Generation and storage of spin-nematic squeezing in a spinor Bose-Einstein condensate

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Generation and storage of spin-nematic squeezing in a spinor Bose-Einstein condensate

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We propose a scheme to store the spin-nematic squeezing in a spinor Bose-Einstein condensate by applying periodic microwave pulses. For a proper pulse period and phase shift, the squeezing can be enhanced and maintained for a long time, which realizes the storage of the maximal squeezing. We also propose a method to generate the spin-nematic squeezed vacuum, which is associated with negligible occupation of the squeezed modes, through an adiabatic sweep of the magnetic field. It is shown that the method can be readily implemented for both ferromagnetic and antiferromagnetic condensates.

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I. INTRODUCTION

Improving measurement sensitivity beyond the standard quantum limit (SQL) has been attracted much attention for many years due to its important applications in quantum metrology. By using the squeezed states, the SQL of measurement uncertainty can be surpassed. Due to such a property, squeezed state has a widely applications in atom interferometers and high-precision atom clocks. In the experiment the spinor BEC, i.e., Bose-Einstein condensate of ultracold atomic gases with internal degrees of freedom, provide an ideal platform for exploring the spin squeezing [1–6], and the squeezed states could be generated via nonlinear interaction such as one axis twisting and two-axis counter twisting [7, 8].

Spin squeezing of the spin-1/2 systems has been widely studied by utilizing two appropriate magnetic sublevels or two mode condensate [9–14]. For spin-1/2 particles, the state can be uniquely specified by different component of the total spin vector $\mathbf{S} = (S_x,\ S_y,\ S_z)$. For higher spin particles, future degrees of freedom beyond the spin vector are required to express the state. In the case of spin-1 atomic Bose-Einstein condensates [15–26], the multipolar moments can be specified in both terms of the spin vector and nematic tensor [5, 27–32]. $Q_{i,j}(\{i, j\} \in \{x, y, z\})$ which constitute SU(3) Lie algebra. In matrix form, the nematic moments $Q_{i,j}$ can be expressed $Q_{i,j} = S_iS_j + S_jS_i - (4/3)\delta_{ij}$ with δ_{ij} being the Kronecker delta. The SU(3) Lie algebra suggests new trade-off relations between the spin operator $\mathbf{S}$ and the nematic tensor operator $Q_{i,j}$, which indicates squeezing can be caused by other types of correlations beyond the spin-spin correlation, such as spin-nematic and internematic correlations. The spin-nematic quadrature squeezing was observed in the recent experiment [5] and improved on the SQL by up to 8-10 dB. However, does the stronger spin-nematic squeezing can be generated in such a system is unknown. How to further improve and control the squeezing is an important goal in the experimental frontier. In addition, improving and controlling the squeezing may also have an important application for the quantum metrology and quantum information processing.

In this paper we propose a scheme for storage of the maximal spin-nematic squeezing in a quantum many-body spin system by periodically manipulating the phases of the states. We employ a system of spin-1 Bose condensate with initial state of all atoms in the state of $m_F = 0$. The free dynamical process gives rise to quantum spin mixing and spin-nematic squeezing. By manipulating the external periodic microwave pulses, we find the system could finally be stabilized and the spin-nematic squeezed vacuum will never disappear. With proper pulse period and phase shift, the spin-nematic squeezing can be enhanced and stored for a long time. The results indicate the spin nematic squeezing can be improved up to −20 dB and maintained for about 100ms. We also propose a method to generate the spin-nematic squeezed vacuum through an adiabatic sweep of the magnetic field and show that the optimal spin-nematic squeezed vacuum is obtained at the phase transition point.

This paper is organized as follows. In Sec. II, we introduce the model of spin-1 condensates under an external magnetic field. In Sec. III, we propose a scheme for storage of the spin-nematic squeezing. In Sec. IV, a method is proposed to generate the spin-nematic squeezed vacuum by slowly sweeping the magnetic field. Finally, our conclusions and some remarks on our results are presented in Sec.V.

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$\{S_x, Q_{yz}, Q_+\}$	Q_{yz}	Q_+	$\{S_y, Q_{xz}, Q_-\}$	Q_{xz}	Q_-
S_x	iQ_+	$-2iQ_{yz}$	S_y	iQ_-	$-2iQ_{xz}$
Q_{yz}		$2iS_x$	Q_{xz}		$2iS_y$

TABLE I: Commutation relationship of the two subspaces $\{S_x, Q_{yz}, Q_+\}$ and $\{S_y, Q_{xz}, Q_-\}$.

