# Precautionary Self-Insurance-cum-Protection *

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Precautionary self-insurance-cum-protection (SICP) arises when an individual spends more on SICP when background risk is introduced. We develop a two-period model wherein additive/multiplicative background risk prevails in the second period. Using the theory of monotone comparative statics and risk apportionment, we derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk deteriorates via higher-order stochastic dominance. Prudence is called for to create a precautionary motive that induces the individual to shift his wealth in a way to reduce the loss of expected utility caused by the addition of background risk, thereby giving rise to the precautionary SICP.

*JEL classification:* D81; G22

*Keywords:* Background risk; Risk apportionment; Self-insurance; Self-protection

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# Precautionary Self-Insurance-cum-Protection


### Abstract

Precautionary self-insurance-cum-protection (SICP) arises when an individual spends more on SICP when background risk is introduced. We develop a two-period model wherein additive/multiplicative background risk prevails in the second period. Using the theory of monotone comparative statics and risk apportionment, we derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk deteriorates via higher-order stochastic dominance. Prudence is called for to create a precautionary motive that induces the individual to shift his wealth in a way to reduce the loss of expected utility caused by the addition of background risk, thereby giving rise to the precautionary SICP.



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## 1. Introduction

The seminal work of Ehrlich and Becker (1972) examines how individuals facing insurable risk of a loss can invest in activities that either reduce the size of the loss (self-insurance) or the probability of the loss (self-protection). Lee (1998) points out that many actions taken by individuals for risk management purposes may provide both self-insurance and self-protection at the same time, which he refers to as self-insurance-cum-protection (SICP).[1] Examples of SICP include the use of high quality brakes that reduces both the probability of an automobile accident and the resulting damages, the practice of regular medical checkups that decreases the probability and severity of an illness, and many others.

Precautionary SICP arises when individuals spend more on SICP upon the addition of background risk. In this paper, we develop a two-period model wherein an individual invests in SICP in the first period to manage insurable risk of a loss that occurs in the second

[1] Indeed, Ehrlich and Becker (1972) recognize SICP and give an example wherein good lawyers are able to reduce not only the probability of conviction but also the punishment for crime.

period.[2] We introduce background risk in the second period, where this risk is independent of the insurable risk, and can be either additive (e.g., random wealth) or multiplicative (e.g., inflation risk) in nature. Using the theory of monotone comparative statics (Milgrom and Shannon, 1994) and risk apportionment (Eeckhoudt and Schlesinger, 2006; Eeckhoudt et al., 2009a, 2009b), we derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk deteriorates via higher-order stochastic dominance. We show that prudence is required for the precautionary SICP. There is a precautionary motive that induces the prudent individual to shift his wealth from the first period to the second period when the background risk is introduced. Doing so reduces the "pain" caused by the background risk in the second period, where "pain" is defined as the loss of expected utility (Eeckhoudt and Schlesinger, 2006).

Our results generalize those of Eeckhoudt et al. (2012) and Lee (2012) to the case of precautionary SICP under additive background risk, without relying on the first- and second-order conditions for the individual's decision problem. We further derive novel results of precautionary SICP under multiplicative background risk, which contributes to the understanding of multiplicative risk apportionment (Wang and Li, 2010).

The rest of this paper is organized as follows. Section 2 delineates our two-period model of SICP in the presence of additive/multiplicative background risk. Sections 3 and 4 derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk deteriorates via higher-order stochastic dominance. The final section concludes.

## 2. The model

Consider a two-period model of an individual who has initial wealth $w_0$ in the first

[2] Our two-period model can also be interpreted as a single-period model as in Lee (2012) by treating the first-period utility as the utility cost arising from SICP.

period and $w_1$ in the second period. The individual has utility functions, $v(w)$ and $u(w)$, defined over his first-period and second-period final wealth, respectively, where both $v(w)$ and $u(w)$ are continuously differentiable functions of $w$. Let $T$ be a positive integer and define $U_T \equiv \{u(w) : (-1)^{n+1}u^{(n)}(w) > 0 \text{ for } n = 1, \ ..., T\}$, where $u^{(n)}(w) = \mathrm{d}^n u(w)/\mathrm{d}w^n$ denotes the $n$th derivative of $u(w)$. Hence, $U_T$ is the set of utility functions that exhibit mixed risk aversion up to order $T$ (Caballé and Pomansky, 1996).

