Aspiring for Change:

A Theory of Middle Class Activism*

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Abstract. We propose a regime change model in which people are uncertain about both the quality of a specific regime and governance in general. The poor perceive the current regime as bad, rationally infer that all governments are bad, and therefore believe mass movements are futile. The middle class are more sanguine about the prospect of good government, and believe that collective action is effective because they expect many fellow citizens to share the same view. This coordination game with incomplete information does not admit monotone equilibrium but exhibits multiple interval equilibria, where middle class people are more likely to attack the regime.

Keywords: global game, model uncertainty, interval equilibrium, political passivity

JEL Classification. D74, D83, D84

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The middle class in modern societies often cause political ferment and spearhead mass movements against the status quo. The protests in Brazil, Bulgaria, and Turkey in 2013 provide the most recent examples of such political unrest. Although these protests occurred for distinct reasons in each country, the common fundamental strength of the demonstrations came from the involvement of the middle class.[1] The phenomenon whereby the middle classes take collective action and demand political changes is anything but new in history. For example, the 1987 demonstrations against Chun Doo-hwan in South Korea were initiated mostly by the urban middle class.[2] More generally, in what is called the "third wave" of democratization that occurred between the 1970s and the early 1990s, the most active participants were middle class, while the rich, the peasants, and even the industrial workers remained relatively inactive or indifferent (Huntington, 1993). The World Values Survey provides some evidence that confirms this well-known hump-shaped relationship between participation in political collective actions and the social class of the participants. Specifically, the lower and upper classes are significantly less likely to participate in demonstrations than those in the middle social strata.[3]

This phenomenon is often referred to as "middle class activism," i.e., the middle classes are among the most active strata in society to exert pressure on governments by participating in collective action, such as demonstrations, protests, or even revolutions. The purpose of this mass action is not necessarily to bring down the government. Rather, the common theme is a demand for reform and a higher quality of governance, such as better public services and less corruption.

It is interesting that the middle class, typically considered the keystone of stability in a modern polity, often spearhead social movements. Compared with the middle

[1]The demonstrations in Turkey grew from a protest against plans for a construction project. An increase in public transport prices triggered the protests in Brazil. Government corruption and cronyism set off the protests in Bulgaria. See "Middle-class rage sparks protest movement in Turkey, Brazil, Bulgaria and beyond," *Washington Post*, June 29, 2013.

[2]See "What happens when tear gas meets the middle class in Seoul?" *The Economist*, June 20, 1987.

[3]The survey asks respondents whether they have participated in demonstrations in the past, and to which social class they belong. We compute the fraction of participants in each class, after controlling for the demographic characteristics of the respondents, using the third, fourth, and fifth waves of the World Values Survey, which cover 78 countries from 1994 to 2007.

class, the rich have little incentive to upset the political system because they are its beneficiaries. However, why are the poor less active than the middle class? If anything, they should be more dissatisfied with the current regime than the middle class are, and their opportunity costs of taking action are lower. This puzzling fact of the poor being politically passive has long been known and documented by political scientists such as Huntington (1968). Berry et al. (1991) also find that political involvement is highly correlated with social class, and that the poor, who need help the most, make the least demands of government. Hoffer (1951) observes that the poor "are not hospitable to change. [. . .] There is thus a conservatism of the destitute as profound as the conservatism of the privileged, and the former is as much a factor in the perpetuation of a social order as the latter." These scholars typically take the passivity of the poor as a behavioral or psychological trait.

In this paper, we seek to provide a rational explanation for why the poor are more "pessimistic" than the middle class. The key is that mass political action is driven as much by hope as it is by discontent. To illustrate this idea, we assume in the benchmark model that the outcome of regime change is uncertain and that people may adopt two contrasting world views about it.[4] Under the *hopeful* view, governments can be a force for good. The quality of governance is potentially high, even though the quality of the existing regime may not be high. Under the *pessimistic* view, the quality of governance is low in general, regardless of who is in power. If the existing regime is unsatisfactory, collective action—even when it succeeds—does not help that much because it merely replaces one set of corrupt leaders or inefficient policies with another. Individuals assess the plausibility of these two world views based on their own personal experiences. The poor believe that the quality of the current regime is low. Because the quality of the current regime is an indicator of the quality of governance in general, the poor also rationally give more weight to the pessimistic world view. In contrast, the middle class attach greater weight to the hopeful world view. Even though they do think that the existing government is of low quality (but not as bad as

[4]Section 3.1 offers a generalized version of this model, where we do not restrict the number of the world views and allow for a continuum of descriptions of how the world works.

the poor believe it is), they are still optimistic that governance can be improved if the current leaders or their policies are replaced.

We embed this mechanism in a regime change framework in which heterogeneous citizens decide whether to participate in a mass protest or not, and the regime is replaced (or reforms implemented) if the number of participants is sufficiently high.[5] Strategic complementarities imply that people are more likely to participate in protests if they estimate that the current economic conditions support a large number of hopeful individuals who are willing to revolt. Interestingly, the poor and the middle class differ in their assessments of the probability of success. This is because people make inferences about economic conditions based on their own experiences and they believe that most of their fellow citizens are in a similar situation as theirs. Citizens at the bottom of society tend to underestimate the crowd of revolters, because they believe that most people are as pessimistic as they are.

In equilibrium, the poor do not participate because they not only hold a pessimistic view about the outcome of regime change, but also they consider the chances of success to be rather slim. The rich refrain from participating because they consider regime change, and the potential re-shuffling of economic status associated with it, undesirable. The middle class, who see the possibility of better governance and are optimistic about the chances of success, form the core participants of mass movements. Therefore, this model does not admit the standard monotone equilibrium in regime change models featuring coordination. Instead, the decision to participate in collective action is non-monotone in economic status; we label it an *interval equilibrium*.

In Section 2, we develop an approach for establishing the existence of interval equilibrium and characterizing equilibrium multiplicity and comparative statics. In Section 3, we show that the same approach can be extended to characterize equilibria when citizens differ in multiple dimensions.

[5]The regime change framework with a continuum of agents is particularly useful for understanding mass political action. A distinctive feature of the protests that we describe is that they are not organised by unions or other established political or interest groups, but are mostly spontaneous and leaderless. See, for example, "The march of protest," *The Economist*, June 29, 2013.

The mechanism of model uncertainty that drives the coordination of collective action in our model adds to the existing economics literature on regime change. To demonstrate its empirical relevance, we provide some evidence in Section 4 to show that the poor are indeed more pessimistic about politics in general, and that more pessimistic individuals are less likely to participate in mass political action. Further, in our model, it is important that individuals estimate the likelihood of success by making inferences about the distribution of the population. They make such inferences based on their own experiences, which naturally lead to the so-called "towards the median bias," i.e., agents tend to believe that they are closer to the median than they actually are. In Section 4, we provide some evidence to corroborate this mechanism.

We do not claim that our theory is the only one that can explain middle class activism, given that it is an inherently multifaceted issue. In the literature, there exist various popular and plausible hypotheses about mass movements, which shed light on the different characteristics of middle class activism, such as modernization theory, economic interest, the education hypothesis, and several behavioral explanations. In Section 5, we discuss the differences between our new hypothesis and the existing ones and highlight our contribution to the literature.

Three types of uncertainty exist in our regime change model: fundamental uncertainty (i.e., the underlying quality of the regime), payoff uncertainty (i.e., the benefit from participating), and model uncertainty (i.e., the description of how the world operates). The first two and their role in coordination games have been studied extensively in the literature. In Morris and Shin (1998) and later studies, such as Chen et al. (2014) and Edmond (2013), agents are uncertain about the fundamental strength of the regime. In Bueno de Mesquita (2010) and Shadmehr and Bernhardt (2011), agents are uncertain about the benefits of participating in collective action. One important difference between the two types of stochastic structure is that the former features two-sided limit dominance, whereas the latter induces one-sided limit dominance (Bueno de Mesquita, 2011). Our work adds to the existing literature by introducing model uncertainty, such that agents are uncertain about the structure of the game. By interacting with the two other types of uncertainty, model uncertainty induces one-sided limit dominance and leads to the possibility of interval equilibria in this model.

Shadmehr and Bernhardt (2010) show that non-monotone equilibria can exist without two-sided limit dominance in a two-player coordination game with uncertain payoffs. In their work, they tackle three analytical challenges, i.e., non-monotone expected net payoff to participation, the absence of global strategic complementarities, and the presence of one-sided limit dominance. Both monotone and interval equilibria can exist in their model and they show the existence of non-monotone equilibrium by using the variation diminishing property. In our model, both model uncertainty and payoff uncertainty are present. However, the former is the driving force for our results, without which one-sided limit dominance does not obtain.[6] Similar to Shadmehr and Bernhardt (2010), the expected net payoff for participation is not monotone in the type of agents; but unlike their model, strategic complementarities still remain. Monotone equilibria do not exist, but we can characterize the existence and multiplicity of non-monotone equilibria and their rankings by using lattice-theoretical methods (Vives, 1990).[7]

It has been shown that multiple equilibria can arise both in global game models (e.g., Angeletos et al. 2007) and in settings with uncertain payoffs (e.g., Bueno de Mesquita, 2010; Shadmehr and Bernhardt, 2011). One common feature of these models is that the net benefit of actions for the *marginal attacker* crosses zero more than once. That feature gives rise to the equilibrium multiplicity, but the decision rule still remains monotone. In our model, the net benefit of taking action for *any* individual citizen crosses zero twice, which leads to an interval decision rule. We elaborate on why multiple interval equilibria exist in our model in Section 2.3.

The formulation of model uncertainty is related to that in Meirowitz and Tucker

[6] In our model, there exists no state in which the dominant strategy for some agents is to attack the regime.

[7] Note that it is not implied that model uncertainty necessarily leads to one-sided dominance or non-monotone equilibrium. Chen and Suen (2014) construct a dynamic global game model to study why crises that are rare tend to be contagious. In that model, both model uncertainty and fundamental uncertainty are present, but it still features two-sided limit dominance, as well as unique monotone equilibrium.

(2013), in which citizens observe the performance of the existing government and make inferences not only about the quality of this particular government, but also about the pool of governments in general. Our work differs in two key aspects. First, their model abstracts from coordination concerns among citizens, which is the focus of our analysis. Second, they study a model with homogenous citizens, whereas agents are heterogeneous in our model. Based on their own circumstances, the agents have different observations or interpretations of the performance of the existing government. Therefore, they make different assessments about the same government and about how the world operates in general, which leads to different participation decisions.

The remainder of this paper is organised as follows. In Section 1, we present the benchmark model and discuss in detail the three types of uncertainty. In Section 2, we characterize interval equilibria and develop a method for establishing existence, multiplicity and comparative statics. In Section 3, we generalize and extend our benchmark model and further demonstrate that our key results still survive in more general environments. In Section 4, we provide evidence for the new mechanisms characterized in our model. In Section 5, we offer broader interpretations of our model insights and highlight the differences of our theory from the existing hypotheses in the literature. The last section outlines the possible directions for extending our theory.

