ALL NONCAUSAL QUANTUM PROCESSES

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Workshop on Nonlocality, Causal Structures,
and Device-Independent Quantum Information
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MOTIVATION

- Defining a model of
  higher-order quantum computation
  where functions can be treated as variables.
  → quantum functional programming

- Extending quantum theory to scenarios
  with indefinite causal structure.
  What is the most general physical theory that is
  compatible with ordinary quantum theory?
  → kinematics for quantum gravity

A WARM-UP GAME

FORGET EVERYTHING, EXCEPT QUANTUM STATES

Promise: there exists a quantum systems.

Quantum state space = space of density matrices

$$\rho \in \mathsf{M}_d(\mathbb{C})\,, \quad \rho \geq 0\,, \quad \mathrm{Tr}[\rho] = 1$$

Question: what are the most general maps transforming quantum states into quantum states?

ADMISSIBLE MAPS

Admissible map: must send states into states,
even when acting locally on one part of
a composite system

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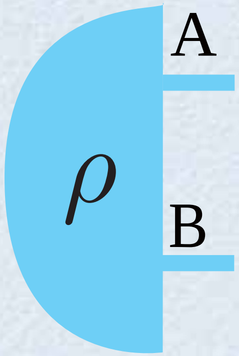

input
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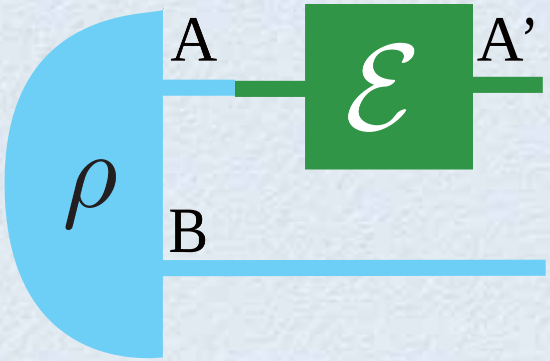

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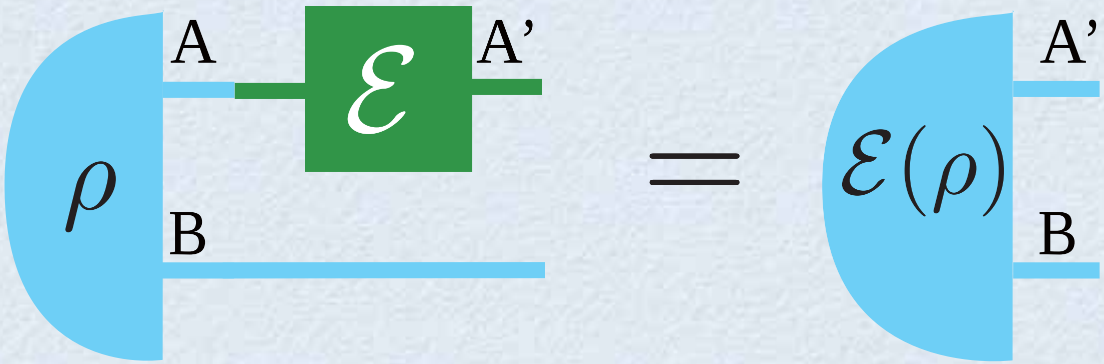

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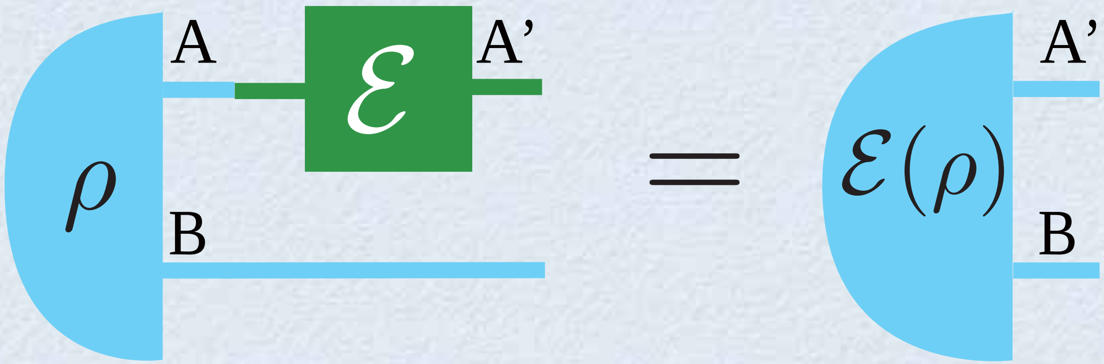

input state

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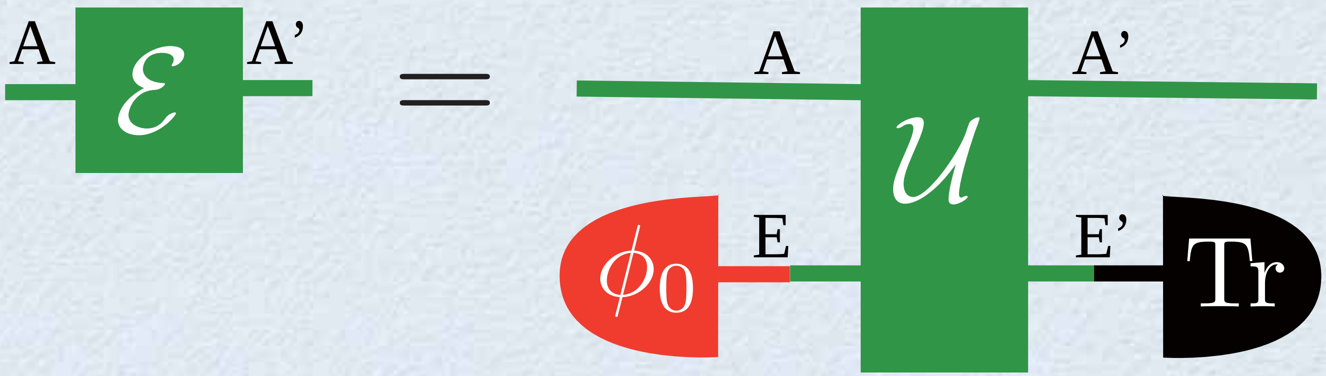
In quantum theory: admissible map = completely positive,
trace-preserving,
linear map
(quantum channel)

ALL ADMISSIBLE MAPS ARE “PHYSICAL”

In quantum theory,
all admissible maps can be realized via reversible evolutions



(Stinespring-Kraus)

SECOND-ORDER QUANTUM THEORY

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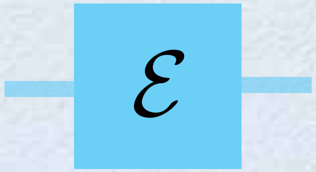

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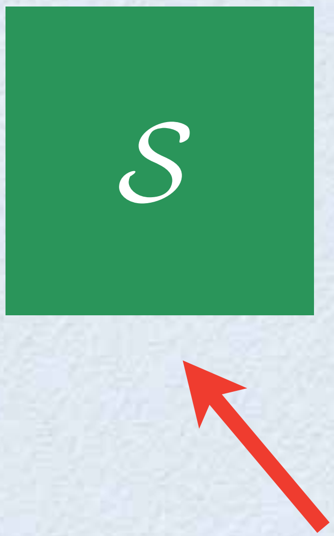

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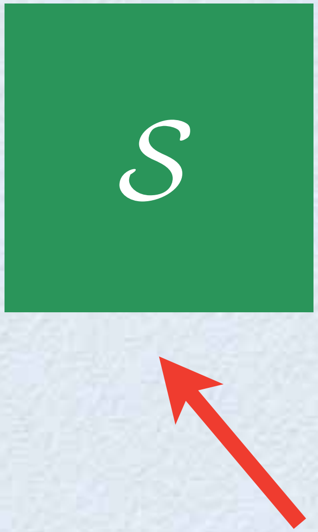

