

New results on stabilization of networked control systems with packet disordering[☆]

Andong Liu^{a,*}, Wen-an Zhang^a, Li Yu^a, Steven Liu^b, Michael Z. Q. Chen^c

^aDepartment of Automation, Zhejiang University of Technology, Zhejiang Provincial United Key Laboratory of Embedded Systems, Hangzhou 310023, P.R. China

^bDepartment of Electrical and Computer Engineering, University of Kaiserslautern, Kaiserslautern 67663, Germany

^cDepartment of Mechanical Engineering, The University of Hong Kong, Hong Kong

Abstract

In this paper, the stabilization problem is studied for a class of networked control systems (NCSs) with delays, packet disordering and packet dropouts. A new packet reordering method is presented to deal with packet disordering and choose the newest control input. A relationship between the reordered packet over two consecutive sampling intervals is given for the NCS with both time delays and packet dropouts. A sufficient condition for the NCS to be exponentially stable is presented by using the average dwell-time method. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed method.

Keywords: Networked control systems, Network-induced delays, Packet disordering, Packet dropouts

1. Introduction

Network-induced delay and packet dropout are two main problems in networked control system (NCS), and have attracted much research interest, see for example [1], [2], [3], [4], [5] and the references therein. Compared with constant delays, the time-varying one is more difficult to treat, especially, when the delay is larger than one sampling period (long delay). Since the delay may be larger than one sampling period, more than one control signals may arrive at the actuator during one sampling interval. Moreover, the transmission of data packets may not necessarily follow a “first send first arrive” principle [6]. This means that the newest control signal may arrive at the actuator before the older one, this is the so-called packet disordering problem [7], [8]. The modeling and stability analysis of NCSs with packet disordering has attracted much research attention, see for example [9], [10], [11], [12] and [13]. Due to limited network transmission capacity, packet dropouts are usually inevitable. Some results on NCSs concerning both delay and packet dropout issues were presented in [14], [15], [16], [17] and [18]. It should be pointed out that the packet disordering problem is not considered in the aforementioned results for NCSs in the presence of delays and packet dropouts. In [19], H_∞ filtering problem has been investigated for NCSs with delays, packet disordering and packet dropouts. However, the explicit expression for how to choose the newest data signal was not given in [19].

On the other hand, due to the time-varying delays and packet dropouts, the number of available control signals at the ac-

tuator vary over different sampling intervals. Therefore, the NCS is naturally a switched system with a group of subsystems describing various system dynamics on different sampling intervals ([20], [21], [22]). In response to the above discussion, we investigate the stability analysis problem for a class of NCSs with delays and packet dropouts, and focus on solving the packet disordering problem and the switched dynamic caused by long delay and packet dropout. The main contributions of the paper are as follows: 1) A new packet reordering method is proposed to deal with packet disordering and choose the newest control signal. 2) A relationship between the reordered packet over two consecutive sampling intervals is given for the NCS with both network-induced delays and packet dropouts. 3) A sufficient condition for exponential stability of the NCS is derived by using the average dwell-time method. Finally, an example is given to demonstrate the effectiveness of the proposed method.

2. Modeling of the NCS

The plant in the NCS is described by the following continuous-time linear system model

$$\dot{x}(t) = A_p x(t) + B_p u(t) \quad (1)$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input, A_p and B_p are constant matrices with appropriate dimensions. The sensor is time-driven with sampling period T , the controller is event-driven, the actuator is time-driven and has a receiving buffer containing the most recent data packet from the controller, time delay τ_k is assumed to be bounded by $0 \leq \tau_k \leq dT$, where d is a known finite integer. Since the actuator is time-driven, one has $\tau_k \in \mathcal{N}_1 = \{0, 1, 2, \dots, d\}T$.

For simplicity, we first consider only the delay issue. Since the network-induced delay may be larger than one sampling

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*Corresponding author. Tel.: +86 571 85290531; fax: +86 571 85290567.