1. The Model

The economy is populated by a continuum of agents indexed by $i \in [0,1]$. The well-being or income of agent i is x_i. We assume that it is log-normally distributed, so that it is non-negative. Denote $y_i \equiv \log x_i$, and let $y_i = \theta + \epsilon_i$. The first component θ represents a regime-related aggregate factor that affects citizens' welfare.[8] Nature selects it from a normal distribution $N(M, \sigma_\theta^2)$. The second component ϵ_i is idiosyncratic and

[8] For example, a well-managed economy with growth friendly policies is characterized by a high θ, whereas a badly-managed economy with severe market distortions and resource misallocation is characterized by a low θ.

regime-independent, which is drawn from another normal distribution $N(0, \sigma_\epsilon^2)$.[9] Citizens only observe their own income x_i, which represents their own life experiences but not the regime quality θ.[10]

Each citizen i chooses to participate in collective action ($a_i = 1$) or not ($a_i = 0$). The aggregate mass of the population that joins the protest is $A = \int_0^1 a_i \, \mathrm{d}i$. If the mass of participants exceeds the threshold $T \in (0, 1)$, the regime will be replaced by a new one or new reform policies will be implemented; otherwise, the existing regime survives. A citizen incurs a positive cost c if he participates. The perceived payoff from participation depends on whether the regime survives, on whether he participates, and on the operation of the world.

There are two alternative models of how the world works. Under the *hopeful* world view, social movements can make a positive difference. This view holds that the quality of governance is *potentially* high, even though the quality of the existing regime may not be high. Therefore, when the current regime is not satisfactory, collective action or pressure from society can either push the government to reform itself and improve its quality, or replace the existing regime with a better one. The hopeful world view can be captured by two assumptions. On the aggregate level, if success is achieved, the replacement is another independent draw θ' from the distribution $N(M, \sigma_\theta^2)$, with $M = m_H$.[11] On the individual level, the welfare consequence for agents is that they can experience a "re-shuffling" of economic status in the new society, as a byproduct of regime change. Specifically, with some probability, citizens can "start a new chap-

[9]This normality assumption is made so that the distribution of income in this economy is single-peaked, which captures one of the key features of modern middle class societies. This model may not be applicable to rural societies where most of the population is poor farmers. Further, y_i can be interpreted as "status" in society. For the worst-off agents with y_i going to minus infinity, their welfare or income is still bounded, which prevents agents at the bottom of society from obtaining an infinite gain from participating in mass movements.

[10]In the benchmark model, we only consider two relevant determinants of income. In reality, personal characteristics such as cognitive skills, human capital and education can also affect one's economic status. In Section 3.2, we explicitly characterize such a case by allowing for this additional determinant and show that the key qualitative results continue to hold. Further, implicitly, we assume that agents do not observe θ and therefore do not have sufficient knowledge about the income distribution. In Section 4, we offer some direct evidence that justifies this assumption and discusses why it is the case in reality.

[11]Note that θ' can be either higher or lower than θ. A higher θ' implies that the quality of the regime improves once the old one is replaced. If θ' is lower, then the mass action is successful but the quality of the new regime is even worse.

Table 1. Payoffs under the Hopeful World View

	$A \geq T$	$A < T$
$a_i = 0$	$p_0 x_i' + (1 - p_0) x_i$	x_i
$a_i = 1$	$p_1 x_i' + (1 - p_1) x_i - c$	$x_i - c$

ter in life," and obtain a new status x_i', where $\log x_i' = \theta' + \epsilon_i'$. This probability is p_1 for participants, and $p_0 < p_1$ for bystanders.[12] If the current regime survives, there will be no re-shuffling in society. The payoffs to agent i under the hopeful view are summarized in Table 1.

There is also a *pessimistic* world view, which holds that governments do not do any good regardless of who is in power. If the existing regime is unsatisfactory, changing it would merely replace one set of corrupt leaders or inefficient policies with another. According to this world view, society is in the grip of vested interests, which results in a low quality of governance, and efforts to improve it by the masses are in vain. Specifically, we assume that in this pessimistic world, $M = m_L < m_H$; and that the quality of the regime cannot be improved by social movements and there is no re-shuffling opportunity open to society. In other words, according to this view, mass movements are valueless for both society and individuals.

Several aspects of our assumptions on the two world views merit further comment. First, in this model, the purpose of collective action is to remove the status quo, in the hope of a better replacement. The random reshuffling mechanism, which is assumed to follow a successful revolution in the hopeful world, is a byproduct of regime change. For example, policies pursued by the new government may affect the economic opportunities of individuals, even though they are not in direct contact with the previous government. However, we stress that this reshuffling assumption is made only for the purpose of realism and can be relaxed in a more general environment (see Section 3.1).

[12]In a model with a continuum of agents, agents have no incentive to participate if $p_1 = p_0$. The assumption that $p_1 > p_0$ reflects that individuals care about the collective outcome and their personal role in bringing about the outcome, so that the free-riding problem does not completely eliminate participation.

Second, regarding the pessimistic world, we assume that the average regime quality is lower and that a change of government does not help at either the aggregate or individual levels. Note that both can be joint consequences of corrupt politics. In such a corrupt world, the government is tightly controlled by a group of elites. Specifically, one particular leader may be ousted, because of internal political fighting or pressure from the public, but the successor is still selected from the same group of elites or they are powerful enough to assimilate anyone who assumes office. This elite-manipulated government prefers implementing policies favorable to the interests of the elite, which may distort the allocation of resources and prevent the economy from operating in an efficient manner. The policies are implemented at the expense of the economic interests of the vast majority of the population.

We stress that the key characteristic of the pessimistic world is that the average regime quality is lower than that in the hopeful world. The assumption that the regime quality cannot be changed even upon success simplifies the baseline model but is not crucial for our key results. In Section 3.1, we offer a generalized model in which a new regime quality is drawn following a successful collective action.

Third, note that neither of the two alternative views is necessarily correct in general, and that this type of uncertainty cannot be easily resolved for a particular country. The popularity of both world views in reality and in the literature lends support to this observation.[13] Therefore, it is realistic to assume that people are uncertain about which view is the true description of how the world works.

Finally, the participation decision of the rich is simplified in both worlds, given that the key question in this paper concerns the contrast between the passivity of the poor and activism of the middle class and its explanation. Our model is reasonable for studying mass movements in which individuals take to the street to protest against the regime. In these events the rich rarely show up for various reasons, but we simply model them as beneficiaries of the existing regime, who do not want to upset it.[14]

[13] For example, Meirowitz and Tucker (2013) argue that citizens may not be sure about the quality of the pool of governments, especially in young democracies.

[14] Specifically, in the benchmark case, the benefit of participating relative to standing by is negative

2. Equilibrium Analysis

2.1. The equilibrium

In this model, three types of uncertainty matter for the decisions of individuals: model uncertainty, fundamental uncertainty, and payoff uncertainty. First, agents are not certain about the true model of the world. We assume that they attach a common prior probability α_0 that the hopeful view is true. Based on their personal experience, they rationally update their beliefs about the two world views. Since $y_i = \theta + \epsilon_i$ and θ is drawn either from $N(m_H, \sigma_\theta^2)$ under the hopeful view or from $N(m_L, \sigma_\theta^2)$ under the pessimistic view, the relative likelihood of observing y_i given these two world views is $\phi((y_i - m_H)/\sigma_y)/\phi((y_i - m_L)/\sigma_y)$, where $\phi(\cdot)$ is the standard normal density function and $\sigma_y^2 \equiv \sigma_\theta^2 + \sigma_\epsilon^2$. Therefore, by Bayes' rule, the updated probability of the hopeful world for agent i is:

$$\alpha(y_i) = \frac{\alpha_0 \phi((y_i - m_H)/\sigma_y)}{\alpha_0 \phi((y_i - m_H)/\sigma_y) + (1 - \alpha_0)\phi((y_i - m_L)/\sigma_y)}. \tag{1}$$

Second, the underlying regime quality θ is unknown to the citizens. Since θ affects the distribution of y_i in the population, it determines the total size of participants in the collective action, and hence its eventual success or failure. Because each agent forms a belief about the underlying regime quality based on his own life experience, each agent also attaches a different probability that the collective action will succeed. We let

$$\pi(y_i) = \Pr[A(\theta) \geq T \mid y_i, H] \tag{2}$$

represent the probability that the current regime collapses on the condition that the hopeful world view is the true description and the status index is y_i. This probability depends on the equilibrium decision rule adopted by agents in the economy.

for the rich in this model, despite of coordination concerns. As a result, they have a dominant strategy of not participating. But this assumption is not essential for our main results and it can be generalized such that the benefit can be positive for the rich.

Third, the payoff reward from the new society is also not certain. Conditional on the hopeful view, success will lead to a new regime quality, which can be better or worse, and the re-shuffling of status following success implies both upward and downward possibilities for the agents. The expected gain from participation relative to standing by for agent i is:

$$\rho(y_i) = (p_1 - p_0)\mathrm{E}[e^{\theta' + \epsilon_i'} - e^{y_i} \mid y_i, H] = (p_1 - p_0)(e^{\hat{y}} - e^{y_i}), \tag{3}$$

where $\hat{y} \equiv m_H + \sigma_y^2/2$.[15] When the free-riding problem is severe ($p_1 - p_0$ is small), the reward from successful mass action for an individual is diminished. Note that $\rho(y_i)$ may be positive or negative, depending on whether y_i is less than or greater than $\hat{y}$.[16]

Equilibrium in this model is characterized by a *participation set* Y^* and a *success set* Θ^*, such that agent i participates if and only if $y_i \in Y^*$, and the collective action is successful if and only if $\theta \in \Theta^*$. Since an agent participates in mass action when the expected benefit exceeds the cost, we require that for any $y_i \in Y^*$,

$$B(y_i; \Theta^*) \equiv \alpha(y_i)\pi(y_i; \Theta^*)\rho(y_i) \geq c. \tag{4}$$

Since the mass action is successful when the size of participants exceeds T, we require that for any $\theta \in \Theta^*$,

$$A(\theta; Y^*) = \int_{y_i \in Y^*} \phi\left(\frac{y_i - \theta}{\sigma_\epsilon}\right) \frac{1}{\sigma_\epsilon} \mathrm{d}y_i \geq T. \tag{5}$$

[15] In this model, $\rho(y_i)$ is bounded from above when y_i goes to minus infinity. This feature is the consequence of our assumption that income is log-normally distributed (and therefore bounded above zero). Intuitively, this formulation prevents agents at the bottom of society from obtaining an infinite gain from participating in social movements.

[16] In the benchmark model, the expected benefit for agents with $y_i > \hat{y}$ is negative and they have a dominant strategy of not participating. In Section 3.1, we provide an extended model in which the expected benefit from participation is positive for all agents.

2.2. The existence of interval equilibria

In this model, the expected reward $\rho(y_i)$ from successful mass action is decreasing in y_i. The rich find the potential re-shuffling in a new social order costly and have no incentive to upset the existing regime. Indeed, for any agent with $y_i > \hat{y}$, $\rho(y_i)$ is strictly negative, so he never participates. In contrast, poorer individuals desire social mobility and have relatively little to lose, therefore they are the natural candidates to participate. We say that there is a *monotone equilibrium* if agents adopt a monotone decision rule; i.e., the equilibrium participation set Y^* takes the form of $(-\infty, \overline{y}]$, for some finite $\overline{y}$.

If there were no model uncertainty (i.e., everyone adopts the hopeful world view), it would be easy to see how a monotone equilibrium arises as an equilibrium outcome of this model. For any agent, the estimated chance of success $\pi(y_i)$ depends on the equilibrium construction. Consider a standard construction as follows. Agents participate when y_i is below a threshold $\overline{y}$. As a result of this decision rule, the mass of participants decreases in θ. Thus, the regime is replaced if the regime quality θ is lower than a threshold $\overline{\theta}$. The poor believe that the regime quality is low, that a large fraction of the population also believe so, and that the mass of protesters must be high. This implies that both $\pi(y_i)$ and $\rho(y_i)$ are higher for poorer agents, which supports a monotone decision rule. However, this conclusion does not hold when we incorporate model uncertainty into the analysis.