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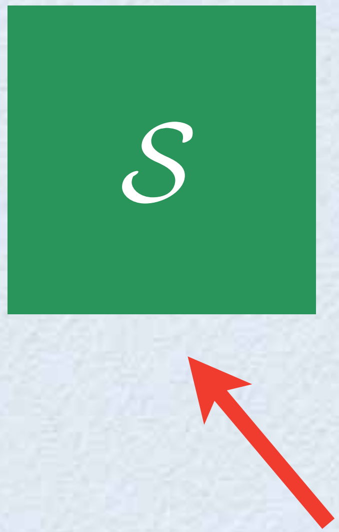

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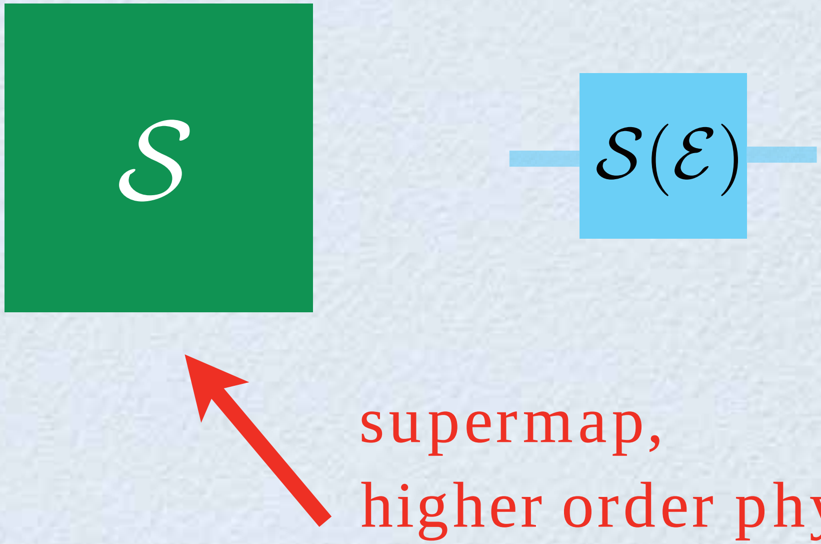

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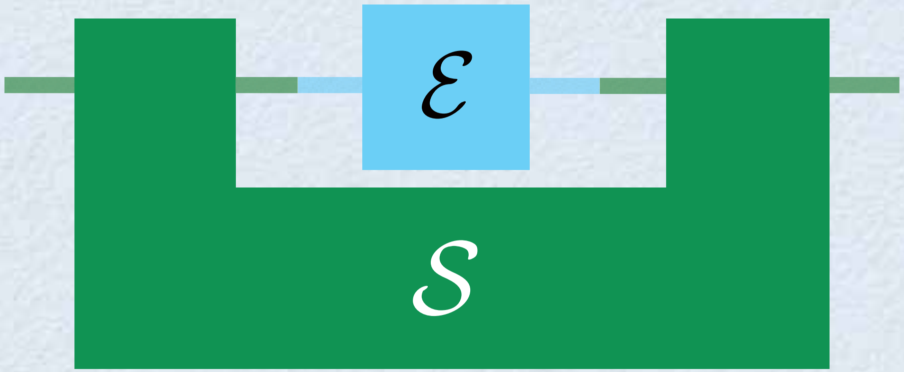

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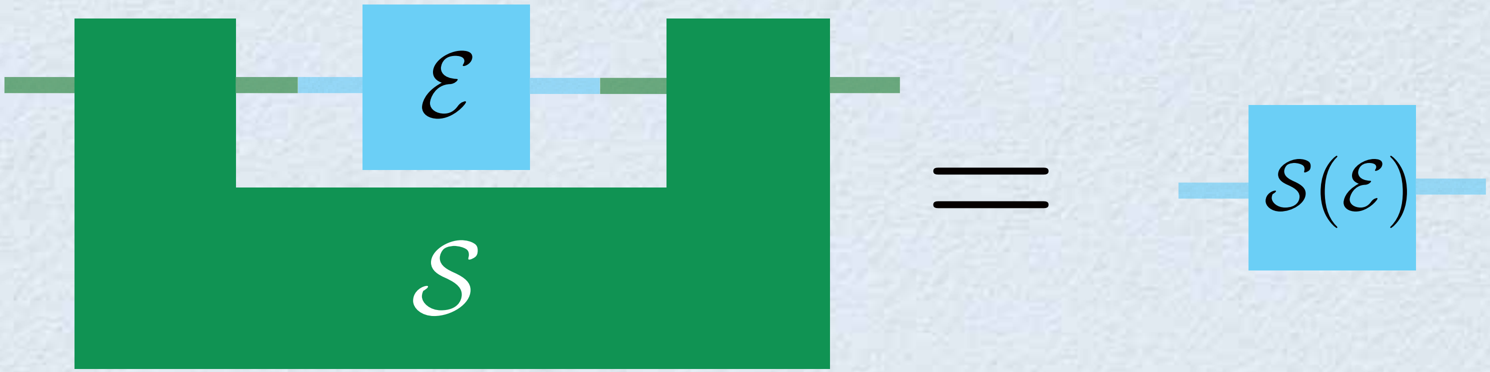

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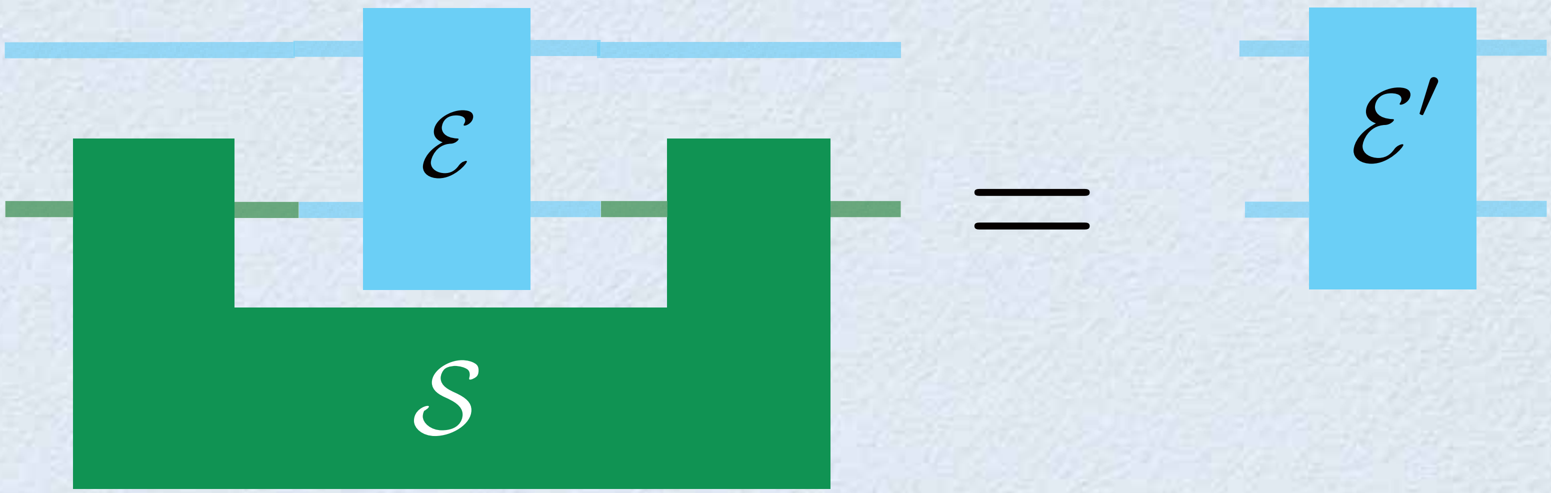
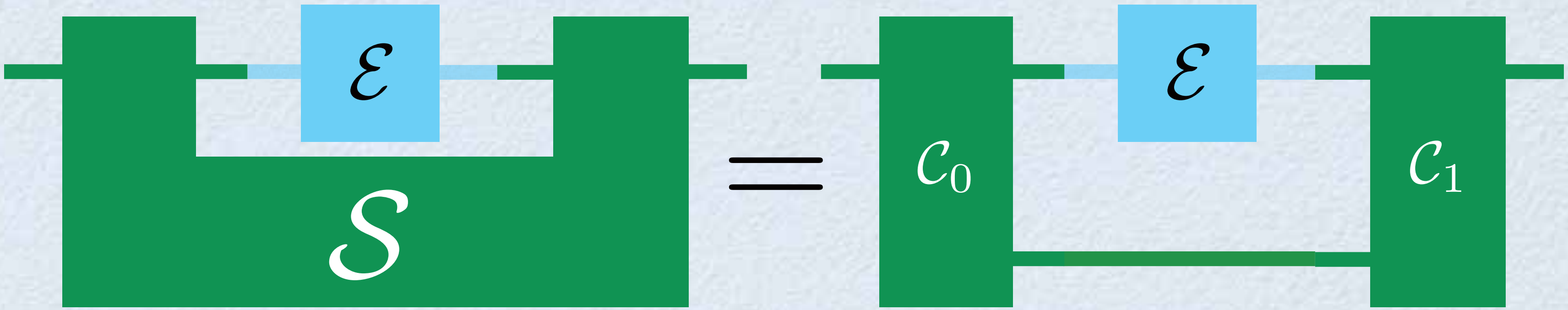


