Regret Theory and the Competitive Firm

Kit Pong Wong

School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong

Abstract

This paper examines the production decision of the competitive firm under uncertainty when the firm is not only risk averse but also regret averse. Regret-averse preferences are characterized by a modified utility function that includes disutility from having chosen ex-post suboptimal alternatives. The extent of regret depends on the difference between the actual profit and the maximum profit attained by making the optimal production decision had the firm observed the true realization of the random output price. If the firm is not too regret averse, we show that the conventional result that the optimal output level under uncertainty is less than that under certainty holds. Using a simple binary model wherein the random output price can take on either a low value or a high value with positive probability, we show the possibility that the firm may optimally produce more, not less, under uncertainty than under certainty, particularly when the firm is sufficiently regret averse and the low output price is very likely to prevail.

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1. Introduction

Since the seminal work of Sandmo (1971), the behavior of the competitive firm has been the subject of considerable research in decision making under uncertainty (Batra and Ullah, 1974; Broll, 1992; Chavas, 1985; Viaene and Zilcha, 1998; Wong, 1996; to name just a few). The extant literature assumes that the firm’s preferences admit the standard
von Neumann-Morgenstern expected utility representation. One notable result is that the risk-averse firm optimally produces less under uncertainty than under certainty.

In reality, firms may have desires to avoid consequences wherein ex-post suboptimal decisions appear to have been made even though these decisions are ex-ante optimal based on the information available at that time. To account for this consideration, Bell (1982, 1983) and Loomes and Sugden (1982) propose regret theory that defines regret as the disutility arising from not having chosen the ex-post optimal alternative, which is later axiomatized by Quiggin (1994) and Sugden (1993). Regret theory is supported by a large body of experimental literature that documents regret-averse preferences among individuals (see, e.g., Loomes, 1988; Loomes et al., 1992; Loomes and Sugden, 1987; Starmer and Sugden, 1993).

The purpose of this paper is to incorporate regret theory into Sandmo’s (1971) model of the competitive firm under uncertainty. To this end, we characterize the firm’s regret-averse preferences by a modified utility function that includes additive separable disutility from having chosen ex-post suboptimal alternatives. The extent of regret depends on the difference between the actual profit and the maximum profit attained by making the optimal production decision had the firm observed the true realization of the random output price. We are particularly interested in examining the impact of regret on the firm’s production decision as compared to the benchmark case of certainty.

We show that the firm optimally produces less under uncertainty than under certainty should the firm be not too regret averse. In this case, the risk-sharing motive remains first-order important to the firm. Hence, the conventional result of the extant literature that the optimal output level under uncertainty is smaller than that under certainty applies. This finding suggests that it is quite possible that the firm may optimally produce more, not less, under uncertainty than under certainty should the firm be sufficiently regret averse.

To verify this conjecture, we develop a binary model wherein the random output price can take on either a low value or a high value with positive probability. In such a binary model, we show that the conventional result does not hold if the firm is sufficiently regret averse and the low output price is very likely to prevail. In this case, the optimal output level under certainty is very close to the one that is ex-post optimal at the low output price. The sufficiently regret-averse firm as such optimally adjusts its output level upward so as to limit the potential regret when the high output price is actually revealed, thereby rendering the optimal output level under uncertainty to exceed that under certainty.

This paper is closely related to the early work of Paroush and Venezia (1979) who examine the competitive firm under uncertainty with a bivariate utility function defined on profits and regret. Our modified regret-theoretical utility function is a tractable version of theirs in that the bivariate utility function is specified as an additive separable function such that the degree of regret can be measured by a constant coefficient. Paroush and Venezia (1979) show that the competitive firm optimally produces less under uncertainty than under certainty if the firm is more risk averse to profits than to regret and the price risk is sufficiently small. This is tantamount to restricting the regret coefficient in our model to be sufficiently small, which is consistent with our findings. Paroush and Venezia (1979) also provide a necessary and sufficient condition under which the optimal output level under uncertainty exceeds that under certainty. However, their condition is based on endogenous variables and thus is not informative. In contrast, we use a binary model to derive sufficient conditions based on exogenous parameters such that the firm indeed optimally produces more under uncertainty than under certainty, which is a novel result in the literature of the competitive firm under uncertainty.

