)}{r+\chi}}{V_X(\Delta t)}=\frac{e^{-r\Delta t}(1-e^{-\chi\Delta t})}{1-e^{-(\chi+r)\Delta t}}$$

➔ $$\lim_{\Delta t\to 0}\frac{V_E(\Delta t)-\frac{(w^{\beta}\bar{\lambda}-\gamma w^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2)}{r+\chi}}{V_X(\Delta t)}=\lim_{\Delta t\to 0}\frac{e^{-r\Delta t}(1-e^{-\chi\Delta t})}{1-e^{-(\chi+r)\Delta t}}$$, and by L'Hôpital's Rule

$$=\lim_{\Delta t\to 0}\frac{[(-r)e^{-r\Delta t}+(r+\chi)e^{-(r+\chi)\Delta t}]}{(r+\chi)e^{-(r+\chi)\Delta t}}=\frac{\chi}{r+\chi}$$

➔ $$V_E=\frac{(w^{\beta}\bar{\lambda}-\gamma w^{\beta}\bar{\lambda}^2-\alpha\bar{\lambda}^2)+\chi V_X}{r+\chi}$$, which is equation (7).

E. Value function for an employed and shirking worker (equation (8)):

$$V_S(\Delta t) = \int_{t=0}^{\Delta t} \left\{ e^{-\chi t} e^{-rt} [0 + w^\beta \overline{\lambda} - \gamma w^\beta (0)^2 - \alpha (0)^2] \right\} dt + e^{-r\Delta t} [e^{-(\chi+\rho)\Delta t} V_S(\Delta t) + (1 - e^{-(\chi+\rho)\Delta t}) V_X(\Delta t)]$$

$$= (w^\beta \overline{\lambda}) \int_{t=0}^{\Delta t} \left[e^{-(\chi+r)t} \right] dt + e^{-r\Delta t} [e^{-(\chi+\rho)\Delta t} V_S(\Delta t) + (1 - e^{-(\chi+\rho)\Delta t}) V_X(\Delta t)]$$

$$= (w^\beta \overline{\lambda}) [\frac{1}{r+\chi} - \frac{e^{-(\chi+r)\Delta t}}{r+\chi}] + e^{-r\Delta t} [e^{-(\chi+\rho)\Delta t} V_S(\Delta t) + (1 - e^{-(\chi+\rho)\Delta t}) V_X(\Delta t)]$$

$$\Rightarrow \quad V_S(\Delta t) = \frac{w^\beta \overline{\lambda} [1 - e^{-(\chi+r)\Delta t}]}{(r+\chi)[1 - e^{-(\chi+r+\rho)\Delta t}]} + \frac{[e^{-r\Delta t} - e^{-(r+\chi+\rho)\Delta t}] V_X(\Delta t)}{1 - e^{-(\chi+r+\rho)\Delta t}}$$

$$\Rightarrow \quad \lim_{\Delta t \to 0} \frac{V_S(\Delta t) - \frac{w^\beta \overline{\lambda} [1 - e^{-(\chi+r)\Delta t}]}{(r+\chi)[1 - e^{-(\chi+r+\rho)\Delta t}]}}{V_X(\Delta t)} = \lim_{\Delta t \to 0} \frac{e^{-r\Delta t} - e^{-(r+\chi+\rho)\Delta t}}{1 - e^{-(\chi+r+\rho)\Delta t}}$$

$$\Rightarrow \quad \frac{V_S - \lim_{\Delta t \to 0} \frac{w^\beta \overline{\lambda} [(\chi+r) e^{-(\chi+r)\Delta t}]}{(r+\chi)[(\chi+r+\rho) e^{-(\chi+r+\rho)\Delta t}]}}{V_X} = \lim_{\Delta t \to 0} \frac{-r e^{-r\Delta t} + (r+\chi+\rho) e^{-(r+\chi+\rho)\Delta t}}{(\chi+r+\rho) e^{-(\chi+r+\rho)\Delta t}}$$

$$\Rightarrow \quad \frac{V_S - \frac{w^\beta \overline{\lambda}}{\chi+r+\rho}}{V_X} = \frac{-r + (r+\chi+\rho)}{\chi+r+\rho} \Rightarrow V_S = \frac{w^\beta \overline{\lambda} + (\chi+\rho) V_X}{r+\chi+\rho}$$, which is equation (8).

