Expectation dependence: The banking firm under risk *

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This paper examines the optimal lending and hedging decisions of a bank facing uncertain returns on its loans. The bank's preferences are state-dependent in that the utility function depends on a state variable, i.e. the business cycle of the economy. The purpose of this paper is to complement the results of the banking literaure. To characterize the bank's optimal use of financial instruments to hedge, we show that the concept of expectation dependence (Wright, 1987) is useful. While the current hedging literature specifies price risk as a monotonically increasing or decreasing function of the state-variable plus noise, expectation dependence provides much more general bivariate dependence structure. The bank's optimal futures position is an under-hedge or an over-hedge, depending on whether the random return on loans is positively or negatively correlated with the business cycle of the economy in the sense of expectation dependence, respectively. The bank as such takes dependencies into consideration when devising its optimal hedging strategy.

JEL classification: D81; G21; E32

Keywords: Expectation dependence; Hedging; Lending; State-dependent preferences

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The banking firm under risk

Abstract This paper examines the optimal lending and hedging decisions of a bank facing uncertain returns on its loans. The bank's preferences are state-dependent in that the utility function depends on a state variable, i.e. the business cycle of the economy. The purpose of this paper is to complement the results of the banking literaure. To characterize the bank's optimal use of financial instruments to hedge, we show that the concept of expectation dependence (Wright, 1987) is useful. While the current hedging literature specifies price risk as a monotonically increasing or decreasing function of the state-variable plus noise, expectation dependence provides much more general bivariate dependence structure. The bank's optimal futures position is an under-hedge or an over-hedge, depending on whether the random return on loans is positively or negatively correlated with the business cycle of the economy in the sense of expectation dependence, respectively. The bank as such takes dependencies into consideration when devising its optimal hedging strategy.

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1 Introduction

Financial risks are at the core of financial intermediation. Banks taking deposits and holding portfolios of loans and other financial contracts need to manage liquidity risk, credit risk, market risk, and operational risk. Management of risk is an important topic in many fields of economics, insurance, banking, and finance (see, for example, Pausch and Welzel, 2013, Baker and Filberg, 2015). In the economics of banking and finance, the question of how financial intermediaries react to changes in random and non-random parameters of their market environment has a long tradition.

Broll and Wong (2010) have recently examined the hedging decision of a bank facing uncertain returns on its loans. The bank's preferences are state-dependent in that the utility function depends on a state variable that denotes the business cycle of the economy. There are unbiased futures contracts that the bank can trade to hedge against its risk exposure. It is shown that the bank takes the business cycle of the economy seriously when devising its optimal hedging strategy.

In a seminal contribution to the literature on futures markets, Benninga et al. (1983) addressed the issue of optimal futures hedging in the presence of unbiased futures prices. They derived conditions for the optimal hedge ratio to be a fixed proportion of the open position, regardless of the agent's utility function. This result is important because of the sizeable research on theoretical and empirical risk management that abstracts from the particular utility functions of risk averse, expected utility maximizers.

The purposes of this study are to complement the results in the literature by endogenizing the bank's lending decision in general, and by incorporating a tractable covariance structure into the underlying uncertainty in particular. We show that the optimal amount of loans extended by the bank depends neither on the risk attitude of the bank, nor on the joint distribution of the random return on loans and the business cycle of the economy. We show further that the optimality of a full-hedge holds if, and only if, the random return on loans is conditionally independent of the business cycle of the economy. In the more realistic case that these two sources of uncertainty are *de facto* correlated, we show that the bank optimally opts for an under-hedge or an over-hedge, depending on whether the random return on loans is positively or negatively correlated with the business cycle of the economy in the sense of expectation dependence *à la* Wright (1987), respectively. Expectation dependence as such completely determines the bank's optimal futures position (see Hong et al. (2011), Li (2011), Wong (2012, 2013) and Broll et al. (2015) for other applications of expectation dependence).

The rest of this paper is organized as follows. Section 2 delineates our model of a competitive bank facing uncertain returns on loans and possessing state-dependent preferences. Section 3 derives the optimal lending and hedging decisions of the competitive bank. The final section concludes.