The rest of this paper is organized as follows. Section 2 delineates the model of the competitive firm under uncertainty when the firm’s preferences exhibit not only risk aversion but also regret aversion. Section 3 solves the model and provides sufficient conditions under which the regret-averse firm’s optimal output level under uncertainty is less than that under
certainty. Section 4 develops a binary model to show the possibility that introducing regret aversion to the firm may induce the firm to optimally produce more under uncertainty than under certainty. The final section concludes.

2. The model

Consider the competitive firm under uncertainty à la Sandmo (1971). There is one period with two dates, 0 and 1. To begin, the firm produces a single commodity according to a deterministic cost function, \( C(Q) \), where \( Q \geq 0 \) is the output level, and \( C(Q) \) is compounded to date 1 with the properties that \( C(0) = C'(0) = 0 \), and \( C'(Q) > 0 \) and \( C''(Q) > 0 \) for all \( Q > 0 \).\(^2\) The firm sells its entire output, \( Q \), at the per-unit price, \( \tilde{P} \), at date 1.\(^3\) The firm regards \( \tilde{P} \) as a random variable that is distributed according to a known cumulative distribution function, \( F(P) \), over support \([P_0, P]\), where \( 0 < P_0 < P \).\(^4\) The firm’s profit at date 1 is therefore uncertain and given by \( \Pi(\tilde{P}) = \tilde{P}Q - C(Q) \).

Paroush and Venezia (1979) define the firm to be regret-averse if its preferences are represented by a bivariate utility function, \( V(\Pi, R) \), defined on profits and regret, where \( \Pi \geq 0 \) is the firm’s profit at date 1, and \( R = \Pi^{\text{max}} - \Pi \geq 0 \) is the regret that is equal to the difference between the actual profit, \( \Pi \), and the maximum profit, \( \Pi^{\text{max}} \), that the firm could have earned if the firm had made the optimal production decision based on knowing the realized output price. Paroush and Venezia (1979) assume that \( V_{\Pi}(\Pi, R) > 0 \), \( V_{R}(\Pi, R) < 0 \), \( V_{\Pi R}(\Pi, R) < 0 \), \( V_{\Pi^2}(\Pi, R)V_{RR}(\Pi, R) > V_{\Pi R}(\Pi, R) \), and \( V_{\Pi^2}(\Pi, R) + V_{RR}(\Pi, R) - 2V_{\Pi R}(\Pi, R) < 0 \), where subscripts denote partial derivatives. For tractability, we adopt the following specification of \( V(\Pi, R) \) as proposed by Braun and

\(^2\)The strict convexity of the cost function reflects the fact that the firm’s production technology exhibits decreasing returns to scale.

\(^3\)Throughout the paper, random variables have a tilde (\( \sim \)) while their realizations do not.

\(^4\)An alternative way to model the output price uncertainty is to apply the concept of information systems that are conditional cumulative distribution functions over a set of signals imperfectly correlated with \( \tilde{P} \) (Broll et al., 2012).
Muermann (2004) and Muermann et al. (2006):

\[ V(\Pi, R) = U(\Pi) - \beta G(R), \]

where \( U(\Pi) \) is a von Neumann-Morgenstern utility function with \( U'(\Pi) > 0 \) and \( U''(\Pi) < 0 \), \( \beta > 0 \) is a constant regret coefficient, and \( G(R) \) is a regret function such that \( G(0) = 0 \), and \( G'(R) > 0 \) and \( G''(R) > 0 \) for all \( R \geq 0 \). It is easily verified that all the assumptions made by Paroush and Venezia (1979) are satisfied by the additive separable utility function given by Eq. (1).

To characterize the regret-averse firm’s optimal production decision, we have to first determine the maximum profit, \( \Pi_{\text{max}} \). If the firm could have observed the realized output price, \( P \), the maximum profit would be achieved by choosing \( Q(P) \) that solves \( C'[Q(P)] = P \). This ex-post optimal output level is increasing in \( P \) since \( Q'(P) = 1/C''[Q(P)] > 0 \). The maximum profit as a function of \( P \) is given by \( \Pi_{\text{max}}(P) = PQ(P) - C[Q(P)] \), which is increasing in \( P \) since \( \Pi_{\text{max}}'(P) = Q(P) > 0 \).