F. Value function for an unemployed worker (equation (9)):

$$V_X(\Delta t) = \int_{t=0}^{\Delta t} \left\{ e^{-\chi t} e^{-rt} [M + w^\beta (0) - \gamma w^\beta (0)^2 - \alpha (0)^2] \right\} dt + e^{-r\Delta t} [e^{-\theta\Delta t} V_X(\Delta t) + (1 - e^{-\theta\Delta t}) V_E(\Delta t)]$$

$$= M \int_{t=0}^{\Delta t} \left[e^{-(\chi+r)t} \right] dt + e^{-r\Delta t} [e^{-\theta\Delta t} V_X(\Delta t) + (1 - e^{-\theta\Delta t}) V_E(\Delta t)]$$

$$= M [\frac{1}{r+\chi} - \frac{e^{-(\chi+r)\Delta t}}{r+\chi}] + e^{-r\Delta t} [e^{-\theta\Delta t} V_X(\Delta t) + (1 - e^{-\theta\Delta t}) V_E(\Delta t)]$$

$$\Rightarrow \quad V_X(\Delta t) = \frac{M [1 - e^{-(\chi+r)\Delta t}]}{(r+\chi)[1 - e^{-(r+\theta)\Delta t}]} + \frac{[e^{-r\Delta t} - e^{-(r+\theta)\Delta t}] V_E(\Delta t)}{1 - e^{-(r+\theta)\Delta t}}$$

$$\Rightarrow \quad \lim_{\Delta t \to 0} \frac{V_X(\Delta t) - \frac{M [1 - e^{-(\chi+r)\Delta t}]}{(r+\chi)[1 - e^{-(r+\theta)\Delta t}]}}{V_E(\Delta t)} = \lim_{\Delta t \to 0} \frac{e^{-r\Delta t} - e^{-(r+\theta)\Delta t}}{1 - e^{-(r+\theta)\Delta t}}$$

$$\Rightarrow \quad \frac{V_X - \lim_{\Delta t \to 0} \frac{M [(\chi+r) e^{-(\chi+r)\Delta t}]}{(r+\chi)[(r+\theta) e^{-(r+\theta)\Delta t}]}}{V_E} = \lim_{\Delta t \to 0} \frac{-r e^{-r\Delta t} + (r+\theta) e^{-(r+\theta)\Delta t}}{(r+\theta) e^{-(r+\theta)\Delta t}}$$

$$\Rightarrow \quad \frac{V_X - \frac{M}{r+\theta}}{V_E} = \frac{-r + (r+\theta)}{r+\theta} \quad \Rightarrow \quad V_X = \frac{M}{r+\theta} + \frac{\theta}{r+\theta} V_E$$, which is equation (9).

G. No-Shirking Condition curve and its derivative (equations (12) and (13)):

Equation (10) can be written as:

$$\overline{\lambda} w^\beta = \frac{r\rho V_X}{\rho - (r+\chi+\rho)\gamma\overline{\lambda}} + \frac{\alpha (r+\chi+\rho) \overline{\lambda}^2}{\rho - (r+\chi+\rho)\gamma\overline{\lambda}} \tag{G.1}$$

Solving for V_E and V_X in equations (7) & (9):

$$V_E = \frac{(w^\beta \bar{\lambda} - \gamma w^\beta \bar{\lambda}^2 - \alpha\bar{\lambda}^2)(r+\theta) + M\chi}{r(r+\chi+\theta)} \tag{G.2}$$

$$V_X = \frac{(w^\beta \bar{\lambda} - \gamma w^\beta \bar{\lambda}^2 - \alpha\bar{\lambda}^2)\theta + M(r+\chi)}{r(r+\chi+\theta)} \tag{G.3}$$

