Superfluid density of a spin-orbit-coupled Bose gas

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We discuss the superfluid properties of a uniform, weakly interacting Bose-Einstein condensed gas with spin-orbit coupling, realized recently in experiments. We find a finite normal fluid density ρ_n at zero temperature which turns out to be a function of the Raman coupling. In particular, the entire fluid becomes normal at the transition point from the zero momentum to the plane wave phase, even though the condensate fraction remains finite. We emphasize the crucial role played by the breaking of Galilean invariance and by the gapped branch of the elementary excitations whose contribution to various sum rules is discussed explicitly. Our predictions for the superfluid density are successfully compared with the available experimental results based on the measurement of the sound velocities.

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I. INTRODUCTION

Bose-Einstein condensation and superfluidity are two distinct but intimately related phenomena [1,2]. Usually, the existence of a finite condensate fraction implies that the system should behave as a superfluid, characterized by a nonzero superfluid density ρ_s. This is the case, for example, of superfluid ^{4}He where the condensate fraction is about 10%, while the superfluid density coincides with the total density ($\rho_s = \rho$) at zero temperature. A convenient quantity to define is the so-called normal density ρ_n, which quantifies the amount of the fluid in equilibrium with a moving wall [3]. For system with Galilean invariance, it can be shown that $\rho_n + \rho_s = \rho$. As a result, for ^{4}He at zero temperature, $\rho_n = 0$. The vanishing of the normal density in ^{4}He is related to the scarcity of long-wavelength excitations, for which only phonons are available [4].

Recently, a new type of Bose-Einstein condensate (BEC) has been realized in ultracold atomic gases with synthetic spin-orbit coupling [5–14]. Two novel features stand out in comparison with the usual Bose-Einstein condensates. First, the Galilean invariance of this novel system is broken due to spin-orbit coupling [15–17]. This has important consequences on the behavior of the dipolar oscillation in a harmonic trap [6], and in particular on its hybridized density and magnetic nature [18]. Second, in spin-orbit coupled BECs, the long-wavelength excitations basically consist of two branches [19]: a phononic excitation with linear dispersion like that in ^{4}He, and a gapped branch dominated by spin excitations, in accordance with experiments [14]. The new structure of elementary excitations and the breaking of Galilean invariance are expected to introduce new effects in the superfluid properties of spin-orbit coupled systems (for recent reviews see, for example, Refs. [20,21]).

In this paper, we show that, for a spin-1/2 Bose gas subjected to spin-orbit coupling with equal superposition of the Rashba and Dresselhaus terms, the normal density ρ_n remains finite even at zero temperature. This is an important consequence of the breaking of Galilean invariance. An analogous situation takes place in the presence of disorder or an external periodic potential. With respect to these latter cases, the spin-orbit Hamiltonian employed in the present work has the peculiarity of being translational invariant. The paper is organized as follows. In Sec. II, we discuss the relevant properties of spin-orbit coupled Bose gas that will be relevant for the discussion of its superfluid behavior. In Sec. III, we carry out explicit calculation of the normal density and in particular, relate it to the magnetic properties of the system. In Sec. IV, we carry out the sum rule analysis of both longitudinal and transverse current-current correlation function and show how the existence of the gapped branch in the long-wavelength excitation is responsible for the finite value of the normal density and discuss its role in the calculation of various sum rules. In particular, we derive a new formula for the superfluid density that can be implemented directly in experiments. In Sec. V, we give an independent definition of the superfluid density using the phase twist method and show that, despite the lack of Galilean invariance, the equality $\rho_s + \rho_n = \rho$ still holds. Finally, we conclude in Sec. VI.

II. SPIN-ORBIT COUPLED BOSE CONDENSATE

The single-particle Hamiltonian of a spin-orbit coupled Bose gas is given by (for simplicity, we set $\hbar = m = 1$) [22–25]

$$h_0 = \frac{1}{2}\left[(p_x - k_0\sigma_z)^2 + p_y^2 + p_z^2\right] + \frac{\Omega}{2}\sigma_x + \frac{\delta}{2}\sigma_z, \quad (1)$$

where k_0 is the momentum transfer from the two Raman lasers, which we assume to be oriented along the $\hat{x}$ direction, and $\mathbf{p} = -i\nabla$ is the canonical momentum, not to be confused with the physical momentum whose x component reads $P_x = p_x - k_0\sigma_z$. The quantity Ω is the two-photon Rabi frequency determined by the intensity of the Raman lasers and δ is the Raman detuning, which we will set equal to zero in the following discussions. Finally the operators $\boldsymbol{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$

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are the usual Pauli matrices describing the two internal states of the atoms. The single-particle Hamiltonian h_0 is translational invariant ($[p_x,h_0]=0$), but breaks Galilean invariance since it does not commute with the physical momentum ($[P_x,h_0]\neq 0$).

