

Controlling Fusion of Majorana Fermions in one-dimensional systems by Zeeman Field

L. B. Shao,^{1,2,*} Z. D. Wang,^{2,†} R. Shen,¹ L. Sheng,¹ B. G. Wang,¹ and D. Y. Xing¹

¹ *National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing 210093, China*

² *Department of Physics and Center of Theoretical and Computational Physics, University of Hong Kong, Pokfulam Road, Hong Kong, China*

We propose to realize Majorana fermions (MFs) on an edge of a two-dimensional topological insulator in the proximity with s -wave superconductors and in the presence of transverse exchange field h . It is shown that there appear a pair of MFs localized at two junctions and that a reverse in direction of h can lead to permutation of two MFs. With decreasing h , the MF states can either be fused or form one Dirac fermion on the π -junctions, exhibiting a topological phase transition. This characteristic can be used to detect physical states of MFs when they are transformed into Dirac fermions localized on the π -junction. A condition of decoupling two MFs is also given.

PACS numbers: 73.23.-b, 74.50.+r, 71.10.Pm

In recent several years, how to realize, manipulate, and detect Majorana fermions (MFs) is one of the most active topics of research in condensed matter physics. [1–5]. The nonabelian character of the MF makes it to be a promising candidate for topological quantum computation [6–8]. There are many systems that manifest the MF, such as half-quantum vortices of p -wave superconductors [6, 9, 10], the hexagonal spin lattice model [7], the one-dimensional (1D) p -wave lattice [11], the topological surface state with proximity to an s -wave superconductor [1, 12], ultracold atom systems [13, 14] and so on. Since Majorana bound states are superpositions of electrons and holes in the middle of superconducting gap, they are neutral zero-energy excitations, and the particle-hole symmetry causes the antiparticle of an MF to be itself in the field-theory framework [15]. Many proposals have been suggested to explore novel properties of MFs, such as the electrically detected Majorana interferometry [3], the Andreev reflection induced by MFs [4], the charge transport with Majorana edge modes [16], and the teleportation by Majorana bound state [17]. For a superconducting system only Cooper pairs can be created and annihilated, and so nonabelian statistics of the MFs can only be formulated in subspaces of same fermion parity [6]. Also, manipulating MFs in 1D systems can be achieved by using assistant quantum systems such as Coulomb blockaded quantum dots [18] and semiconducting wire networks composed of trijunctions [19]. Although it has been reported recently that signature of experiment supports the existence of MFs [20], how to detect MFs still remains as an open question.

In this paper, we propose to realize the MFs on the edge of a 2D topological insulator. Superconducting order parameters with amplitude Δ_0 are introduced to the edge states by proximity effect of s -wave superconducting junctions, and a local transverse Zeeman field h is also applied there. It is found that existence of a pair of MFs depends explicitly upon the relative magnitude of h and Δ_0 [21]. As $|h| > \Delta_0$, two MFs in different spin components emerge at the two junctions, respectively,

and an inverse Zeeman field will lead to permutation of MFs. At $|h| = \Delta_0$, there will be a topological transition. For $|h| < \Delta_0$, the MFs can either be fused or form Dirac fermions localized on the junctions, depending on the phase differences of the junctions. When the phase difference is unequal to $(2N + 1)\pi$, the wavefunctions of MFs are extended into the bulk and fused; otherwise, one additional MF is created at the junction and combined with the original one to form one Dirac fermion. Therefore, when the phase difference of only one junction is equal to $(2N + 1)\pi$, the MF on the junction is effectively driven to the other junction and forms one Dirac fermion there. This character can be used to detect physical states of two MFs. The coupling between two MFs will vanish under some conditions. In the present proposal, all the processes allow us to realize, manipulate, and detect the MFs readily.

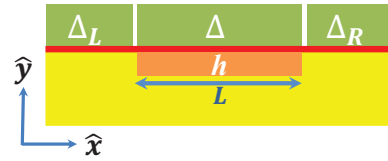


FIG. 1: (Color online) A half-infinite 2D topological insulator (yellow region) and its 1D edge (red line) along the \hat{x} direction. Three blocks of superconductors (green blocks) are fabricated on its top to form two superconducting junctions at $x = 0$ and $x = L$. A local Zeeman field along the \hat{y} direction is applied to the middle (brown) region of $0 < x < L$.