Email addresses: lad-andrew@hotmail.com (Andong Liu), wazhg@hotmail.com (Wen-an Zhang), lyu@zjut.edu.cn (Li Yu), sliu@eit.uni-kl.de (Steven Liu), and mzqchen@gmail.com (Michael Z. Q. Chen)

period, more than one control signals may arrive at the actuator during one sampling interval, but only one control signal is adopted by the actuator. Then, the problem is how to choose the newest control signal. Moreover, packet disordering problem arises when the newest packet arrive at the actuator before the older ones. A time diagram of the signal transmission is illustrated in Figure 1, in which it is assumed that $d = 3$, and at most four control signals may arrive at the actuator during one sampling interval. Furthermore, control signal $u(k+2)$ arrives at the actuator earlier than $u(k+1)$. Thus, $u(k+2)$ is adopted at time $(k+3)T$. This phenomenon is called packet disordering. In order to use the newest control signal, $u(k+1)$ will be discarded.

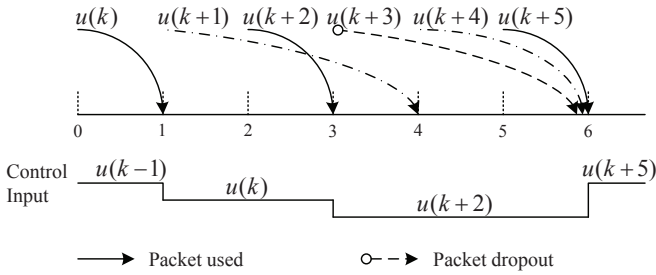


Fig.1. Time diagram of signal transmitting in the NCS.

By the aforementioned analysis, it can be seen that the adopted control signal may take values in $\{u(k-d+1), \dots, u(k-1), u(k)\}$ at sampling instant kT , which will result in $d+1$ different dynamics of the system. Moreover, during the sampling interval $[kT, (k+1)T)$, the system dynamics are actually determined by $\{\tau_{k-d+1}, \dots, \tau_{k-1}, \tau_k\}$. Therefore, we use a vector $\tau(k) = [\tau_{k-d+1}, \dots, \tau_{k-1}, \tau_k]$ to represent the control signal that is applied at the actuator, and define a vector-valued function $f : \tau(k) \rightarrow \sigma(k)$ to map $\tau(k)$ into a scalar $\sigma(k) \in \mathcal{N}_2 = \{0, 1, \dots, d\}$. The expression of $\sigma(k)$ is given in detail as follows

$$\sigma(k) = \begin{cases} 0, & \tau_k = 0, \tau_{k-j} \in \mathcal{N}_1, j = 1, 2, \dots, d-1 \\ 1, & \tau_k \geq T, \tau_{k-1} \leq T, \tau_{k-j} \in \mathcal{N}_1, \\ & j = 2, \dots, d-1 \\ 2, & \tau_{k-i} \geq (i+1)T, i = 0, 1, \tau_{k-2} \leq 2T, \\ & \tau_{k-j} \in \mathcal{N}_1, j = 3, \dots, d-1 \\ \vdots & \vdots \\ d, & \tau_{k-i} \geq (i+1)T, i = 0, 1, \dots, d-1 \end{cases} \quad (2)$$

and then the adopted control signal at time kT is $u(k-\sigma(k))$. Equation (2) presents an explicit logic expression for how to choose the newest control signal and eliminating the impact of packet disordering. Also, $\sigma(k)$ can be given concisely as follows

$$\sigma(k) = \min \{i | \tau_{k-i} - iT \leq 0, i = 0, 1, \dots, d\} \quad (3)$$

Then, we have the following proposition.

Proposition 1. *If $\sigma(k) = r$, then $\sigma(k+1) \leq r+1$.*

Proof. Let $\sigma(k+1) = \min \{j | \tau_{k+1-j} - jT \leq 0, j = 0, 1, \dots, d\}$. If $\sigma(k) = r$, it follows from (3) that $\tau_{k-r} - rT \leq 0$ and $\tau_{k-i} - iT > 0$ for $i < r$. From the above analysis, it can be obtained that $\tau_{k+1-(r+1)} - (r+1)T < 0$, which infers that $\sigma(k+1) \leq r+1$. This completes the proof. \square

Remark 1. *The packet disordering can be effectively eliminated by applying the mechanism given in (3). Assuming that the packets of time $(k-i-1)T$ and $(k-i)T$ are disordering, then one obtains $\tau_{k-i-1} \geq \tau_{k-i} + 2T$. Therefore, if $\tau_{k-i-1} - (i+1)T \leq 0$, it must be $\tau_{k-i} - iT < 0$, which means that the actuator will use the newest packet of time $(k-i)T$ and discard packet of time $(k-i-1)T$.*