For ^{87}Rb atoms employed in current experiments, the interaction between atoms can be written as $V_{\rm int}=1/2\sum_{\alpha\beta}\int d\mathbf{r}g_{\alpha\beta}n_\alpha(\mathbf{r})n_\beta(\mathbf{r})$, where $g_{\alpha\beta}=4\pi a_{\alpha\beta}$ are the various coupling constants in different spin channels, with $a_{\alpha\beta}$ the corresponding scattering lengths ($\alpha,\beta=\uparrow,\downarrow$ label the relevant internal hyperfine-Zeeman states). In the following we will assume $g_{\uparrow\uparrow}=g_{\downarrow\downarrow}\equiv g$, while $g_{\uparrow\downarrow}=g_{\downarrow\uparrow}\equiv g'$ is not necessarily equal to g. In the present work, we shall only consider uniform gases in the absence of external harmonic traps.

The Hamiltonian (1) has been implemented experimentally and both the phase diagram [7] and the elementary excitations [14] have been investigated. Following Ref. [23], let us define the interaction paramters $G_1=n(g+g')/4$ and $G_2=n(g-g')/4$, where $n=N/V$ is the average density. Then one can predict three different quantum phases. For small Rabi frequency Ω and $G_2>0$, a stripe phase with density modulation in the ground state exists. The low frequency elementary excitations of the stripe phase consist of two gapless modes associated with the spontaneous breaking of translational and gauge symmetries. For relatively larger values of Ω, two new phases emerge where the condensate wave function can be written in the form

$$|0\rangle=\sqrt{n}\begin{bmatrix}\cos\theta\\-\sin\theta\end{bmatrix}\exp(ik_1x). \tag{2}$$

Minimizing the mean field energy, one finds that, for $\Omega<2(k_0^2-2G_2)$, the ground state configuration is characterized by $k_1=k_0\sqrt{1-\Omega^2/[2(k_0^2-2G_2)]^2}$ and $\cos 2\theta=k_1/k_0$. This state breaks the Z_2 symmetry of the Hamiltonian and features a nonzero magnetization in the ground state, given by $M/N\equiv\langle\sigma_z\rangle=k_1/k_0$. This phase is usually referred to as the plane wave phase. For $\Omega>2(k_0^2-2G_2)$, one has $k_1=0$ and $\theta=\pi/4$. This gives rise to the zero momentum phase. The phase transition between these two phases is of second-order nature with a divergent thermodynamic magnetic susceptibility χ_M at the critical point [18]. The elementary excitations of these two phases consist of two branches: a gapless phonon branch $\omega_1(\mathbf{q})$ corresponding, in the small q limit, to the sound propagation and a gapped branch $\omega_2(\mathbf{q})$ dominated by spin excitations. The sound velocity along the $\hat{x}$ direction is strongly quenched by spin-orbit coupling and phonons exhibit a hybridized density and spin nature [19]. In fact, at the transition point, taking place at the Rabi frequency $\Omega_c\equiv 2(k_0^2-2G_2)$, the sound velocity vanishes, even though the compressibility of the gas remains finite. This implies that the Landau critical velocity along the $\hat{x}$ direction becomes zero, according to the usual Landau criterion. On the other hand, the condensate density remains finite, the quantum depletion being small even at the transition point [16].

III. NORMAL DENSITY

According to the usual concept of two-fluid hydrodynamics, the normal density is the fraction of the fluid which is dragged by the wall of a moving cylindrical container, while the superfluid component can move with respect to the container without friction. In our case the role of the wall is played by the Raman lasers which block the motion of the normal component of the gas along the $\hat{x}$ direction. As a result, the normal density is a tensor of the form $\hat{\rho}_n=\rho_{n,\parallel}\hat{x}\hat{x}+\rho_{n,\perp}(\hat{y}\hat{y}+\hat{z}\hat{z})$. The transverse component $\rho_{n,\perp}$ behaves as usual since excitations along the transverse directions remain the same in the long-wavelength limit, thereby yielding $\rho_{n,\perp}=0$ at zero temperature. On the other hand, the longitudinal component $\rho_{n,\parallel}$ (hereafter denoted as ρ_n) is modified significantly due to spin-orbit coupling.