It has been shown theoretically and experimentally that the quantum spin Hall effect can be realized in HgTe/(Hg,Cd)Te quantum wells and the gapless edge state is protected by time reversal symmetry [22, 23]. In Fig. 1, we consider a half-infinite 2D topological insulator, and its edge is in the proximity to three blocks of s -wave superconductors with different phases. The

Hamiltonian of this system may be written as [3]

$$H = \int dx \{ \psi_{\alpha}^{\dagger}(x) (\hat{p}\sigma_1 + h(x)\sigma_2 - \mu)_{\alpha\beta} \psi_{\beta}(x) + \Delta \psi_{\uparrow}^{\dagger}(x) \psi_{\downarrow}^{\dagger}(x) + \Delta^* \psi_{\downarrow}(x) \psi_{\uparrow}(x) \}. \quad (1)$$

Here the first term is the Hamiltonian of the 1D edge state with a uniform Zeeman field h along the the y direction for $0 < x < L$, in which $\hat{p} = -i\partial_x$ is the momentum operator, σ_i are the spin Pauli matrices, μ is the Fermi energy. Also, $\hbar = 1$ s and Fermi velocity $v_F \equiv 1$ have been taken. Hamiltonian (1) is not invariant under time reversal because of the presence of Zeeman field h . The superconducting order parameter is given by

$$\Delta(x) = \begin{cases} \Delta_0 e^{i\varphi_L} & x < 0, \\ \Delta_0 & 0 < x < L, \\ \Delta_0 e^{i\varphi_R} & x > L, \end{cases} \quad (2)$$

as shown in Fig. 1, with $\Delta_0 > 0$ and φ 's as the phase of each superconducting region. Obviously, the charge conjugation symmetry is preserved. The quasiparticle operator in the Nambu representation $|\Psi(x)\rangle = [\psi_{\uparrow}(x), \psi_{\downarrow}(x), \psi_{\downarrow}^{\dagger}(x), -\psi_{\uparrow}^{\dagger}(x)]^T$ is defined as $\gamma = \int dx \{ u_{\uparrow}^* \psi_{\uparrow}(x) + u_{\downarrow}^* \psi_{\downarrow}(x) + v_{\downarrow}^* \psi_{\downarrow}^{\dagger}(x) - v_{\uparrow}^* \psi_{\uparrow}^{\dagger}(x) \}$. When the quasiparticle has relation $\gamma = \gamma^{\dagger}$, it is a neutral MF. The fact that quasiparticle annihilates itself leads to $u_{\uparrow} = -v_{\uparrow}^*$ and $u_{\downarrow} = v_{\downarrow}^*$. By calculating the equation of motion given by $E\gamma = [\gamma, H]$, we recover the form of Bogoliubov-de Gennes (BdG) Hamiltonian in $(u_{\uparrow}, u_{\downarrow}, v_{\downarrow}, v_{\uparrow})^T$ as

$$\mathcal{H}_{BdG} = \hat{p}\sigma_1\tau_3 + h(x)\sigma_2 - \mu\tau_3 + \mathbf{Re}\Delta\tau_1 - \mathbf{Im}\Delta\tau_2. \quad (3)$$

Here τ is the Pauli matrices in the Nambu representation. It has been pointed out that the Dirac field with the s -wave superconducting order parameter is equivalent to that of a $p + ip$ superconductor that has zero-mode MFs [24–26]. In Eq. (3), the existence of zero modes relies only on vanishing of the determinant for Hamiltonian (3). As a result, the wave vector is readily solved as $k = \pm\sqrt{\mu^2 - h^2} \pm i\Delta_0$ in the middle region. The imaginary wave vectors indicate that there are localized states that may give rise to the MFs. It should be noted that the first term of Hamiltonian (1) in the absence of Δ_0 breaks the time reverse symmetry and yields two Fermi zero modes localized at $x = 0$ and $x = L$, which decay as $e^{-|h|x}$. If we choose that superconducting order parameters are introduced at $\mu = 0$, only two Fermi zero modes have contribution to superconducting condensation. When $\mu = 0$, the continuous spectrum of Eq. (3) in the middle region is given by $E = \sqrt{k^2 + (h \pm \Delta_0)^2}$ with two energy gaps $\Delta_{\pm} = |h \pm \Delta_0|$. Obviously, the topological phase transition happens when the gap is closed at $h = \pm\Delta_0$.

