Multinationals and futures hedging: An optimal stopping approach

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ABSTRACT

This paper examines the optimal design of a futures hedge program for a risk-averse multinational firm (MNF) under exchange rate uncertainty. All currency futures contracts are marked to market and require interim cash settlement of gains and losses. The MNF commits to prematurely liquidating its futures position on which the interim loss incurred exceeds a threshold level (i.e., the liquidation threshold). When the liquidation threshold is exogenously given, we show that the MNF optimally opts for an under-hedge (an over-hedge) should the futures exchange rates be not too (sufficiently) positively autocorrelated. When the liquidation threshold is endogenously determined, we show that the MNF voluntarily chooses to prematurely liquidate its futures position only if the futures exchange rates are positively autocorrelated. In the case that the futures exchange rates are uncorrelated or negatively autocorrelated, the MNF prefers not to commit to any finite liquidation thresholds.

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1. Introduction

Multinational firms (MNFs) take liquidity risk seriously when devising their risk management strategies. Ignorance of liquidity risk is likely to result in fatal consequences for even technically solvent MNFs. Prominent examples of this sort include the disaster at Metallgesellschaft and the debacle of Long-Term Capital Management.[1]

In a recent article in this Journal, Lien and Wong (2005) have examined the effect of liquidity risk on the behavior of a risk-averse MNF facing exchange rate uncertainty. In the two-period model of Lien and Wong (2005), the MNF trades unbiased currency futures contracts that are marked to market for hedging purposes. In order to meet the indeterminate interim loss from its futures position due to the marking-to-market process,

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[1]See Jorion (2007) for details of these cases.

the MNF puts aside a fixed amount of capital earmarked for its futures hedge program. Succinctly, the MNF commits to prematurely liquidating its futures position on which the interim loss incurred exhausts the earmarked capital. The capital commitment as such constitutes a liquidity constraint that affects the MNF's optimal hedging decision. Focusing on the case that the futures exchange rates follow a random walk, Lien and Wong (2005) show that the MNF optimally under-hedges its exchange rate risk exposure in response to the exogenous liquidity constraint. The coexistence of the exchange rate risk and the liquidity risk makes an under-hedge optimal.

The purpose of this paper is to characterize the optimal design of a futures hedge program for a risk-averse MNF within the two-period model of Lien and Wong (2005) with two caveats. First, we allow the futures exchange rate dynamics to be a first-order autoregression, rendering a random walk to simply be a special case. Second, we endogenize an provision for premature termination of the MNF's futures hedge program. When both extensions are incorporated into the model of Lien and Wong (2005), we essentially study the MNF's futures hedging decision in an optimal stopping approach, which possibly depends on how the futures exchange rates are autocorrelated.[2]

Asset pricing models that consider speculative interactions between rational and irrational (noise) traders result in price bubbles and fads (see, e.g., Black, 1986; Cutler et al., 1990, 1991; DeLong et al., 1990; and Shiller, 1984). Since bubbles and fads are transitory components of asset prices, they decay exponentially over a long period of time, thereby inducing negative autocorrelations at higher lags and forcing prices to revert to their fundamental levels. At shorter lags, however, bubbles and fads give an impression of persistence of price deviations from equilibrium values, thereby giving rise to positive autocorrelations. It is well-documented that short horizon returns in stock markets possess positive autocorrelations (see, e.g., French and Roll, 1986; Lo and MacKinlay, 1988; and Poterba and Summers, 1988). Puri et al. (2002) find that short horizon returns in currency futures markets also exhibit this empirical regularity. Specifically, the average autocorrelations observed for weekly and monthly returns across different currencies and various lengths of holding periods are mostly of the order of 0.015 and 0.04 per lag, respectively, and are statistically significant. These empirical findings lead to rejection of the random walk hypothesis and

[2]We would like to thank an anonymous referee for pointing out that the MNF's futures hedging decision contains an optimal stopping problem, i.e., the optimal liquidation timing for the futures position.

motivate us to specify the futures exchange rate dynamics as a first-order autoregression.[3]

We first consider the case that the MNF faces an exogenous liquidity constraint as in Lien and Wong (2005), but allow the futures exchange rates to be autocorrelated. We show that the MNF optimally under-hedges or over-hedges its exchange rate risk exposure in response to the exogenous liquidity constraint, depending on whether the autocorrelation coefficient of the futures exchange rate dynamics is lower or higher than a positive critical value, respectively. These results thus generalize those of Lien and Wong (2005) in the random walk case to the case of autocorrelated futures exchange rates.