In terms of the transverse current response function [3], the normal density ρ_n at zero temperature is given by [26]

$$\frac{\rho_n}{\rho}=\frac{1}{N}\lim_{\mathbf{q}\to 0}\left[\sum_{n\neq 0}\frac{\left|\langle 0|J_x^T(\mathbf{q})|n\rangle\right|^2}{E_n-E_0}+(\mathbf{q}\to-\mathbf{q})\right], \tag{3}$$

where $J_x^T(\mathbf{q})$ is the transverse current operator along the $\hat{x}$-direction ($\mathbf{q}\perp\hat{x}$). Here $|n\rangle$ is the set of exact many-body eigenstate with energy E_n. We take $\mathbf{q}=q\hat{y}$ so that the transverse current operator takes the explicit form

$$J_x^T(q)=\sum_k(p_{k,x}-k_0\sigma_{k,z})e^{iqy_k}, \tag{4}$$

where k enumerates the number of particles. Since the transverse current operator does not excite the gapless phonon mode, which is of longitudinal nature, the only contribution to Eq. (3) comes from the gapped branch. Let us denote the gap in the limit $q\to 0$ as $\Delta\equiv\omega_2(q=0)$, then we can write Eq. (3) as

$$\begin{aligned}\frac{\rho_n}{\rho}&=\frac{1}{N\Delta^2}\lim_{q\to 0}\left[\sum_{n\neq 0}|\langle 0|J_x^T(q)|n\rangle|^2(E_n-E_0)+(q\to -q)\right]\\&=\frac{1}{N\Delta^2}\lim_{q\to 0}\langle 0|\left[J_x^T(-q),\left[H,J_x^T(q)\right]\right]|0\rangle.\end{aligned} \tag{5}$$

In the limit $q\to 0$, the only noncommutating term between the current operator and the Hamiltonian is the spin term; the contribution of the canonical component of the current vanishing when $q\to 0$, as a consequence of the translational invariance of the Hamiltonian. One can consequently write

$$\lim_{q\to 0}\langle 0|\left[J_x^T(-q),\left[H,J_x^T(q)\right]\right]|0\rangle=k_0^2\langle 0|[\Sigma_z,[H,\Sigma_z]]|0\rangle, \tag{6}$$

where $\Sigma_z=\sum_k\sigma_{k,z}$ is the total spin operator along the $\hat{z}$ direction. The double commutator only receives contribution from the Raman term proportional to Ω in the single-particle Hamiltonian, yielding $[\Sigma_z,[H,\Sigma_z]]=-2N\Omega\sigma_x$. One finally obtains the result

$$\frac{\rho_n}{\rho}=-\frac{2k_0^2\Omega}{\Delta^2}\langle\sigma_x\rangle. \tag{7}$$

A further important connection between the superfluid and the magnetic properties of the system is given by the nontrivial identity

$$\frac{\rho_n}{\rho_s}=k_0^2\chi_M, \tag{8}$$

for the ratio between the normal density (ρ_n) and the superfluid ($\rho_s = \rho - \rho_n$) density of the system. The quantity χ_M entering Eq. (8) is the thermodynamic magnetic susceptibility of the system, determined by the energy cost $\delta E = N(\delta\langle\sigma_z\rangle)^2/(2\chi_M)$ associated with the change $\delta\langle\sigma_z\rangle$ in the polarization of the medium. In the following we will show that the identity (8) holds both in the plane wave phase and in the zero momentum phase.

(i) Plane wave phase ($\Omega \leqslant \Omega_c = 2(k_0^2 - 2G_2)$). The transverse spin polarization is given by $\langle\sigma_x\rangle = -\Omega/2(k_0^2 - 2G_2)$ and the excitation gap is given by $\Delta^2 = 4(k_0^2 - 2G_2)(k_0^2 - 2G_2k_1^2/k_0^2)$ [19]. As a result, we have

$$\frac{\rho_n}{\rho} = \frac{k_0^2\Omega^2}{4\left(k_0^2 - 2G_2\right)^3 + 2G_2\Omega^2}. \tag{9}$$

In the plane wave phase $\chi_M = \Omega^2/\{(k_0^2 - 2G_2)[4(k_0^2 - 2G_2)^2 - \Omega^2)]\}$ [19], thus confirming the relation (8).