2 The model

Consider a competitive bank that makes decisions in a single period horizon with two dates, 0 and 1. At date 0, the bank has the following balance sheet:

$$L = D + E, \tag{1}$$

where $L > 0$ is the amount of loans, $D > 0$ is the quantity of deposits, and $E > 0$ is the stock of equity capital. By regulation, the bank is subject to the following capital adequacy requirement:

$$\alpha L \leq E, \tag{2}$$

where $\alpha \in (0, 1)$ is the minimum capital-to-loan ratio.[1]

The bank's loans belong to a single homogeneous class, which mature at date 1. The gross return on loans, $\tilde{R}$, is stochastic and distributed according to a known cumulative distribution function (CDF), $F(R)$, over support $[\underline{R}, \overline{R}]$ with $0 < \underline{R} < \overline{R}$.[2] The bank can hedge this risk exposure by selling (buying if negative) Z units of futures contracts at the prespecified gross return, $R_F \in (\underline{R}, \overline{R})$, at date 0. The futures contracts are settled at date 1 based on the realized value of $\tilde{R}$ at that time. To focus on the bank's pure hedging motive, we assume throughout the paper that the futures contracts are unbiased in that R_F is set equal to the unconditional expected value of $\tilde{R}$.

The bank's deposits are insured by a government-funded deposit insurance scheme. The supply of deposits is perfectly elastic at the fixed one-plus deposit rate, $R_D \geq 1$. The

[1] This specification of capital adequacy requirement is consistent with those of Basel I and the standardized approach of Basel II, both of which set $\alpha = 8\%$. See also Wong (1997, 2011).

[2] Throughout the paper, random variables have a tilde ($\tilde{}$) while their realizations do not.

bank's shareholders contribute equity capital with a required gross return, R_E, on their investment, where $R_E > R_D$, reflecting the scarcity of shareholders' wealth. We assume that the unconditional expected gross return on loans is no less than R_E, and thereby is greater than R_D, so that the bank has incentives to extend loans by raising deposits and equity capital.

The bank's shareholders receive the following net worth at date 1:

$$\tilde{W} = \tilde{R}L - R_D D - R_E E - C(L) + (R_F - \tilde{R})Z, \tag{3}$$

where $C(L)$ is the cost function of servicing loans, and $(R_F - \tilde{R})Z$ is the gain (loss if negative) from the futures position, Z. We refer to Z as an over-hedge, a full-hedge, or an under-hedge, depending on whether Z is larger than, equal to, or smaller than the amount of loans, L, respectively. We assume that $C(L)$ is continuously differentiable such that $C(0) = C'(0) = 0$, and $C'(L) > 0$ and $C''(L) > 0$ for all $L > 0$.

The bank is risk averse and possesses a bivariate state-dependent utility function, $U(W, S)$, defined over the net worth of its shareholders at date 1, W, and the realization of a state variable, $\tilde{S}$, that denotes the business cycle of the economy. Let $G(S)$ be the known CDF of $\tilde{S}$ over support $[\underline{S}, \overline{S}]$ with $\underline{S} < \overline{S}$. To allow the random gross return on loans, $\tilde{R}$, to be correlated with $\tilde{S}$, we denote $H(R, S)$ as their joint CDF over support $[\underline{R}, \overline{R}] \times [\underline{S}, \overline{S}]$. We assume that the state-dependent utility function, $U(W, S)$, is continuously differentiable with respect to both arguments such that $U_W(W, S) > 0$, $U_{WW}(W, S) < 0$, and $U_{WS}(W, S) > 0$ for all $W \geq 0$ and $S \in [\underline{S}, \overline{S}]$, where the subscripts indicate partial derivatives.