We first focus on the left junction at $x = 0$ in the case of $h > \Delta_0$. Since the left domain in Fig. 1 is

free of the Zeeman field, the quasiparticle spectrum for plane waves is simply given by $E = \sqrt{k^2 + \Delta_0^2}$. For the bound state of $E = 0$, Hamiltonian (3) can be solved to yield $(u_{\uparrow(\downarrow)}, v_{\uparrow(\downarrow)})^T = e^{\pm x\Delta_0}(1, \pm ie^{-i\varphi_L})^T$ for $x < 0$. The wavefunction proportional to $e^{-x\Delta_0}$ is invalid because it diverges as $x \rightarrow -\infty$, so that we have $(u_{\uparrow(\downarrow)}, v_{\uparrow(\downarrow)})^T = e^{x\Delta_0}(1, ie^{-i\varphi_L})^T$. For the zero-energy mode of $x > 0$, one finds that solutions are decoupled into the spin-up and spin-down components in Eq.(3), yielding $(u_{\uparrow}, v_{\uparrow})^T = Ae^{x\lambda_+}(1, i)^T + Be^{x\lambda_-}(1, -i)^T$ and $(u_{\downarrow}, v_{\downarrow})^T = Ce^{-x\lambda_+}(1, -i)^T + De^{-x\lambda_-}(1, i)^T$ with $\lambda_{\pm} = (h \pm \Delta_0)/\hbar v_F$. Since the solution in spin-up component diverges as $e^{x\lambda_{\pm}}$ ($\lambda_{\pm} > 0$ as $h > \Delta_0$) for $x \rightarrow \infty$, we have $A = B = 0$, and coefficients C and D are determined by the boundary condition of wavefunction continuousness at $x = 0$. As a result, after gauging away the phase factor, the wavefunction of $E = 0$ in the left junction is obtained as

$$|\psi_L\rangle = e^{x\Delta_0}(0, e^{i(\frac{\varphi_L}{2} - \frac{\pi}{4})}, e^{-i(\frac{\varphi_L}{2} - \frac{\pi}{4})}, 0)^T \quad (4)$$

for $x < 0$ and

$$|\psi_L\rangle = e^{-x\lambda_+} \sin \frac{\varphi_L}{2} (0, e^{\frac{\pi}{4}i}, e^{-\frac{\pi}{4}i}, 0)^T + e^{-x\lambda_-} \cos \frac{\varphi_L}{2} (0, e^{-\frac{\pi}{4}i}, e^{\frac{\pi}{4}i}, 0)^T \quad (5)$$

for $x > 0$. It can be readily shown that the quasiparticle operator defined above as $\gamma_L = \langle \psi_L | \Psi \rangle$ satisfies relation $\gamma_L = \gamma_L^{\dagger}$, and so there is an MF in spin-down component localized at $x = 0$.

The same approach can be applied to the right junction at $x = L$. It is found that there exists an MF in spin-up component located at $x = L$, whose wavefunctions are obtained as

$$|\psi_R\rangle = e^{(x-L)\lambda_+} \sin \frac{\varphi_R}{2} (e^{\frac{\pi}{4}i}, 0, 0, -e^{-\frac{\pi}{4}i})^T + e^{(x-L)\lambda_-} \cos \frac{\varphi_R}{2} (e^{-\frac{\pi}{4}i}, 0, 0, -e^{\frac{\pi}{4}i})^T \quad (6)$$

for $x < L$, and

$$|\psi_R\rangle = e^{-(x-L)\Delta_0} (e^{(\frac{1}{2}\varphi_R - \frac{\pi}{4})i}, 0, 0, -e^{-(\frac{1}{2}\varphi_R - \frac{\pi}{4})i})^T \quad (7)$$

for $x > L$. Obviously, $\gamma_R = \langle \psi_R | \Psi \rangle$ satisfies $\gamma_R = \gamma_R^{\dagger}$. Away from each interface, the MF has two decay lengths: $1/\lambda_+$ and $1/\lambda_-$, which are closely related to two energy gaps $\Delta_{\pm} = |h \pm \Delta_0|$. Which one dominates the decay rate of the MF depends upon the phase difference of the junction. For example, if $\varphi_L = 2N\pi$ with N an arbitrary integer, the left MF decays as $e^{-x\lambda_-}$ for $x > 0$, while at $\varphi_L = (2N + 1)\pi$, it decays as $e^{-x\lambda_+}$ for $x > 0$.