(ii) Zero momentum phase ($\Omega \geqslant \Omega_c$). The transverse spin polarization is given by $\langle\sigma_x\rangle = -1$ and the excitation gap is given by $\Delta^2 = \Omega(\Omega + 4G_2)$ [19]. Thus we find

$$\frac{\rho_n}{\rho} = \frac{2k_0^2}{\Omega + 4G_2}. \tag{10}$$

In the zero momentum phase, $\chi_M = 2/(\Omega + 4G_2 - 2k_0^2)$ [19], thus confirming again the relation (8).

Equations (9) and (10) show that the normal density takes the maximum value at the transition point ($\Omega = \Omega_c$) between the two phases, where $\rho_n/\rho = 1$ and hence $\rho_s = 0$, consistent with the divergent behavior of χ_M [19], which was confirmed experimentally [6] through the measurement of the amplitude ratio of the spin and momentum dipole oscillation of a harmonically trapped spin-orbit coupled condensate. According to our predictions, the entire fluid becomes normal at $\Omega = \Omega_c$, even though the condensate fraction is finite [16]. The explicit dependence of ρ_s/ρ on the Raman coupling Ω is shown in Fig. 1.

In both the zero momentum and the plane wave phases, the normal density depends explicitly on the interaction parameters G_2, which quantifies the breaking of the SU(2) invariance of the interatomic force. In the limit $G_2 = 0$, one finds $\rho_n/\rho = \Omega^2/4k_0^4$ for the plane wave phase and $\rho_n/\rho = 2k_0^2/\Omega$ for the zero momentum phase. These results can also be written in the useful form

$$\frac{\rho_n}{\rho} = 1 - \frac{m}{m^*}, \tag{11}$$

where m^* is the effective mass of atoms close to the single-particle energy minimum of the Hamiltonian (1), given by $m/m^* = 1 - (\Omega/2k_0^2)^2$ in the plane wave phase and $m/m^* = 1 - 2k_0^2/\Omega$ in the zero momentum phase [16,20]. These results show that if the interaction is SU(2) invariant, the normal density in the ground state is independent of interparticle interaction and is controlled entirely by the Raman lasers.

IV. SUM-RULE ANALYSIS

It is by now clear that the existence of the gapped branch in the elementary excitation spectrum is responsible for the finite normal density even at zero temperature. To gain further insight into the problem, we investigate the moments [2]

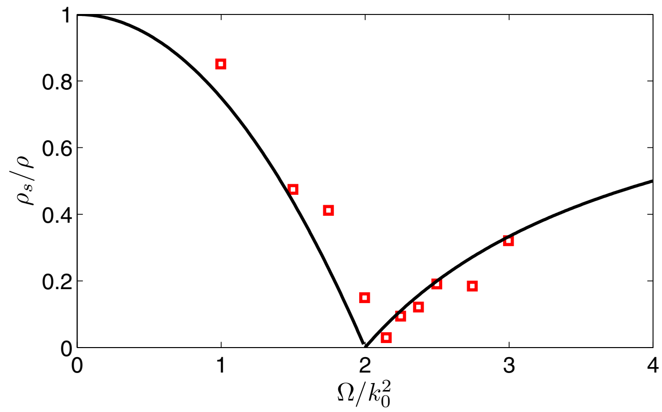

FIG. 1. Dependence of ρ_s/ρ on the Raman coupling strength at $T = 0$. Full line: Theoretical prediction [Eqs. (9) and (10)] with $G_1 = 0.12k_0^2$ and $G_2/G_1 = 10^{-3}$, appropriate for the experiment [14]. The experimental values (red squares) are based on Eq. (15), with the measured values of the sound velocities taken from [14] and the value of the compressibility calculated in [19]. The figure reveals the strong quenching of the superfluid density near the transition between the plane wave and the zeroth momentum phase.

$$m_p = \int d\omega\, \omega^p S(\mathbf{q},\omega) = \sum_n (E_n - E_0)^p |\langle n|\rho_{\mathbf{q}}|0\rangle|^2 \tag{12}$$

of the density dynamical structure factor $S(\mathbf{q},\omega)$, where $\rho_{\mathbf{q}}$ is the density fluctuation operator. It can be verified directly that, even in the presence of spin-orbit coupling, the $p = 1$ moment obeys the well known f-sum rule,

$$m_1(\mathbf{q}) + m_1(-\mathbf{q}) = \langle 0|[[\rho_{\mathbf{q}}, H], \rho_{\mathbf{q}}^\dagger]|0\rangle = Nq^2. \tag{13}$$