Let θ_k be the number of consecutive packet dropout with $\theta_k \in [0, s]$, $s \leq d$ at time kT , and $\tau_k = dT$ if the packet of time kT is lost. A time diagram of the signal transmitting with both time delay and packet dropout is illustrated in Figure 1. In view of (2), it is difficult to handle packet dropout in a unified framework because $\sigma(k) \in \{0, 1, \dots, d+s\}$. Inspired by Equation (3), the following result can be derived with both time delays and packet dropouts

$$\sigma(k) = r + \theta_{k-r} \quad (4)$$

where $r = \min \{i | \tau_{k-i} - (i + \theta_{k-i})T \leq 0, i = 0, 1, \dots, d\}$.

Remark 2. *If there is no packet dropout during sampling interval $[(k-d)T, kT]$, one has $\theta_{k-i} = 0$ for $i \in \{0, 1, \dots, d\}$. Then (4) equals to (3). If $\theta_{k-r} \neq 0$, the packets of time $(k-r)T$ to $(k-r-\theta_{k-r})T$ are dropout and $\tau_{k-i} - (i + \theta_{k-i})T > 0$ for $0 \leq i < r$, which means that the packet of time $(k-i)T$ for $0 \leq i < r$ is not successfully delivered during interval $[(k-r)T, kT]$. Therefore, Equation (4) shows that the actuator always uses the newest packet during interval $[(k-d-s)T, kT]$. Similar to Proposition 1, the following proposition can be obtained.*

Proposition 2. *If $\sigma(k) = r + \theta_{k-r}$, $0 \leq \theta_{k-r} \leq s$, then $\sigma(k+1) \leq r + \theta_{k-r} + 1$ and $\sigma(k+1) \neq r+2, \dots, r + \theta_{k-r}$.*

Proof. By using a similar method to the proof of Proposition 1, it can be easily obtained that $\sigma(k+1) \leq r + \theta_{k-r} + 1$. Next, we will prove $\sigma(k+1) \neq r+2, \dots, r + \theta_{k-r}$.

If the packet of time $(k-r)T$ is successfully delivered during interval $[(k-d)T, kT]$, it means $\theta_{k-r} = 0$. Thus, $\sigma(k+1) \leq r+1$. If the packet of time $(k-r)T$ is dropout, then one has

$$r + \theta_{k-r} = r + 1 + \theta_{k-r-1} = \dots = r + \theta_{k-r} + \theta_{k-r-\theta_{k-r}} \quad (5)$$

which implies that $\theta_{k-r-\theta_{k-r}} = 0$. If the packet at time $(k-r+1)T$ is dropout, the following result can be obtained by (5)

$$r - 1 + \theta_{k-r+1} = r + \theta_{k-r} \quad (6)$$

It follows from (6) that $\sigma(k) = r - 1 + \theta_{k-r+1}$. However, it is inconsistent with condition $\sigma(k) = r + \theta_{k-r}$. Therefore, if the packet of time $(k-r)T$ is dropout, the packet at time $(k-r+1)T$ must be successfully delivered.

Let $\sigma(k+1) = \bar{j} + \theta_{k+1-\bar{j}}$. If $\tau_{k+1-j} - (j + \theta_{k+1-j})T > 0$ for $0 \leq j \leq r$, then $\bar{j} = r+1$ and $\sigma(k+1) = r + \theta_{k-r} + 1$. If there

exists a j for $0 \leq j \leq r$ satisfied $\tau_{k+1-j} - (j + \theta_{k+1-j})T \leq 0$, it follows from (5) and (6) that $\bar{j} + \theta_{k+1-\bar{j}} \leq r$ because the packet at time $(k - r + 1)T$ must be successfully delivered. Therefore, $\sigma(k + 1) \leq r$ and $\sigma(k + 1) \neq r + 1, r + 2, \dots, r + \theta_{k-r}$. The proof is thus completed. \square

Remark 3. If there is no delay in the communication network, then $\tau_k = 0$. Thus, Equation (4) becomes $\sigma(k) = \theta_k$. It can be known that if the packet of time kT is successfully delivered, then $\sigma(k) = 0$, otherwise $\sigma(k) = \gamma_k$. Therefore, if $\sigma(k) = \theta_k$, $0 \leq \theta_k \leq m$, then $\sigma(k + 1) = 0$ or $\sigma(k + 1) = \theta_k + 1$.