While there is no uncertainty in the first period, the individual faces insurable risk, $\tilde{z}$, and background risk, $\tilde{\varepsilon}$, in the second period. The background risk, $\tilde{\varepsilon}$, is either additive (e.g., random wealth) or multiplicative (e.g., inflation risk) in nature. The two random variables, $\tilde{z}$ and $\tilde{\varepsilon}$, are independent of each other.

In contrast to the background risk, $\tilde{\varepsilon}$, which is neither hedgeable nor insurable, the insurable risk, $\tilde{z}$, can be managed by the individual by spending an amount, $e$, on self-insurance-cum-protection (SICP). The expenditure, $e$, on SICP is endogenously chosen from the compact set, $[0, w_0]$, by the individual in the first period. There are two loss events in the second period, "loss" and "no loss," which partition the state space into the "loss states" and the "no-loss states," respectively. Given that $e$ has been spent on SICP, $\tilde{z} = \ell(e)$ in the loss states, which occurs with probability $p(e)$, and $\tilde{z} = 0$ in the no-loss states, which occurs with probability $1 - p(e)$, where $0 < \ell(e) < w_1$ and $0 < p(e) < 1$ for all $e \in [0, w_0]$. We assume that the individual's SICP is effective in that more expenditure on SICP reduces both the magnitude of loss and the probability of the loss event, i.e., both $\ell(e)$ and $p(e)$ are decreasing functions of $e$.

## 3. Additive background risk

In this section, we examine the case that the background risk, $\tilde{\varepsilon}$, is additive in nature such that $\tilde{\varepsilon}$ is a zero-mean random variable. The individual's expected utility over the two periods is given by

$$f(e) = v(w_0 - e) + p(e)\mathrm{E}\{u[w_1 - \ell(e) + \tilde{\varepsilon}]\} + [1 - p(e)]\mathrm{E}[u(w_1 + \tilde{\varepsilon})]. \tag{1}$$

where $\mathrm{E}(\cdot)$ is the expectation operator. The individual's ex-ante decision problem is to choose $e \in [0, w_0]$ so as to maximize $f(e)$. Since $f(e)$ is a continuous function of $e$, the set, $\arg\max_{e\in[0,w_0]} f(e)$, is non-empty, and plausibly not a singleton. Let $e^*$ be an element in $\arg\max_{e\in[0,w_0]} f(e)$.

The individual is said to demonstrate precautionary SICP if he spends more on SICP in the presence than in the absence of the zero-mean additive background risk. To derive conditions under which precautionary SICP prevails, we examine the case that the background risk, $\tilde{\varepsilon}$, changes to $\tilde{\xi}$, where $\tilde{\xi}$ is a random variable that is dominated by $\tilde{\varepsilon}$ via the $N$th-order stochastic dominance and $N \geq 1$. In this case, the individual's two-period expected utility becomes

$$g(e) = v(w_0 - e) + p(e)\mathrm{E}\{u[w_1 - \ell(e) + \tilde{\xi}]\} + [1 - p(e)]\mathrm{E}[u(w_1 + \tilde{\xi})]. \tag{2}$$

The individual's ex-ante decision problem is to choose $e \in [0, w_0]$ so as to maximize $g(e)$. Since $g(e)$ is a continuous function of $e$, the set, $\arg\max_{e\in[0,w_0]} g(e)$, is non-empty, and plausibly not a singleton. Let $e^{**}$ be an element in $\arg\max_{e\in[0,w_0]} g(e)$.

To derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk becomes more risky, i.e., $e^{**} \geq e^*$, we have to compare the two sets, $\arg\max_{e\in[0,w_0]} f(e)$ and $\arg\max_{e\in[0,w_0]} g(e)$. This falls into a principal concern in the theory of monotone comparative statics (Milgrom and Shannon, 1994). We state and prove the following lemma.