In Fig. 2, we plot zero-energy quasiparticle probability $|u|^2$ for $h > \Delta_0$ (a) and $h < -\Delta_0$ (b) as $\varphi_L = \varphi_R = 0$. As shown in Fig. 2(a), there are two MFs localized at $x = 0$ and $x = L$ for $h > \Delta_0$. As the magnitude of the Zeeman field is turned down, the MFs become

more and more extended. And when $|h| < \Delta_0$, the two MFs are annihilated on the junctions and fused. Such a fusion of MFs arises from the sign reverse of λ_- at $h = \Delta_0$ (λ_+ at $h = -\Delta_0$) due to the closing of energy gap Δ_- (Δ_+). For $|h| < \Delta_0$, since $\lambda_+ > 0$ and $\lambda_- < 0$, the second terms in Eqs. (5) and (6) would diverge as $x \rightarrow \infty$, and so the solution for the MFs would be an unphysical result. Therefore, there is no MF for $|h| < \Delta_0$, and there appears a topological transition at $|h| = \Delta_0$ from the MF phase to the trivial one without MF. If the direction of the Zeeman field ($h > \Delta_0$) is reversed, we have $h < -\Delta_0$ and so $\lambda_{\pm} < 0$, with the result being shown in Fig. 2(b). In this case, using the same procedure of calculation, the wavefunctions on both sides of each interface at $x = 0$ or $x = L$ can be obtained as follow. The MF located at $x = 0$ is now obtained in the spin-up component, and its wavefunction is given by $|\psi'_L\rangle = e^{x\lambda_-} \sin \frac{\varphi_L}{2} (-e^{-\frac{\pi}{4}i}, 0, 0, e^{\frac{\pi}{4}i})^T + e^{x\lambda_+} \cos \frac{\varphi_L}{2} (e^{\frac{\pi}{4}i}, 0, 0, -e^{-\frac{\pi}{4}i})^T$ for $x > 0$. Compared with Eq. (5), one finds that the direction reverse of the exchange field leads to that the MF in the spin-down component is replaced by an MF in the spin-up component, accompanied with an exchange of $\exp(-x\lambda_{\pm})$ and $\exp(x\lambda_{\mp})$ due to the sign reverse of λ_{\pm} . Similarly, the wavefunction of the MF located at $x = L$ is obtained in the spin-down component as $|\psi'_R\rangle = -e^{-(x-L)\lambda_-} \sin \frac{\varphi_R}{2} (0, e^{-\frac{\pi}{4}i}, e^{\frac{\pi}{4}i}, 0)^T + e^{-(x-L)\lambda_+} \cos \frac{\varphi_R}{2} (0, e^{\frac{\pi}{4}i}, e^{-\frac{\pi}{4}i}, 0)^T$ for $x < L$. It can also be obtained by performing a symmetry operation to recover the solution of a reversed exchange field. Hamiltonian (1) satisfies $UH(h)U^\dagger = H(-h)$ with $U = \hat{\mathcal{R}}\hat{\mathcal{O}}$ where $\hat{\mathcal{R}}$ is a spin-rotation of π around \hat{x} and $\hat{\mathcal{O}}$ is the central inversion with $x \rightarrow L - x$. It is obvious that $|\psi'_{L,R}\rangle \sim U|\psi_{L,R}\rangle$.