Remark 4. In [19], an H_∞ filtering problem has been investigated for NCSs both with random delays and packet dropouts. A logical data packet processor is introduced to choose the newest data signal from the network to the filter. However, the explicit expression for how to choose the newest data signal was not given in [19]. In this paper, the explicit expression for how to choose the newest data signal is given in (4), which leads to a new modeling approach, and some new results are also given in Proposition 2.

Let $\mathcal{N}_3 = \{0, 1, \dots, d + s\}$, it can be known from (4) that $\sigma(k) \in \mathcal{N}_3$. Denote $A = e^{A_p T}$ and $B = \int_0^T e^{A_p t} dt B_p$, then the NCS under consideration is modeled as the following discrete-time delay system

$$x(k + 1) = Ax(k) + Bu(k - \sigma(k)) \quad (7)$$

Let $\Lambda_{i\sigma(k)} = \begin{cases} I, & i = \sigma(k) \\ 0, & i \neq \sigma(k) \end{cases}$, $i \in \mathcal{N}_3$, then system (7) can be represented as follows

$$x(k + 1) = Ax(k) + \sum_{i=0}^{d+s} \bar{B}_{i\sigma(k)} u(k - i) \quad (8)$$

where $\bar{B}_{i\sigma(k)} = \Lambda_{i\sigma(k)} B$. Let $X(k) = \begin{bmatrix} x^T(k) & u^T(k - 1) & \dots \\ u^T(k - d - s + 1) & u^T(k - d - s) \end{bmatrix}^T$, system (8) can be rewritten as the following switched system model

$$X(k + 1) = G_{\sigma(k)} X(k) + H_{\sigma(k)} u(k) \quad (9)$$

where

$$G_{\sigma(k)} = \begin{bmatrix} A & \bar{B}_{1\sigma(k)} & \dots & \bar{B}_{(d+s-1)\sigma(k)} & \bar{B}_{(d+s)\sigma(k)} \\ 0 & 0 & \dots & 0 & 0 \\ 0 & I & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I & 0 \end{bmatrix},$$

$$H_{\sigma(k)} = \begin{bmatrix} \bar{B}_{0\sigma(k)}^T & I & 0 & \dots & 0 \end{bmatrix}^T.$$

Then, the objective is to design a stabilizing controller $u(k) = KX(k)$ for the NCS (9) via switched system approach. Thus, the closed-loop system of (9) is given as follows

$$X(k + 1) = \Phi_{\sigma(k)} X(k) \quad (10)$$

where $\Phi_{\sigma(k)} = G_{\sigma(k)} + H_{\sigma(k)} K$.

Remark 5. The subsystem of (10) activated during the sampling interval $[kT, (k + 1)T)$ is determined by $\tau(k)$. When $\tau(k)$ takes a certain value, the system (10) resides in the corresponding subsystem. Therefore, system (10) is essentially a switched system with a finite number of subsystems. By using the vector-valued function $f : \tau(k) \rightarrow \sigma(k)$, $\sigma(k)$ is used as the switching signal of the closed-loop NCS (10).

3. Stability analysis and controller design

Before proceeding further, some useful definitions are introduced.

Definition 1. The NCS (10) is said to be exponentially stable, if there exist positive constants c and $\alpha < 1$ such that the solution of system (10) satisfies

$$\|X(k)\| = c\alpha^k \|X_0\|$$

for any initial state $X_0 = X(0) \in R^{(d+1)n}$, where $X(0) = \begin{bmatrix} x^T(0) & u^T(-1) & \dots & u^T(-d - s + 1) & u^T(-d - s) \end{bmatrix}^T$, $u(i) = 0$, $i = -1, \dots, -d - s + 1, -d - s$.

Definition 2 [12]. For any switching signal $\sigma(k)$ and any $k \geq 1$, let $N_\sigma[0, k)$ be the number of switching of $\sigma(k)$ over the interval $[0, k)$. If $N_\sigma[0, k) \leq N_0 + k/T_\alpha$ holds for $N_0 \geq 0$ and $T_\alpha \geq 0$, then T_α is called the average dwell time and N_0 is the chatter bound.

For simplicity, but without the loss of generality, we choose $N_0 = 0$ in the following development. A sufficient condition for the exponential stability of the closed-loop NCS (10) is now presented in the following theorem.