**Lemma 1.** *Consider two random variables, $\tilde{\varepsilon}$ and $\tilde{\xi}$, such that $\tilde{\varepsilon}$ dominates $\tilde{\xi}$ via the $N$th-order stochastic dominance. The following condition holds:*

$$\mathrm{E}[u(w + k + \tilde{\xi})] - \mathrm{E}[u(w - k + \tilde{\xi})] > \mathrm{E}[u(w + k + \tilde{\varepsilon})] - \mathrm{E}[u(w - k + \tilde{\varepsilon})], \tag{3}$$

*for all scalars, $k > 0$, if, and only if, the utility function, $u(w)$, exhibits mixed risk aversion up to order $N+1$, i.e., $u(w) \in U_{N+1}$.*

*Proof.* According to Theorem 3 of Eeckhoudt et al. (2009b), the 50-50 binary lottery, $[\tilde{\varepsilon} - k; \tilde{\xi} + k]$, dominates the 50-50 binary lottery, $[\tilde{\varepsilon} + k; \tilde{\xi} - k]$, in the sense of $(N+1)$th-order stochastic dominance. Hence, we have

$$\frac{1}{2}\mathrm{E}[u(w - k + \tilde{\varepsilon})] + \frac{1}{2}\mathrm{E}[u(w + k + \tilde{\xi})] > \frac{1}{2}\mathrm{E}[u(w + k + \tilde{\varepsilon})] + \frac{1}{2}\mathrm{E}[u(w - k + \tilde{\xi})], \quad (4)$$

if, and only if, $u(w) \in U_{N+1}$. Rearranging terms of inequality (4) yields condition (3). □

Lemma 1 shows preferences for harm disaggregation (Eeckhoudt and Schlesinger, 2006) in that individuals with $u(w) \in U_{N+1}$ prefer the 50-50 binary lottery, $[\tilde{\varepsilon} - k; \tilde{\xi} + k]$, to the 50-50 binary lottery, $[\tilde{\varepsilon} + k; \tilde{\xi} - k]$. The two harms, replacing $\tilde{\varepsilon}$ by $\tilde{\xi}$ and $k$ by $-k$, are better apportioned in the former lottery than in the latter lottery in the sense that they never jointly appear in each of the two states of nature.

Using Lemma 1 and the theory of monotone comparative statics, we derive necessary and sufficient conditions under which $e^{**} \geq e^*$ in the following proposition.

**Proposition 1.** *Given that the zero-mean additive background risk, $\tilde{\varepsilon}$, experiences an increase in risk to $\tilde{\xi}$ in the sense of $N$th-order stochastic dominance, the individual spends more on SICP, i.e., $e^{**} \geq e^*$, if, and only if, the individual's utility function, $u(w)$, exhibits mixed risk aversion up to order $N+1$, i.e., $u(w) \in U_{N+1}$.*

*Proof.* For any $e_1 > e_0$, it follows from Eq. (1) that $f(e_1) - f(e_0) \geq 0$ is equivalent to

$$p(e_1)\mathrm{E}\{u[w_1 - \ell(e_1) + \tilde{\varepsilon}]\} - p(e_0)\mathrm{E}\{u[w_1 - \ell(e_0) + \tilde{\varepsilon}]\} + [p(e_0) - p(e_1)]\mathrm{E}[u(w_1 + \tilde{\varepsilon})]$$

$$\geq v(w_0 - e_0) - v(w_0 - e_1). \quad (5)$$

Setting $w = w_1 - [\ell(e_0) + \ell(e_1)]/2 > 0$ and $k = [\ell(e_0) - \ell(e_1)]/2 > 0$, we can write condition (3) as

$$\mathrm{E}\{u[w_1 - \ell(e_1) + \tilde{\xi}]\} - \mathrm{E}\{u[w_1 - \ell(e_0) + \tilde{\xi}]\}$$

$$> \mathrm{E}\{u[w_1 - \ell(e_1) + \tilde{\varepsilon}]\} - \mathrm{E}\{u[w_1 - \ell(e_0)]\tilde{\varepsilon}]\}. \tag{6}$$

Likewise, we set $w = w_1 - \ell(e_0)/2 > 0$ and $k = \ell(e_0)/2 > 0$ in condition (3) to yield

$$\mathrm{E}[u(w_1 + \tilde{\xi})] - \mathrm{E}\{u[w_1 - \ell(e_0) + \tilde{\xi}]\} > \mathrm{E}[u(w_1 + \tilde{\varepsilon})] - \mathrm{E}\{u[w_1 - \ell(e_0) + \tilde{\varepsilon}]\}. \tag{7}$$