II. MODEL

We consider the system with spin-1 Bose-Einstein condensate under an external magnetic field. In the single mode approximation, the Hamiltonian of the system can be written as [5, 15]

$$\hat{H} = \lambda(\hat{\mathbf{S}}^2 - 2\hat{N}) + p\hat{S}_z + \frac{q}{2}\hat{Q}_{zz}, \tag{1}$$

where λ is the inter-spin interaction energy, p and $q(\propto B^2)$ are the linear and quadratic Zeeman energy, respectively. $\hat{Q}_{zz} = (2/3)\hat{a}_1^\dagger\hat{a}_1 - (4/3)\hat{a}_0^\dagger\hat{a}_0 + (2/3)\hat{a}_{-1}^\dagger\hat{a}_{-1}$ is an element of spin-1 nematic tensor, and $\hat{a}_i$ is the annihilation operator of the ith spin mode. The Hamiltonian conserves the magnetization $S_z = \hat{a}_1^\dagger\hat{a}_1 - \hat{a}_{-1}^\dagger\hat{a}_{-1}$ and the total number $\hat{N} = \hat{a}_1^\dagger\hat{a}_1 + \hat{a}_0^\dagger\hat{a}_0 + \hat{a}_{-1}^\dagger\hat{a}_{-1}$, and thus the dynamical evolution then occurs only in the internal spin degrees of freedom. Then the effective Hamiltonian for the evolution problem becomes [5]

$$\hat{H}_{\text{eff}} = \lambda\hat{\mathbf{S}}^2 + \frac{q}{2}\hat{Q}_{zz}. \tag{2}$$

For a high magnetic field, the system favors a nematic ordering of the spins with $\langle\hat{\mathbf{S}}\rangle = 0$. Such a nematic phase brokes the rotational symmetry with the anisotropy of the spin fluctuation. In the Fock basis, $|N_1, N_0, N_{-1}\rangle$, this state can be written as $|0, N, 0\rangle$ which corresponds to all of the atoms stay at $m_F = 0$ state. For a low magnetic field, the ground state of the system is determined by the sign of λ, i.e., $\lambda < 0$ (^{87}Rb) and > 0 (^{23}Na) means the system favors a ferromagnetic or antiferromagnetic phase, respectively. For an intermediate field, the system exhibits a quantum transition at the critical point $q_c = -4N\lambda$ for the ferromagnetic case [33].

For our system with spin-1 atoms, squeezing is generated by the nonlinear collisional spin interaction $H_s = \lambda\hat{\mathbf{S}}^2$, which contains a four-wave mixing terms $H_{\text{FWM}} = 2\lambda(\hat{a}_0^{\dagger^2}\hat{a}_1\hat{a}_{-1} + \hat{a}_1^\dagger\hat{a}_{-1}^\dagger\hat{a}_0^2)$. In this case, the squeezing can be described in terms of a two-mode formalism in which $a_{\pm1}$ modes represent the signal and idler, respectively. The dynamical behavior of the spin-1 condensates have been widely investigated in the experiments [34–37] and exhibits many interesting quantum phenomena [5, 37, 38]. Here we investigate the squeezing using the commutators of the SU(3) group and consider the ferromagnetic case, $\lambda < 0$.

III. SPIN-NEMATIC SQUEEZING

Based on the definition of the operator $\hat{Q}_{i,j}$, such as [5]

$$\begin{aligned}
\hat{Q}_{yz} &= \frac{i}{\sqrt{2}}(\hat{a}_0^\dagger\hat{a}_{-1} - \hat{a}_1^\dagger\hat{a}_0 + \hat{a}_0^\dagger\hat{a}_1 - \hat{a}_{-1}^\dagger\hat{a}_0),\\
\hat{Q}_{xz} &= \frac{1}{\sqrt{2}}(\hat{a}_1^\dagger\hat{a}_0 - \hat{a}_0^\dagger\hat{a}_{-1} + \hat{a}_0^\dagger\hat{a}_1 - \hat{a}_{-1}^\dagger\hat{a}_0),\\
\hat{Q}_{xx} &= \frac{2}{3}\hat{a}_0^\dagger\hat{a}_0 - \frac{1}{3}(\hat{a}_1^\dagger\hat{a}_1 + \hat{a}_{-1}^\dagger\hat{a}_{-1}) + \hat{a}_1^\dagger\hat{a}_{-1} + \hat{a}_{-1}^\dagger\hat{a}_1,\\
\hat{Q}_{yy} &= \frac{2}{3}\hat{a}_0^\dagger\hat{a}_0 - \frac{1}{3}(a_1^\dagger\hat{a}_1 + \hat{a}_{-1}^\dagger\hat{a}_{-1}) - \hat{a}_1^\dagger\hat{a}_{-1} - \hat{a}_{-1}^\dagger\hat{a}_1,
\end{aligned}$$