Using the continuity equation $[\rho_{\mathbf{q}}, H] = \omega\rho_{\mathbf{q}} = \mathbf{q}\cdot\mathbf{J}^L(\mathbf{q})$, where $J_x^L(\mathbf{q}) = \sum_k (p_{k,x}e^{iqx_k} + e^{iqx_k}p_{k,x})/2 - k_0\sum_k \sigma_{k,z}e^{iqx_k}$ is the longitudinal current operator with $\mathbf{q} = q\hat{x}$, we can rewrite the f-sum rule Eq. (13) in terms of matix elements of $\mathbf{J}^L(\mathbf{q})$ as

$$Nq^2 = \sum_{n\neq 0} \frac{\left|\langle n|J_x^L(\mathbf{q})|0\rangle\right|^2}{E_n - E_0} q^2 + (q \to -q), \tag{14}$$

where the summation over n includes both the phonon branch and the gapped branch. For the phonon branch $(E_n - E_0) \sim cq$ as $q \to 0$, with c being the sound velocity. The corresponding matrix element $\langle n|J_x^L(\mathbf{q})|0\rangle$ vanishes like $\sqrt{q}$. On the other hand, for the gapped branch, $(E_n - E_0) \to \Delta$ as $q \to 0$ and the matrix element $\langle n|J^L(\mathbf{q})|0\rangle$ approaches a constant value since the operator $P_x = \sum_k (p_{k,x} - k_0\sigma_{k,z})$ does not commute with the Hamiltonian. As a result, both the gapped and the phonon branch contribute to the f-sum rule in the long-wavelength limit. This differs from the usual situation (see, for example, liquid ^{4}He) where the phonon contribution dominates at small values of q and exhausts the f-sum rule. Actually, the contributions $|\langle n|J_x^T(\mathbf{q})|0\rangle|^2$ and $|\langle n|J_x^L(\mathbf{q})|0\rangle|^2$, arising from the gapped branch and entering the transverse and longitudinal sum rules (3) and (14), coincide in the $q \to 0$ limit. Consequently the normal density fraction ρ_n/ρ is fixed by the contribution of the gapped branch to the f-sum rule.

Analogously, we can investigate the contributions arising from the phonon branch and from the gapped branch to the other sum rules. In the $q \to 0$ limit, the phonon contribution is of order q^{p+1}, while for the gapped branch it is always q^2. As a result, the dominant contribution to the compressibility sum rule m_{-1} and to the static structure factor m_0 arises from the phonon branch. The higher order sum rules m_p with $p > 1$ are instead exhausted by the gapped branch.

An important consequence of the above analysis is that the superfluid density along the $\hat{x}$ direction corresponds to the phonon contribution to the inverse energy weighted sum rule (14) relative to the longitudinal current operator. On the other hand, using the continuity equation $qJ_x(\pm q) = \omega_\pm(q)\rho_q$, where $\omega_\pm(q)$ labels the excitation frequency along the positive (negative) $\hat{x}$ direction with sound velocity c_+ (c_-): $\omega_\pm(q) = c_\pm q$ [27], and the fact that the m_{-1} sum rule is exhausted by the phonon branch, we can write [28]

$$\rho_s = \rho c_- c_+ \kappa, \tag{15}$$

where κ is the thermodynamic compressibility. This equation shows that the measurements of the sound velocities along the $\pm\hat{x}$ direction and the knowledge of the static compressibility are enough for a direct determination of the superfluid density. In the absence of spin-orbit coupling, $\rho_s = \rho$ at zero temperature and Eq. (15) reduces to the standard relation $\kappa c^2 = 1$. The sound propagating along the $\hat{x}$ direction with velocities $c_\pm$ can be regarded as a manifestation of fourth sound [29], characterized by the motion of the superfluid component, while the normal component ρ_n remains at rest.

In Fig. 1 we plot our theoretical prediction for the ratio $\rho_s/\rho \equiv 1 - \rho_n/\rho$ derived in Eqs. (9) and (10) with solid line. To compare with experimental results, according to Eq. (15), we can use the of the sound velocities $c_\pm$ measured in [14]. However, the compressibility $\kappa = n\partial\mu/\partial n$ has not been measured experimentally, and we use the calculated values in [20] (Eqs. (29) and (30) in [20]). In fact, for configurations considered in Fig. 1, the compressibility is practically unaffected by the spin-orbit coupling, so that the quenching of the superfluid density is directly related to the observed quenching of the sound velocities.