Multiplying $p(e_1) > 0$ to inequality (6) and $p(e_0) - p(e_1) > 0$ to inequality (7), we have

$$p(e_1)\mathrm{E}\{u[w_1 - \ell(e_1) + \tilde{\xi}]\} - p(e_0)\mathrm{E}\{u[w_1 - \ell(e_0) + \tilde{\xi}]\} + [p(e_0) - p(e_1)]\mathrm{E}[u(w_1 + \tilde{\xi})]$$

$$> p(e_1)\mathrm{E}\{u[w_1 - \ell(e_1) + \tilde{\varepsilon}]\} - p(e_0)\mathrm{E}\{u[w_1 - \ell(e_0) + \tilde{\varepsilon}]\}$$

$$+[p(e_0) - p(e_1)]\mathrm{E}[u(w_1 + \tilde{\varepsilon})]. \tag{8}$$

It follows from inequalities (5) and (8) that

$$p(e_1)\mathrm{E}\{u[w_1 - \ell(e_1) + \tilde{\xi}]\} - p(e_0)\mathrm{E}\{u[w_1 - \ell(e_0) + \tilde{\xi}]\} + [p(e_0) - p(e_1)]\mathrm{E}[u(w_1 + \tilde{\xi})]$$

$$> v(w_0 - e_0) - v(w_0 - e_1). \tag{9}$$

Using Eq. (2), inequality (9) is equivalent to $g(e_1) - g(e_0) > 0$. Hence, we conclude that if $f(e_1) - f(e_0) \geq 0$ for any $e_1 > e_0$, then $g(e_1) - g(e_0) > 0$.

Since $e^* \in \arg\max_{e\in[0,w_0]} f(e)$, we have $f(e^*) - f(e^{**}) \geq 0$. If $e^* > e^{**}$, we must have $g(e^*) - g(e^{**}) > 0$ from the above conclusion, which is contradictory to the fact that $e^{**} \in \arg\max_{e\in[0,w_0]} g(e)$. Hence, it must be true that $e^{**} \geq e^*$. $\square$

It is straightforward to verify that if $\tilde{\xi}$ has more $N$th-degree risk than $\tilde{\varepsilon}$ in the sense of Ekern (1980), which is a special case of $N$th-order stochastic dominance, Proposition 1 holds if, and only if, $(-1)^{N+1}u^{(N)}(w) > 0$.[3] When $\tilde{\varepsilon} \equiv 0$, the zero-mean random variable, $\tilde{\xi}$, has more second-degree risk than $\tilde{\varepsilon}$, i.e., $N = 2$. It then follows from Proposition 1 that the individual demonstrates precautionary SICP upon the introduction of the zero-mean additive background risk if, and only if, the individual's preferences exhibit prudence, i.e., $u'''(w) > 0$. To see the underlying intuition, we follow Eeckhoudt and Schlesinger (2006) to define the utility premium for the zero-mean additive background risk, $\tilde{\xi}$, as

$$\pi(e) = p(e)\mathrm{E}\{u[w_1 - \ell(e) + \tilde{\xi}]\} + [1 - p(e)]\mathrm{E}[u(w_1 + \tilde{\xi})]$$

$$-p(e)u[w_1 - \ell(e)] - [1 - p(e)]u(w_1). \tag{10}$$

The utility premium, $\pi(e)$, which is negative if, and only if, the individual is risk averse, i.e., $u''(w) < 0$, measures the "pain" caused by $\tilde{\xi}$ in terms of the loss of expected utility. Differentiating Eq. (10) with respect to $e$ yields

$$\pi'(e) = -p'(e)\Big\{\mathrm{E}[u(w_1 + \tilde{\xi})] - \mathrm{E}\{u[w_1 - \ell(e) + \tilde{\xi}]\} - u(w_1) + u[w_1 - \ell(e)]\Big\}$$

$$-p(e)\Big\{\mathrm{E}\{u'[w_1 - \ell(e) + \tilde{\xi}]\} - u'[w_1 - \ell(e)]\Big\}\ell'(e). \tag{11}$$

Setting $w = w_1 - \ell(e)/2$, $k = \ell(e)/2$, and $\tilde{\varepsilon} \equiv 0$, Lemma 1 implies that the expression inside the curly brackets of the first term on the right-hand side of Eq. (11) is positive if, and only if, $u'''(w) > 0$. The expression inside the curly brackets of the second term on the right-hand side of Eq. (11) is positive if, and only if, $u'''(w) > 0$. Since $p'(e) < 0$ and $\ell'(e) < 0$, we conclude that $\pi'(e) > 0$ if, and only if, $u'''(w) > 0$. The prudent individual as such has a precautionary motive that induces him to spend more on SCIP so as to reduce the "pain" caused by the introduction of $\tilde{\xi}$ in the second period. This gives rise to the precautionary SICP, i.e., $e^{**} \geq e^*$.