We then consider the case that the MNF is able to choose a threshold of the interim loss incurred on its futures position such that the MNF commits to prematurely terminating its futures hedge program whenever the actual loss exceeds the chosen liquidation threshold. The MNF's ex-ante decision problem as such contains an optimal stopping problem, i.e., the choice of optimal timing for the termination of the futures hedge program. We show that the MNF voluntarily chooses to prematurely liquidate its futures position only if the futures exchange rates are positively autocorrelated. A positive autocorrelation implies that a loss from a futures position tends to be followed by another loss from the same position. The MNF as such finds premature liquidation of its futures position to be ex-post optimal. The liquidation threshold is thus chosen by the MNF to strike a balance between ex-ante and ex-post efficient risk sharing. In the case that the futures exchange rates are uncorrelated or negatively autocorrelated, premature liquidation of the futures position is never ex-post optimal, thereby making the MNF prefer not to commit to any finite liquidation thresholds.

Finally, we follow Korn (2004) and Adam-Müller and Panaretou (2009) to consider an alternative setting in which the MNF is allowed to adjust its futures positions over time. The MNF, however, is cash-constrained and has to borrow at a rate over and above the riskless rate of interest to cover any interim losses that may arise from its initial futures position. Such a borrowing constraint makes hedging with the currency futures contracts costly, thereby inducing the MNF to adopt an under-hedge as its optimal initial futures position. This is in line with the findings by Korn (2004) and Adam-Müller and Panaretou (2009). Our results thus suggest that the source of liquidity risk plays a pivotal role in determining the optimal futures hedging strategies for MNFs facing exchange rate uncertainty.

[3]Positive autocorrelations in futures exchange rates may also be explained in a partial adjustment model wherein central banks lean against the wind with their interventions.

The rest of the paper is organized as follows. Section 2 describes the two-period model of Lien and Wong (2005) in which a risk-averse MNF facing both liquidity risk and exchange rate risk. Section 3 characterizes the MNF's optimal futures position for an exogenous liquidity constraint. Section 4 goes on to endogenize the choice of optimal timing for the termination of the MNF's futures hedge program. Section 5 offers some numerical examples. Section 6 examines an alternative setting in which the MNF can adjust its futures position in the interim period. The final section concludes.

2. The model

Consider the two-period model of Lien and Wong (2005). A multinational firm (MNF), which lasts for two periods with three dates (indexed by $t = 0$, 1, and 2), has an operation domiciled in a foreign country. Interest rates in both periods are known at $t = 0$ with certainty. To simplify notation, we henceforth suppress the interest factors by compounding all cash flows to their future values at $t = 2$.

To begin, the MNF expects to receive a net cash inflow, x, from its foreign operation at $t = 2$, where x is denominated in the foreign currency. While the actual amount of x is known at $t = 0$, the MNF does not know the then prevailing spot exchange rate at $t = 2$, which is denoted by e_2 and is expressed in units of the domestic currency per unit of the foreign currency. The MNF is risk averse and possesses a von Neumann-Morgenstern utility function, $u(\pi)$, defined over its domestic currency income at $t = 2$, π, with $u'(\pi) > 0$ and $u''(\pi) < 0$.[4]

To hedge its exposure to the exchange rate risk, the MNF trades infinitely divisible currency futures contracts at $t = 0$. We follow Lien and Wong (2005) to restrict the MNF to trade the currency futures contracts at $t = 0$ only. In Section 6, we consider an alternative setting in which futures positions can be adjusted at $t = 1$ through trading at that time as in Korn (2004) and Adam-Müller and Panaretou (2009).

Each of the currency futures contracts calls for delivery of the domestic currency against

[4]The risk-averse behavior of the MNF can be motivated by managerial risk aversion (Stulz, 1984), corporate taxes (Smith and Stulz, 1985), costs of financial distress (Smith and Stulz, 1985), and capital market imperfections (Froot, Scharfstein, and Stein, 1993; Stulz, 1990). See Tufano (1996) for evidence that managerial risk aversion is a rationale for corporate risk management in the gold mining industry.

the foreign currency at $t = 2$, and is marked to market at $t = 1$. Let f_t be the futures exchange rate at date t ($t = 0$, 1, and 2) expressed in units of the domestic currency per unit of the foreign currency. While the initial futures exchange rate, f_0, is predetermined at $t = 0$, the other futures exchange rates, f_1 and f_2, are regarded as positive random variables. By convergence, the futures exchange rate at $t = 2$ must be set equal to the spot exchange rate at that time. Thus, we have $f_2 = e_2$.