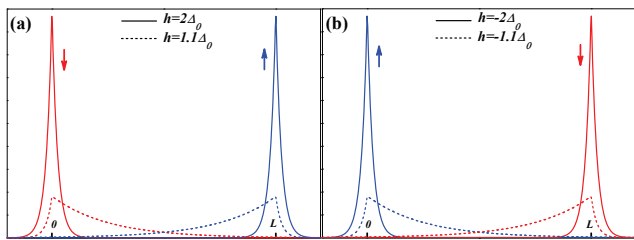


FIG. 2: (Color online) Space distribution of probability density $|u|^2$ of the MFs for different parities (red and blue lines) with $\varphi_L = \varphi_R = 0$. The exchange field is taken to be $h = 2\Delta_0$ (solid line) and $h = 1.1\Delta_0$ (dashed line) in (a), and $h = -2\Delta_0$ (solid line) and $h = -1.1\Delta_0$ (dashed line) in (b).

The above discussion is suitable to the case of $\varphi_{L,R} \neq (2N + 1)\pi$. For $\varphi_{L,R} = (2N + 1)\pi$, the second terms of Eqs. (5) and (6) vanish, and the calculated results for zero-energy quasiparticle distributions are plotted in Fig. 3. It is found that there exist still two MFs on the junctions for $h > \Delta_0$. The essential difference is that the MF peaks have merely a very slight extension with

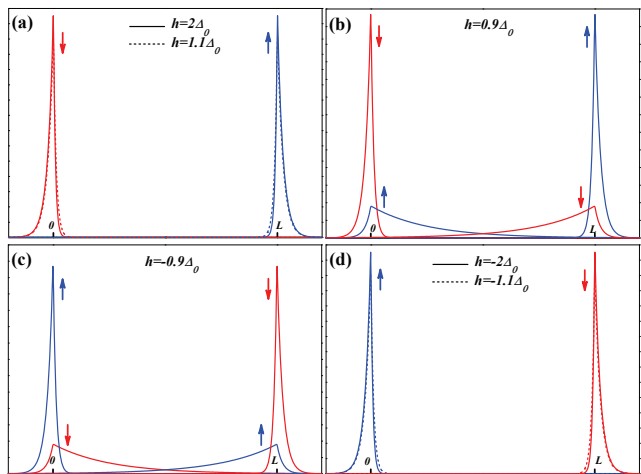


FIG. 3: (Color online) Space distribution of $|u|^2$ for different parities (red and blue lines) with $\varphi_L = \varphi_R = \pi$. The exchange field is taken to be $h = 2\Delta_0$ (solid line) and $h = 1.1\Delta_0$ (dashed line) in (a), $h = 0.9\Delta_0$ in (b), $h = -0.9\Delta_0$ in (c), and $h = -2\Delta_0$ (solid line) and $h = -1.1\Delta_0$ (dashed line) in (d).

decreasing h , as shown in Fig. 3(a) and (b). At the same time, another pair of MFs are generated just after closing energy gap $\Delta_- = |h - \Delta_0|$ so as to form two Dirac fermions at $x = 0$ and $x = L$. Such a novel behavior can be understood by the following argument. Taking the left junction for example again, the MF state in Eq. (7) decays as $e^{-x\lambda_+}$ for $x > 0$. For $|h| < \Delta_0$, regardless of the closing of gap Δ_- , $\Delta_+ = h + \Delta_0$ makes the original MF survive, for this MF is protected only by gap Δ_+ . More interestingly, there appears an additional MF in the spin-up component at $x = 0$, whose wavefunction is proportional to $e^{x\lambda_-}$ with $\lambda_- < 0$. This MF must be combined with the original one, forming a Dirac fermion. Since a change of the phase for a superconductor will not close energy gap of the bulk, the present phase with the Dirac fermion located at the junction for $\varphi_{L,R} = (2N + 1)\pi$ is topologically equivalent to that without Dirac fermion there for $\varphi_{L,R} \neq (2N + 1)\pi$. We wish to point out here that the MF in the spin-up component at $x = 0$ exists only for $\Delta_0 > h$, and annihilates for $h > \Delta_0$ due to the sign reverse of λ_- at $h = \Delta_0$. The same argument is also applied to the right junction. As a result, a pair of Dirac fermions are formed in Fig. 3 (b). For $h < 0$, the particle distribution is shown in Figs. 3(c) and (d). With increasing the magnitude of $h < 0$, the topological phase transition occurs once again at $h = -\Delta_0$.