We can now state the regret-averse firm’s ex-ante decision problem. At date 0, the firm chooses an output level, \( Q \), so as to maximize the expected value of its regret-theoretical utility function:

\[ \max_{Q \geq 0} \mathbb{E}\{U[\Pi(\tilde{P})] - \beta G[\Pi_{\text{max}}(\tilde{P}) - \Pi(\tilde{P})]\}, \]

where \( \Pi(P) = PQ - C(Q) \) and \( \Pi_{\text{max}}(P) = PQ(P) - C[Q(P)] \) for all \( P \in [\underline{P}, \overline{P}] \), and \( \mathbb{E}(\cdot) \) is the expectation operator with respect to the cumulative distribution function, \( F(P) \).

3. Solution to the model

\(^{5}\)Braun, and Muermann (2004) and Muermann et al. (2006) consider a regret function that depends on the difference between the utility level of the actual profit and that of the maximum profit, \( U(\Pi_{\text{max}}) - U(\Pi) \) (see also Wong, 2011, 2012). Since such a specification is simply a monotonic transformation of ours, none of the qualitative results are affected if we adopt this alternative approach.
The first-order condition for program (2) is given by
\[ E \left\{ \{ U'[\Pi^* (\tilde{P})] + \beta G'[\Pi^{\text{max}} (\tilde{P}) - \Pi^* (\tilde{P})]\} \{ \tilde{P} - C'(Q^*)\} \right\} = 0, \tag{3} \]
where an asterisk (*) indicates an optimal level. The second-order condition for program (2) is given by
\[ E \left\{ \left\{ U''[\Pi^* (\tilde{P})] - \beta G''[\Pi^{\text{max}} (\tilde{P}) - \Pi^* (\tilde{P})]\right\} \{ \tilde{P} - C'(Q^*)\} \right\} - E \left\{ \left\{ U'[\Pi^* (\tilde{P})] + \beta G'[\Pi^{\text{max}} (\tilde{P}) - \Pi^* (\tilde{P})]\right\} C''(Q^*) \right\} < 0, \tag{4} \]
which is satisfied given the assumed properties of \( U(\Pi) \), \( C(Q) \), and \( G(R) \).

As a benchmark, suppose that the uncertain output price, \( \tilde{P} \), is fixed at its expected value, \( E(\tilde{P}) \). In this benchmark case of certainty, Eq. (3) reduces to \( C'(Q^n) = E(\tilde{P}) \), which is the usual optimality condition that the optimal output level, \( Q^n \), is the one that equates the marginal cost of production, \( C'(Q^n) \), to the known output price, \( E(\tilde{P}) \).

We are interested in comparing \( Q^* \) with \( Q^n \). To this end, we differentiate the objective function of program (2) with respect to \( Q \) and evaluate the resulting the derivative at \( Q^* = Q^n \) to yield
\[ \frac{\partial E\{ V[\Pi(\tilde{P}), \Pi^{\text{max}} (\tilde{P}) - \Pi(\tilde{P})]\}}{\partial Q} \bigg|_{Q=Q^n} = E \left\{ \{ U'[\Pi^n (\tilde{P})] + \beta G'[\Pi^{\text{max}} (\tilde{P}) - \Pi^n (\tilde{P})]\} \{ \tilde{P} - C'(Q^n)\} \right\}, \tag{5} \]
where \( \Pi^n(P) = PQ^n - C(Q^n) \) for all \( P \in [\underline{P}, \overline{P}] \). If the right-hand side of Eq. (5) is negative (positive), Eqs. (3) and (4) imply that \( Q^* < (>) Q^n \). The following proposition provides sufficient conditions under which \( Q^* < Q^n \).