PROPOSITION 1. *Monotone equilibrium does not exist in this model.*

Proof. From equation (1), we have $\lim_{y_i \to -\infty} \alpha(y_i) = 0$, whereas both $\pi(y_i)$ and $\rho(y_i)$ are bounded above. Therefore, for y_i sufficiently low, $B(y_i) < c$, which violates condition (4). ∎

Individuals in this model make two inferences based on y_i. They need to form expectations about the quality of the existing regime θ, and the quality of governance in general, which is captured by the mean M of the distribution from which θ is drawn.

Sufficiently poor agents infer that the current regime is bad (i.e., θ is low), but they also infer that all governments are bad (i.e., M is low). The extreme pessimism of the very poor accounts for the failure to obtain a monotone equilibrium in this model. This mechanism is consistent with the classical view of Huntington (1968), who points out that the poor in underdeveloped countries "do not seriously expect their government to do anything to alleviate the situation." We provide some further evidence that the poor do, indeed, take a pessimistic view of governance in general in Section 4.

Generally, $\alpha(y_i)$ is increasing in y_i, that is, better-off agents are more hopeful. This follows from the fact that the ratio of the likelihood of observing y_i in the hopeful world relative to that in the pessimistic world, $\phi((y_i - m_H)/\sigma_y)/\phi((y_i - m_L)/\sigma_y)$, is increasing in y_i. Compared with the poor, the middle classes are less dissatisfied with the present government because they believe that θ is not as low as that perceived by the poor. Therefore, they are more hopeful, i.e., middle class citizens tend to believe that they live in a world where their situation can be changed for the better through collective action because the average regime quality M is likely high.

Since we have shown that individuals who are very poor or very rich do not participate in collective action, we look for interval equilibria, in which Y^* takes the form of $[\underline{y}, \overline{y}]$, for finite $\underline{y} \leq \overline{y}$. A special case is a *degenerate interval equilibrium*, in which no one participates. This corresponds to the case $\underline{y} = \overline{y}$, which we can represent by $Y^* = \emptyset$.

For any interval participation set $Y = [\underline{y}, \overline{y}]$, the mass of participants $A(\theta; Y)$ is hump-shaped in θ. When θ is low, most individuals have low y_i; and when θ is high, most individuals have high y_i. In either case, the mass of agents with intermediate values of y_i is small. Let $\Phi(\cdot)$ represent the cumulative standard normal distribution function. We have

$$A(\theta; Y) = \Phi\left(\frac{\overline{y} - \theta}{\sigma_\epsilon}\right) - \Phi\left(\frac{\underline{y} - \theta}{\sigma_\epsilon}\right).$$

This function reaches a maximum at $\theta = (\underline{y} + \overline{y})/2$, and is strictly decreasing toward 0 as θ goes to positive or negative infinity. Thus, the set of θ for which $A(\theta; Y) \geq T$ is either empty (if T is high) or an interval, $[\underline{\theta}, \overline{\theta}]$, for some finite $\underline{\theta} < \overline{\theta}$ (if T is low). We conclude that if the equilibrium participation set Y^* is an interval, then the equilibrium success set Θ^* must also be an interval.

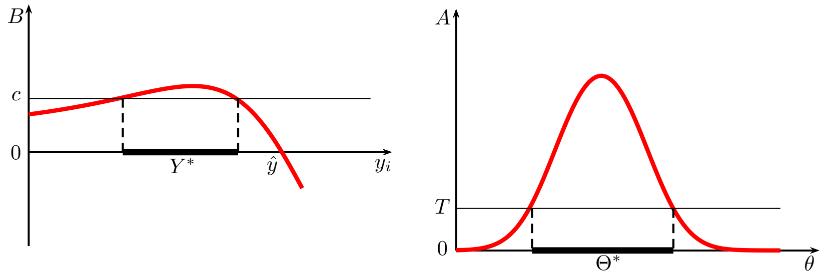
Fix any interval success set $\Theta = [\underline{\theta}, \overline{\theta}]$. The probability of success in equation (2) can be written as:

$$\pi(y_i; \Theta) = \Phi\left(\frac{\overline{\theta} - \beta m_H - (1-\beta)y_i}{\sqrt{\beta}\sigma_\theta}\right) - \Phi\left(\frac{\underline{\theta} - \beta m_H - (1-\beta)y_i}{\sqrt{\beta}\sigma_\theta}\right). \tag{6}$$

In this formula, $\beta m_H + (1-\beta)y_i$ is the posterior mean of θ given y_i and the hopeful view, and $\beta\sigma_\theta^2$ is the posterior variance, where $\beta \equiv \sigma_\epsilon^2/\sigma_y^2$. It is straightforward to observe that $\pi(y_i; \Theta)$ is hump-shaped in y_i. This non-monotonicity is another key feature of our model.

The distribution of income in the economy is normal with mean θ. Because individuals do not observe the true θ, each citizen has to infer the location of the center of the distribution based on his own circumstances. Therefore, citizens tend to believe that they are close to the center of the distribution, i.e., people at the bottom of society tend to overestimate their relative position in society, while those at the top tend to underestimate their relative status. Citizens with a low y_i think that θ is low, and therefore most fellow citizens are similarly situated. They expect that the size of participants (i.e., citizens with $y_i \in [\underline{y}, \overline{y}]$) who attack the regime is small, and the probability of success is small. Likewise, citizens with a high y_i think that the center of the distribution is near their own y_i, which is to the right of the participation set $[\underline{y}, \overline{y}]$. They also believe that there will not be enough participants to achieve success. Only the population of the intermediate segment attaches a high enough probability to the event of a successful attack and is thus motivated to participate. We provide evidence consistent with this mechanism in Section 4.

In sum, individuals care not only about the probability of success but also about what difference success would make to their welfare. Under this construction, the payoff from participation in collective action is low for poor agents because both $\alpha(y_i)$ and $\pi(y_i; \Theta)$ are low. The payoff is also low (or even negative) for rich agents because



(a) The $B(y_i;\Theta)$ function at $\Theta = \Theta^*$ (b) The $A(\theta;Y)$ function at $Y = Y^*$

Figure 1. *The benefit function $B(\cdot;\Theta)$ is single-crossing and hump-shaped when positive, given the success set Θ is an interval. The size of attack $A(\cdot;Y)$ is hump-shaped, given the participation set Y is an interval.*

both $\pi(y_i;\Theta)$ and $\rho(y_i)$ are low. In other words, pessimism and the perceived low chance of success override the potential gain from a new society and prevent the poor from participating. The perceived low chance of success and the concern about potential downward mobility discourage well-off agents from upsetting the system. Only a group of middle class citizens actively engages in this movement. They are hopeful enough about the upcoming change of regime, sufficiently optimistic about the chance of success, and expect upward mobility in their individual life chances.

LEMMA 1. *Fix any non-empty interval success set Θ. The expected benefit from participating, $B(y_i;\Theta)$, crosses zero once at $\hat{y}$ and from above, is hump-shaped in y_i when $y_i < \hat{y}$, and approaches zero when y_i approaches $-\infty$ or $\hat{y}$.*

In Lemma 1, we prove (see the Appendix) that $\alpha(y_i)$ is increasing, $\pi(y_i;\Theta)$ is hump-shaped, and $\rho(y_i)$ is decreasing in y_i. Moreover, each of these functions is log-concave in y_i when positive. Therefore, $B(y_i;\Theta)$ is increasing then decreasing for $y_i < \hat{y}$. This shows that individuals with y_i in the intermediate segment of the population distribution have the greatest incentive to participate in collective action. See Figure 1(a) for an illustration.

A participation set $Y^* = [\underline{y}, \overline{y}]$ and a success set $\Theta^* = [\underline{\theta}, \overline{\theta}]$ constitute a (non-degenerate) interval equilibrium of the model if and only if

$$B(\underline{y};\Theta^*) = B(\overline{y};\Theta^*) = c; \tag{7}$$

$$A(\underline{\theta};Y^*) = A(\overline{\theta},Y^*) = T. \tag{8}$$

Since $B(\cdot;\Theta^*)$ and $A(\cdot;Y^*)$ are both hump-shaped, conditions (7) and (8) imply equilibrium conditions (4) and (5). See Figure 1.

Let $\mathcal{I} = \{[\underline{y},\overline{y}] : \underline{y} < \overline{y}\} \cup \emptyset$ be the set of all finite participation intervals (including degenerate ones). Consider the following mapping $f : \mathcal{I} \to \mathcal{I}$. For any participation interval Y, solve the equation $A(\theta;Y) = T$. If no solution exists or there is only one solution, assign $f(Y) = \emptyset$; when there are two solutions, label them $\underline{\theta}$ and $\overline{\theta}$ and let $\Theta = [\underline{\theta},\overline{\theta}]$. For such Θ, solve the equation $B(y_i;\Theta) = c$. If no solution exists or there is only one solution, assign $f(Y) = \emptyset$; otherwise label the two solutions $\underline{y}'$ and $\overline{y}'$ and assign $f(Y) = [\underline{y}',\overline{y}']$. An equilibrium participation interval Y^* is a fixed point of f.

LEMMA 2. *The mapping f is monotone according to the set-inclusion order, i.e., $Y_1 \supseteq Y_0$ implies $f(Y_1) \supseteq f(Y_0)$.*

Since the empty set $\emptyset$ is a fixed point of f, a degenerate interval equilibrium always exists in this model. If no one attacks the regime, then $A(\theta;\emptyset) < T$ for any θ, which implies that the set of successful states is empty. If the success interval is empty, then $B(y_i;\emptyset) < c$ for any y_i, so no one attacks. It is also obvious that when the cost of participation c is sufficiently high, the equation $B(y_i;\Theta) = c$ has no solution for any Θ, because the benefit from participation is bounded above. In that case, a degenerate equilibrium is the only equilibrium. We next provide a proof that shows that when participation costs c are low enough, there are indeed non-degenerate interval equilibria.

PROPOSITION 2. *For any $T \in (0,1)$, there is a critical value $\hat{c}(T)$ such that non-degenerate interval equilibria exist if and only if $c \leq \hat{c}(T)$.*

Proof. Find a large enough finite participation interval $\tilde{Y}$ such that the upper boundary of this interval is strictly less than $\hat{y}$ and $\max_\theta A(\theta;\tilde{Y}) > T$. Such $\tilde{Y}$ exists because $\max_\theta A(\theta;\tilde{Y})$ goes to 1 as the lower boundary of $\tilde{Y}$ goes to $-\infty$. Let $\tilde{\Theta}$ represent the interval of θ for which $A(\theta;\tilde{Y}) \geq T$. Pick a sufficiently small c such that $B(y_i;\tilde{\Theta}) > c$ for any $y \in \tilde{Y}$. Such c exists because $B(y_i;\tilde{\Theta})$ is hump-shaped when $y_i < \hat{y}$ and approaches 0 when y_i approaches $-\infty$ or $\hat{y}$. Recall that $f(\tilde{Y})$ is the set of y_i for which $B(y_i;\tilde{\Theta}) \geq c$. By construction, we have $f(\tilde{Y}) \supset \tilde{Y}$. Let $Y_{\max}$ represent the interval of y_i for which $\alpha(y_i)\rho(y_i) \geq c$. Since $\alpha(y_i)\rho(y_i) > B(y_i;\Theta)$ for any finite success interval Θ, we have $Y_{\max} \supset f(Y)$ for any Y. Therefore,

$$Y_{\max} \supset f(Y_{\max}) \supset f(\tilde{Y}) \supset \tilde{Y}.$$

Denote the restricted domain $\mathcal{I}^* = \{Y : Y_{\max} \supseteq Y \supseteq \tilde{Y}\}$. Any element of $\mathcal{I}^*$ is a non-degenerate interval. Moreover, the partially ordered set $(\mathcal{I}^*, \supseteq)$ is a complete lattice, with supremum $Y_{\max}$ and infimum $\tilde{Y}$. Since both $f(Y_{\max})$ and $f(\tilde{Y})$ belong to $\mathcal{I}^*$, and since f is monotone, we have $f(Y) \in \mathcal{I}^*$ for any $Y \in \mathcal{I}^*$. In other words, f is a monotone mapping from $\mathcal{I}^*$ to $\mathcal{I}^*$. By Tarski's fixed point theorem, a fixed point of f in $\mathcal{I}^*$ exists.