CIRCUIT REALIZATION OF QUANTUM SUPERMAPS

Theorem (Chiribella, D'Ariano, Perinotti, EPL 2008)

in quantum theory

every admissible supermap can be realized by a network of gates



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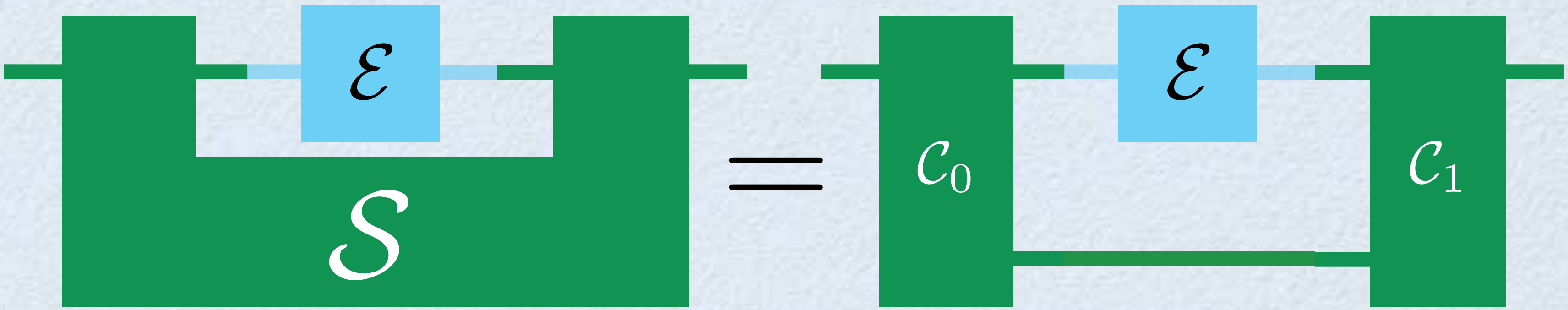


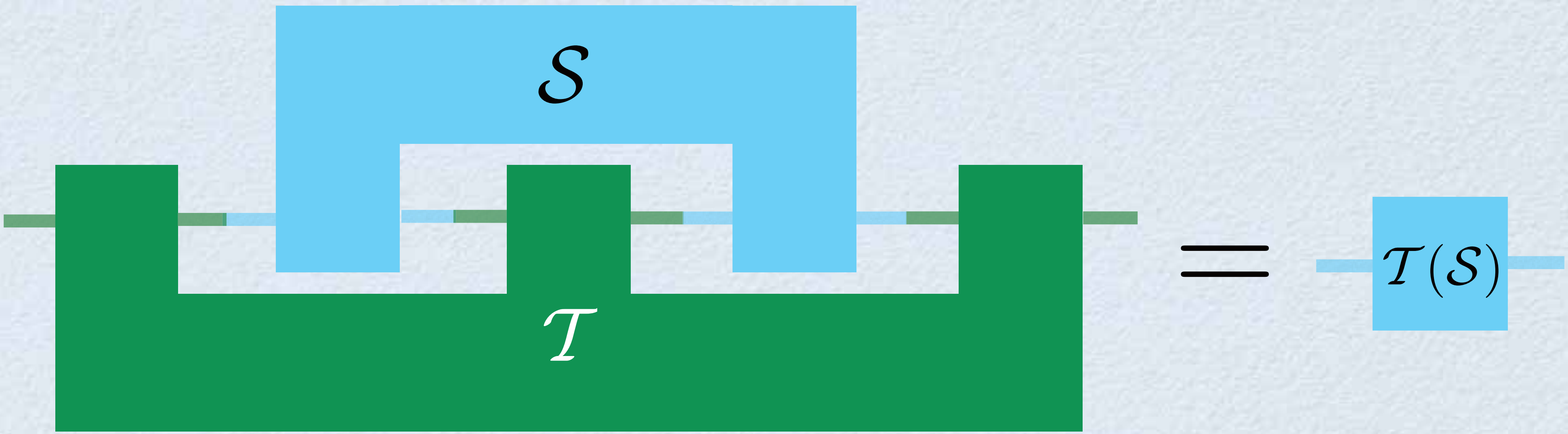
CAVEAT: In a general theory,

some supermaps may not be implemented by circuits.

CLIMBING
UP
THE HIERARCHY
OF
HIGHER ORDER MAPS

THIRD ORDER MAPS

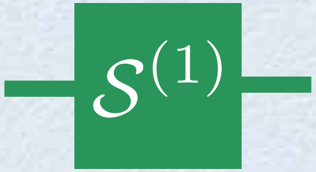
What is the most general transformation that transforms a second-order map into a first-order map?



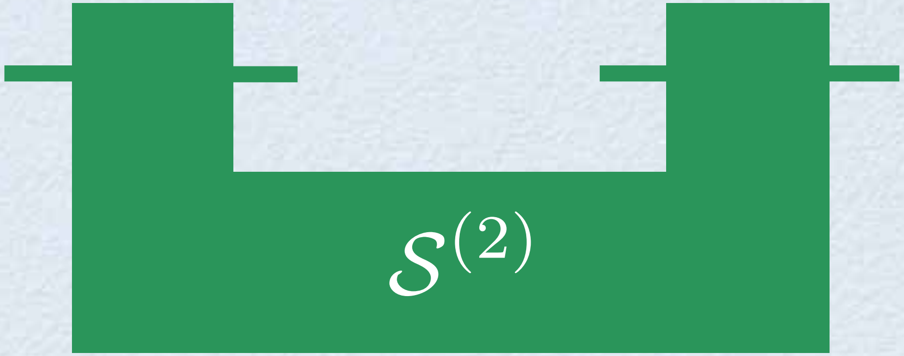
must send valid maps into valid maps, even when acting locally.