Theorem 1. For given positive scalars $\lambda < 1$ and $\mu > 1$, if there exist matrices $P_i > 0$, K , $i = 0, 1, \dots, d + m$ of appropriate dimensions such that the following inequalities

$$\Phi_i^T P_i \Phi_i - \lambda^2 P_i \leq 0 \quad (11)$$

$$P_a \leq \mu P_b, \quad \forall a, b \in \mathcal{N}_3 \quad (12)$$

$$T_\alpha > T_\alpha^* = \frac{\ln \mu}{2 \ln \lambda^{-1}} \quad (13)$$

hold, then the closed-loop NCS (10) is exponentially stable and ensures a decay rate $\alpha = \lambda \mu^{\frac{1}{2T_\alpha}}$.

Proof. The proof can be followed by applying some similar procedures to those in [12], and is thus omitted. \square

Theorem 2. For given positive scalars $\lambda < 1$ and $\mu > 1$, if there exist matrices Z , V , $Q_i > 0$, $i = 0, 1, \dots, d + s$ of appropriate dimensions such that (13) is true and that the following LMIs

$$\begin{bmatrix} -\lambda^2 Q_i & * \\ G_i Z + H_i V & -Z - Z^T + Q_i \end{bmatrix} < 0 \quad (14)$$

$$Q_a \leq \mu Q_b, \quad \forall a, b \in \mathcal{N}_3 \quad (15)$$

hold, then the closed-loop NCS (10) is exponentially stable and ensure a decay rate $\alpha = \lambda \mu^{\frac{1}{2T_\alpha}}$ with controller gain $K = VZ^{-1}$.

By choosing the same parameters $x(0)$, μ and λ , one obtains $T_\alpha = 100/65 = 1.5385$. Therefore, the condition $T_\alpha > T_\alpha^*$ is also satisfied. By Theorem 2, one obtains the controller gain $K = \begin{bmatrix} 25.6369 & 4.5785 & -0.3927 & -0.3254 & -0.2552 \end{bmatrix}$. Thus, the NCS controlled by the designed controller via network is exponentially stable and has a decay rate $\alpha = 0.992$. The simulation results are shown in Figure 5, where the dotted line and solid line describe the state trajectories of the NCS without packet reordering and with packet reordering, respectively. It can be seen from Figure 5 that the packet disordering has a detrimental effect on the performance of the NCS.

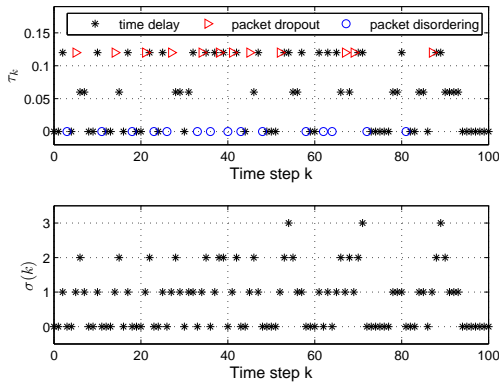


Fig.4. Distribution of τ_k , $\sigma(k)$ and packet dropout.

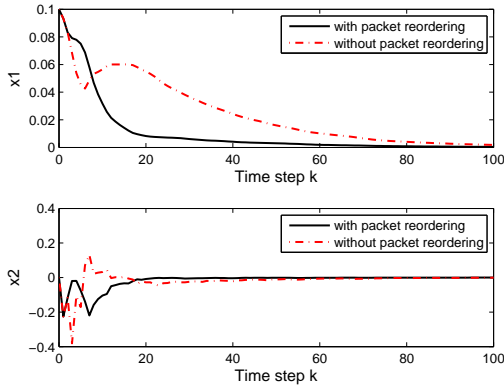


Fig.5. State trajectories of the NCS.

5. Conclusion

In this paper, a switched system approach was presented for a class of NCSs with time delays, packet disordering and packet dropouts. A new approach was proposed to deal with the detrimental effect of packet disordering, and the NCS was modeled as a switched system, which can fully describe the phenomenon of packet disordering. Two propositions are proposed for the relationship of the reordered packet. The condition for exponential stability and controller design procedure were presented for the closed-loop NCS. The effectiveness of the proposed method was illustrated by a numerical example.

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