Next, we show that if a non-degenerate interval equilibrium exists for some $\hat{c}$, then a non-degenerate interval equilibrium exists for any $c < \hat{c}$. To see this, let $\hat{Y}^*$ be an equilibrium participation interval when $c = \hat{c}$. Since $f(\hat{Y}^*;\hat{c}) = \hat{Y}^*$ and $c < \hat{c}$, we must have $f(\hat{Y}^*;c) \supset \hat{Y}^*$. Moreover we have already established that $Y_{\max} \supset f(Y_{\max};c)$. This means that for any $c < \hat{c}$, $f(\cdot;c)$ is a monotone mapping from $\mathcal{I}^*(c)$ to $\mathcal{I}^*(c)$, where $\mathcal{I}^*(c) = \{Y : Y_{\max} \supseteq Y \supseteq \hat{Y}^*\}$. Tarski's theorem guarantees the existence of a fixed point of $f(\cdot;c)$. ■

Our approach to establishing the existence of interval equilibrium is different from that of Morris and Shin (1998). Their method of showing the existence of monotone equilibrium and eliminating non-monotone equilibria depends on the fact that the ranking of agents by their expected benefit from participating is always monotone and invariant to any monotone strategy. In contrast, in our model, the ranking of agents by their expected benefit is not monotone and is not invariant to their decision rule, because the $B(y_i; \Theta)$ function depends on the success set, or how agents coordinate.

2.3. Equilibrium multiplicity

To show that multiple non-degenerate equilibria exist in this model, we extend our approach that establishes existence. Let Y^* be the largest equilibrium participation interval in the restricted domain $\mathcal{I}^*$, defined in the proof of Proposition 2. We construct another restricted domain $\mathcal{I}^{**}$ such that each element in $\mathcal{I}^{**}$ is a non-degenerate strict subset of Y^*, and show that there exists another equilibrium in $\mathcal{I}^{**}$.

To achieve this, we use the inverse mapping $f^{-1} : \mathcal{I} \to \mathcal{I}$, defined as follows. Take any participation interval Y. Find the success interval Θ such that $\{y_i : B(y_i; \Theta) \geq c\} = Y$. From the Θ so obtained, find the participation interval Y' such that $\{\theta : A(\theta; Y') \geq T\} = \Theta$. Assign $f^{-1}(Y) = Y'$ if a non-degenerate solution exists in each step; otherwise assign $f^{-1}(Y) = \emptyset$. Note that f^{-1} is monotone according to the set-inclusion order. The proof of Proposition 2 implies that, for $c < \hat{c}(T)$, there exists some interval Y_m such that $Y^* \supset Y_m$ and $f(Y_m) \supset Y_m$. Because f^{-1} is monotone, the latter is equivalent to $Y_m \supset f^{-1}(Y_m)$. We can also show that there always exists some non-degenerate interval Y_l such that $f^{-1}(Y_l) \supset Y_l$. We choose Y_l such that:

$$Y_m \supset f^{-1}(Y_m) \supset f^{-1}(Y_l) \supset Y_l.$$

This guarantees the existence of a fixed point of f^{-1} (and hence a fixed point of f) in $\mathcal{I}^{**} = \{Y : Y_m \supseteq Y \supseteq Y_l\}$.

PROPOSITION 3. *For any $T \in (0, 1)$, multiple non-degenerate interval equilibria exist if $c < \hat{c}(T)$.*

While it is common for models with complementarities to exhibit multiple equilibria, Proposition 3 implies that whenever a non-degenerate interval equilibrium exists, there must be more than one of them (except in the non-generic case of $c = \hat{c}(T)$). To better understand this point, it is useful to reduce the dimensionality of the problem. Because $A(\theta; Y)$ is symmetric about the point $\theta = (\underline{y} + \overline{y})/2$, in equilibrium we must have $\underline{\theta} + \overline{\theta} = \underline{y} + \overline{y}$. Let $q \equiv \overline{y} - \underline{y}$ represent the width of the participation interval and let $\omega \equiv \overline{\theta} - \underline{\theta}$ represent the width of the success interval. Then, condition (8) can be written as:

$$\Phi\left(\frac{\omega + q}{2\sigma_\epsilon}\right) - \Phi\left(\frac{\omega - q}{2\sigma_\epsilon}\right) = T. \tag{9}$$

Condition (7) can be written as:

$$B\left(\underline{y}; \left[\frac{\overline{y} + \underline{y} - \omega}{2}, \frac{\overline{y} + \underline{y} + \omega}{2}\right]\right) = B\left(\overline{y}; \left[\frac{\overline{y} + \underline{y} - \omega}{2}, \frac{\overline{y} + \underline{y} + \omega}{2}\right]\right) = c. \tag{10}$$

Define $g : [0, \infty) \to [0, \infty)$ in the following manner. Fix $q \geq 0$, and find the ω that solves (9). Using such ω, find the $\underline{y}$ and $\overline{y}$ that solve (10). Assign $g(q) = \overline{y} - \underline{y}$ if a solution exists in each of these two steps; otherwise assign $g(q) = 0$. If $[\underline{y}, \overline{y}]$ is an equilibrium participation interval, $\overline{y} - \underline{y}$ must be a fixed point of g.

Observe that there exists a $\underline{q}$ such that $g(q) = 0$ for all $q \leq \underline{q}$. If q is very small, the associated ω that solves (9) is too small to provide sufficient incentive for any agent to participate. When q goes to infinity, the associated ω also goes to infinity. However, although the probability of success is one, the expected benefit is still hump-shaped in y_i when it is positive. Therefore, $g(q)$ remains finite. These two facts imply that $g(q)$ is strictly below q both for q at $\underline{q}$ and for q sufficiently large. Therefore $g(q)$ is either completely below the 45-degree line, or there are multiple non-degenerate fixed points. In the latter case, $g(q)$ crosses the 45-degree line from below at the smallest fixed point, and from above at the largest fixed point. See Figure 2.

To gain intuition, suppose that agents conjecture that the participation interval is very small. As a result, they believe that it is impossible to have a sufficient number of protestors to surpass the hurdle T and nobody participates. When they conjecture that almost everybody participates, then the chance of success can be very high. However, the very poor and the rich still find that it is not worth taking to the street. This is

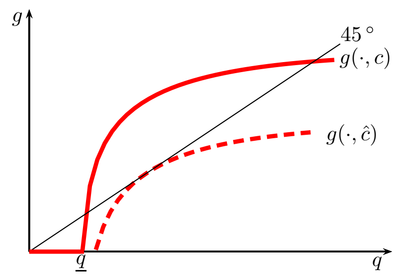


Figure 2. *The mapping $g(q)$ is below the 45-degree line when q (the width of the participation interval) approaches $\underline{q}$ or ∞. When $c < \hat{c}$, g has more than one non-degenerate fixed point.*

because $B(\cdot;\Theta)$ is still hump-shaped, even when Θ is arbitrarily large. Further, given the complementarities in action (that lead to Lemma 2), multiple non-empty equilibria may arise, or when the cost is too high, only an empty equilibrium exists.

2.4. Non-interval equilibria, discontinuity, and comparative statics

So far, our analysis has focused on interval equilibria. Recall that when the participation set is an interval, both the attack function $A(\cdot;Y)$ and the associated benefit function $B(\cdot;\Theta)$ are hump-shaped. The logic of strategic complementarity suggests that other forms of equilibria are possible. For example, suppose that the participation set Y consists of two non-overlapping intervals. Then, it is possible that the attack function $A(\cdot;Y)$ may have two peaks (near the mid-points of the two participation intervals). A twin-peaked $A(\cdot;Y)$ function may support a success set Θ that consists of two non-overlapping intervals. Such a success set Θ may in turn produce a $B(\cdot;\Theta)$ that is twin-peaked in y_i, so that two non-overlapping intervals of agents indeed have the greatest incentive to participate.

PROPOSITION 4. *The participation set in any non-interval equilibrium is strictly contained in that of the largest interval equilibrium.*

The equilibrium with the largest set of participants must be an interval equilibrium and cannot be any other type. In other words, the largest interval equilibrium contains all other equilibria, either interval or non-interval. The key logic underpinning this result is similar to that in section 2.3 and it is the consequence of complementaries in action, or precisely, the monotonicity of the mapping f. It provides an upper bound

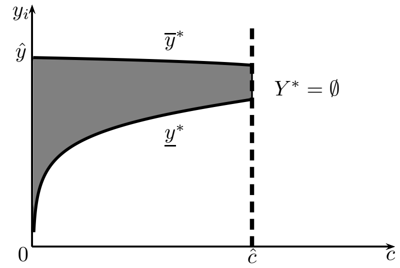


Figure 3. *The participation interval Y^* in the largest equilibrium shrinks ($\underline{y}^*$ increases and $\overline{y}^*$ decreases) as the cost of participation c rises. The only equilibrium is the degenerate equilibrium when c exceeds $\hat{c}$.*

for the size of a protest for any given θ and also implies that the very rich and the very poor, who are distributed outside the largest equilibrium participation interval, would never participate. Even when such equilibria exist, our central insight on the passivity of the poor still survives.

PROPOSITION 5. *In the largest equilibrium, both Y^* and Θ^* decrease (according to the set-inclusion order) in c. Both sets have a strictly positive measure at $c = \hat{c}(T)$ and change discontinuously to an empty set when c exceeds $\hat{c}(T)$. Further, $\hat{c}(T)$ decreases in T.*

Proposition 5 provides the comparative statics result for the cost of participation c. Interestingly, the comparative statics exhibit discontinuity: the largest equilibrium participation set shrinks (i.e., $\underline{y}$ rises and $\overline{y}$ falls) as c increases, until c reaches the critical level $\hat{c}$, after which the interval equilibrium collapses and the degenerate equilibrium becomes the only equilibrium. See Figure 3 for illustration. Clearly, the equilibrium participation interval will not shrink continuously to an empty set when the cost increases to the maximum of the expected benefit function. This is because the benefit function $B(y_i; \Theta)$ depends on the success interval Θ, which is endogenously determined by (8). When the participation interval Y is small enough, the associated success interval Θ is so small that the expected benefit of participating $B(y_i; \Theta)$ is less than c for any y_i. Therefore, the participation interval implodes to an empty set.