**Proposition 1.** If \( U'''(\Pi) \geq 0 \) and \( G'''(R) \geq 0 \), then a sufficient condition that ensures the regret-averse firm to produce less than the optimal output level under certainty, i.e.,
is that the constant regret coefficient, \( \beta \), is sufficiently small such that

\[
\beta \leq \frac{U'\{\Pi^n[E(\tilde{P})]\} - U'\{\Pi^n(P)\}}{G'\{\Pi_{\text{max}}(P) - \Pi^n(P)\} - G'(0)}.
\]

(6)

**Proof.** Let \( \Psi(P) = U'[\Pi^n(P)] + \beta G'[\Pi_{\text{max}}(P) - \Pi^n(P)] \). Since \( C'(Q^n) = E(\tilde{P}) \), we can write Eq. (5) as

\[
\frac{\partial E\{V[\Pi(\tilde{P}), \Pi_{\text{max}}(\tilde{P}) - \Pi(\tilde{P})]\}}{\partial Q} \bigg|_{Q = Q^n} = E\left\{ \{\Psi(\tilde{P}) - \Psi[E(\tilde{P})]\}[\tilde{P} - E(\tilde{P})] \right\},
\]

(7)

where \( \Psi[E(\tilde{P})] = U'[\Pi^n[E(\tilde{P})]] + \beta G'(0) \) since \( \Pi_{\text{max}}[E(\tilde{P})] = \Pi^n[E(\tilde{P})] = E(\tilde{P})Q^n - C(Q^n) \).

Differentiating \( \Psi(P) \) twice with respect to \( P \) yields

\[
\Psi'(P) = U''[\Pi^n(P)]Q^n + \beta G''[\Pi_{\text{max}}(P) - \Pi^n(P)][Q(P) - Q^n] + \beta G'''[\Pi_{\text{max}}(P) - \Pi^n(P)]Q'(P).
\]

(8)

Since \( U''(\Pi) \geq 0 \) and \( G''(R) \geq 0 \), Eq. (9) implies that \( \Psi''(P) > 0 \) for all \( P \in [P, \overline{P}] \).

Since \( Q(P) < (>) Q^n \) for all \( P < (>) E(\tilde{P}) \), it follows from Eq. (8) that \( \Psi'(P) < 0 \) for all \( P \leq E(\tilde{P}) \). Hence, \( \Psi(P) > \Psi[E(\tilde{P})] \) for all \( P < E(\tilde{P}) \). Condition (6) ensures that \( \Psi[E(\tilde{P})] \geq \Psi(\overline{P}) \). Since \( \Pi^n[E(\tilde{P})] < \Pi^n(\overline{P}) \) and \( U''(\Pi) < 0 \), we have \( U''[\Pi^n[E(\tilde{P})]] > U''[\Pi^n(\overline{P})] \).

Furthermore, \( \Pi_{\text{max}}(\overline{P}) > \Pi^n(\overline{P}) \) and \( G''(R) > 0 \) so that \( G'[\Pi_{\text{max}}(\overline{P}) - \Pi^n(\overline{P})] > G'(0) \).

The right-hand side of condition (6) as such is strictly positive so that condition (6) is non-trivial.

Since \( \Psi(P) \) is strictly convex in \( P \) and \( \Psi'[E(\tilde{P})] < 0 \), it follows from condition (6) that \( \Psi(P) < \Psi[E(\tilde{P})] \) for all \( P > E(\tilde{P}) \). The right-hand side of Eq. (7) as such is negative so that \( Q^* < Q^n \). □
The intuition for Proposition 1 is as follows. In the limiting case that $\beta = 0$, the firm is purely risk averse. It is well-known that the risk-averse firm produces less than $Q^n$ so as to limit its exposure to the price uncertainty (Sandmo, 1971). For $\beta$ sufficiently small, introducing regret aversion to the firm would not substantially change such a risk-sharing motive, thereby rendering $Q^* < Q^n$.\(^6\)