N-TH ORDER MAPS

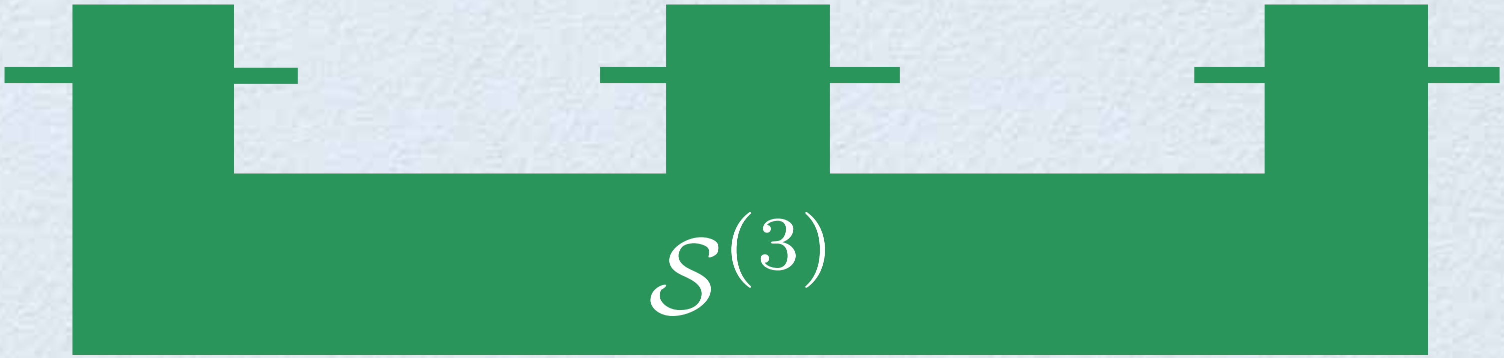
N=1 quantum channel



N=2



N=3



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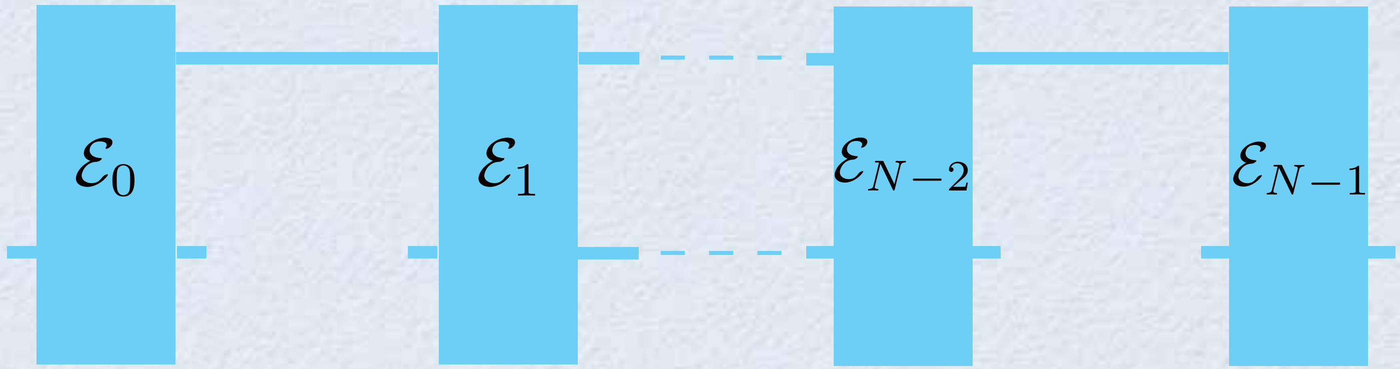

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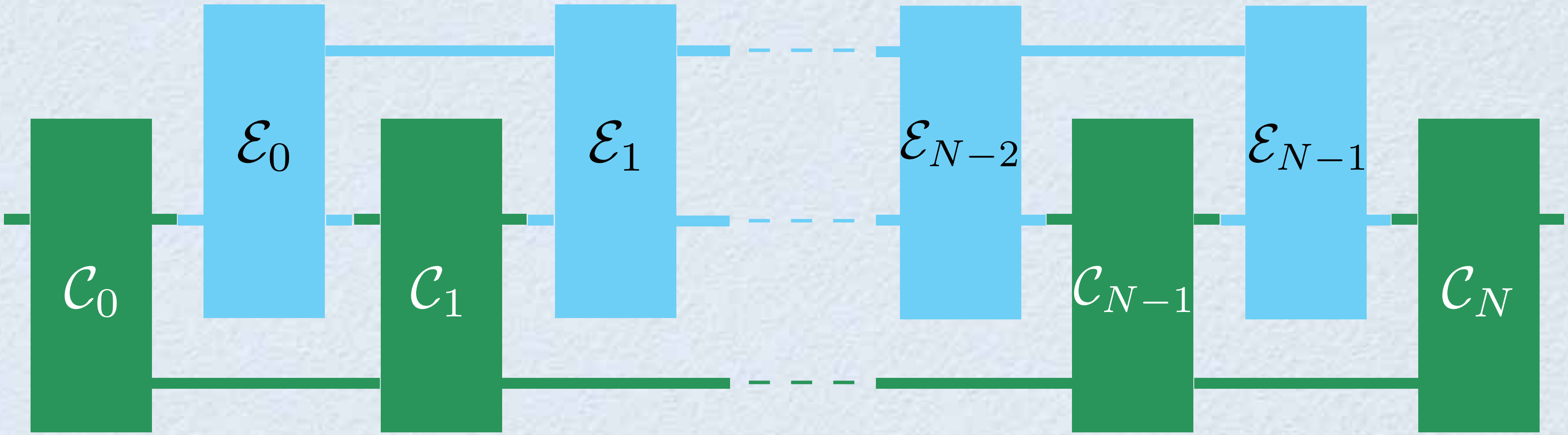


RECONSTRUCTING CAUSAL CIRCUITS

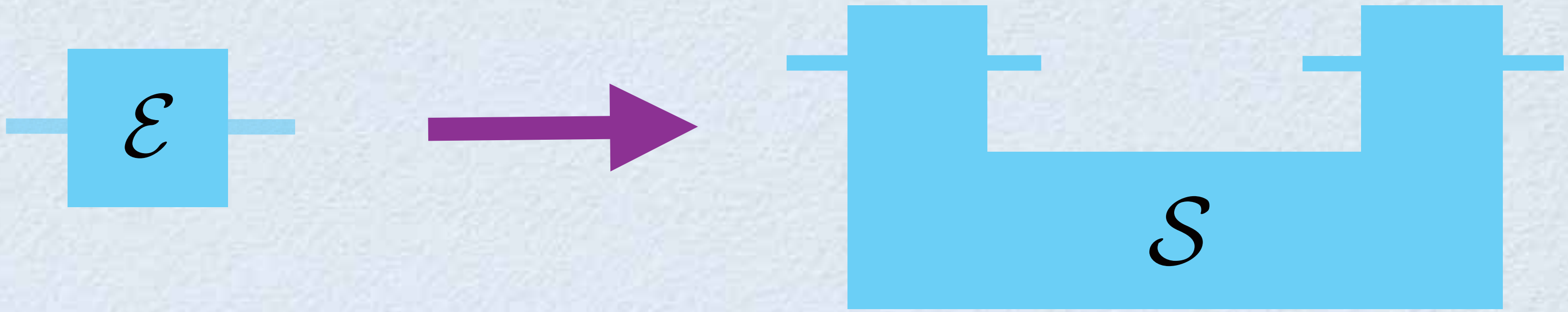
Remember that quantum channels are the most general transformations of quantum states.

Now, just by pure reasoning about higher order computation, we reconstructed causal sequences of quantum channels.