Similarly, we can show that an increase in the critical mass for success T decreases both Y^* and Θ^* in the largest equilibrium. Moreover, an increase in $p_1 - p_0$ increases both Y^* and Θ^*. The bigger is $p_1 - p_0$, the smaller is the incentive to free ride. Because each agent expects a greater benefit from participating, the equilibrium participation set and the corresponding equilibrium success set become larger.

An exogenous shift in the perceived average regime quality m_H under the hopeful world view affects the incentive to participate through three channels. First, for the worst-off agents, $\alpha(y_i)$ decreases in m_H, because the discordance between what the hopeful world view entails about regime quality and their own personal experiences becomes greater if m_H is higher. Second, those agents tend to believe that success is more likely, because the quality of the regime θ has a higher chance of falling into the success interval $[\underline{\theta}, \overline{\theta}]$, according to their estimate. That is, $\pi(y_i; \Theta)$ may increase in m_H. For agents on the top, they may observe the opposite of these two effects. Third, a higher m_H implies that the reward from participation $\rho(y_i)$ in the hopeful world is larger for anyone.

The combined net effect of these three mechanisms on the expected benefit is generally ambiguous. However, it can be the case that the effect of updating the world view (the first channel) dominates the other two. Specifically, when m_H rises, the lower middle class find it even harder to believe that the mean regime quality is high. Therefore, the expected benefit becomes smaller, despite the other two opposing effects. As a result, the middle class at the lower end tend to drop out from protests; that is, the lower boundary $\underline{y}$ of the participation interval increases. In contrast, the upper middle classes find that a higher m_H under the hopeful world view is more congruent with their own personal experiences. This effect of updating the world view, together with the higher reward for participation, dominates the effect of a possibly lower chance of success. Therefore, the upper middle class are more inclined to participate; that is, the upper boundary $\overline{y}$ for the participation interval increases. Moreover, interval equilibria collapse into a degenerate equilibrium with no attack as m_H becomes too small. Intuitively, when the reward for participation $\rho(y_i)$ is too low, nobody considers involvement in social movements an attractive option. Therefore, everybody believes that the chance of winning the battle is very low, which, in turn, justifies the quiescent outcome.

3. Generalization and Extension

3.1. A generalized model

We make three specific assumptions in the baseline case: (1) there are only two alternative worlds; (2) in the pessimistic world, the regime quality cannot be changed even if the revolution is successful; and (3) the opportunity cost of participation depends on one's income only in so far as successful collective action brings about a reshuffling of income. We show that the key results in our benchmark model still hold when we relax this structure.

First, suppose that there is a continuum of possible world views, ranging from extremely pessimistic to extremely hopeful. Each world is described by a normal distribution $N(M, \sigma_\theta^2)$ from which the regime quality is drawn. Instead of assuming that the mean quality of governance M is a two-point distribution with realisation m_H or m_L, assume that agents possess a common prior belief of the distribution of M, which is a normal distribution $N(\mu, \sigma_m^2)$, where μ is the mean of M, and σ_m^2 captures the degree of agents' prior uncertainty over the possible world views. Each agent revises his belief about the possible world views based on his own experience. The posterior probability (density) that the world is $M = m$ is:

$$\alpha(y_i, m) = \frac{1}{\sqrt{\gamma}\sigma_m}\phi\left(\frac{m - \gamma\mu - (1-\gamma)y_i}{\sqrt{\gamma}\sigma_m}\right),$$

where $\gamma = \sigma_y^2/(\sigma_y^2 + \sigma_m^2)$. The expected probability of success in world m is denoted by:

$$\pi(y_i, m) = \Pr[A(\theta) \geq T | y_i, M = m].$$

Second, assume that in each world m, when the collective action succeeds, a new regime with quality θ' drawn from $N(m, \sigma_\theta^2)$ will replace the old one. All agents in the economy are affected by the change in regime, but participants receive an extra reward that is increasing in the new regime quality θ' if the revolt is successful. Specifically, the extra gain from participation in a successful revolt in world m is:

$$\rho(m) = (p_1 - p_0)\mathrm{E}[e^{\theta' + \epsilon_i'} | y_i, M = m] \equiv k e^m,$$

where $k \equiv (p_1 - p_0)e^{(\sigma_\theta^2 + \sigma_\epsilon^2)/2}$. Note that, in contrast to the baseline model, a new draw of θ' is taken upon successful collective action in all possible worlds, including the relatively pessimistic worlds (where the average regime quality m is relatively low). Since θ' centers around m, the expected extra gain will be lower in more pessimistic worlds and higher in more hopeful worlds. In those pessimistic worlds, successfully removing the status quo and replacing it with a new regime does not bring that much gain in expectation because the average quality of governance is low.

Third, agents' cost of participation may depend directly on their economic status. This may reflect considerations such as the time cost of taking action, or the income loss resulting from reprisal by the regime. We let $C(y_i) = c_0 + c_1 e^{y_i}$ represent the cost of participation for agent i, where $c_0 > 0$ is a fixed cost component and $c_1 e^{y_i}$ is a variable cost component with $c_1 > 0$, which is higher for individuals with higher income.

In this extended model, agents participate if and only if the expected benefit exceeds the cost:

$$B(y_i) = \int_{-\infty}^{+\infty} \alpha(y_i, m)\pi(y_i, m)\rho(m)\, \mathrm{d}m \geq C(y_i).$$

PROPOSITION 6. *Given any finite success interval Θ, the benefit function $B(y_i;\Theta)$ in the extended model is hump-shaped in y_i and approaches zero when y_i approaches positive or negative infinity. For any $T \in (0,1)$, a monotone equilibrium does not exist. However, there exists a critical value $\hat{c}_0(T) > 0$, such that non-degenerate interval equilibria exist for all $c < \hat{c}_0(T)$.*

Interval equilibria exist in this model because the benefit-cost ratio $B(y_i;\Theta)/C(y_i)$ is hump-shaped in y_i for any success set Θ that takes the form of an interval. Similar to the benchmark model, $\hat{c}_0(T)$ cannot be too high and it is decreasing in T. Moreover, in this extended model, the mapping f is also monotone. Therefore, our key results concerning the multiplicity of equilibria (Proposition 3), the comparison of interval and non-interval equilibria (Proposition 4), and comparative statics and discontinuity with respect to fixed cost (Proposition 5) are still valid.

3.2. Education

In the benchmark model, an implicit assumption is that agents only differ in income; and income is determined only by the regime quality and personal luck. In reality, individuals also differ in terms of occupation, human capital, and education, which directly affect their income. In fact, economic status and education are among the most important characteristics of the social identity of individuals: the underclasses are typically poorer and less educated, and the middle classes are usually richer and better educated. In this section, we extend our benchmark model by allowing for human capital as an additional determinant of agents' incomes, so as to capture the positive correlation between these two key characteristics. Moreover, this section also demonstrates how the approach that establishes the interval equilibria in the benchmark case can be extended to characterize models with multi-dimensional heterogeneity.

Specifically, we assume that the income of individual i is determined not only by the regime quality θ and idiosyncratic experience ϵ_i, but also by his human capital or education level h_i, which is known to individual i. That is,

$$y_i = \theta + \delta h_i + \epsilon_i,$$

where $\delta > 0$ can be interpreted as returns to human capital.[17]

In this setting, both y_i and h_i matter for agent i to update his world view, estimate the likelihood of success in the hopeful world, and calculate the expected gain of participation relative to standing by. Denote $r_i \equiv y_i - \delta h_i$, which can be interpreted as

[17] In this extension, we assume that the human capital h_i is exogenously given and independent of the luck component ϵ_i. This assumption is reasonable in this static model. However, in a dynamic version of this revolution game, the choice of education or human capital could be correlated with ϵ_i.

the residual income after partialling out the known effect of human capital or education. Since $r_i = \theta + \epsilon_i$, an individual's inferences about the quality of the regime and about the quality of governance in general depends on r_i but not on h_i. Therefore, the updated probability of the world being hopeful for agent i is $\alpha(r_i)$ and his estimate of the probability of success is $\pi(r_i)$. However, the relative gain from participation, $(p_1 - p_0)\mathrm{E}[e^{\theta' + \delta h_i + \epsilon'_i} - e^{r_i + \delta h_i} \mid r_i, H] = \rho(r_i)e^{\delta h_i}$, is affected by the amount of human capital. Therefore, the expected benefit of agent i can be written as:

$$B(r_i, h_i) = \alpha(r_i)\pi(r_i)\rho(r_i)e^{\delta h_i} = B(r_i)e^{\delta h_i}.$$

According to this formulation, for agents with the same residual income, on expectation, the more educated ones benefit more from participating in collective action and the change in the regime. Therefore, they have more incentive to participate. Accordingly, it follows from the fact that, in this model, individuals with more human capital have more to gain in a society with a higher quality of governance. This is largely consistent with the empirical finding that educated individuals tend to participate in political events.

In this extended model, for each particular level of human capital or education, it is still the case that individuals with income in the middle range have the largest incentive to participate in mass protest to push for a political change. The following proposition formalizes this result. Specifically, let $G(h_i)$ be the cumulative distribution function of h_i, and let $Y = \{(y_i, h_i) : a_i = 1\}$ represent the set of individuals who choose to participate.

PROPOSITION 7. *If $1 - G(\cdot)$ is log-concave, then for any $c < \hat{c}(T)$, there exist non-degenerate equilibria such that Θ^* is an interval. The set of y_i for which $(y_i, h_i) \in Y^*$ is an interval, and this interval increases (in the sense of set-inclusion) as h_i increases.*

4. Evidence for the Model Mechanisms and Implications

In this model, we rationalize the passivity of the poor and the activism of the middle class using two mechanisms. First, we characterize the attitudes of each class toward social movements (i.e., the pessimism of the poor and the optimism of the middle class) with Bayesian inferences under model uncertainty, and further explain how these attitudes matter for their coordination in a collective action setting. Second, we highlight that the poor tend to underestimate the chances of success, which also prevents them from participating in social movements. In this section, we offer some corroborating evidence for these mechanisms.

Regarding the effect of pessimism, our model specifies two links, i.e., from economic status to beliefs, and from beliefs to action. The World Values Survey provides information about individuals' beliefs or attitudes regarding governments and politics. In one set of questions, the survey describes four types of governments to respondents—having a strong leader, having experts make decisions, having the army rule, and having a democratic political system—and then asks what they think about each as a way of governing their country. If the respondents consider one or more types of government to be good and the others bad, then this set of answers reveals their political preferences. However, some people dislike all four types of government, which likely means that they do not think governments can do any good and do not believe that their countries can be made better off by changing the government. This type of negative attitude towards governments and changes in politics is fairly close to the notion of pessimism in our model. Therefore, we regard a respondent to be *pessimistic* only if he views all four types of government as "very bad" or "fairly bad."