4. A binary model

To gain more insights, we consider in this section a simple binary model such that $\tilde{P}$ takes on the low value, $P$, with probability $p$ and the high value, $\bar{P}$, with probability $1 - p$, where $0 < p < 1$. In such a binary model, the right-hand side of Eq. (5) becomes

$$p[U'[\Pi^n(P)] + \beta G'[\Pi^{\text{max}}(P) - \Pi^n(P)]][P - C'(Q^n)]$$

$$+ (1 - p)[U'[\Pi^n(\bar{P})] + \beta G'[\Pi^{\text{max}}(\bar{P}) - \Pi^n(\bar{P})]][\bar{P} - C'(Q^n)]$$

$$= p(1 - p)(\bar{P} - P)[U'[\Pi^n(P)] + \beta G'[\Pi^{\text{max}}(P) - \Pi^n(P)]]$$

$$- U'[\Pi^n(P)] - \beta G'[\Pi^{\text{max}}(P) - \Pi^n(P)],$$

(10)

where the equality follows from $pP + (1 - p)\bar{P} = C'(Q^n)$. If right-hand side of Eq. (10) is negative (positive), it then follows from Eqs. (3) and (4) that $Q^* < (>) Q^n$.

Define the following output level, $Q^\circ$, that solves

$$U'[\Pi^\circ(\bar{P})] + \beta G'[\Pi^{\text{max}}(\bar{P}) - \Pi^\circ(\bar{P})] = U'[\Pi^\circ(P)] + \beta G'[\Pi^{\text{max}}(\bar{P}) - \Pi^\circ(P)],$$

(11)

where $\Pi^\circ(P) = P Q^\circ - C(Q^\circ)$. Since $U''(\Pi) < 0$, we have $U'[\Pi^\circ(\bar{P})] < U'[\Pi^\circ(P)]$. Eq. (11)\(^6\)Paroush and Venezia (1979) show that $Q^* < Q^n$ if the firm is more risk averse to profits than to regret, i.e., $V_\Pi(\Pi, R) < V_{RR}(\Pi, R)$ and if the price risk, $\tilde{P}$, is sufficiently small. Condition (6) is consistent with their conditions.
as such implies that $G'[\Pi_{\text{max}}(P) - \Pi_{\text{max}}(\bar{P})] < G'[\Pi_{\text{max}}(\bar{P}) - \Pi_{\text{max}}(\bar{P})]$. Differentiating Eq. (11) with respect to $\beta$ yields

$$
\frac{dQ^o}{d\beta} = \left\{ \beta G''[\Pi_{\text{max}}(\bar{P}) - \Pi_{\text{max}}(\bar{P})] - U''[\Pi_{\text{max}}(\bar{P})] \left[ \bar{P} - C'(Q^o) \right] \right\}^{-1} > 0.
$$

(12)

As $\beta$ approaches infinity, $Q^o$ converges to $Q^* = \left[ \Pi_{\text{max}}(\bar{P}) - \Pi_{\text{max}}(\bar{P}) \right] / (\bar{P} - P).$\(^7\) We state and prove the following proposition.

**Proposition 2.** Suppose that the random output price, $\tilde{P}$, can take on the low value, $P$, with probability $p$ and the high value, $\bar{P}$, with probability $1 - p$, where $0 < p < 1$. There exists a critical value, $p^*$, given by

$$
p^* = \frac{\bar{P} - C'(Q^o)}{P - \bar{P}} \in (0, 1),
$$

(13)

such that the regret-averse firm optimally produces less (more) than the optimal output level under certainty, i.e., $Q^* < (>) Q^n$, for all $p < (>) p^*$. Furthermore, $p^*$ decreases with an increase in the regret coefficient, $\beta$, and converges to $[\bar{P} - C'(Q^*)] / (\bar{P} - P)$ as $\beta$ approaches infinity.

**Proof.** Let $\Phi(p) = U'[\Pi^n(\bar{P})] + \beta G'[\Pi_{\text{max}}(\bar{P}) - \Pi^n(\bar{P})] - U'[\Pi^n(P)] - \beta G'[\Pi_{\text{max}}(P) - \Pi^n(P)]$. Differentiating $\Phi(p)$ with respect to $p$ yields\(^8\)

$$
\Phi'(p) = \left\{ p \beta G''[\Pi_{\text{max}}(\bar{P}) - \Pi^n(\bar{P})] + \beta G''[\Pi_{\text{max}}(\bar{P}) - \Pi^n(\bar{P})] - U''[\Pi^n(\bar{P})] \left[ \bar{P} - \Pi^n(\bar{P}) \right] \right\} > 0,
$$

(14)

\(^7\)The limiting output level, $Q^*$, solves $G'[\Pi_{\text{max}}(\bar{P}) - \bar{P}Q^* + C(Q^*)] = G'[\Pi_{\text{max}}(\bar{P}) - \bar{P}Q^* + C(Q^*)]$.

