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DO THE MATHEMATICALLY GIFTED AND TALENTED SENIOR PRIMARY SCHOOL STUDENTS IN HONG KONG UNDERSTAND MATHEMATICS?

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Students in Hong Kong have been consistently performing well in international comparative studies of mathematics achievement such as the Trend International Mathematics and Science Study (TIMSS) (Beaton et al, 1996; Mullis et al, 1997; Mullis et al, 2000) and the OECD Program for International Student Assessment (PISA) (OECD, 2001; 2003). This research intended to study the mathematically gifted and talented senior primary school students in Hong Kong and to investigate their performances in Mathematical Olympiad. Top-performing students were chosen for interview and they are required to generate explicit principle-based explanations on their answers. It was found that most of the gifted and talented students in Hong Kong solved the mathematical Olympiad problems using conceptual knowledge, while a few of them adopted procedural knowledge.
Keywords: Mathematical Olympiad, gifted and talented students, conceptual and procedural knowledge

INTRODUCTION

Students in Hong Kong have been consistently performing well in international comparative studies of mathematics achievement such as the Trend International Mathematics and Science Study (TIMSS) (Beaton et al, 1996; Mullis et al, 1997; Mullis et al, 2000) and the OECD Program for International Student Assessment (PISA) (OECD, 2001; 2003). However, standardized achievement tests may not identify students who are gifted and talented in mathematics. One reason is that these tests often focused on low level tasks (Sheffield, 1994). In contrast to solving problems using ready-made schemes, Samovol and Applebaum (2006) found that the problems in Mathematical Olympiad "differ in quality from the standard textbook ones, primarily, by the presence of vivid mathematical idea, peculiar folklore grace, hard-to achieve but surprisingly brilliant, sometime insight solution". They also suggested that the Olympiad problems "possessed genuine mathematical grace and profound contents". This research intended to study the mathematically gifted and talented senior primary school students in Hong Kong and to investigate their performances in Mathematical Olympiad.

BACKGROUND: MATHEMATICAL OLYMPIAD IN HONG KONG

Different Mathematical Olympiad games are offered in Hong Kong for different levels of students. We will only focus on the contests in primary schools. The contests are classified according to the number of participants (Ng, 2003) as follows: Worldwide and regional, Hong Kong, Districts in Hong Kong, Others (e.g. charitable and religious organizations). There are currently many Mathematics Olympiads organized by different associations for primary school students in Hong Kong. Different Mathematics Olympiads have different rules. Some of the Mathematics Olympiads are by invitation. Students are chosen by the school teachers or they have to take part in some preliminary tests before they participate in a Mathematics Olympiad. Other Mathematics Olympiads are open to all primary 4 (normally aged 10) or primary 6 (normally aged 12) school students. Students can join them voluntarily.

The researcher has been teaching Mathematics Olympiad for primary school students in Hong Kong for seven years and she has also set the papers for Sheng Kung Hui (Anglican Church in Hong Kong) Primary Mathematics Olympiad for three years. This year, she will investigate the results from this Mathematics Olympiad and this paper presents the findings. Assume that the mathematically gifted and talented senior primary school students in Hong Kong will be selected in the Mathematics Olympiad, the researcher intended to analyze their understanding in the Mathematics Olympiad questions.

LITERATURE REVIEW

It is not an easy task to define "mathematical understanding". Wu (1999) suggested that, "in mathematics, skills and understanding are completely intertwined." Fluency and precision in executing the basics are essential for further progress in the course of one's mathematics education. Instead of using the phrase "conceptual understanding", Devlin (2007) proposed to call it "functional understanding." He suggested that,

> "It means, roughly speaking, understanding a concept sufficiently well to get by for the present. Because functional understanding is defined in terms of what the learner can do with it, it is possible to test if the learner has achieved it or not, which avoids my uncertainty about full conceptual understanding."

Matthews and Rittle-Johnson (2009) defined conceptual knowledge as explicit or implicit knowledge of the principles that govern a domain and their interrelations. In contrast, they defined procedural knowledge as the ability to execute action sequences to solve problems. Greater conceptual and procedural knowledge are associated with better performance on a variety of problem types. They also suggested that "successful learners tend to give more principle-based explanations, to consider the goals of operators and procedures more frequently, and to show illusions of understanding less frequently."