[3] Compared with the $N$th-order stochastic dominance, the $N$th-degree increase in risk in the sense of Ekern (1980) has the additional restrictions that $\tilde{\varepsilon}$ and $\tilde{\xi}$ must have identical first $N - 1$ moments.

When $\ell(e) \equiv \ell$, our model reduces to the one that focuses on self-protection. Eeckhoudt et al. (2012) and Lee (2012) show that prudence is required for the precautionary self-protection (effort) when $p(e)$ is a convex function and $u(w)$ is a concave function. Since we prove the results of Proposition 1 using the theory of monotone comparative statics and risk apportionment, the precautionary effort arises even for risk lovers who are prudent (Crainich et al., 2013). Wang and Li (2014) draw the same conclusion and thereby refer to the precautionary effort as another trait for prudence.

Proposition 1 also tells us the marginal effect of changes in the additive background risk on precautionary SICP. The individual with mixed risk aversion up to order $N+1$ always spends more on precautionary SICP when the additive background risk deteriorates via the $N$th-order stochastic dominance, which is consistent with the findings of Eeckhoudt et al. (2012) and Wang and Li (2014).

## 4. Multiplicative background risk

In this section, we examine the case that the background risk, $\tilde{\varepsilon}$, is multiplicative in nature such that $\tilde{\varepsilon}$ is a positive random variable with a mean equal to unity. The individual's expected utility over the two periods is given by

$$f(e) = v(w_0 - e) + p(e)\mathrm{E}\Big\{u\{[w_1 - \ell(e)]\tilde{\varepsilon}\}\Big\} + [1 - p(e)]\mathrm{E}[u(w_1\tilde{\varepsilon})]. \tag{12}$$

When the background risk, $\tilde{\varepsilon}$, changes to $\tilde{\xi}$, where $\tilde{\xi}$ is a positive random variable that is dominated by $\tilde{\varepsilon}$ via the $N$th-order stochastic dominance and $N \geq 1$, the individual's two-period expected utility becomes

$$g(e) = v(w_0 - e) + p(e)\mathrm{E}\Big\{u\{[w_1 - \ell(e)]\tilde{\xi}\}\Big\} + [1 - p(e)]\mathrm{E}[u(w_1\tilde{\xi})]. \tag{13}$$

Let $e^*$ and $e^{**}$ be elements in $\arg\max_{e\in[0,w_0]} f(e)$ and $\arg\max_{e\in[0,w_0]} g(e)$, respectively.

We state and prove the following lemma.

**Lemma 2.** *Consider two positive random variables, $\tilde{\varepsilon}$ and $\tilde{\xi}$, such that $\tilde{\varepsilon}$ dominates $\tilde{\xi}$ via the Nth-order stochastic dominance. For any utility function, $u(w) \in U_{N+1}$, the following condition holds:*

$$\mathrm{E}\{u[(w+k)\tilde{\xi}]\} - \mathrm{E}\{u[(w-k)\tilde{\xi}]\} > \mathrm{E}\{u[(w+k)\tilde{\varepsilon}]\} - \mathrm{E}\{u[(w-k)\tilde{\varepsilon}]\}, \tag{14}$$

*for all scalars, $k > 0$, if, and only if, the nth-degree relative risk aversion, $R^{(n)}(w) = -wu^{(n+1)}(w)/u^{(n)}(w)$, exceeds $n$ for $n = 1, ..., N$.*

*Proof.* Let $\phi(w) = \mathrm{E}[u(w\tilde{\xi})] - \mathrm{E}[u(w\tilde{\varepsilon})]$. We can write condition (14) as $\phi(w+k) > \phi(w-k)$. To prove this lemma, it suffices to show that $\phi'(w) > 0$ if, and only if, $R^{(n)}(w) > n$ for $n = 1, ..., N$.