THE NON-CAUSAL LEVELS OF THE HIERARCHY

THE EASIEST NON-CAUSAL EXAMPLE

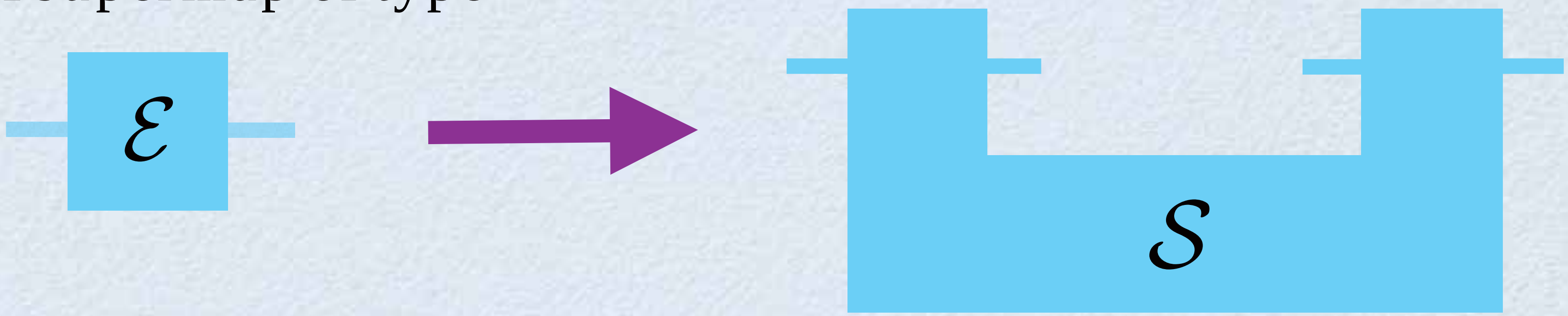
Question: what is the most general transformation that maps a quantum channel into a quantum supermap?



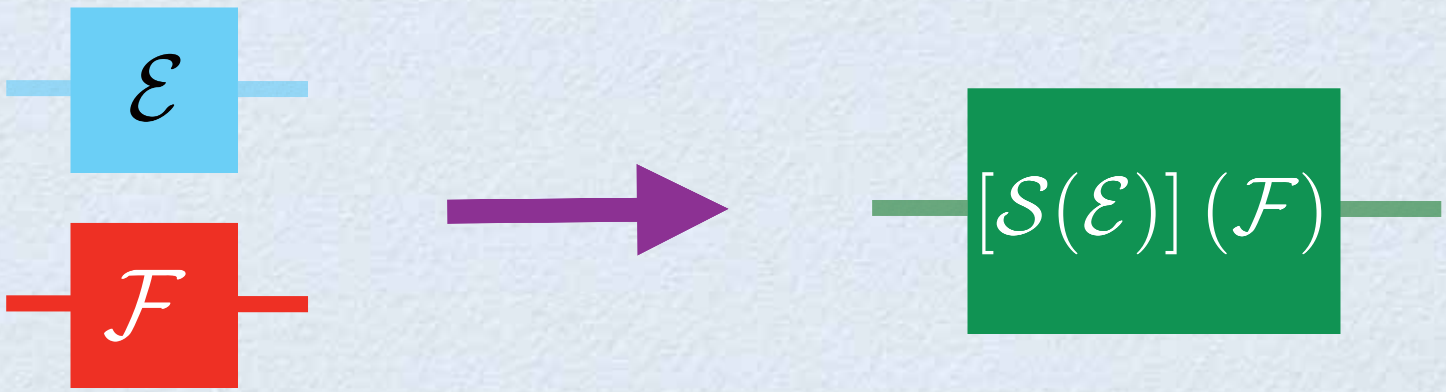
EQUIVALENT FORMULATION

Easy proposition

A supermap of type



is equivalent to a supermap of type



TWO COMPLEMENTARY ORDERS

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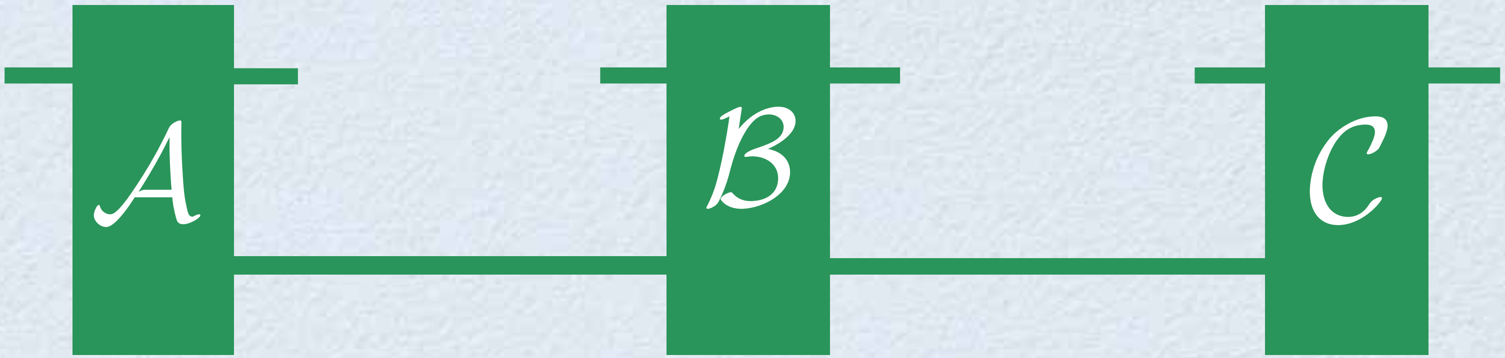

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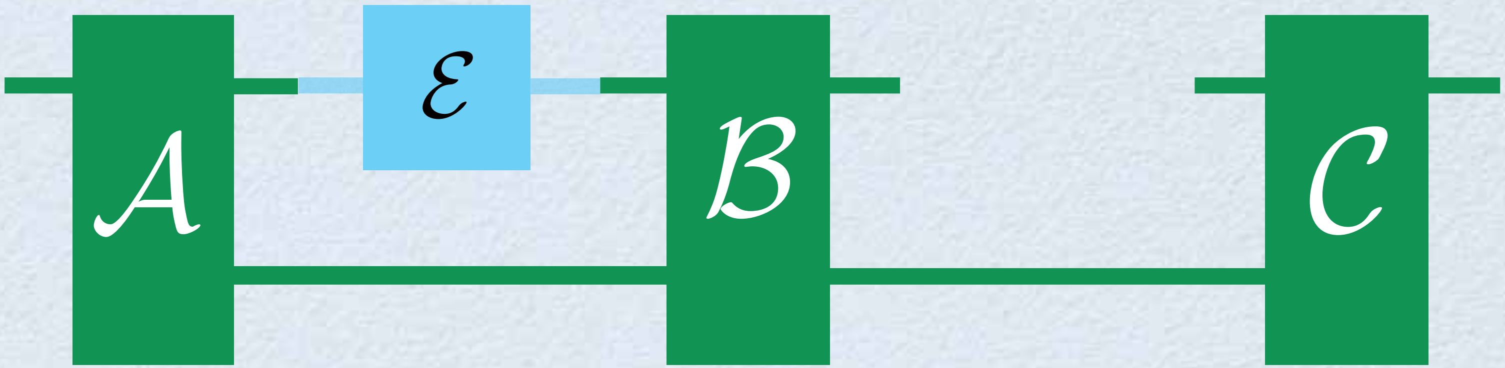