In Table 2, we show regressions of a dummy variable for *pessimistic* on the income group of the respondent, using the lowest income quintile as the omitted category. Columns (1) and (2) of the table show that the coefficients for the higher income quintiles are all negative and statistically significant, with or without controlling for personal characteristics (education, age, and sex). Moreover, the magnitudes of the coeffi-

Table 2. Pessimistic View of Governance and Income Quintiles

	pessimistic		*uninterested*		*pessimistic* × *uninterested*	
	(1)	(2)	(3)	(4)	(5)	(6)
Lowest Quintile (1–20%)	0	0	0	0	0	0
Second Quintile	-0.214^{*}	-0.301^{**}	-2.51^{***}	-4.49^{***}	-0.292^{***}	-0.361^{***}
(21–40%)	(−2.08)	(−3.00)	(−8.27)	(−15.42)	(−3.54)	(−4.46)
Third Quintile	-0.242^{*}	-0.424^{***}	-4.67^{***}	-8.60^{***}	-0.257^{**}	-0.416^{***}
(41–60%)	(−2.22)	(−4.03)	(−14.35)	(−28.02)	(−2.94)	(−4.93)
Fourth Quintile	-0.415^{**}	-0.675^{***}	-6.09^{***}	-11.8^{***}	-0.380^{***}	-0.597^{***}
(61–80%)	(−3.28)	(−5.59)	(−15.92)	(−33.09)	(−3.74)	(−6.18)
Highest Quintile	-0.467^{**}	-0.856^{***}	-7.52^{***}	-15.2^{***}	-0.524^{***}	-0.834^{***}
(81–100%)	(−2.86)	(−5.54)	(−14.85)	(−32.40)	(−3.99)	(−6.71)
demographic characteristics	yes		yes		yes	
wave fixed effect	yes	yes	yes	yes	yes	yes
country fixed effect	yes	yes	yes	yes	yes	yes
Waves	3–5	3–5	2–5	1–5	3–5	3–5
No. countries	82	83	85	86	82	83
No. observations	148140	151063	195217	214164	143587	146412

Demographic characteristics include education, age, and sex. Levels of statistical significance are indicated by astericks (* for 5%, ** for 1%, and *** for 0.1%), and *t*-statistics are in parentheses. Source: World Values Survey.

cients are larger for higher income quintiles, which implies that people tend to be less pessimistic about governance in general when they enjoy a higher level of welfare in society. It is consistent with our first model mechanism which maps economic status to beliefs about the operation of the world, i.e., $\alpha(y_i)$ is increasing in y_i.

Columns (3) and (4) of Table 2 use an alternative dependent variable to capture the pessimistic world view. The World Values Survey asks the respondents how interested they are in politics. We create a dummy variable *uninterested*, which takes the value of 1 if the response is “not very interested” or “not at all interested.” It is reasonable that people show little interest in politics if they hold a pessimistic view of governance in general and believe that changing governments will not bring real changes to society and their own lives. We also consider a stricter definition in columns (5) and (6), which requires both *pessimistic* and *uninterested* to be equal to 1. The results are similar to

Table 3. The Degree of Pessimism and Participation in Demonstrations

	pessimistic = 1	*pessimistic* = 0
uninterested = 1	9.08%	11.57%
uninterested = 0	17.16%	21.54%

Data source: The third, fourth and fifth waves of the World Values Survey, covering 80 countries from 1994 to 2007.

those in columns (1) and (2).

Furthermore, the evidence suggests that people who hold more pessimistic world views about governance are less likely to participate in mass political action. This is consistent with our second link which connects the beliefs of individuals to their actions. To establish the relationship between the degree of pessimism and the likelihood of participation, we compute the fraction of participants among individuals who are *pessimistic* only, *uninterested* only, both *pessimistic* and *uninterested*, or neither, by using past participation in demonstrations as an indicator of political activism. The results are reported in Table 3, after controlling for demographic characteristics, and country and wave fixed effects. The fraction of participants in each group is statistically significantly different from the others, with the most pessimistic group being the least likely to participate in demonstrations. The results are similar if we do not control for demographics. This finding is in line with our model predictions, and further validates our constructed pessimism measures.

The second mechanism in our model is that the poor and the rich believe that the chances of success are small, i.e., $\pi(y_i;\Theta)$ is low when y_i is small or large in equilibrium. This follows from the fact that individuals, who use their own circumstances to make inferences on the distribution of income in society tend to believe that they are located near the median. This causes the poor to overestimate the size of the poor population and the rich to overestimate the size of the population who are well-off. Both groups underestimate the mass of middle-income individuals who will attack the regime. Figure 4 illustrates this mechanism. In this figure, $\tilde{y}_i$ is the median of the

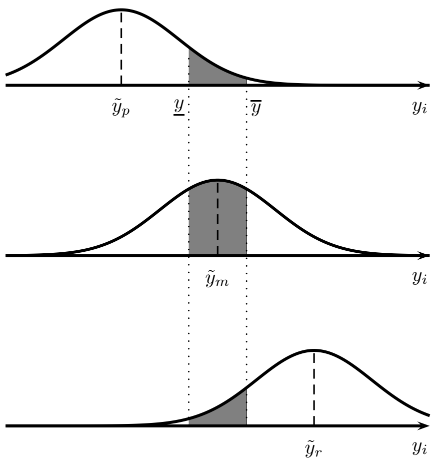


Figure 4. *Poor, middle-income and rich citizens perceive that the distribution of well-being in society is centered at different points, i.e., $\tilde{y}_p$, $\tilde{y}_m$ and $\tilde{y}_r$. The estimated size of protestors is largest if the center of the distribution is perceived to be at $\tilde{y}_m$.*

distribution of well-being perceived by agent i. Since $\tilde{y}_i = \mathrm{E}[\theta | y_i, H]$ in the hopeful world, $\tilde{y}_i$ is increasing in y_i. For example, a poor citizen tends to believe that the distribution is centered near $\tilde{y}_p$, while a rich citizen tends to believe that it is centered near $\tilde{y}_r$. Both believe that the mass of attackers (the shaded region) is small. However, a middle-income citizen who believes that the distribution is centered near $\tilde{y}_m$ is more optimistic about having a large number of like-minded middle-income citizens who are prepared to attack the regime.

Data from the International Social Survey Programme provide information regarding both the perceived relative position and the actual income of individuals, which can be useful for corroborating the aforementioned mechanism in our model. The respondents were asked to place themselves in an income decile of their own society, where 10 represents the top ten percent and 1 the bottom ten percent. We also identify to which income decile they actually belong in their own society, based on the actual income that they reported and information regarding income distribution provided by the World Income Distribution database.[18] Figure 5 shows the average self-reported relative position in each income group.

[18]Specifically, we use the 2009 wave of Social Inequality Survey in the International Social Survey Programme for information on the actual income of the respondents. These income figures are converted to actual income deciles based on the cutoff points available from the World Income Distribution database.

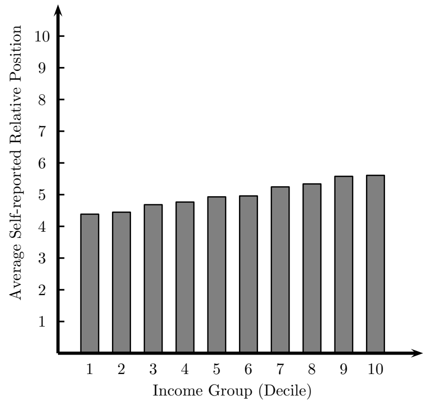


Figure 5. *The average perceived relative position in each income decile group. The perceived relative position is on average increasing in their actual income status but individuals from all of income groups tend to believe that they are close to the median. Individuals at the lower and upper ends of the income spectrum overestimate and underestimate their relative position, respectively. Source: International Social Survey Programme and World Income Distribution database, covering 26 countries.*

Although higher-income individuals tend to place themselves in higher income deciles, it is evident from the figure that on average the lower-income group tend to overestimate their relative position while richer individuals tend to underestimate theirs.[19] This evidence indicates that citizens do not have sufficient knowledge about the actual income distribution and tend to believe that they are closer to the median than they actually are.

In our model, the lack of information on actual income distribution is assumed. In reality, such lack of knowledge may result from spatial segregation, which reduces the availability of information to individuals, who typically can only update from a sample of similarly situated people.[20] This explanation is supported both empirically and theoretically in Cruces et al. (2013) that demonstrate that the perception bias of income distribution is indeed correlated with an individual's relative position within the reference group (proxied by area of residence).

[19]A regression analysis that we conduct with country fixed effects and various sets of controls also confirms this pattern. Cruces et al. (2013) find the same pattern with survey data from Argentina.

[20]We thank one of the anonymous referees for offering this explanation.

5. Model Interpretation and Alternative Hypotheses

Political aspiration. In our model, the middle class aspire and fight for a better society, because they expect that such a goal is achievable through coordinated effort and that they can benefit from it. The lower classes, struggling at the bottom of society, would potentially have even more to gain but choose to standby, because they care little about the likely impending changes. We demonstrate both theoretically and empirically that this type of political aspiration is affected by economic status.

However, our theory of political aspiration can be interpreted more broadly. First, economic status, such as income, can be considered as an approximation of the experience of individuals in society. Downtrodden individuals hold unfavorable views about the possibility of real change, based on their *own* circumstances. This implies that their interaction with individuals from other strata or more generally, new information about how the world operates, may change their local pessimistic beliefs. That is actually one essential point made by Lenin in his influential political pamphlet, *What is to be Done?*, in which he argues that to motivate the working class to take part in political action, their political consciousness "would have to be brought to them from without" (Lenin, 1988).

Second, some variant of our model can also be useful to shed light on mass movements that push for political changes relevant to non-economic issues, such as political rights or justice. The Hong Kong "Umbrella Movement" in 2014 is a good case in point. The politically disadvantaged (mostly the poor) are inclined to hold a politically cynical views about how their country is governed, and remain skeptical about what would be brought to them by political rights promised by the movement. More generally, our theory can explain the contrast between the passivity of the poor and activism of the middle class in many other political settings, such as voting, signing petitions, and working for a political party.[21] Consistent with our theory, this contrast

[21] For example, Jackson et al. (1998) empirically demonstrate that the poor are less inclined to vote in elections and are more responsive to registration obstacles.

may stem from the lack of desire and aspiration of lower class citizens, because they perceive the political system as distant and unresponsive to their interests.

The theory of political aspiration that we put forth in this paper adds to the existing literature on mass movements. However, while stressing its importance, we do not claim that our theory is the only plausible explanation for this phenomenon. In the reminder of this section, we discuss other hypotheses on the role that the middle class play in modern polities and contrast these hypotheses with our theory to explicitly highlight the key differences.

Modernization and economic interest. According to modernization theory (Lipset 1959), a growing middle class demands and serves democracy, which facilitates democratization, because this social stratum is more receptive to democratic values. Barro (1999) offers evidence that there is a positive relationship between the size of the middle class and the extent of democracy. However, Acemoglu and Robinson (2009) argue that the middle class are driven mainly by economic interests in the process of the creation and consolidation of democracy. In their framework, the middle class can choose to side with the poor or the rich, depending purely on which action promotes their economic interest.

Our work differs from these investigations and debates in two dimensions. First, our focus is not on democratization in developing countries, but the phenomenon of middle class activism, which is observed both in democratic and non-democratic countries.[22] As shown in the literature, the middle classes, although active in politics, do not necessarily embrace democracy (Huntington, 1993; Chen and Lu, 2011).

Second, we specifically model how agents from different income groups coordinate their actions and challenge the status quo, whereas the modernization theory treats each income group as a single decision maker. It should be stressed that by incorporating the coordination features, our model can address a range of interesting characteristics of mass movements. For example, our model can shed light on why

[22]We stress that the demands for higher quality governance and democracy are conceptually distinct. We focus on collective action against the status quo that seeks for change, not necessarily democracy.

middle class protests are usually sudden in timing and large in scale, see Section 2.4.