we can define $\{S_x, Q_{yz}, Q_+\}$ and $\{S_y, Q_{xz}, Q_-\}$ as two subspaces of SU(3). Here, Q_+ and Q_- are defined as $Q_+ = Q_{zz} - Q_{yy}$, $Q_- = Q_{xx} - Q_{zz}$, respectively. The commutation relationship for these two subspaces is shown in Table I. According to the generalized Heisenberg uncertainty relation $\Delta A\Delta B \geqslant \frac{1}{2}|\langle[A, B]\rangle|$, squeezing occurs only for the operator pairs with non-zero expectation values for their commutation relations. For the initial state of the condensate with all atoms in the $m_F = 0$ state, only two operators in the above two subspaces have nonzero expectation values, i.e., $\langle Q_\pm\rangle \neq 0$. Then the two different spin-nematic squeezing parameters in a SU(2) subspace are defined by [2]

$$\xi^2_{x(y)} = \left\langle(\Delta(\cos\theta S_{x(y)} + \sin\theta Q_{yz(xz)}))^2\right\rangle / \left\langle Q_{+(-)}/2\right\rangle, \tag{3}$$

θ is the quadrature angle. For a proper θ, we can obtain a minimum value of $\xi^2_{x(y)}$. If the state of the system with negligible populations of the $m_F = \pm1$ states, we have $\langle[S_x, Q_{yz}]\rangle = -2iN$ and $\langle[S_y, Q_{xz}]\rangle = 2iN$. In this case, the relevant uncertainty relations between a quadrupole nematic operator and a spin operator is given by $\Delta S_x Q_{yz} \geqslant N$ and $\Delta S_y Q_{xz} \geqslant N$. Then the squeezing parameter $\xi^2_{x(y)}$ is the ratio between the variance of the quadrature operator to the standard quantum limit of N and $\xi^2_{x(y)} < 1$ indicates spin-nematic squeezed vacuum.

With the above discussion, we now turn on our main task. Considering the initial state of the system with all of the atoms in $m_F = 0$ state, and then let the state become free dynamic evolution with the system in a lower magnetic field. During this processing, the spin mixing Hamiltonian conserves both the total particle number N and magnetization, the evolution state of the system in vector form is

$$|\psi(t)\rangle = \sum_{k=0}^{N/2} a_k(t)\,|N, k\rangle\,, \tag{4}$$

where $|N, k\rangle$ is so-called pairs basis with N the total particle number and k the number of pairs of atom in the $m_F = \pm1$ states. In Fig. 1 (a) we plot the spin-nematic squeezing parameter $(10\log_{10}\xi^2_x)$ as a function of t with the spinor interaction energy $\lambda = -7.2\pi\hbar/N$ Hz and the magnetic field $B = 60$ mG, which determines the

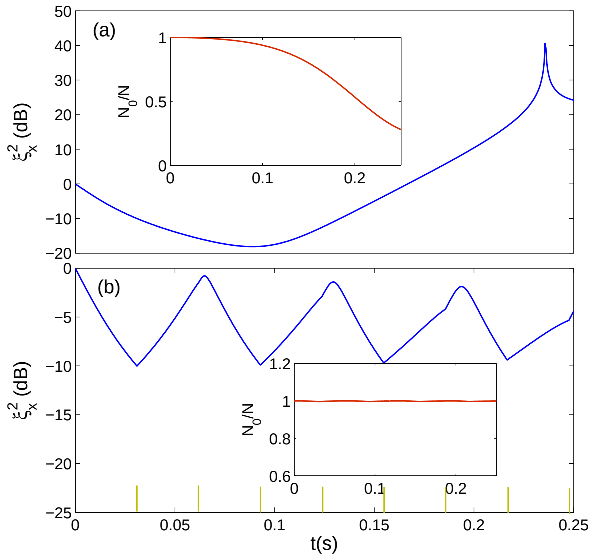

FIG. 1: (Color online) Evolution of spin-nematic squeezing parameter ξ_x^2 for different applied phase shifts (a) $\Delta\theta = 0$ (free dynamic evolution) and (b) $\Delta\theta = -0.9\pi$. The green ticks represent the pulses. The phase period is 31 ms and the total particle number $N = 10^3$. The inset shows the time evolution of N_0/N.