The above discussion has emphasized the deep difference, typical of superfluids, between the longitudinal and the transverse current response functions. In particular the transverse static response [Eq. (3)], differently from the longitudinal one [Eq. (13)], is proportional to the normal (nonsuperfluid) component of the gas. At high frequencies, however, both response functions instead exhibit, when $q \to 0$, the leading $1/\omega^2$ dependence with the coefficients given by

$$\lim_{q\to 0} \sum_n \left|\langle 0|J_x^{T,L}(q)|n\rangle\right|^2 (E_n - E_0) = \frac{k_0^2}{2}\langle 0|[\Sigma_z,[H,\Sigma_z]]|0\rangle, \tag{16}$$

determined by the energy weighted sum rule $\langle 0|[\Sigma_z,[H,\Sigma_z]]|0\rangle = -2N\Omega\langle\sigma_x\rangle$.

V. PROOF OF THE IDENTITY $\rho_n + \rho_s = \rho$

The superfluid density can be defined microscopically by employing the phase twist method [30]. The phase twist can be generated by the unitary transformation (Galilean boost)

$$|\Psi'\rangle = \exp\left(i\theta \sum_k x_k/L_x\right)|\Psi\rangle, \tag{17}$$

applied to the wave function $|\Psi\rangle$ where L_x is the length of the system and $|\Psi\rangle$ obeys the usual periodic boundary conditions. The many-body wave function $|\Psi'\rangle$ is then characterized by a phase twist $\varphi(L_x) - \varphi(0) = \theta$. At $T = 0$, minimization of the energy for a fixed and small value of θ defines the superfluid density ρ_s according to

$$E' - E \equiv \frac{1}{2}N\frac{\rho_s}{\rho}\left(\frac{\theta}{L_x}\right)^2, \tag{18}$$

where E' and E are the ground state energies in the presence and in the absence of the twist constraint, respectively. According to (17) the physical momentum operator P_x acts on $|\Psi'\rangle$ as $P_x|\Psi'\rangle = \exp(i\theta\sum_k x_k/L_x)(P_x + N\theta/L_x)|\Psi\rangle$ and consequently the calculation of E' corresponds to minimizing the energy with respect to $|\Psi\rangle$ with a modified Hamiltonian:

$$\langle\Psi'|H|\Psi'\rangle = \langle\Psi|\left[H + \frac{1}{2}\left(\frac{\theta}{L_x}\right)^2 N + \frac{\theta}{L_x}P_x\right]|\Psi\rangle. \tag{19}$$

The energy difference $E' - E$, and hence ρ_s, can be easily calculated by second-order perturbation theory. Noting that $\langle 0|P_x|0\rangle = 0$, since there is no net current in the ground state, we find that the superfluid density is eventually given by

$$\rho_s = \rho\left(1 - \frac{2}{N}\sum_{n\neq 0}\frac{|\langle 0|P_x|n\rangle|^2}{E_n - E_0}\right). \tag{20}$$

Since only the spin component of the operator P_x gives rise to nonvanishing matrix elements (and in particular only the upper branch can be excited), one finds that the sum entering the above equation coincides with Eq. (3) in the small-q limit, which defines the normal density. This completes the proof that the identity $\rho_s + \rho_n = \rho$, where both ρ_s and ρ_n are defined microscopically in an independent way, holds also in the absence of Galilean invariance.

VI. CONCLUSIONS

In this paper, we have derived explicit results for the normal density ρ_n of a Bose-Einstein condensed gas with spin-orbit coupling. We show that ρ_n does not vanish even at zero temperature as a consequence of the breaking of Galilean invariance. We further demonstrate that the effect is largest at the transition between the plane wave and the zero momentum phase, where the normal density ρ_n, associated with the flow along the direction of the momentum transferred by the Raman lasers, coincides with the total density of the gas, with the consequent vanishing of the superfluid density. Our results set the stage for constructing a two fluid model of a spin-orbit coupled Bose gas whose behavior, here investigated at zero temperature, is expected to exhibit intriguing features also at finite temperature, in particular as concerns the propagation of second sound. Further important questions concern the moment of inertia [31] of the gas in trapped configurations and the nature of the vortical configurations. Extensions to pure Rashba spin-orbit coupling in both bosonic [32–34] and fermionic [35,36] systems require further investigations.

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[27] As discussed in [19] the sound velocities c_+ and c_- are different in the plane wave phase if $G_2 \neq 0$. In the zero momentum phase the identity $c_+ = c_-$ always holds.
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