Education. The middle class are typically both economically better off and more educated than the poor. In fact, economic status and education are two key characteristics of the middle class. A popular idea concerning the observed middle class activism is that people in this social stratum are more educated than the poor, and educated individuals tend to be more politically involved. Therefore, the rise of the middle class leads to greater political involvement among the masses and demand for political change. Glaeser et al. (2007) provide empirical evidence for the conjecture that education raises the perceived benefits of political participation.[23]

While acknowledging that this hypothesis is very reasonable, we focus on the other characteristic of this social stratum and demonstrate how economic status affects political participation. This focus is justified, because we have shown empirically that the political aspiration of individuals is related to their economic status, even when controlling for the effect of education (see our discussion in Section 4). Our view is that the theory offered in this paper and the education nexus are complementary, rather than mutually exclusive. The complementarity of these dimensions is explicitly characterized and highlighted in Section 3.2.

Disappointment and the tunnel effect. Hirschman and Rothschild (1973) outline a hypothesis regarding the relationship between social mobility and political stability, which may also shed some light on middle class activism. In a changing society, the upper middle class, who have acquired a considerable amount of marketable skills, may have risen in social status by accumulating wealth. However, they remain disappointed since they may encounter obstacles in many other social dimensions, which frustrates their continued ascent. The lower middle class, who have similar skills, may initially derive satisfaction from observing their peers rise in status, but then lose their earlier hope and turn to the enemies of the status quo when they do not advance as

[23]On the one hand, Glaeser et al. (2007) show that individuals acquire civic skills through education, which facilitates their participation in politics. On the other hand, Kam and Palmer (2008) show that education may be a proxy for cognitive skills. Both mechanisms imply that more educated individuals tend to be more politically involved.

much as they expect (the tunnel effect).

This hypothesis essentially argues that disappointment of the middle class may lead to action against the existing order, based on the implicit behavioral assumption that clamoring their disappointment is a necessary action for them. In contrast, we demonstrate that the middle class may expend costly effort in demanding change, because they are hopeful and optimistic, or in other words, they believe that the changes are both beneficial and likely, while the poor are inactive because they do not believe that the new regime would deliver anything substantially better and collective action is unlikely to be successful. More importantly, we rationalize the formation of such beliefs for each individual with Bayes' rule and build an explicit connection from economic status to beliefs and to participation choices.

Psychological attribute. The flip side of the activism of the middle class is the passivity of the poor. Both Huntington (1968) and Fukuyama (2013) hypothesize that there are behavioral differences between the poor and the middle class. They argue that the poor do not care about politics because they are concerned mostly about day-to-day survival. However, that observation should be the result of the political passivity instead of the cause. If the poor believe that changes in politics brought about by mass movements can significantly improve their life quality, it would be most natural for them to participate, given that the opportunity costs are low and the potential benefits are high. Fukuyama (2013) also argues that the failure of governments to meet the expectations of the middle class leads to their collective action against authorities, but does not explain why this social stratum, instead of the poor, hold such high expectations for the quality of governance.

Our paper can be seen as an attempt to provide a rationale for the pessimism of the poor instead of just postulating it as a behavioral or psychological attribute. Such a theoretical foundation is useful because it helps explain, among other things, why the size of the middle class affects their optimism, or why people who used to be politically quiescent when they were poor become active when their economic status improves.

6. Conclusion

In this paper, we rationalize the pessimism of the poor and the hopefulness of the middle class with model uncertainty, and provide evidence to corroborate our model mechanisms. Although our theory is formalized in an environment of political economy, a substantial extension of our work can potentially contribute to the economics literature on the behavior of the poor and the poverty trap. One line of this development economics literature is based on the hypothesis that the poor, who live under scarcity, may make very different choices from the non-poor. This view is labeled as "poor but behavioral" (Ghatak 2014). A variant of the mechanism that we put forth in this paper to explain the political pessimism of the poor can be extended to rationalize their "economic pessimism." The poor, who receive bad outcomes from their market participation, may make negative inferences about the opportunities that the market offers, which further causes the lack of information and their poverty. In other words, poverty leads to pessimism, which, in turn, reinforces it.

Our work also explicates how individuals with heterogeneous beliefs about the underlying structure of the game coordinate in taking collective action, thereby enriching the literature on coordination games by adding model uncertainty. We also develop a workable approach to establishing the existence and multiplicity of interval equilibria, which can be applied to coordination games with similar features.

Appendix

Proof of Lemma 1. The function $B(y_i;\Theta)$ has a single-crossing from above because $\rho(y_i) > 0$ if $y_i < \hat{y}$ and $\rho(y_i) < 0$ if $y_i > \hat{y}$. It goes to 0 when y_i approaches $-\infty$ because $\lim_{y_i \to -\infty} \alpha(y_i) = 0$; and it goes to 0 when y_i approaches $\hat{y}$ because $B(\hat{y};\Theta) = 0$ and $B(\cdot;\Theta)$ is continuous. Finally, it is straightforward to verify from equations (1) and (3) that $\alpha(y_i)$ is log-concave and $\rho(y_i)$ is log-concave when positive. For $\pi(y_i;\Theta)$, we rewrite equation (6) as:

$$\begin{aligned}\pi(y_i;\Theta) &= \int_{\frac{\underline{\theta}-\beta m_H}{\sqrt{\beta}\sigma_\theta}}^{\frac{\overline{\theta}-\beta m_H}{\sqrt{\beta}\sigma_\theta}} \phi\left(t - \frac{1-\beta}{\sqrt{\beta}\sigma_\theta} y\right) \mathrm{d}t \\ &= \int_{-\infty}^{+\infty} \frac{1-\beta}{\sqrt{\beta}\sigma_\theta} \phi\left(\frac{1-\beta}{\sqrt{\beta}\sigma_\theta}(y-t)\right) \mathrm{I}(t)\, \mathrm{d}t,\end{aligned}$$

where the indicator function $\mathrm{I}(t)$ is equal to 1 if $t \in [(\underline{\theta} - \beta m_H)/(1-\beta), (\overline{\theta} - \beta m_H)/(1-\beta)]$ and is equal to 0 otherwise. Since $\phi(\cdot)$ and $\mathrm{I}(\cdot)$ are both log-concave, their convolution is log-concave (Prekopa-Leindler inequality) as well. Therefore, $B(y_i;\Theta) = \alpha(y_i)\pi(y_i;\Theta)\rho(y_i)$ is log-concave when positive, which implies that it is increasing then decreasing for $y < \hat{y}$. ■

Proof of Lemma 2. If $f(Y_0) = \emptyset$, then the result is trivially true. Assume $f(Y_0) \neq \emptyset$, and consider $Y_1 \supseteq Y_0$. It is obvious that $A(\theta;Y_1) \geq A(\theta;Y_0)$ for any θ. Because $A(\theta;Y)$ is hump-shaped, an upward shift in this function lowers the smaller root and raises the larger root to the equation $A(\theta;Y) = T$. Denote the respective solutions to the inequalities $A(\theta;Y_1) \geq T$ and $A(\theta;Y_0) \geq T$ by Θ_1 and Θ_0. We have $\Theta_1 \supseteq \Theta_0$. A wider success interval raises the estimated probability of success for every agent. Thus, $B(y_i;\Theta_1) \geq B(y_i;\Theta_0)$ for any $y_i < \hat{y}$. Since $B(y_i;\Theta)$ is hump-shaped, the respective solutions to the inequalities $B(y_i;\Theta_1) \geq c$ and $B(y_i;\Theta_0) \geq c$, denoted Y_1' and Y_0', satisfy $Y_1' \supseteq Y_0'$. ■

Proof of Proposition 3. Observe that $\alpha(y_i)\rho(y_i)$ is hump-shaped in y_i. Denote $y^P \equiv \operatorname{argmax}_{y_i} \alpha(y_i)\rho(y_i)$ and $\bar{c} \equiv \alpha(y^P)\rho(y^P)$. Further, observe that $\pi(y_i;[y_i - \omega/2, y_i +$

$\omega/2]$) is also hump-shaped in y_i and attains a maximum at m_H for any width of success interval $\omega > 0$.

Claim 1. For any $c < \bar{c}$, there exist y^B and ω^B such that $B(y_i; [y^B - \omega^B/2, y^B + \omega^B/2])$ attains its maximum at y^B and the maximum value is equal to c. Further, y^B is between y^P and m_H.

Proof. Define $\tilde{B}(y_i, \omega) \equiv B(y_i; [y_i - \omega/2, y_i + \omega/2])$. For any $\omega > 0$, this function is hump-shaped in y_i and attains a maximum at some $y^*(\omega)$, which is between y^P and m_H. The function $\tilde{B}(y^*(\omega), \omega)$ is strictly increasing in ω. Further, it approaches 0 when ω approaches 0, and goes to $\bar{c}$ when ω approaches ∞. Therefore, for any $c < \bar{c}$, there exists an ω^B such that $\tilde{B}(y^*(\omega^B), \omega^B) = c$. Let $y^B \equiv y^*(\omega^B)$, then the claim is shown.

Claim 2. For any $c < \bar{c}$, there exists an arbitrarily small interval Y_ε such that $f^{-1}(Y_\varepsilon) \supset Y_\varepsilon$.

Proof. We pick $Y_\varepsilon = [y^B - \varepsilon, y^B + \varepsilon]$ for some small $\varepsilon > 0$. Let Θ_ε satisfy $\{y_i : B(y_i; \Theta_\varepsilon) \geq c\} = Y_\varepsilon$. By construction, Θ_ε contains the interval $[y^B - \omega^B/2, y^B + \omega^B/2]$ and is arbitrarily close to this interval when ε approaches 0. Therefore, the mid-point of Θ_ε can be made arbitrarily close to y^B. Let Y'_ε satisfy $\{\theta : A(\theta; Y'_\varepsilon) \geq T\} = \Theta_\varepsilon$. The mid-point of Y'_ε and that of Θ_ε must be the same, since $A(\cdot; Y'_\varepsilon)$ is symmetric about the mid-point of the interval Y'_ε. Therefore, the mid-point of Y'_ϵ can be also made arbitrarily close to y^B. For any $T \in (0, 1)$, Y'_ε must have a strictly positive measure, but the measure of Y_ε can be made arbitrarily small. Therefore, for ε sufficiently small, Y_ε collapses around the mid-point of Y'_ε. As a result, we have $Y'_\epsilon \supset Y_\varepsilon$, and the claim follows.

For c sufficiently small, we can always find a $\tilde{Y}$ such that it contains the interval between y^P and m_H and $f(\tilde{Y}) \supset \tilde{Y}$. That is because m_H is always smaller than $\hat{y}$ and $f(\tilde{Y})$ is arbitrarily close to the half interval $(-\infty, \hat{y}]$ when c is very small. Then such a $\tilde{Y}$ will satisfy $\tilde{Y} \supset Y_\varepsilon$ for small ε. Therefore,

$$\tilde{Y} \supset f^{-1}(\tilde{Y}) \supset f^{-1}(Y_\varepsilon) \supset Y_\varepsilon.$$

The second inequality holds because the mapping f^{-1} is monotone; the last inequality is implied by Claim 2.

Denote the set of intervals $\mathcal{I}^{**} = \{Y : \tilde{Y} \supseteq Y \supseteq Y_\varepsilon\}$. Any element of $\mathcal{I}^{**}$ is a non-degenerate interval. Moreover, the partially ordered set $(\mathcal{I}^{**}, \supseteq)$ is a complete lattice, with supremum $\tilde{Y}$ and infimum Y_ε. Since both $f^{-1}(\tilde{Y})$ and $f^{-1}(Y_\varepsilon)$ belong to $\mathcal{I}^{**}$, and since f^{-1} is monotone, we have $f^{-1}(Y) \in \mathcal{I}^{**}$ for any $Y \in \mathcal{I}^{**}$. In other words, f^{-1} is a monotone mapping from $\mathcal{I}^{**}$ to $\mathcal{I}^{**}$. By Tarski's fixed point theorem, a fixed point of f^{-1} in $\mathcal{I}^{**}$ exists.