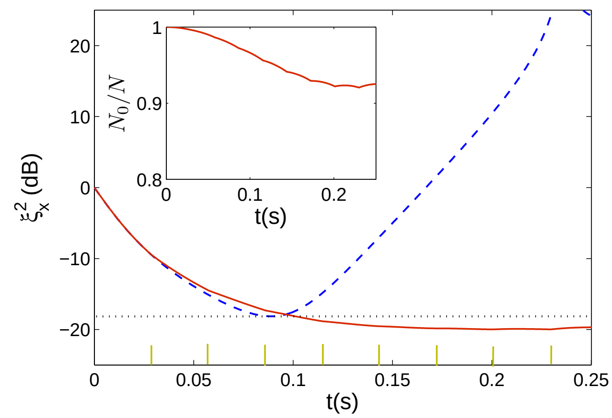

FIG. 2: (Color online) Evolution of spin-nematic squeezing parameter ξ_x^2 for different quadrature phase shifts $\Delta\theta = 0$ (blue dashed) and $\Delta\theta = -0.99\pi$ (red). The dotted line represents the minimum value for the free evolution of ξ_x^2. The phase period is 28.7 ms and the green ticks represent the pulses.

quadratic Zeeman effect $q = 143.2\pi\hbar \times B^2$ Hz/G^2. From Fig. 1, we can see the free evolution gives rise to spin-nematic squeezing and quantum spin mixing. For the initial short time, the inset plot in Fig. 1(a) shows that there is essentially no population transfer from the state $m_F = 0$ to the other two states $m_F = \pm 1$ ($N_0/N > 99\%$), which corresponds to squeezed vacuum for the $m_F = \pm 1$ modes. However, the spin nematic squeezed vacuum only keeps for a short time due to the spin mixing dynamic of atom population transfer to $m_F = \pm 1$ modes ($N_1/N = N_{-1}/N > 0.5\%$).

To maintain the phenomenon of the spin-nematic squeezed vacuum, we need to stabilize the evolution of state $m_F = 0$. It can be achieved by using periodic phase shifts of the spinor wave function, which is manifested as a rotation about a polar axis in the spin-nematic phase space and represented as the operator $\exp(i\Delta\theta Q_{zz})$ [15]. In the experiment, such a rotation can be implemented by using 2π Rabi pluses on the $|f = 1, m_F = 0\rangle \leftrightarrow |f = 2, m_F = 0\rangle$ microwave clock transition, which can effectively shift the phase of the state $|f = 1, m_F = 0\rangle$ with an amount $\Delta\theta_0 = \pi(1+\Delta/\sqrt{1 + \Delta^2})$, where Δ is the detuning normalized to the on-resonance Rabi rate [5]. After the phase shift of $\Delta\theta$, the wave function is

$$|\psi(t)\rangle_{\Delta\theta} = e^{iQ_{zz}\Delta\theta}|\psi(t)\rangle = \sum_{k=0}^{N/2} e^{4i\Delta\theta(-N/3+k)} a_k |N, k\rangle . \quad (5)$$

As shown in Fig. 1(b), the time evolution of the spin-nematic squeezing is plotted with a microwave pulse with phase shift $\Delta\theta = -0.9\pi$. The pulse period is 31 ms with the first pulse at 31 ms after the quench. For a proper size of quadrature phase shift, the value of the spin population $N_0/N \approx 1$ (inset of Fig. 1(b)), which corresponds to stabilized dynamics. From Fig. 1(b) we can see ξ_x^2 oscillates as a function of t and keeps the corresponding value smaller than 0, which indicates that in the evolution process, the system always exhibits a spin-nematic squeezed vacuum phenomenon.