The ex-ante decision problem of the bank is given by

$$\max_{L>0, D>0, E>0, Z} \mathrm{E}[U(\tilde{W}, \tilde{S})] \quad \text{s.t.} \quad L = D + E \quad \text{and} \quad \alpha L \leq E, \tag{4}$$

where $\mathrm{E}(\cdot)$ is the expectation operator with respect to the joint CDF, $H(R, S)$, and $\tilde{W}$ is given by Eq. (3). It follows from $R_E > R_D$ that the bank would like to rely on deposits rather than equity capital to finance loans, thereby rendering the capital adequacy requirement to be binding. Substituting the initial balance sheet constraint, Eq. (1), and the binding capital adequacy requirement, Eq. (2), into Eq. (3) yields the following net worth of the bank's shareholders at date 1:

$$\tilde{W} = [\tilde{R} - \alpha R_E - (1-\alpha)R_D]L - C(L) + [\mathrm{E}(\tilde{R}) - \tilde{R}]Z, \tag{5}$$

since $R_F = \mathrm{E}(\tilde{R})$. We can interpret $\alpha R_E + (1-\alpha)R_D$ as the bank's weighted average cost of capital, where the weights are based on the capital-to-loan ratio, α, and the deposit-to-loan ratio, $1-\alpha$. The ex-ante decision problem of the bank as such reduces to

$$\max_{L>0,Z} \ \mathrm{E}[U(\tilde{W}, \tilde{S})], \tag{6}$$

where $\tilde{W}$ is given by Eq. (5).

3 Optimal lending and hedging decisions

The first-order conditions for program (6) are given by

$$\mathrm{E}\{U_W(\tilde{W}^*, \tilde{S})[\tilde{R} - \alpha R_E - (1-\alpha)R_D - C'(L^*)]\} = 0, \tag{7}$$

and

$$\mathrm{E}\{U_W(\tilde{W}^*, \tilde{S})[\mathrm{E}(\tilde{R}) - \tilde{R}]\} = 0, \tag{8}$$

where an asterisk (*) signifies an optimal level. The second-order conditions for program (6) are satisfied given the assumed properties of $U(W, S)$ and $C(L)$.

Adding Eq. (7) to Eq. (8) yields

$$\mathrm{E}[U_W(\tilde{W}^*, \tilde{S})][\mathrm{E}(\tilde{R}) - \alpha R_E - (1-\alpha)R_D - C'(L^*)] = 0. \tag{9}$$

Since $U_W(W, S) > 0$, our first proposition is an immediate consequence of Eq. (9).

Proposition 1. *The competitive bank that hedges its risk exposure by using the unbiased futures contracts optimally chooses the amount of loans, L^*, that is the unique solution to*

$$C'(L^*) = \mathrm{E}(\tilde{R}) - \alpha R_E - (1-\alpha)R_D. \tag{10}$$

Given the availability of the unbiased futures contracts, the bank can perfectly lock in the random gross return on loans, $\tilde{R}$, at its expected value, $\mathrm{E}(\tilde{R})$. At the optimum, the bank must equate the known lock-in marginal revenue, $\mathrm{E}(\tilde{R})$, from lending out the last unit to the sum of the marginal cost of servicing that unit, $C'(L^*)$, and the weighted average cost of capital, $\alpha R_E + (1-\alpha)R_D$, thereby yielding the usual optimality condition, Eq. (10). An immediate implication of Proposition 1 is that the bank's optimal lending decision depends neither on the state-dependent utility function, $U(W, S)$, nor on the joint CDF, $H(R, S)$.

We now examine the bank's optimal hedging decision. To this end, we write Eq. (8) as[3]

$$\mathrm{Cov}[U_W(\tilde{W}, \tilde{S}), \tilde{R}] = 0, \tag{11}$$

where $\mathrm{Cov}(\cdot, \cdot)$ is the covariance operator with respect to the joint CDF, $H(R, S)$. According to Ingersoll (1987), a random variable, $\tilde{X}$, is said to be conditionally independent of another random variable, $\tilde{Y}$, if $\mathrm{E}(\tilde{X}|Y) = \mathrm{E}(\tilde{X})$ for all realizations of $\tilde{Y}$, where $\mathrm{E}(\tilde{X}|Y)$ is the expected value of $\tilde{X}$ conditional on $\tilde{Y} = Y$. Ingersoll (1987) proves that $\tilde{X}$ is conditionally independent of $\tilde{Y}$ if, and only if, $\mathrm{Cov}[\tilde{X}, f(\tilde{Y})] = 0$ for all functions, $f(\cdot)$.[4] Equipped with the concept of conditional independence, we can state and prove the following proposition.