Substituting V_X in (G.3) into equation (G.1):

$$\bar{\lambda} w^\beta = \frac{r\rho}{\rho - (r+\chi+\rho)\gamma\bar{\lambda}} \frac{(w^\beta \bar{\lambda} - \gamma w^\beta \bar{\lambda}^2 - \alpha\bar{\lambda}^2)\theta + M(r+\chi)}{r(r+\chi+\theta)} + \frac{\alpha(r+\chi+\rho)\bar{\lambda}^2}{\rho - (r+\chi+\rho)\gamma\bar{\lambda}}$$

$$\Rightarrow [\rho-(r+\chi+\rho)\gamma\bar{\lambda}](r+\chi+\theta)\bar{\lambda}w^\beta = \rho\theta\bar{\lambda}w^\beta(1-\gamma\bar{\lambda}) + \rho(r+\chi)M + \alpha[(r+\chi+\rho)(r+\chi+\theta) - \rho\theta]\bar{\lambda}^2 \Rightarrow$$

$$\{[\rho-(r+\chi+\rho)\gamma\bar{\lambda}](r+\chi+\theta) - \rho\theta(1-\gamma\bar{\lambda})\}\bar{\lambda}w^\beta = \rho(r+\chi)M + \alpha[(r+\chi+\rho)(r+\chi+\theta) - \rho\theta]\bar{\lambda}^2$$

$$\Rightarrow [(r+\chi)\rho\bar{\lambda} - (r+\chi)(r+\chi+\rho+\theta)\gamma\bar{\lambda}^2]w^\beta = (r+\chi)[\rho M + (r+\chi+\rho+\theta)\alpha\bar{\lambda}^2]$$

$$\Rightarrow w(n)^\beta = \frac{\rho M + (r+\chi+\rho+\theta)\alpha\bar{\lambda}^2}{\rho\bar{\lambda} - (r+\chi+\rho+\theta)\gamma\bar{\lambda}^2}$$

$$\Rightarrow \hat{w}(n) = [\frac{\rho M + (r+\chi+\rho+\theta)\alpha\bar{\lambda}^2}{\rho\bar{\lambda} - (r+\chi+\rho+\theta)\gamma\bar{\lambda}^2}]^{\frac{1}{\beta}}$$ which is equation (12).

$$\Rightarrow \hat{w}(n) = [\frac{\rho M + (r+\chi+\rho+\frac{\chi n}{n_F - n})\alpha\bar{\lambda}^2}{\rho\bar{\lambda} - (r+\chi+\rho+\frac{\chi n}{n_F - n})\gamma\bar{\lambda}^2}]^{\frac{1}{\beta}}$$ (From equation (11): $\theta = \frac{\chi n}{n_F - n}$)

$$\Rightarrow \hat{w}(n) = \{\frac{\rho M(n_F - n) + [(r+\chi+\rho)(n_F - n) + \chi n]\alpha\bar{\lambda}^2}{\rho\bar{\lambda}(n_F - n) - [(r+\chi+\rho)(n_F - n) + \chi n]\gamma\bar{\lambda}^2}\}^{\frac{1}{\beta}}$$

$$\Rightarrow \hat{w}(n) = \{\frac{[\rho M + (r+\chi+\rho)\alpha\bar{\lambda}^2]n_F - [\rho M + (r+\rho)\alpha\bar{\lambda}^2]n}{[\rho\bar{\lambda} - (r+\chi+\rho)\gamma\bar{\lambda}^2]n_F - [\rho\bar{\lambda} - (r+\rho)\gamma\bar{\lambda}^2]n}\}^{\frac{1}{\beta}} \tag{G.4}$$ $\Rightarrow$

$$\frac{d\hat{w}(n)}{dn} = \frac{\hat{w}^{1-\beta}\begin{bmatrix}\{[\rho\bar{\lambda} - (r+\chi+\rho)\gamma\bar{\lambda}^2]n_F - [\rho\bar{\lambda} - (r+\rho)\gamma\bar{\lambda}^2]n\}\{-[\rho M + (r+\rho)\alpha\bar{\lambda}^2]\} \\ -\{[\rho M + (r+\chi+\rho)\alpha\bar{\lambda}^2]n_F - [\rho M + (r+\rho)\alpha\bar{\lambda}^2]n\}\{-[\rho\bar{\lambda} - (r+\rho)\gamma\bar{\lambda}^2]\}\end{bmatrix}}{\beta\{[\rho\bar{\lambda} - (r+\chi+\rho)\gamma\bar{\lambda}^2]n_F - [\rho\bar{\lambda} - (r+\rho)\gamma\bar{\lambda}^2]n\}^2} \Rightarrow$$