Let $h(x) = u'(wx)x$. Differentiating $h(x)$ $n$ times with respect to $x$ yields

$$h^{(n)}(x) = \frac{\mathrm{d}^n h(x)}{\mathrm{d}x^n} = w^{n-1}u^{(n)}(wx)[n - R^{(n)}(wx)]. \tag{15}$$

It follows from Eq. (15) that $-h(x) \in U_N$ if, and only if, $R^{(n)}(w) > n$ for $n = 1, ..., N$. Since $\tilde{\varepsilon}$ dominates $\tilde{\xi}$ via the $N$th-order stochastic dominance and $-h(x) \in U_N$, we have $\mathrm{E}[-h(\tilde{\varepsilon})] > \mathrm{E}[-h(\tilde{\xi})]$. Since $\phi'(w) = \mathrm{E}[u'(w\tilde{\xi})\tilde{\xi}] - \mathrm{E}[u'(w\tilde{\varepsilon})\tilde{\varepsilon}] = \mathrm{E}[h(\tilde{\xi})] - \mathrm{E}[h(\tilde{\varepsilon})]$, we have $\phi'(w) > 0$. $\square$

When the $n$th-degree relative risk aversion is sufficiently high for $n = 1, ..., N$, Lemma 2 shows preferences for harm disaggregation (Eeckhoudt and Schlesinger, 2006) in that individuals with $u(w) \in U_{N+1}$ prefer the 50-50 binary lottery, $[(w-k)\tilde{\varepsilon}; (w+k)\tilde{\xi}]$, to the 50-50 binary lottery, $[(w+k)\tilde{\varepsilon}; (w-k)\tilde{\xi}]$:

$$\frac{1}{2}\mathrm{E}\{u[(w-k)\tilde{\varepsilon}]\} + \frac{1}{2}\mathrm{E}\{u[(w+k)\tilde{\xi}]\} > \frac{1}{2}\mathrm{E}\{u[(w+k)\tilde{\varepsilon}]\} + \frac{1}{2}\mathrm{E}\{u[(w-k)\tilde{\xi}]\}, \tag{16}$$

where inequality (16) is equivalent to condition (14). As pointed out by Eeckhoudt et al. (2009a), the idea of preferences for harm disaggregation is more subtle when it is applied to multiplicative lotteries vis-à-vis additive lotteries.[4] To see this, suppose that $\tilde{\varepsilon} \equiv 1$ and $\tilde{\xi} \equiv c$, where $0 < c < 1$. In this case, $\tilde{\varepsilon}$ dominates $\tilde{\xi}$ via the first-order stochastic dominance, i.e., $N = 1$. While the variance of the 50-50 binary lottery, $[w - k; (w + k)c]$, is smaller than that of the 50-50 binary lottery, $[w + k; (w - k)c]$, by $wk(1 - c^2)$, the expected value of the former lottery is smaller than that of the latter lottery by $k(1 - c)$. Risk aversion alone does not guarantee that individuals would always prefer the former lottery, which is less risky but less valuable, to the latter lottery, which is more risky but more valuable. A more stringent condition that requires individuals to be sufficiently risk averse is called for to ensure preferences for harm disaggregation, thereby justifying the condition that the Arrow-Pratt measure of relative risk aversion exceeds unity, i.e., $-wu''(w)/u'(w) > 1$.

Using Lemma 2 and the theory of monotone comparative statics, we derive necessary and sufficient conditions under which $e^{**} \geq e^{*}$ in the following proposition.[5]

**Proposition 2.** *Given that the unit-mean multiplicative background risk, $\tilde{\varepsilon}$, experiences an increase in risk to $\tilde{\xi}$ in the sense of $N$th-order stochastic dominance, the individual with $u(w) \in U_{N+1}$ spends more on SICP, i.e., $e^{**} \geq e^{*}$, if, and only if, the individual's $n$th-degree relative risk aversion, $R^{(n)}(w) = -wu^{(n+1)}(w)/u^{(n)}(w)$, exceeds $n$ for $n = 1, \ldots, N$.*

It is straightforward to verify that if $\tilde{\xi}$ has more $N$th-degree risk than $\tilde{\varepsilon}$ in the sense of Ekern (1980), Proposition 2 holds if, and only if, $R^{(N)}(w) > N$. When $\tilde{\varepsilon} \equiv 1$, the unit-mean random variable, $\tilde{\xi}$, has more second-degree risk than $\tilde{\varepsilon}$, i.e., $N = 2$. It then follows from Proposition 2 that the individual with mixed risk aversion up to the third-order demonstrates precautionary SICP upon the introduction of the unit-mean multiplicative background risk if, and only if, the individual's measure of relative prudence exceeds two, i.e., $-wu'''(w)/u''(w) > 2$. Unlike the case of additive background risk, prudence alone is not enough to give rise to the precautionary SICP. To see the underlying intuition, we

[4] Wang and Li (2010) generalize the setting of Eeckhoudt et al. (2009a) and refer to it as multiplicative risk apportionment.