In order to investigate whether students in Hong Kong have an understanding to the Mathematical Olympiad problems, they were interviewed to see if they can give principle-based explanations. Are they working on the problems using their conceptual knowledge or procedural knowledge? Our research question in this study is: Do the mathematically gifted and talented senior primary school students in Hong Kong understand mathematics?

METHODOLOGY

This research based on the Sheng Kung Hui Primary Mathematics Olympiad which was held on 14th January, 2012. Four hundred and eight students from 54 schools participated. All participants are selected by their teachers in schools and different schools may have different screening procedures.

Design of the tasks

The researcher set the tasks. In addition to the Mathematics Olympiads held in Hong Kong, the researcher had studied some of the popular mathematics competitions for senior primary school students. They included Gauss Contest organized by Waterloo Mathematics Foundation in Canada, Mathematical Olympiads for Elementary and Middle Schools (MOEMS) in the US, and Mathematical Challenge problem-solving competition organized by the Scottish Mathematical Council. Tasks would be set on different areas in Mathematics. The research classified the tasks into five categories:

- Arithmetic, which includes four basic operations in whole numbers, fractions and decimals, evaluating finite continued fractions, converting recurring decimals to fractions, and cryptarithm.
- Number Theory, which includes problems on divisibility, remainder, common factor, common multiple, parity, Chinese Remainder Theorem, place value, and perfect square.
- Geometry, which included angle, perimeter and area of different shapes, volume and surface area of solids.
- Probability and Statistics, which includes counting by listing and using Venn diagrams, pigeonhole principle, inclusion–exclusion principle, permutation and combination.
- Applications which require skills on algebraic operations, looking for patterns, working backwards and solving simultaneous equations. The tasks include speed (catching up and encountering problems), excess-and-shortage problems, age problems, counting (problems from planting trees and problems related to page numbers), rate and ratio (problems of concentration), sequence with common difference, other operations, logic, average problems, mathematics on time.

Format of the Mathematics Olympiad

The format of the Sheng Kung Hui Primary Mathematics Olympiad was set by the school principals of the Sheng Kung Hui primary schools. The paper was divided into three sections. Section A was calculation and there were 10 questions. Each question carried 4 marks. Section B was Application I. Ten medium-high level questions were set and each of them carried 5 marks. Section C was Application II, which included five high level problems and five very high level problems. Each of them carried 6 marks. The full mark in this Mathematics Olympiad would be 150. Students would be allowed to finish Part (I) within 2 hours. It was also required that there should be one and only one answer for each question and students were not expected to explain their answers.

Case Study

After the Mathematics Olympiad, the papers were marked. According to the rules or format of this Mathematics Olympiad, students were not expected to explain their answers.

Therefore, the results were analyzed quantitatively. Twelve (out of 408) students who scored the highest (with range from 128 to 145 out of 150) were selected for interview. In the interview, their background information was first asked. And each student was asked to work on two questions, which have appeared in the question set of the Mathematics Olympiad they attempted on 14th January, 2012. The researcher recorded the interviews and then analyzed their understanding on the questions. Students were required to generate explicit principle-based explanations for their answers. Their understanding depends on their logical arguments. The interview for each student lasted for 15 minutes. It is also worth mentioning that, in order to conduct the interview in a more naturalistic manner, the researcher did not follow strictly both the format and the sequence of the interview questions.

FINDINGS

Demography

Eleven of the twelve students were born in 2000 (aged 11 or 12 at the time of interview), while the remaining one was born in 2002 (aged 9).

In contrast to the popular shallow education in Hong Kong, all the twelve students have never had private tutors. Three of them were taught by their fathers, while the rest were trained by their school teachers. All the courses provided by the schools were free of charge. Talented students were chosen for the training courses regardless their family backgrounds.

Since the twelve students were from different schools, their training started at different time. Some of the schools started the training courses as early as in primary one, while one of the schools only provided the training course since primary five. Different schools also provided different training sessions. It ranged from once per week to three times per week. Each session lasted from 45 minutes to 3 hours.

The following three questions were asked in the interview and the answers given by the students were analyzed.

Question 1

Evaluate 2012+2008 – 2004 – 2000+1996+1992 – 1988 – 1984+…+8 – 4.