$$\frac{d\hat{w}(n)}{dn} = \frac{\hat{w}^{1-\beta}\begin{bmatrix}\{[\rho\bar{\lambda} - (r+\chi+\rho)\gamma\bar{\lambda}^2]n_F\}\{-[\rho M + (r+\rho)\alpha\bar{\lambda}^2]\} \\ -\{[\rho M + (r+\chi+\rho)\alpha\bar{\lambda}^2]n_F\}\{-[\rho\bar{\lambda} - (r+\rho)\gamma\bar{\lambda}^2]\}\end{bmatrix}}{\beta\{[\rho\bar{\lambda} - (r+\chi+\rho)\gamma\bar{\lambda}^2]n_F - [\rho\bar{\lambda} - (r+\rho)\gamma\bar{\lambda}^2]n\}^2} \Rightarrow$$

$$\frac{d\hat{w}(n)}{dn} = \frac{n_F\hat{w}^{1-\beta}\begin{bmatrix}\{-\rho M\cdot\rho\bar{\lambda} + \rho M\cdot(r+\chi+\rho)\gamma\bar{\lambda}^2 - (r+\rho)\alpha\bar{\lambda}^2\cdot\rho\bar{\lambda} + (r+\rho)\alpha\bar{\lambda}^2\cdot(r+\chi+\rho)\gamma\bar{\lambda}^2\} \\ -\{-\rho\bar{\lambda}\cdot\rho M - \rho\bar{\lambda}\cdot(r+\chi+\rho)\alpha\bar{\lambda}^2 + \rho M\cdot(r+\rho)\gamma\bar{\lambda}^2 + (r+\rho)\gamma\bar{\lambda}^2\cdot(r+\chi+\rho)\alpha\bar{\lambda}^2\}\end{bmatrix}}{\beta\{[\rho\bar{\lambda} - (r+\chi+\rho)\gamma\bar{\lambda}^2]n_F - [\rho\bar{\lambda} - (r+\rho)\gamma\bar{\lambda}^2]n\}^2}$$

$$\Rightarrow \frac{d\hat{w}(n)}{dn} = \frac{n_F\hat{w}^{1-\beta}\begin{bmatrix}\rho M\cdot(r+\chi+\rho)\gamma\bar{\lambda}^2 - (r+\rho)\alpha\bar{\lambda}^2\cdot\rho\bar{\lambda} \\ +\rho\bar{\lambda}\cdot(r+\chi+\rho)\alpha\bar{\lambda}^2 - \rho M\cdot(r+\rho)\gamma\bar{\lambda}^2\end{bmatrix}}{\beta\{[\rho\bar{\lambda} - (r+\chi+\rho)\gamma\bar{\lambda}^2]n_F - [\rho\bar{\lambda} - (r+\rho)\gamma\bar{\lambda}^2]n\}^2}$$

$$\rightarrow \frac{d\hat{w}(n)}{dn} = \frac{n_F \rho \chi \bar{\lambda}^2 \hat{w}^{1-\beta}(\alpha\bar{\lambda} + \gamma M)}{\beta\{[\rho\bar{\lambda} - (r+\chi+\rho)\gamma\bar{\lambda}^2]n_F - [\rho\bar{\lambda} - (r+\rho)\gamma\bar{\lambda}^2]n\}^2} > 0 \text{, which is (13).}$$

An increase in γ affects $\hat{w}(n)$ in two ways:

Firstly, the numerator term $\dfrac{w^\beta \bar{\lambda} - \gamma w^\beta \bar{\lambda}^2 - \alpha\bar{\lambda}^2}{r+\chi}$ will reduce V_E. Hence $\hat{w}$ must be raised to maintain the $V_E \geq V_S$ condition.

Secondly, V_E will be reduced by $\dfrac{(r+\theta)w^\beta \bar{\lambda}^2}{r(r+\chi+\theta)}$ and V_X by only $\dfrac{\theta w^\beta \bar{\lambda}^2}{r(r+\chi+\theta)}$, causing $\hat{w}(n)$ to rise (see equations (G.2) and (G.3)). In other words, an increase in γ reduces the value of being employed by an amount greater than that of being unemployed.