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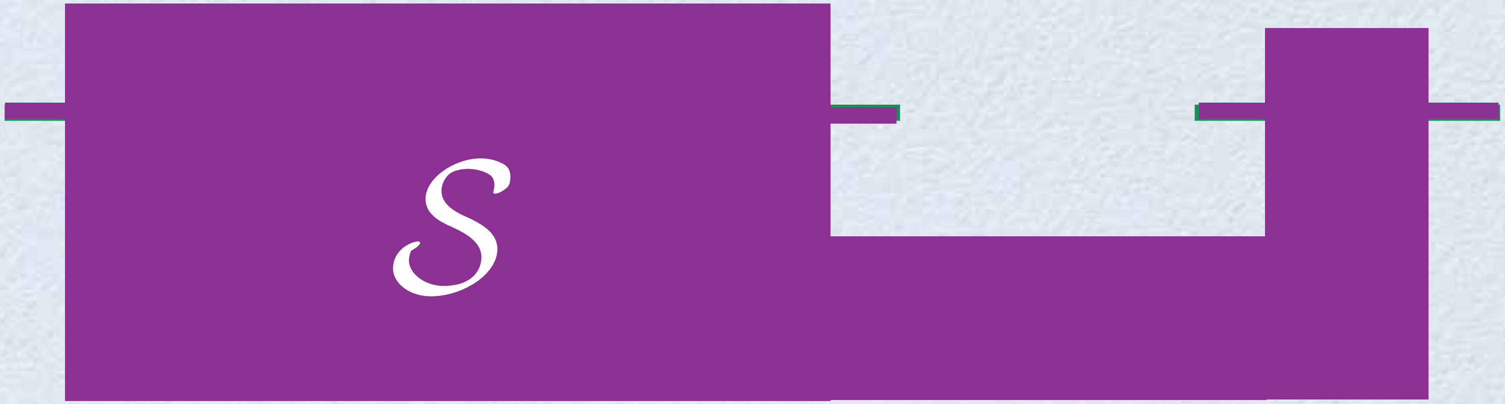

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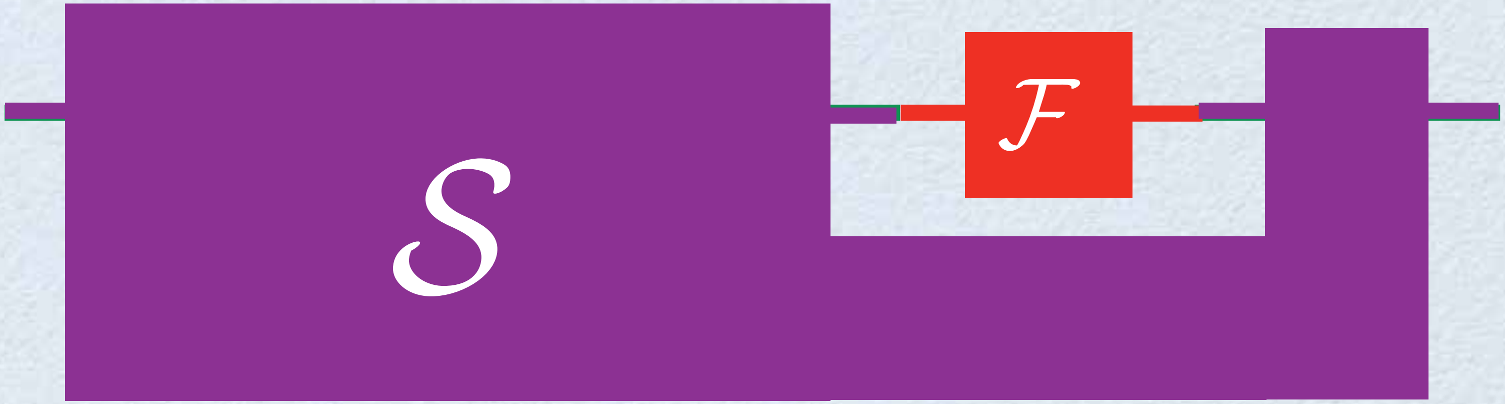

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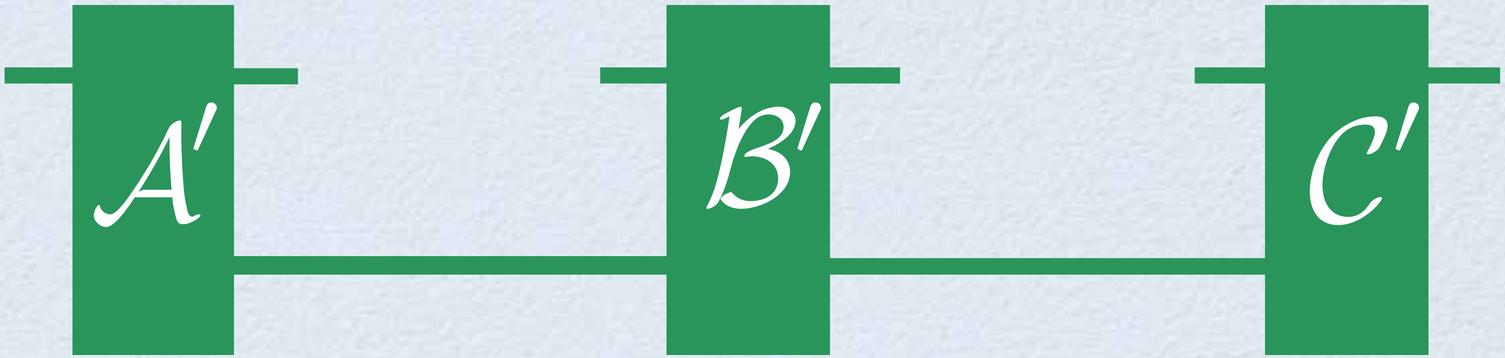

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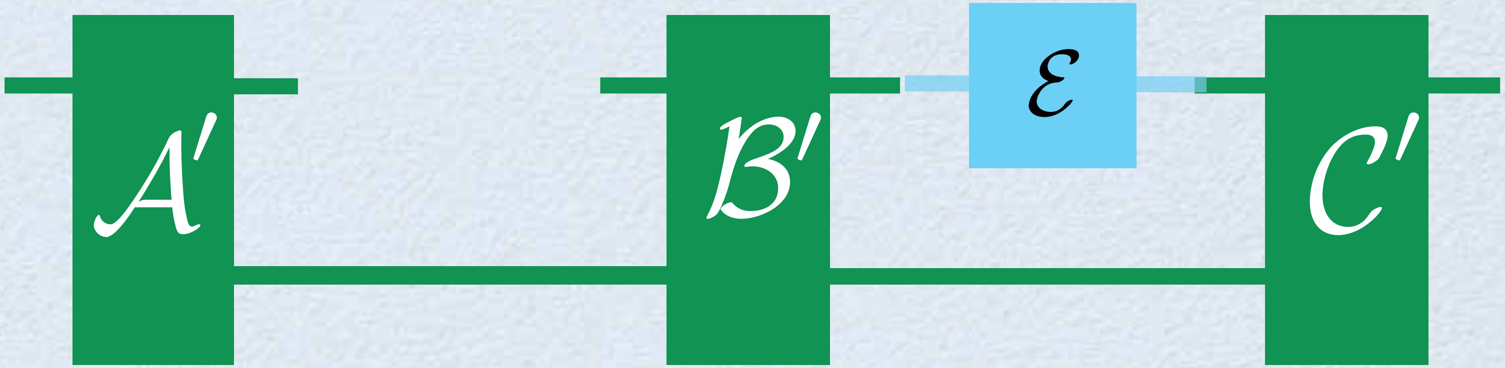