\(^8\)In this binary model, we have $dQ^* / dp = (\bar{P} - \bar{P}) / C''(Q^*) < 0$. 

since $U''(\Pi) < 0$, $G''(R) > 0$, and $p \bar{P} + (1 - p) \bar{P} = C'(Q^n)$. At $p = 0$, we have $Q^n = Q(\bar{P})$. In this case, $\Phi(0) = U'[\Pi^{\text{max}}(\bar{P})] - U'[\bar{P}Q(\bar{P}) - C[Q(\bar{P})]] + \beta \{ G'(0) - G'(\Pi^{\text{max}}(\bar{P}) - \bar{P}Q(\bar{P}) + C[Q(\bar{P})]) \} < 0$ since $U''(\Pi) < 0$ and $G''(R) > 0$. On the other hand, at $p = 1$, we have $Q^n = Q(\bar{P})$. In this case, $\Phi(1) = U'(\bar{P})Q(\bar{P}) - C[Q(\bar{P})] - U'[\Pi^{\text{max}}(\bar{P})] + \beta \{ G'(\Pi^{\text{max}}(\bar{P}) - \bar{P}Q(\bar{P}) + C[Q(\bar{P})]) \} - G'(0) > 0$ since $U''(\Pi) < 0$ and $G''(R) > 0$. It then follows from Eq. (14) that there exists a unique point, $p^* \in (0, 1)$, such that $\Phi(p) < (>) 0$ and $Q^* < (>) Q^n$ for all $p < (>) p^*$. At $p^*$, we have $\Phi(p^*) = 0$. It then follows from Eqs. (3) and (10) that $Q^* = Q^n = Q^*$ since $Q^*$ solves Eq. (11). Eq. (13) then follows from $p^* \bar{P} + (1 - p^*) \bar{P} = C'(Q^*)$. From Eq. (12), we have $dp^*/d\beta < 0$ and thus $p^*$ converges to $[\bar{P} - C'(Q^*)]/(\bar{P} - P)$ as $\beta$ approaches infinity. $\square$

The intuition for Proposition 2 is as follows. When $\bar{P}$ is very likely to be seen at date 1, $Q^n$ is closer to $Q(\bar{P})$ and further way from $Q(\bar{P})$. Introducing regret aversion, which is sufficiently severe, to the firm makes the firm take into account the substantial disutility from the large discrepancy of its output level, $Q^n - Q(\bar{P})$, when the low output price is revealed. To avoid regret, the regret-averse firm optimally adjusts its output level downward from $Q^n$ to move closer to $Q(\bar{P})$ so that $Q^* < Q^n$ when $p$ is small. On the other hand, when $\bar{P}$ is very likely to be seen at date 1, in this case $Q^n$ is close to $Q(\bar{P})$. The regret-averse firm as such optimally adjusts its output level upward from $Q^n$ to reduce the discrepancy of its output level, $Q(\bar{P}) - Q^*$, when the high output price is revealed. Hence, we have $Q^* > Q^n$ when $p$ is large.

5. Conclusion

In this paper, we incorporate regret theory into Sandmo’s (1971) model of the competitive firm under uncertainty. Regret-averse preferences are characterized by a modified utility function that includes additive separable disutility from having chosen ex-post sub-
optimal alternatives. The extent of regret depends on the difference between the actual profit and the maximum profit attained by making the optimal production decision had the firm observed the true realization of the random output price. We show that the conventional result of the extant literature that the optimal output level under uncertainty is less than that under certainty holds if the firm is not too regret averse. This suggests that it is possible that the firm may optimally produces more, not less, under uncertainty than under certainty. We verify such a conjecture by using a simple binary model wherein the random output price can only take on a low value or a high value with positive probability. We show that the non-conventional result holds in the binary model if the firm is sufficiently regret averse and the low output price is very likely to prevail. Regret aversion as such plays a distinctive role, vis-à-vis risk aversion, in shaping the production decision of the competitive firm under uncertainty.

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