H. U is concave with respect to the income proxy $w^\beta \lambda$

A) $w^\beta \bar{\lambda} > 2(\gamma w^\beta \bar{\lambda}^2 + \alpha\bar{\lambda}^2) \rightarrow$

$$\frac{\partial U}{\partial(w^\beta \lambda)} = 1 - \gamma(w^\beta \lambda \frac{\partial \lambda}{\partial w^\beta \lambda} + \lambda) - 2\alpha\lambda \frac{\partial \lambda}{\partial w^\beta \lambda} = 1 - \gamma(w^\beta \lambda \frac{1}{\frac{\partial w^\beta \lambda}{\partial \lambda}} + \lambda) - 2\alpha\lambda \frac{1}{\frac{\partial w^\beta \lambda}{\partial \lambda}} = 1 - \gamma(w^\beta \lambda \frac{1}{w^\beta} + \lambda) - \frac{2\alpha\lambda}{w^\beta} = 1 - 2\gamma\lambda - \frac{2\alpha\lambda}{w^\beta} > 0.$$

B) $\dfrac{\partial U}{\partial(w^\beta \lambda)} = 1 - 2\gamma\lambda - \dfrac{2\alpha\lambda^2}{w^\beta \lambda} = 1 - 2\gamma\lambda - 2\alpha\lambda^2 \bullet (w^\beta \lambda)^{-1}$ and $\alpha > \dfrac{1}{2}$ $\rightarrow$

$$\frac{\partial^2 U}{\partial(w^\beta \lambda)^2} = -2\gamma \frac{\partial \lambda}{\partial w^\beta \lambda} + 2\lambda^2 (w^\beta \lambda)^{-2} - 4\alpha\lambda \frac{\partial \lambda}{\partial w^\beta \lambda}(w^\beta \lambda)^{-1} = -2\gamma \frac{1}{w^\beta} + \frac{2\lambda^2}{w^{2\beta}\lambda^2} - \frac{4\alpha\lambda}{w^{2\beta}\lambda} = -\frac{2\gamma}{w^\beta} - \frac{4\alpha - 2}{w^{2\beta}} < 0$$

I. Deriving equations (14)

From (B.2), $w = \dfrac{Z}{L}$ (firm's labor demand curve) (I.1)

By the identical firm assumption, all firms are paying w, which makes it also the economy-wide wage rate. Each firm is hiring $L = \dfrac{Z}{w}$, making the total employment level $n = NL$. This implies $w = \dfrac{NZ}{n}$.

From (11) and (12), $\hat{w} = \left[\dfrac{\rho M + (r+\chi+\rho+\frac{\chi n}{n_F - n})\alpha\bar{\lambda}^2}{\rho\bar{\lambda} - (r+\chi+\rho+\frac{\chi n}{n_F - n})\gamma\bar{\lambda}^2}\right]^{\frac{1}{\beta}}$.

At equilibrium $w = \hat{w}$ $\rightarrow$

$$\frac{NZ}{n} = \left[\frac{\rho M + (r+\chi+\rho+\frac{\chi n}{n_F - n})\alpha\bar{\lambda}^2}{\rho\bar{\lambda} - (r+\chi+\rho+\frac{\chi n}{n_F - n})\gamma\bar{\lambda}^2}\right]^{\frac{1}{\beta}} \quad \text{(I.2)}$$

Denote $\overset{*}{n}_{E0}$ as the solution to (I.2).

When Z is increased to Z_1, in order to hold n at $\overset{*}{n}_{E0}$, γ must be raised to γ_1 such that:

$$\frac{NZ_1}{\overset{*}{n}_{E0}} = \left[\frac{\rho M + (r+\chi+\rho+\frac{\chi \overset{*}{n}_{E0}}{n_F - \overset{*}{n}_{E0}})\alpha\bar{\lambda}^2}{\rho\bar{\lambda} - (r+\chi+\rho+\frac{\chi \overset{*}{n}_{E0}}{n_F - \overset{*}{n}_{E0}})\gamma_1\bar{\lambda}^2}\right]^{\frac{1}{\beta}} \quad \text{(I.3)}$$