In contrast to Lien and Wong (2005), we model the futures exchange rate dynamics by assuming that $f_t = f_{t-1} + \varepsilon_t$ for $t = 1$ and 2, where $\varepsilon_2 = \rho\varepsilon_1 + \delta$, ρ is a scalar, and ε_1 and δ are two random variables independent of each other. To focus on the MNF's hedging motive, vis-à-vis its speculative motive, we further assume that ε_1 and δ have means of zero so that the initial futures exchange rate, f_0, is unbiased and set equal to the unconditional expected value of the random spot exchange rate at $t = 2$, e_2. The futures exchange rate dynamics as such is a first-order positive or negative autoregression, depending on whether ρ is positive or negative, respectively (see also Wong, 2008). If $\rho = 0$, the futures exchange rate dynamics becomes a random walk, which is the case considered by Lien and Wong (2005).

Let h be the number of the currency futures contracts sold (purchased if negative) by the MNF at $t = 0$. Due to marking to market at $t = 1$, the MNF enjoys a gain (or suffers a loss if negative) of $(f_0 - f_1)h$ from its futures position, h, at that time. The MNF is liquidity constrained in that it is obliged to prematurely liquidate its short futures position on which the loss incurred at $t = 1$ exceeds a predetermined threshold level, k, where $0 < k < \infty$. Otherwise, the MNF holds its futures position until $t = 2$. The MNF's random domestic currency income at $t = 2$ is therefore given by

$$\pi = \begin{cases} e_2 x + (f_0 - f_2)h & \text{if } (f_1 - f_0)h \leq k, \\ e_2 x + (f_0 - f_1)h & \text{if } (f_1 - f_0)h > k. \end{cases} \tag{1}$$

While the threshold level, k, is taken as given in the base scenario, we shall relax this assumption in Section 4 by allowing the MNF to choose the optimal values for h and k simultaneously.

Anticipating the liquidity constraint at $t = 1$, the MNF chooses its futures position, h, at $t = 0$ so as to maximize the expected utility of its random domestic currency income at

$t = 2$. In Appendix A, we have shown that the MNF optimally opts for a short futures position. The MNF's ex-ante decision problem can therefore be stated as

$$\max_{h>0} \int_{-\infty}^{k/h} \mathrm{E}_\delta\{u[(f_0+\varepsilon_1+\rho\varepsilon_1+\delta)x-(\varepsilon_1+\rho\varepsilon_1+\delta)h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1$$

$$+\int_{k/h}^{\infty} \mathrm{E}_\delta\{u[(f_0+\varepsilon_1+\rho\varepsilon_1+\delta)x-\varepsilon_1 h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1, \tag{2}$$

where we have used Eq. (1), $\mathrm{E}_\delta(\cdot)$ is the expectation operator with respect to the probability density function of δ, and $g(\varepsilon_1)$ is the probability density function of ε_1 with $g(\varepsilon_1) > 0$ for all ε_1. We say that the MNF's short futures position is an under-hedge, a full-hedge, or an over-hedge, if, and only if, $h < x$, $h = x$, or $h > x$, respectively.

Using Leibniz's rule, the first-order condition for program (2) is given by

$$-\int_{-\infty}^{k/h^*} \mathrm{E}_\delta\{u'[f_0x+(\varepsilon_1+\rho\varepsilon_1+\delta)(x-h^*)](\varepsilon_1+\rho\varepsilon_1+\delta)\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1$$

$$-\int_{k/h^*}^{\infty} \mathrm{E}_\delta\{u'[(f_0+\rho\varepsilon_1+\delta)x+\varepsilon_1(x-h^*)]\}\varepsilon_1 g(\varepsilon_1)\ \mathrm{d}\varepsilon_1$$

$$+\mathrm{E}_\delta\{u[(f_0+\delta)x+(1+\rho)xk/h^*-k]$$

$$-u[f_0x+(1+\rho)(x-h^*)k/h^*+\delta(x-h^*)]\}g(k/h^*)k/h^{*2}=0, \tag{3}$$

where h^* is the MNF's optimal short futures position. We assume that the second-order condition for program (2) is satisfied.[5]

3. Optimal futures hedging

As a benchmark, we consider first the case that the liquidity constraint does not exist, which is tantamount to setting $k = \infty$. In this benchmark case, Eq. (3) becomes

$$\int_{-\infty}^{\infty} \mathrm{E}_\delta\{u'[f_0x+(\varepsilon_1+\rho\varepsilon_1+\delta)(x-h^*)](\varepsilon_1+\rho\varepsilon_1+\delta)\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1 = 0. \tag{4}$$

[5] We numerically verify in Section 5 that this assumption is valid.