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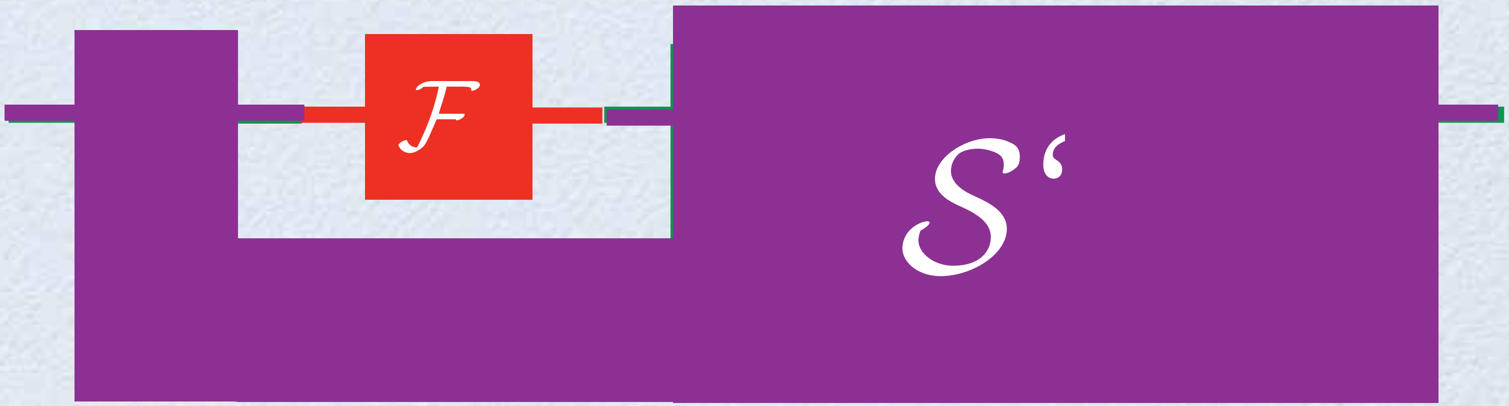
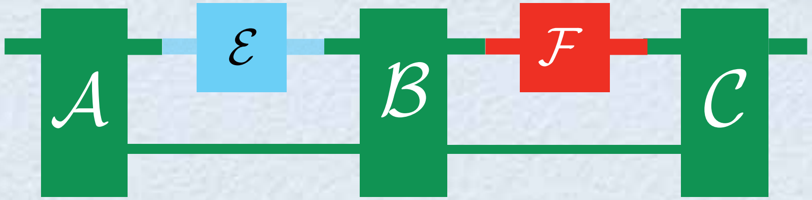


MIXTURE VS SUPERPOSITION OF CAUSAL STRUCTURES

Two complementary choices of circuits:



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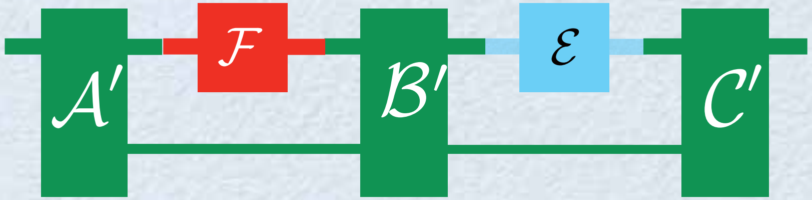

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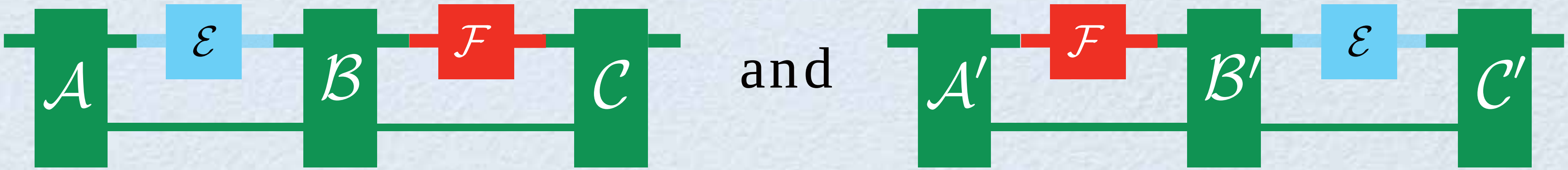

We can choose randomly between these two supermaps.

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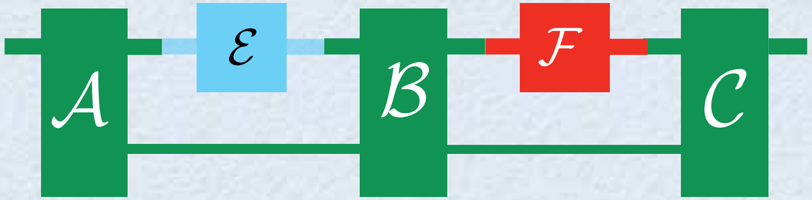

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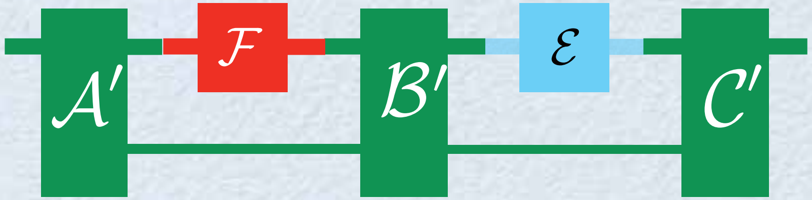

We can choose randomly between these two supermaps.

Furthermore, since quantum mechanics satisfies
the purification principle,
we can find a pure supermap which is
a coherent superposition of the above two.

THE
QUANTUM SWITCH

Chiribella, D'Ariano, Perinotti, Valiron
arXiv:0912.0195 / PRA 2013

THE SUPERMAP "SWITCH"

Suppose we are given two black boxes,
implementing two generic channels $\mathcal{E}$ and $\mathcal{F}$:

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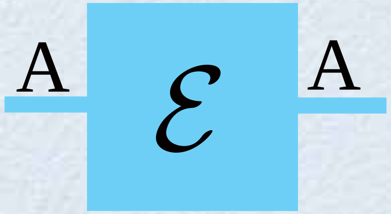
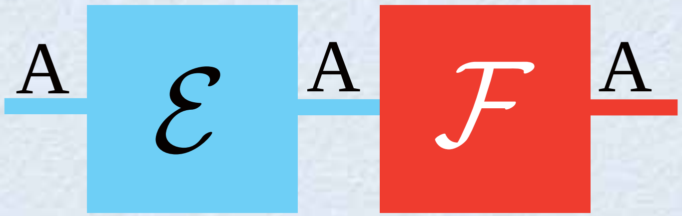
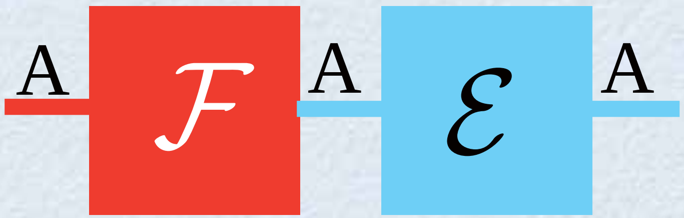


Suppose that we are given a qubit system Q

You want to connect the boxes as



if the state of the qubit is
$\varphi_0 = |0\rangle\langle 0|$



if the state of the qubit is
$\varphi_1 = |1\rangle\langle 1|$

RELATION WITH "TIME TRAVELS"