For $\varphi_L = (2N + 1)\pi$ and $\varphi_R \neq (2N + 1)\pi$, the situation is also interesting, and the space distribution of $|u|^2$ with different Zeeman fields is plotted in Fig. 4. For $h > \Delta_0$ in Fig. 4(a), the right MF extends rapidly into the bulk with decreasing h , whereas the left MF remains almost unchanged. As h is less than Δ_0 , the MF on the right junction is annihilated and at the same time another MF is created on the left junction, as shown in Fig. 4(b).

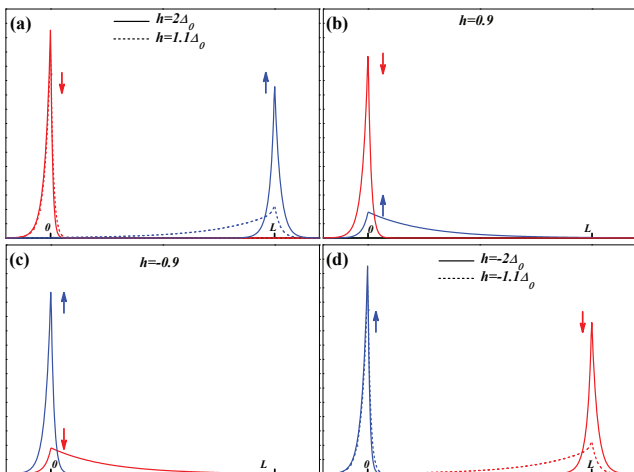


FIG. 4: (Color online) Space distribution of $|u|^2$ for different parities (red and blue lines) with $\varphi_L = \pi$ and $\varphi_R = \pi/3$. The exchange field is set $h = 2\Delta_0$ (solid line) and $h = 1.1\Delta_0$ (dashed line) in (a), $h = 0.9\Delta_0$ in (b), $h = -0.9\Delta_0$ in (c), and $h = -2\Delta_0$ (solid line) and $h = -1.1\Delta_0$ (dashed line) in (d).

This evolution is equivalent to the process that the MF on the right junction is driven to the left junction and two MFs there are combined to form one Dirac fermion. As h is reversed, the two MFs that combine into one Dirac fermion located at $x = 0$ exchanges their magnitudes [see Fig. 4(c)]; and as $h \leq -\Delta_0$, an MF moves back to the right junction and the system reenters the topological phase [see Fig. 4(d)]. The underlying physics has been discussed above, and will not be repeated here. The evolution from MFs to Dirac fermions in Fig. 4 can be used to detect the physical state of MFs. We can drive them to form Dirac fermions for detection and initialize MFs between subspaces of different parities.

The MFs on the right and left junctions are coupled with each other, i.e., $\langle \psi_L | \mathcal{H}_{BDG} | \psi_R \rangle \neq 0$. For $h > \Delta_0$, $|\psi_L\rangle$ and $|\psi_R\rangle$ have been given by Eqs. (4) and (5), and (6) and (7), respectively. It can be shown that such a coupling depends to a great degree upon phase differences of the two junctions, and it will vanish if the following condition is satisfied,

$$\cot \frac{\varphi_L - \varphi}{2} \cot \frac{\varphi - \varphi_R}{2} = e^{-2L/\xi}, \quad (8)$$

where $\xi = \hbar v_F / \Delta_0$ is the superconducting coherent length. In this case, we have a pair of zero-energy MFs decoupled exactly, such as those in Figs. (3) and (4) where $\varphi_L = \pi$ and/or $\varphi_R = \pi$ and $L \gg \xi$. If condition (8) is not satisfied, the coupling will make the MFs have a small departure from zero energy, proportional to $e^{-L/\xi}$.

In summary, we have shown that the edge state of a 2D topological insulator in the proximity with s -wave superconductors and under a vertical Zeeman field may accom-

modate MFs. The MFs can be manipulated by tuning the phase differences of the junction to either be fused or form one Dirac fermion on the π -junction. As the exchange field becomes less than the superconducting order parameter, one MF can be driven from a non- π -junction to a π -junction, forming one Dirac fermion on the π -junction. There exhibits a topological phase transition at $|h| = \Delta_0$. For $|h| < \Delta_0$, there are three topologically equivalent phases with different phase differences: one and two Dirac fermions localized on the π -junctions, and none. We have also illustrated a condition that decouples completely the two MFs at two junctions regardless of the magnitude of the exchange field.