Together with Proposition 2, this argument implies that there exists some small c' such that multiple non-degenerate equilibria exist for $c \leq c'$. Let Y^* denote the largest non-degenerate equilibrium participation interval and Y^{**} denote a smaller one when $c = c'$. By definition, $Y^* \supset Y^{**}$.

Let $Y_l = f(Y^{**}; c' + \eta)$. For small η, Y_l is non-degenerate. Because $f(Y; c)$ is decreasing in c for any Y, we have $f(Y^{**}; c') = Y^{**} \supset Y_l = f(Y^{**}; c' + \eta)$. Let Y_m denote the largest equilibrium participation interval when $c = c' + \eta + \xi < \hat{c}$, where $\xi > 0$. We have $f(Y_m; c' + \eta) \supset Y_m = f(Y_m; c' + \eta + \xi)$. For $\eta + \xi$ sufficiently small, Y_m is sufficiently close to Y^* and thus $Y_m \supset Y^{**} \supset Y_l$. Therefore,

$$Y_m \supset f^{-1}(Y_m; c' + \eta) \supset f^{-1}(Y_l; c' + \eta) \supset Y_l.$$

Using a similar logic as before, a fixed point of $f^{-1}(\cdot; c' + \eta)$ exists in $\mathcal{I}^{**} = \{Y : Y_m \supseteq Y \supset Y_l\}$. Moreover, the proof of Proposition 2 establishes a fixed point of $f(\cdot; c' + \eta)$ exists in $\mathcal{I}^* = \{Y : Y_{\max} \supseteq Y \supseteq Y_m\}$. Therefore, f has at least two non-degenerate fixed points for any $c \leq c' + \eta$.

Proceeding iteratively by replacing $c' + \eta$ for c', this argument can be repeated for any $c' < \hat{c}$. Thus, f has at least two non-degenerate fixed points whenever $c < \hat{c}$. ■

Proof of Proposition 4. Since $B(y_i; \Theta)$ approaches 0 for y_i very low and is negative for $y_i > \hat{y}$, both the participation set and the success set must be unions of non-overlapping finite intervals. Suppose there is a non-interval equilibrium (Y_n^*, Θ_n^*). Let $Y_n^* = \cup_{j=1}^{J_1}[\underline{y}^j, \overline{y}^j]$ and $\Theta_n^* = \cup_{j=1}^{J_2}[\underline{\theta}^j, \overline{\theta}^j]$, where J_1 and J_2 are positive integers and $\overline{y}^j < \underline{y}^{j+1}$ and $\overline{\theta}^j < \underline{\theta}^{j+1}$.

Consider the participation interval $Y_0 = [\underline{y}^1, \overline{y}^{J_1}]$ and the success interval $\Theta_0 = [\underline{\theta}^1, \overline{\theta}^{J_2}]$. We have:

$$A(\underline{\theta}^1; Y_0) > A(\underline{\theta}^1; Y_n^*) = T,$$
$$A(\overline{\theta}^{J_2}; Y_0) > A(\overline{\theta}^{J_2}; Y_n^*) = T;$$

where the inequality follows from $Y_0 \supset Y_n^*$, and the equality follows because $\underline{\theta}_1$ and $\overline{\theta}^{J_2}$ are on the boundary of the equilibrium success set Θ_n^*. Although $A(\theta; Y_n^*)$ may not be hump-shaped in θ, $A(\theta; Y_0)$ is hump-shaped as Y_0 is an interval. Therefore, if Θ' is the set of θ for which $A(\theta; Y_0) \geq T$, the two inequalities above imply that $\Theta' \supset \Theta_0 \supset \Theta_n^*$. This, in turn, implies:

$$B(\underline{y}^1; \Theta') > B(\underline{y}^1; \Theta_n^*) = c;$$
$$B(\overline{y}^{J_1}; \Theta') > B(\overline{y}^{J_1}; \Theta_n^*) = c.$$

Recall that $f(Y_0)$ is given by the set of y_i for which $B(y_i; \Theta') \geq c$. Therefore, we must have $f(Y_0) \supset Y_0$. As in the proof of Proposition 2, there exists at least one fixed point of f in the domain $\mathcal{I}^* = \{Y : Y_{\max} \supseteq Y \supseteq Y_0\}$. Any fixed point in this domain is larger than Y_0, and is therefore larger than Y_n^*. ∎

Proof of Proposition 5. Let (Y_1, Θ_1) and (Y_2, Θ_2) be the largest equilibria when the costs are c_1 and c_2, respectively, with $c_1 < c_2 < \hat{c}$. Since $B(y_i; \Theta_2) - c$ strictly decreases in c for any y_i, we have $f(Y_2; c_1) \supset f(Y_2; c_2) = Y_2$. Thus, $f(\cdot; c_1)$ has a fixed point in the set $\mathcal{I}^* = \{Y : Y_{\max} \supseteq Y \supseteq Y_2\}$, which implies that $Y_1 \supset Y_2$. Furthermore, since $A(\theta; Y_1) > A(\theta; Y_2)$ for any θ, we have $\Theta_1 \supset \Theta_2$.

The second part of the proposition follows from the fact that $f(Y; T)$ is decreasing in T for any Y. Let $\hat{Y}^*$ be a fixed point of f when the threshold is T and when $c = \hat{c}(T)$.

Then, for a lower threshold $T' < T$, we have $f(\hat{Y}^*; T') \supset \hat{Y}^*$. A fixed point of $f(\cdot; T')$ exists in $\mathcal{I}^* = \{Y : Y_{\max} \supseteq Y \supseteq \hat{Y}^*\}$ when the threshold is T' and when $c = \hat{c}(T)$. This implies $\hat{c}(T') \geq \hat{c}(T)$. ■

Proof of Proposition 6. Since $\alpha(y_i, m)$ is the density function of a normal distribution with mean $\gamma\mu + (1-\gamma)y_i$ and variance $\gamma\sigma_m^2$, we have:

$$\alpha(y_i, m)\rho(m) = ke^{\gamma\mu+(1-\gamma)y_i+\gamma\sigma_m^2/2}\frac{1}{\sqrt{\gamma}\sigma_m}\phi\left(\frac{m-(\gamma\mu+(1-\gamma)y_i+\gamma\sigma_m^2)}{\sqrt{\gamma}\sigma_m}\right),$$

which also takes the form of a normal density. Therefore, for any θ_0,

$$\begin{aligned}&\int_{-\infty}^{\infty}\alpha(y_i, m)\rho(m)\Pr[\theta \leq \theta_0 \mid y_i, M = m]\,\mathrm{d}m \\ = \quad & ke^{\gamma\mu+(1-\gamma)y_i+\gamma\sigma_m^2/2}\Phi\left(\frac{\theta_0-\beta\gamma\mu-(1-\beta\gamma)\,y_i-\beta\gamma\sigma_m^2}{\sqrt{\beta\sigma_\theta^2+\beta^2\gamma\sigma_m^2}}\right),\end{aligned}$$

where we make use of the fact that $\mathrm{E}[\Phi(aX+b)] = \Phi(b/\sqrt{1+a^2})$ when X is a standard normal random variable. This expression goes to 0 as y_i goes to negative infinity, while $C(y_i)$ goes to $c_0 > 0$ as y_i goes to negative infinity. Therefore, the very poor never participate. There is no monotone equilibrium in the extended model.

Take any interval success set $\Theta = [\underline{\theta}, \overline{\theta}]$. The benefit function $B(y_i; \Theta)$ is equal to:

$$\begin{aligned}&ke^{\gamma\mu+(1-\gamma)y_i+\gamma\sigma_m^2/2} \\ &\times\left[\Phi\left(\frac{\overline{\theta}-\beta\gamma\mu-(1-\beta\gamma)y_i-\beta\gamma\sigma_m^2}{\sqrt{\beta\sigma^2+\beta^2\gamma\sigma_m^2}}\right)-\Phi\left(\frac{\underline{\theta}-\beta\gamma\mu-(1-\beta\gamma)y_i-\beta\gamma\sigma_m^2}{\sqrt{\beta\sigma^2+\beta^2\gamma\sigma_m^2}}\right)\right].\end{aligned}$$

This function is log-concave in y_i, and goes to 0 as y_i approaches positive or negative infinity. The cost function $C(y_i)$ is log-convex, and is bounded away from 0. This implies that the function $B(y_i; \Theta)/C(y_i)$ is increasing then decreasing. If c_0 is greater than some critical value $\hat{c}_0$, the equation $B(y_i; \Theta)/C(y_i) - 1 = 0$ has no solution; if c_0 is lower than that critical value, it has two solutions. The proof of the existence of an interval equilibrium for $c_0 \leq \hat{c}_0$ follows the same steps as in Proposition 2. ■

Proof of Proposition 7. The key to the proof is to show that there exist equilibria in which Θ^* is a non-empty interval. Consider any success interval Θ. Since $B(r_i)e^{\delta h_i}$ is quasi-concave in (r_i, h_i), the set $R \equiv \{(r_i, t_i) : B(r_i; \Theta)e^{\delta h_i} \geq c\}$ is convex. Let $\hat{h}(r_i)$ be the minimum h_i that satisfies $B(r_i; \Theta)e^{\delta h_i} \geq c$. If no such h_i exists, define $\hat{h}(r_i)$ to be the highest h_i in the support. Since the level set is convex, the function $\hat{h}$ is convex. Given the set R, the mass of attackers can be written as:

$$A(\theta; R) = \int_{-\infty}^{\infty} \left[1 - G\left(\hat{h}(r_i)\right)\right] \phi\left(\frac{r_i - \theta}{\sigma_\epsilon}\right) \frac{1}{\sigma_\epsilon} \, \mathrm{d}r_i.$$

To show that $A(\theta; R)$ is hump-shaped, the following two facts are sufficient. First, $A(\theta; R)$ approaches 0, when θ goes to $+\infty$ and $-\infty$. Second, $A(\theta; R)$ is log-concave, which follows from the log-concavity of $1 - G(\hat{h}(r_i))$ and $\phi(\cdot)$. To see this,

$$\frac{\mathrm{d}^2 \log[1 - G(\hat{h}(r_i))]}{\mathrm{d}r_i^2} = -\left(\frac{g}{1-G}\right)' \left(\frac{\mathrm{d}\hat{h}}{\mathrm{d}r_i}\right)^2 - \frac{g}{1-G} \frac{\mathrm{d}^2 \hat{h}}{\mathrm{d}r_i^2} < 0.$$

This inequality holds because the hazard rate is increasing if $1 - G(\cdot)$ is log-concave and because $\hat{h}(\cdot)$ is convex. Therefore, we may construct a mapping which takes an interval Θ and solve for the convex set R that satisfies $B(r_i; \Theta)e^{\delta h_i} \geq c$, and from such R solve for the interval Θ' that satisfies $A(\theta; R) \geq T$. Following the same logic as the proof of Proposition 2, we can show that a fixed point of this mapping exists for c less than some critical value $\hat{c}$. Further, if $t' > t$, then $(r_i, t) \in R^*$ implies $(r_i, t') \in R^*$. Given the definition of r_i, the proposition follows. ■

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