In addition to the generation of the squeezing state itself, it is desirable to maintain the squeezing for a long time. In Fig. 2 we plot the spin-nematic squeezing parameter ξ_x^2 as a function of time with $\Delta\theta = -0.99\pi$. The pulse period is 28.7 ms with the first pulse at 28.7 ms after the quench. We can see that the spin-nematic squeezing parameter maintain at the value of -20dB for a long time ($\approx$ 100ms). Comparing with the result of the free evolution ($\Delta\theta = 0$), we can find the squeezing can even be enhanced. Here, we emphasize that the maintained squeezing is not squeezed vacuum, as shown in the set of Fig. 2, $m_f = \pm 1$ states are macroscopically populated ($N_1/N > 0.5\%$) when $t > 53$ms. Varying both the parameters of the pulse period and the phase shift over from 8ms to 45ms and 0 to 2π, respectively. We have found the spin-nematic squeezing ($\xi_x^2 \approx -20$dB) can also be maintained for a long time ($\approx$ 100ms) with many other sizes of the microwave pulse parameters, such as $\delta\theta = -0.99\pi$, the pulse period $T = 31$ms, and $\delta\theta = -0.985\pi$, $T = 31$ms. In this way, we realize the storage of spin-nematic squeezing by applying external microwave pulses.

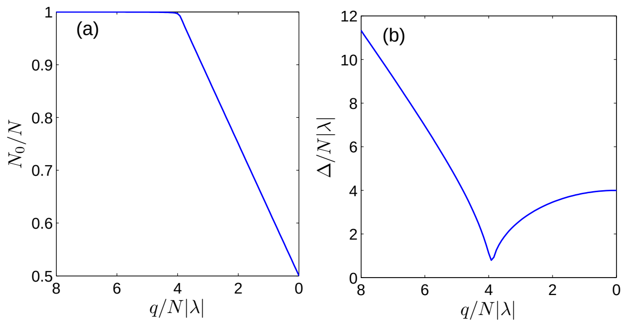

FIG. 3: (Color online) (a) The order parameter N_0/N and (b) The energy gap Δ in the unit of $N|\lambda|$ as a function of $q/N|\lambda|$ with the total atom number $N = 10^3$.

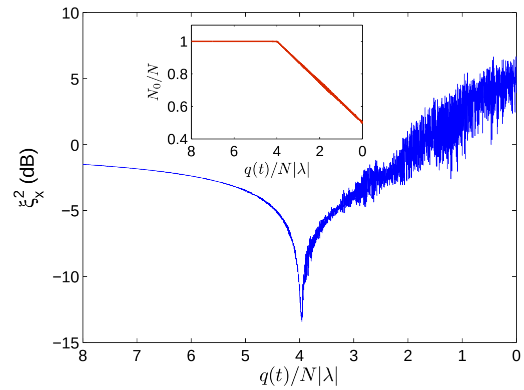

FIG. 4: Spin-nematic squeezing as a function of $q/N|\lambda|$ with a linear field sweeping rate $v = 1 \times 10^{-3}$. The total particle number is chosen $N = 4 \times 10^3$. The inset shows the evolution of N_0/N as a function $q(t)/N|\lambda|$.

IV. ADIABATIC PASSAGE GENERATE SQUEEZING

Now we propose another method to generate the spin nematic squeezing though an adiabatic sweep of the magnetic field. To analyze the squeezing behavior, first we quantitatively calculate the phase transition points during this adiabatic passage and the corresponding energy gap. For our proposed adiabatic passage, we perform the numerical many-body calculation in the Hilbert space with $N_{-1} = N_1$. As shown in Fig. 3(a), numerical result of the condensate fraction in $m_F = 0$ state is plotted as a function of $q/(N|\lambda|)$. It shows a second-order phase transition at the critical point $\frac{q}{N|\lambda|} = 4$, in which the condensate fraction N_0/N drops from 1 to a positive number. This result agrees with the the mean-field prediction [16]. Besides the prediction of the behavior of the condensate fraction at the critical point, the exact numerical calculation can also show the energy gap at the phase transition point. In Fig. 3(b), we plot the energy gap Δ as a function of $q/N|\lambda|$, where Δ is defined as the energy difference between the first excite state and the ground state. At the phase transition point, we can see the energy gap attains its minimum value. In fact the energy gap is dependent on the particle number and the scaling of the energy gap at the phase transition point is given by $\Delta = 7.4N^{-1/3}|\lambda|$ [39].