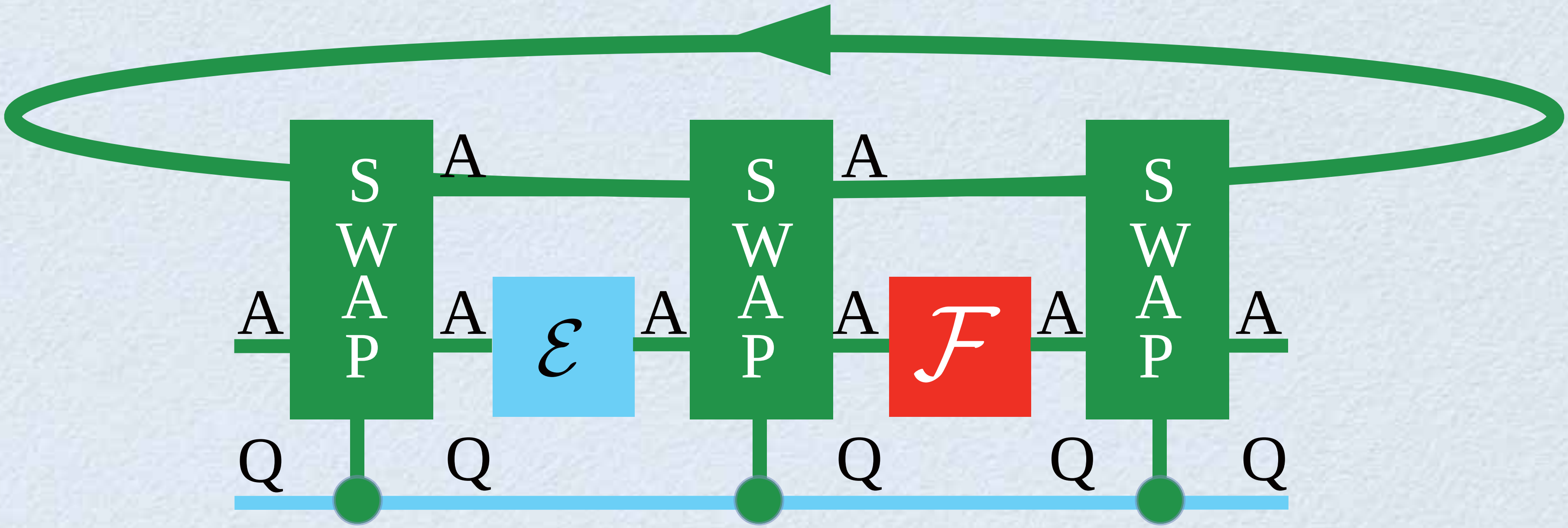
Theorem (GC, D'Ariano, Perinotti, Valiron, 2009)

If a circuit implements the task SWITCH deterministically,
then it must contain a loop.

The converse holds:

If we have access to a circuit with a loop,
then we use it to construct a circuit that implements the task SWITCH.

REALIZATION OF THE SWITCH IN A CIRCUIT WITH LOOP



INFORMATION-THEORETIC ADVANTAGE OF THE QUANTUM SWITCH

GC, PRA 2012

A CLASSIFICATION PROBLEM

Problem: You are given two black boxes

$$\mathcal{E}(\rho) = \sum_i E_i \rho E_i^\dagger \qquad \mathcal{F}(\rho) = \sum_i F_i \rho F_i^\dagger$$

with the following promise:

either (+) $\quad E_i F_j = F_j E_i \qquad \forall i, j$

or (-) $\quad E_i F_j = -F_j E_i \qquad \forall i, j$

Task: Find out whether the two black boxes are of type (+) or type (-)

ADVANTAGE FROM CAUSAL SUPERPOSITION

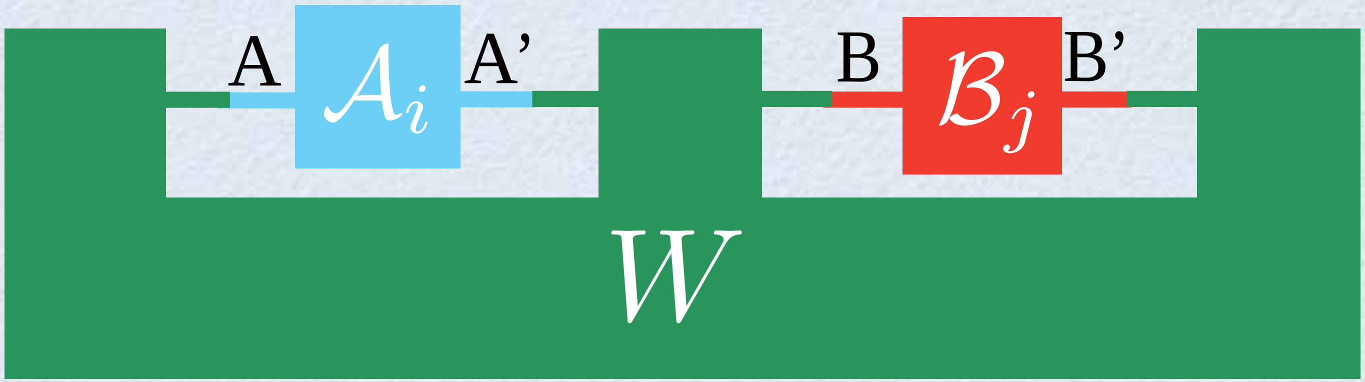
Theorem (GC, PRA 2012): No causal deterministic circuit can perfectly discriminate between the two classes of black boxes (+) and (-) using a single query.

For this classification problem
there is always a non-zero error in the framework of quantum circuits.

cf. Vienna experiment, Nat. Comm. 2015
Innsbruck ion trap proposal, PRA 2014

RELATION WITH W MATRICES

Oreshkov, Costa, Brukner, Nat. Comm. 2011:
there exist correlations that are incompatible with definite causal order.



$$= p(i, j)$$

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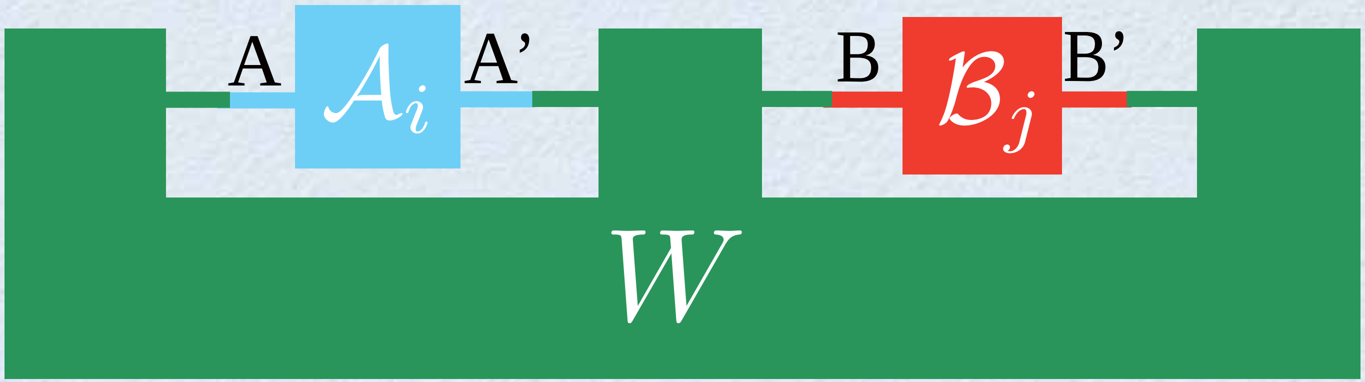

$= p(i, j)$

W matrices = supermaps with trivial output

THE FULL PICTURE

QUANTUM THEORY, BEYOND CAUSAL STRUCTURE

Types of maps:

- Maps of type 0 (quantum states)
- If x and y are allowed types, then (x,y) is an allowed type

Admissible (x,y) maps: all linear maps transforming maps of type x into maps of type y, even when acting locally.

Conjecture: all admissible maps are physically realizable

CONCLUSIONS

- Higher-order computation:
  the notion of admissible supermap

- Reconstructing causal circuits:
  in quantum theory, causal circuits can be retrieved just from the structure of the state space

- Beyond causal circuits:
  higher-order maps incompatible with causal order (e. g. quantum SWITCH),
  complete extension of quantum theory

Conjecture: ALL higher-order maps are physically realizable