Proposition 2. *The competitive bank optimally opts for a full-hedge, i.e., $Z^* = L^*$, if, and only if, the random gross return on loans, $\tilde{R}$, is conditionally independent of the state variable, $\tilde{S}$.*

[3] For any two random variables, $\tilde{X}$ and $\tilde{Y}$, we have $\mathrm{Cov}(\tilde{X}, \tilde{Y}) = \mathrm{E}(\tilde{X}\tilde{Y}) - \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y})$.

[4] Ingersoll (1987) shows that two random variables, $\tilde{X}$ and $\tilde{Y}$, are independent if, and only if, $\mathrm{Cov}[f(\tilde{X}), g(\tilde{Y})] = 0$ for all functions, $f(\cdot)$ and $g(\cdot)$. Thus, conditional independence contributes to a weaker condition than independence.

Proof. Suppose that a full-hedge is optimal. Substituting $Z^* = L^*$ into Eq. (11) yields

$$\mathrm{Cov}\Big\{U_W\{[\mathrm{E}(\tilde{R}) - \alpha R_E - (1-\alpha)R_D]L^* - C(L^*), \tilde{S}\}, \tilde{R}\Big\} = 0. \tag{12}$$

Since Eq. (12) must hold for all $U(W,S)$, thereby $U_W(W,S)$, it follows that $\tilde{R}$ is conditionally independent of $\tilde{S}$. This proves the necessity part.

To prove the sufficiency part, suppose that $\tilde{R}$ is conditionally independent of $\tilde{S}$. Then, we have $\mathrm{Cov}[f(\tilde{S}), \tilde{R}] = 0$ for all functions, $f(\cdot)$. In particular, it is true for $f(S) = U_W\{[\mathrm{E}(\tilde{R}) - \alpha R_E - (1-\alpha)R_D]L^* - C(L^*), S\}$. Eq. (11) as such holds at $Z^* = L^*$. $\square$

Proposition 2 shows that conditional independence of $\tilde{R}$ and $\tilde{S}$ is both necessary and sufficient to make a full-hedge optimal. The intuition underlying Proposition 2 is as follows. Since covariances can be interpreted as marginal variances, inspection of Eq. (11) reveals that the optimal futures position, Z^*, minimizes the variance of the bank's marginal utility. If the bank adopts a full-hedge, the net worth of the bank's shareholders at date 1 is non-stochastic. The variability of the bank's marginal utility as such comes entirely from the randomness of the state variable, which is non-hedgeable should $\tilde{R}$ and $\tilde{S}$ be conditionally independent, thereby rendering the optimality of a full-hedge.

An immediate implication of Proposition 2 is that the bank is likely to deviate from adopting a full-hedge when the two random variables, $\tilde{R}$ and $\tilde{S}$, are *de facto* correlated. In order to characterize the bank's optimal futures position, Z^*, in this more realistic case, we need a tractable covariance structure of $\tilde{R}$ and $\tilde{S}$. To this end, we define the following function:

$$\mathrm{ED}(\tilde{R}|S) = \int_{\underline{R}}^{\overline{R}} \left[\frac{H(R,S)}{G(S)} - F(R)\right] \mathrm{d}R, \tag{13}$$

for all $S \in [\underline{S}, \overline{S}]$. According to Wright (1987), $\tilde{R}$ is said to be positively (negatively) expectation dependent on $\tilde{S}$ if $\mathrm{ED}(\tilde{R}|S) \geq (\leq)\ 0$ for all $S \in [\underline{S}, \overline{S}]$, where the inequality is strict for some non-degenerate intervals. Wright (1987) shows that positive (negative) expectation dependence is a sufficient, but not necessary, condition for positive (negative) correlation. Equipped with the concept of expectation dependence, we can state and prove the following proposition.