This was the type of questions that appeared in many standard textbooks for Mathematics Olympiad. Eight students were required to answer this question. Three of the students identified that every four terms form a group. Two of them recognized that there were three extra term while one of them added the hidden term –0 at the end. The value in each group is 16. There are $\frac{2012-28}{16}+1=125$ groups in the expression. Hence they obtained the answer 16 × 125 + 12 + 8 – 4=2016. Another five students also combined the terms into a group of four, but they started grouping from 2008. Since 2008 – 2004 – 2000+1996=0, all the terms until +8 vanished. Therefore three terms 2012+8 – 4 remained and the answer is 2016.

The formula for counting the number of terms in a series is taught in most of the courses. It is worth knowing whether the students understood the meaning behind the formula or they just memorized it. The “plus one” at the end of the formula is a usual trick in the counting problems. This question tested whether the students learned to find the sum of sequences procedurally or conceptually. Out of the eight students, five of them can give very precise explanations on the formula $\text{number of terms} = \frac{\textit{last term} - \textit{first term}}{\textit{common difference}} + 1.$ One of the students said,

> "The first number in every group 2012, 1996, … have the same common difference. The difference of the first term in the first group and the first term in the last group, i.e. 2012 – 28, gave you the total "gap". However, we only need to count the first term of each group and the common difference is 2012 – 1996 = 16. We did $\frac{2012-28}{16}$ because we wanted to know how many "gaps" are there in the total "gaps". There are always two gaps between three numbers and three spaces between four numbers. We should add one afterwards."

The other three students said that they learned the formula by heart and they could do the question procedurally. One of them tried to explain and two of them said they did not know the rationale behind the formula.

Question 2

There are 8 balls of same shape and size, numbered (1) to (8). Six of them are standard and they have the same weight. One of them is 1g lighter than the standard ball and the remaining ball is 1g heavier than the standard ball. The balls are then weighed three times, with the following results:

1st time: (1)+(2)+(3)=(4)+(5)+(6)

2nd time: (1)+(2)+(7)<(3)+(5)+(8)

3rd time: (3)+(7)+(2)>(4)+(5)+(6)

What is the number of the lighter ball?

This question tests whether students can propose hypotheses and apply logical deductions to their hypotheses. Eight students were required to answer this question. Four of the students compared conditions 1 and 3, and found that ball no. 7 was heavier than ball no. 1. They then applied condition 2 to find out ball no. 1 is the lightest. Another two students made hypotheses from condition 1 and deduced three different possibilities: the lightest ball and the heaviest ball in ball no. 7 or no. 8 or in the same sides of the equation in condition 1. That means they are either in no. 1, 2, 3, or in no. 4, 5, 6. They then checked their hypothesis by using the other two given conditions and found that ball no. 1 is the lightest. Another student started with conditions 2 and 3 and argued that the lightest ball were not no. 3, no. 5, no. 8, no. 2, and no. 7. He then used conditions 1 and 3 and found out the lightest ball were not no. 4 and no.6. Therefore, he concluded that the lightest ball was no. 1. The remaining student gave the correct answer just because he remembered what he wrote in the Olympiad (2 months and 3 weeks before the interview).

Question 3

In a primary school, $\frac{1}{3}$ of the students do not like durian, $\frac{4}{7}$ of the students do not like grapefruit, $\frac{1}{6}$ of the students do not like both durian and grapefruit. If there are 22 students who like both durian and grapefruit, how many students are there?

Eight students were required to answer this question. Five of them found out the fractions of students who like durian and grapefruit respectively and then they got the correct answer by $22 \div \left[1-(\frac{1}{3}+\frac{4}{7}-\frac{1}{6})\right] = 22 \div \frac{11}{42} = 84$. They could explain the inclusion–exclusion principle by drawing Venn diagrams. One girl student solved the problem by finding the LCM of 3, 7 and 6. She then tried to check whether 42 is the total number of students. She found that 42 was not possible and then she checked that 84 was the correct answer. The remaining two students could not finish it in the interview, even they gave correct answers in the Olympiad.

Conclusion

It was found from Question 1 that most gifted and talented primary school students in Hong Kong were able to generate explicit principle-based explanations. They understood the rationales behind the formula in finding the number of terms before they used it. Their answers to the question relied on both conceptual and procedural understanding. In Question 2, all students were able to use logical arguments to find the answers. In Question 3, most students were able to demonstrate their understanding of the inclusion-exclusion principle. They understood the principle before they applied it. To sum up, gifted and talented primary school students in Hong Kong understand mathematics conceptually and procedurally. However, the sample in this research is small and this result cannot be generalized.

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