We now turn to discuss the spin nematic squeezing generation with the adiabatic passage. Suppose the system is initially prepared under a strong magnetic field $q^{(i)}$, which produces state with all the particles in the $m_F = 0$ state. We then sweep q from positive value to 0 with a constant rate $v > 0$ and the corresponding time dependent magnetic field is given by

$$q(t) = q^{(i)} - vt \tag{6}$$

with $t > 0$. At time $t = t_f$, we measure the spin-nematic squeezing. As shown in Fig. 4, the spin-nematic squeezing is plotted as a function of $q/N|\lambda|$ with the total particle number $N = 4 \times 10^3$. We see the squeezing attains its minimum value at $q = 4N|\lambda|$ which corresponds the phase transition point. As the field sweeps over the phase transition point, the condensate wave function bifurcates into the states forming the minimum gap and thus becomes a coherent superposition of states with different particle population in $m_F = 0$ state. To get the optimal squeezing, we can stop the sweeping at the magnetic field $q = 4N|\lambda|$. When $q \leqslant 4N|\lambda|$, as shown the inset of Fig. 4, $N_0/N \simeq 1$, thus the generated squeezing is a spin-nematic squeezed vacuum and the corresponding squeezing is improved on the standard quantum limit by up to 13 dB.

In the above discussion, all the calculation are done for the case of ferromagnetic with $\lambda < 0$ due to the positive quadratic Zeeman shift q. In the recent experiments [34, 40–42], it is found that the value of q can be switched to both the positive and the negative side by using an ac Stark shift from a π-polarized microwave dressing field which couples the hyperfine levels $|F = 1\rangle$ and $|F = 2\rangle$. In such a case, our scheme can also be applied for the antiferromagnetic case. With $\lambda > 0$, we can change the adiabatic sweep along the highest eigenstate of the Hamiltonian in Eq. (2) and then all of the calculation is same with the ferromagnetic case. Here we shall note that the initially parameter needs to be set with a negative value, i.e., $q < 0$ and the atoms are prepared into the state $m_F = 0$.

V. CONCLUSION

In summary, we have investigate the generation and the coherent control of spin-nematic squeezing in a spinor BEC condensate. By applying the periodic microwave pulses, spin-nematic squeezed vacuum can exist all the time. With a proper pulse period and phase shift, the spin-nematic squeezing can be even enhanced to -20dB and maintained for a long time. We also propose another scheme to generate spin-nematic squeezed vacuum through an adiabatic sweep of the magnetic field. We obtain the optimal spin-nematic squeezing vacuum, which improves on the standard quantum limit by up to 13 dB, at the phase transition point.