Proposition 3. *The competitive bank optimally opts for an under-hedge (over-hedge), i.e., $Z^* < (>)\ L^*$, if the random gross return on loans, $\tilde{R}$, is positively (negatively) expectation dependent on the state variable, $\tilde{S}$.*

Proof. Cuadras (2002) proves that $\mathrm{Cov}[f(\tilde{R}), g(\tilde{S})]$ can be written in terms of the CDFs, $F(R)$, $G(S)$, and $H(R, S)$, as follows:

$$\mathrm{Cov}[f(\tilde{R}), g(\tilde{S})] = \int_{\underline{R}}^{\overline{R}} \int_{\underline{S}}^{\overline{S}} [H(R, S) - F(R)G(S)]\, \mathrm{d}f(R)\, \mathrm{d}g(S), \tag{14}$$

for all functions, $f(\cdot)$ and $g(\cdot)$. Evaluating the left-hand side of Eq. (11) at $Z^* = L^*$ and using Eq. (14) with $f(\tilde{R}) = \tilde{R}$ and $g(\tilde{S}) = U_W\{[\mathrm{E}(\tilde{R}) - \alpha R_E - (1-\alpha)R_D]L^* - C(L^*), \tilde{S}\}$ yields

$$\int_{\underline{R}}^{\overline{R}} \int_{\underline{S}}^{\overline{S}} [H(R, S) - F(R)G(S)] U_{WS}\{[\mathrm{E}(\tilde{R}) - \alpha R_E - (1-\alpha)R_D]L^* - C(L^*), S\}\, \mathrm{d}R\, \mathrm{d}S$$

$$= \int_{\underline{S}}^{\overline{S}} \mathrm{ED}(\tilde{R}|S) U_{WS}\{[\mathrm{E}(\tilde{R}) - \alpha R_E - (1-\alpha)R_D]L^* - C(L^*), S\} G(S)\, \mathrm{d}S, \tag{15}$$

where $\mathrm{ED}(\tilde{R}|S)$ is given by Eq. (13). Given that $U_{WS}(W, S) > 0$, the right-hand side of Eq. (15) is positive (negative) should $\tilde{R}$ be positively (negatively) expectation dependent on $\tilde{S}$. It then follows from Eq. (11) and the second-order conditions for program (6) that $Z^* < (>)\ L^*$. □

The intuition for Proposition 3 is as follows. Given that covariances can be interpreted as marginal variances, Eq. (11) implies that the optimal futures position, Z^*, is the one that minimizes the variance of the bank's marginal utility. Suppose that the bank chooses a full-hedge so that the net worth of its shareholders at date 1 is non-stochastic. If $\tilde{R}$ and

$\tilde{S}$ are positively (negatively) correlated in the sense of expectation dependence, the bank should decrease (increase) its futures position, i.e., adopt an under-hedge (over-hedge) since this would raise the net worth of the bank's shareholders at date 1 as the gross return on loans increases (decreases), which is more likely when the realized value of the state variable is higher. Given that $U_{WS}(W,S) > 0$, it follows from $U_{WW}(W,S) < 0$ that such a futures position is more effective in reducing the variability of the bank's marginal utility, thereby rendering the optimality of an under-hedge (over-hedge).

4 Conclusion

This paper extends the results in the literature by endogenizing a competitive bank's lending decision in general, and by incorporating a tractable covariance structure into the random return on loans and the business cycle of the economy in particular. Given that the bank can hedge its risk exposure by trading unbiased futures contracts, we show that the optimal amount of loans extended by the bank depends neither on the risk attitude of the bank nor on the underlying uncertainty. We show further that the concept of expectation dependence *à la* Wright (1987) is useful in determining the bank's optimal futures position. Specifically, the bank optimally opts for an under-hedge or an over-hedge, depending on whether the random return on loans is positively or negatively correlated with the business cycle of the economy in the sense of expectation dependence, respectively.

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