[5] The proof of Proposition 2 is analogous to that of Proposition 1 and thus is omitted.

define the utility premium for the unit-mean multiplicative background risk, $\tilde{\xi}$, as

$$\pi(e) = p(e)\mathrm{E}\Big\{u\{[w_1 - \ell(e)]\tilde{\xi}\}\Big\} + [1 - p(e)]\mathrm{E}[u(w_1\tilde{\xi})]$$
$$-p(e)u[w_1 - \ell(e)] - [1 - p(e)]u(w_1). \quad (17)$$

Differentiating Eq. (17) with respect to $e$ yields

$$\pi'(e) = -p'(e)\Big\{\mathrm{E}[u(w_1\tilde{\xi})] - \mathrm{E}\Big\{u\{[w_1 - \ell(e)]\tilde{\xi}\}\Big\} - u(w_1) + u[w_1 - \ell(e)]\Big\}$$
$$-p(e)\Big\{\mathrm{E}\Big\{u'\{[w_1-\ell(e)]\tilde{\xi}\}\Big\}+\mathrm{Cov}\Big\{u'\{[w_1-\ell(e)]\tilde{\xi}\},\tilde{\xi}\Big\}-u'[w_1-\ell(e)]\Big\}\ell'(e), \quad (18)$$

where $\mathrm{Cov}(\cdot,\cdot)$ is the covariance operator.[6] Setting $w = w_1 - \ell(e)/2$, $k = \ell(e)/2$, and $\tilde{\varepsilon} \equiv 1$, Lemma 2 implies that the expression inside the curly brackets of the first term on the right-hand side of Eq. (18) is positive if, and only if, $-wu'''(w)/u''(w) > 2$, where $u(w) \in U_3$. The expression inside the curly brackets of the second term on the right-hand side of Eq. (18) is positive if, and only if, $u(w) \in U_3$. Since $p'(e) < 0$ and $\ell'(e) < 0$, we conclude that $\pi'(e) > 0$ if, and only if, $-wu'''(w)/u''(w) > 2$, where $u(w) \in U_3$. The sufficiently prudent individual as such has a precautionary motive that induces him to spend more on SCIP so as to reduce the "pain" caused by the introduction of $\tilde{\xi}$ in the second period. This gives rise to the precautionary SICP, i.e., $e^{**} \geq e^*$.

Proposition 2 also tells us the marginal effect of changes in the multiplicative background risk on precautionary SICP. To give rise to additional precautionary SICP spent by the individual with mixed risk aversion up to order $N+1$ when the multiplicative background risk deteriorates via the $N$th-order stochastic dominance, we need to impose $N$ more stringent conditions that require the individual's $n$th-degree relative risk aversion to exceed $n$ for $n = 1, ..., N$. These restrictive conditions are satisfied, for example, by a power utility function, $u(w) = w^{1-\gamma}/(1-\gamma)$, with $\gamma > 1$, which generates $R^{(n)}(w) = \gamma + n - 1 > n$ for $n = 1, ..., N$.

[6]For any two random variables, $\tilde{x}$ and $\tilde{y}$, we have $\mathrm{Cov}(\tilde{x},\tilde{y}) = \mathrm{E}(\tilde{x}\tilde{y}) - \mathrm{E}(\tilde{x})\mathrm{E}(\tilde{y})$.

## 5. Conclusion

Precautionary self-insurance-cum-protection (SICP) arises when an individual spends more on SICP when background risk is introduced. We develop a two-period model wherein additive/multiplicative background risk prevails in the second period. Using the theory of monotone comparative statics (Milgrom and Shannon, 1994) and risk apportionment (Eeckhoudt and Schlesinger, 2006; Eeckhoudt et al., 2009a, 2009b), we derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk deteriorates via higher-order stochastic dominance. Specifically, we show that prudence is called for to create a precautionary motive that induces an individual to spend more on SICP. Doing so allows the individual to shift his wealth from the first period to the second period so as to reduce the loss of utility caused by the addition of background risk, thereby giving rise to the precautionary SICP.

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