VI. ACKNOWLEDGMENTS

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[1] A. Sørensen, L.-M. Duan, J. I. Cirac, and P. Zoller, Nature (London) **409**, 63 (2001).
[2] L.-M. Duan, J. I. Cirac, and P. Zoller, Phys. Rev. A **65**, 033619 (2002).
[3] C. Gross, T. Zibold, E. Nicklas, J. Estève, and M. K. Oberthaler, Nature (London) **464**, 1165 (2010).
[4] M.F. Riedel, P. Böhi, Y. Li, T.W. Hänsch, A. Sinatra, and P. Treutlein, Nature (London) 464, 1170 (2010);
[5] C.D. Hamley, C. S. Gerving, T. M. Hoang, E. M. Bookjans, and M. S. Chapman, Nat. Phys. **8**, 305 (2012).
[6] J. Ma, X. Wang, C. Sun, and F. Nori, Phys. Rep. **509**, 89 (2011).
[7] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993).
[8] D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, Phys. Rev. A **50**, 67 (1994).
[9] H. Pu, and P. Meystre, Phys. Rev. Lett. **85**, 3987 (2000).
[10] L-M. Duan,A. Søensen, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **85**, 3991 (2000).
[11] G. I. Mias, N. R. Cooper, and S. M. Girvin, Phys. Rev. A **77**, 023616 (2008).
[12] M. J. Steel and M. J. Collett, Phys. Rev. A **57**, 2920 (1998).
[13] G. R. Jin and S.W. Kim, Phys. Rev. Lett. **99**, 170405 (2007).
[14] Y. Li, Y. Castin, and A. Sinatra, Phys. Rev. Lett. **100**, 210401 (2008).
[15] T. M. Hoang, C. S. Gerving, B. J. Land, M. Anquez, C. D. Hamley, and M. S. Chapman, Phys. Rev. Lett. **111**, 090403 (2013).
[16] D. M. Stamper-Kurn and M. Ueda, Rev. Mod. Phys. **85**, 1191 (2013).
[17] Tin-Lun Ho, Phys. Rev. Lett. **81**, 742 (1998).
[18] Tetsuo Ohmi and Kazushige Machida, J. Phys. Soc. Jpn **67**, 1822 (1998).
[19] J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur, and W. Ketterle, Nature **396**, 345 (1999).
[20] M.-S. Chang, C. D. Hamley, M. D. Barrett, J. A. Sauer, K. M. Fortier, W. Zhang, L. You, and M. S. Chapman Phys. Rev. Lett. **92**, 140403 (2004).
[21] H. Schmaljohann, M. Erhard, J. Kronjäger, M. Kottke, S. van Staa, L. Cacciapuoti, J. J. Arlt, K. Bongs, and K. Sengstock Phys. Rev. Lett. **92**, 040402 (2004).
[22] C. K. Law, H. Pu, and N. P. Bigelow, Phys. Rev. Lett. **81**, 5257 (1998).
[23] Yixiao Huang, Zhe Sun, and Xiaoguang Wang Phys. Rev. A **89**, 043601 (2014).
[24] S. Yi, L. You, and H. Pu, Phys. Rev. Lett. **93**, 040403 (2004).
[25] S. Yi and H. Pu, Phys. Rev. A **73**, 023602 (2006).
[26] Yixiao Huang, Yunbo Zhang, Rong Lü, Xiaoguang Wang, and Su Yi, Phys. Rev. A **86**, 043625 (2012).
[27] J. Hald, J. L. Sørensen, C. Schori, and E. S. Polzik, Phys. Rev. Lett. **83**, 1319 (1999).
[28] J. D. Sau, S. R. Leslie, M. L. Cohen, and D. M. Stamper-Kurn, New. J. Phys. **12**, 085011 (2010).
[29] A. Kuzmich, K. Mømer, and E. S. Polzik, Phys. Rev. Lett. **79**, 4782 (1997).
[30] J. L. Sørensen, J. Hald, and E. S. Polzik, Phys. Rev. Lett. **80**, 3487 (1998).
[31] R. J. Sewell, M. Koschorreck, M. Napolitano, B. Dubost, N. Behbood, and M. W. Mitchell, Phys. Rev. Lett. **109**, 253605 (2012).
[32] Emi Yukawa, Masahito Ueda, and Kae Nemoto, Phys. Rev. A **88**, 033629 (2013).
[33] L. E. Sadler, J. M. Higbie, S. R. Leslie, M. Vengalattore, and D. M. Stamper-Kurn Nature (London) **443**, 312 (2006).
[34] Y. Liu, E. Gomez, S. E. Maxwell, L. D. Turner, E. Tiesinga, and P. D. Lett, Phys. Rev. Lett. **102**, 225301 (2009).
[35] S. R. Leslie, J. Guzman, M. Vengalattore, Jay D. Sau, Marvin L. Cohen, and D. M. Stamper-Kurn, Phys. Rev. A **79**, 043631 (2009).
[36] C. Klempt, O. Topic, G. Gebreyesus, M. Scherer, T. Henninger, P. Hyllus, W. Ertmer, L. Santos, and J. J. Arlt, Phys. Rev. Lett. **104**, 195303 (2010).
[37] Eva M. Bookjans, Christopher D. Hamley, and Michael S. Chapman, Phys. Rev. Lett. **107**, 210406 (2011).
[38] C. S. Gerving, T. M. Hoang, B. J. Land, M. Anquez, C. D. Hamley, and M. S. Chapman, Nat. Commun. **3**, 1169 (2012).
[39] Z. Zhang and L.-M. Duan, Phys. Rev. Lett. **111**, 180401 (2013).
[40] F. Gerbier, A. Widera, S. Fölling, O. Mandel, and I. Bloch, Phys. Rev. A **73**, 041602 (2006).
[41] E.M. Bookjans, A. Vinit, and C. Raman, Phys. Rev. Lett. **107**, 195306 (2011).
[42] L. Zhao, J. Jiang, T. Tang, M. Webb, and Y. Liu, Phys. Rev. A **89**, 023608 (2014).