Acknowledgment We would like to thank Shi-Liang Zhu for helpful discussions. This work is supported by the GRF (HKU7058/11P) and CRF (HKU-8/11G) of the RGC of Hong Kong, the State Key Program for Basic Researches of China under Grant Nos. 2011CB922103 and 2010CB923400 (DYX), by the National Natural Science Foundation of China under Grant Nos. 10874066, 11074110 (LS), 11174125, 11074109, and 91021003 (DYX), and by a project funded by the PAPD of Jiangsu Higher Education Institutions.

* Electronic address: lbshao@nju.edu.cn

† Electronic address: zwang@hku.hk

- [1] L. Fu, and C. L. Kane, Phys. Rev. Lett. **100**, 096407 (2008).
- [2] Y. Tanaka, T. Yokoyama and N. Nagaosa, Phys. Rev. Lett. **103**, 107002 (2009).
- [3] A. R. Akhmerov, J. Nilsson, and C. W. J. Beenakker, Phys. Rev. Lett. **102**, 216404 (2009).
- [4] K. T. Law, P. A. Lee and T. K. Ng, Phys. Rev. Lett. **103**, 237001 (2009).
- [5] J. D. Sau, R. M. Lutchyn, S. Tewari and S. Das Sarma, Phys. Rev. Lett. **104**, 040502 (2010).
- [6] D. A. Ivanov, Phys. Rev. Lett. **86**, 268 (2001).
- [7] A. Kitaev, Ann. Phys. **321**, 2 (2006).
- [8] A. Stern, F. von Oppen, and E. Mariani, Phys. Rev. B **70**, 205338 (2004).
- [9] G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **70**, 601 [JETP Lett. **70**, 609] (1999).
- [10] G. E. Volovik, *The Universe in a Helium Droplet* (Oxford University Press, New York City) (2003).
- [11] A. Kitaev, Phys. -Usp. **44**, 131 (2001).
- [12] X. L. Qi, T. L. Hughes, and S. C. Zhang, Phys. Rev. B **82**, 184516 (2010).
- [13] M. Sato, Y. Takahashi and S. Fujimoto, Phys. Rev. Lett. **103**, 020401 (2009).
- [14] S. -L. Zhu, L. B. Shao, Z. D. Wang and D. M. Duan, Phys. Rev. Lett. **106**, 100404 (2011).
- [15] C. Chamon, R. Jackiw, Y. Nishida, S. Y. Pi, and L. Santos, Phys. Rev. B **81**, 224515 (2010).
- [16] Liang Fu, and C. L. Kane, Phys. Rev. Lett. **102**, 216403 (2009).
- [17] Liang Fu, Phys. Rev. Lett. **104**, 056402 (2010).
- [18] K. Flensberg, Phys. Rev. Lett. **106**, 090503 (2011).

- [19] J. Alicea, Y. Oreg, G. Refael, F. von Oppen, and M. P. A. Fisher, *Nature Phys.* **7**, 412 (2011).
- [20] V. Mourik, K. Zuo, S. M. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, *Science* **336**, 1003 (2012).
- [21] A. M. Black-Schaffer, and J. Linder, *Phys. Rev. B* **83**, 220511 (2011).
- [22] B. A. Bernevig, T. L. Hughes, and S. C. Zhang, *Science* **314**, 1757 (2006).
- [23] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Büch-
mann, L. W. Molenkamp, X. L. Qi, and S. C. Zhang,
Science **318**, 766 (2007).
- [24] R. M. Lutchyn, J. D. Sau, and S. Das Sarma, *Phys. Rev. Lett.* **105**, 077001 (2010).
- [25] Y. Oreg, G. Refael, and F. Oppen, *Phys. Rev. Lett.* **105**, 177002 (2010).
- [26] X. L. Qi, and S. C. Zhang, *Rev. Mod. Phys.* **83**, 1057 (2011).