If $h^* = x$, the left-hand side of Eq. (4) reduces to

$$(1+\rho)u'(f_0x)\int_{-\infty}^{\infty}\varepsilon_1 g(\varepsilon_1)\ \mathrm{d}\varepsilon_1 = 0, \tag{5}$$

where we have used the fact that ε_1 and δ have means of zero. It follows from Eqs. (4) and (5) that $h^* = x$ is indeed the MNF's optimal futures position when $k = \infty$, thereby invoking our first proposition.

Proposition 1. *If the risk-averse MNF faces no liquidity constraints, its optimal short futures position, h^*, is a full-hedge.*

The intuition of Proposition 1 is as follows. When $k = \infty$, the liquidity unconstrained MNF's ex-ante decision problem is given by

$$\max_{h>0}\int_{-\infty}^{\infty}\mathrm{E}_\delta\{u[f_0x + (\varepsilon_1 + \rho\varepsilon_1 + \delta)(x-h)]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1. \tag{6}$$

It is evident from program (6) that a full-hedge, i.e., $h = x$, eliminates all the exchange rate risk. Thus, the liquidity unconstrained MNF, being risk averse, finds it optimal to opt for a full-hedge. Proposition 1 is analogous with the full-hedging theorem of Danthine (1978), Feder et al. (1980), and Holthausen (1979).[6]

We now resume the original case that the liquidity constraint is present, i.e., $0 < k < \infty$. Let $L(\rho)$ be the left-hand side of Eq. (3) evaluated at $h^* = x$:

$$\begin{aligned}L(\rho) = &-(1+\rho)u'(f_0x)\int_{-\infty}^{k/x}\varepsilon_1 g(\varepsilon_1)\ \mathrm{d}\varepsilon_1\\ &-\int_{k/x}^{\infty}\mathrm{E}_\delta\{u'[(f_0+\rho\varepsilon_1+\delta)x]\}\varepsilon_1 g(\varepsilon_1)\ \mathrm{d}\varepsilon_1\\ &+\Big\{\mathrm{E}_\delta\{u[(f_0+\delta)x+\rho k]\} - u(f_0x)\Big\}g(k/x)k/x^2.\end{aligned} \tag{7}$$

If $L(\rho) > (<)\ 0$, it follows from Eq. (3) and the second-order condition for program (2) that $h^* > (<)\ x$.

[6]See also Adam-Müller (1997), Broll (1992), and Broll and Zilcha (1992) for the case of risk-averse MNFs.

When there are multiple sources of uncertainty, it is well-known that the Arrow-Pratt theory of risk aversion is usually too weak to yield intuitively appealing results (Gollier, 2001). Kimball (1990, 1993) defines $u'''(\pi) \geq 0$ as prudence, which measures the propensity to prepare and forearm oneself under uncertainty, vis-à-vis risk aversion that is how much one dislikes uncertainty and would turn away from it if one could. As shown by Leland (1968), Drèze and Modigliani (1972), and Kimball (1990), prudence is both necessary and sufficient to induce precautionary saving. Moreover, prudence is implied by decreasing absolute risk aversion, which is instrumental in yielding many intuitive comparative statics under uncertainty (Gollier, 2001). The following proposition shows that prudence is indeed useful in unambiguously identifying the sign of $L(\rho)$.[7]

Proposition 2. *If the risk-averse MNF faces an exogenous liquidity constraint and is prudent, its optimal short futures position, h^*, is an under-hedge, a full-hedge, or an over-hedge, depending on whether the autocorrelation coefficient, ρ, is less than, equal to, or greater than ρ^*, respectively, where $\rho^* > 0$ uniquely solves $L(\rho^*) = 0$.*

Proof. See Appendix B. □

To see the intuition of Proposition 2, we refer to Eq. (1). If the liquidity constrained MNF adopts a full-hedge, i.e., $h = x$, its random domestic currency income at $t = 2$ becomes

$$\pi = \begin{cases} f_0 x & \text{if } \varepsilon_1 \leq k/x, \\ (f_0 + \rho\varepsilon_1 + \delta)x & \text{if } \varepsilon_1 > k/x. \end{cases} \tag{8}$$

Eq. (8) implies that a full-hedge is not optimal due to the residual exchange rate risk, $(\rho\varepsilon_1 + \delta)x$, that arises from the premature liquidation of the futures position at $t = 1$. According to Kimball (1990, 1993), the prudent MNF is more sensitive to low realizations of its random domestic currency income at $t = 2$ than to high ones. If ρ is sufficiently positive such that $\rho\varepsilon_1 + \delta > 0$ most of the time for all $\varepsilon_1 > k/x$, the low realizations of the MNF's random domestic currency income at $t = 2$ occur when the futures position is continued until $t = 2$. Thus, to avoid these realizations the prudent MNF has incentives to short

[7] It should be evident that the results of Proposition 2 remain intact when the liquidity constrained MNF has a quadratic utility function, i.e., $u'''(\pi) \equiv 0$.

more of the currency futures contracts, i.e., $h > x$, so as to reduce the interval, $(-\infty, k/h]$, over which the the futures position is continued until $t = 2$. In this case, the prudent MNF optimally opts for an over-hedge, i.e., $h^* > x$. On the other hand, if ρ is not too positive such that $\rho\varepsilon_1 + \delta < 0$ most of the time for all $\varepsilon_1 > k/x$, the low realizations of the MNF's random domestic currency income at $t = 2$ occur when the futures position is prematurely liquidated at $t = 1$. Thus, to avoid these realizations the prudent MNF has incentives to short less of the currency futures contracts, i.e., $h < x$, so as to shrink the interval, $(k/h, \infty)$, over which the premature liquidation of the futures position prevails at $t = 1$. In this case, the prudent MNF optimally adopts an under-hedge, i.e., $h^* < x$. Proposition 2 shows that the MNF's optimal futures position, h^*, is continuous in the autocorrelation coefficient, ρ, such that there is a unique critical value, $\rho^* > 0$, below (above) which under-hedging (over-hedging) is optimal.

Lien and Wong (2005) have thoroughly analyzed the random walk case of $\rho = 0$. They show that an under-hedge is optimal for prudent MNFs under exogenous liquidity constraints. Proposition 2 generalizes their results to the case of autocorrelated futures exchange rates. Specifically, if the autocorrelation of the disturbances across periods is not too positive, Proposition 2 shows that an under-hedge remains optimal for these MNFs. Otherwise, an over-hedge is called for in response to the exogenous liquidity constraints.

4. Optimal endogenous liquidation

In this section, we allow the MNF to choose a short futures position, h, and a liquidation threshold, k, simultaneously so as to maximize the expected utility of its random domestic currency income at $t = 2$. Thus, we formulate the MNF's ex-ante decision problem in an optimal stopping approach:

$$\max_{h>0,k>0} \int_{-\infty}^{k/h} \mathrm{E}_\delta\{u[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x - (\varepsilon_1 + \rho\varepsilon_1 + \delta)h]\}g(\varepsilon_1)\,\mathrm{d}\varepsilon_1$$

$$+ \int_{k/h}^{\infty} \mathrm{E}_\delta\{u[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x - \varepsilon_1 h]\}g(\varepsilon_1)\,\mathrm{d}\varepsilon_1. \tag{9}$$

Let h^{**} and k^{**} be the optimal futures position and the optimal liquidation threshold, respectively. The following proposition characterizes k^{**} for different values of the autocorrelation coefficient, ρ.

Proposition 3. *Suppose that the risk-averse MNF can choose a threshold level, k, to commit to prematurely liquidating its futures position at $t = 1$. The MNF's optimal liquidation threshold, k^{**}, is finite (infinite) if the autocorrelation coefficient, ρ, is positive (non-positive).*

Proof. See Appendix C. □

The intuition of Proposition 3 is as follows. If the MNF chooses $k = \infty$, risk sharing is optimal at $t = 0$ because the MNF can completely eliminate all the exchange rate risk. However, doing so is not optimal at $t = 1$, especially when $\rho > 0$. To see this, note that for any given $k < \infty$ the MNF prematurely liquidates its short futures position at $t = 1$ for all $\varepsilon_1 \in (k/h, \infty)$. Conditional on premature liquidation, the expected value of f_2 is equal to $f_1 + \rho\varepsilon_1$, which is greater (not greater) than f_1 when $\rho > (\leq)\ 0$. Thus, it is optimal at $t = 1$ for the MNF to liquidate its short futures position prematurely in order to limit further losses if $\rho > 0$. In this case, the MNF chooses the optimal threshold level, k^{**}, to be finite so as to strike a balance between optimal risk sharing at $t = 0$ and at $t = 1$. If $\rho \leq 0$, premature liquidation is never optimal at $t = 1$ and thus the MNF chooses $k^{**} = \infty$.

When $\rho \leq 0$, Proposition 3 implies that the MNF never liquidates its short futures position at $t = 1$. In this case, it follows from Proposition 1 that the MNF finds a full-hedge optimal. When $\rho > 0$, Proposition 3 implies that the MNF optimally liquidates its short futures position at $t = 1$ whenever the loss at that time exceeds k^{**}. From Proposition 2, we know that either an under-hedge, a full-hedge, or an over-hedge can be optimal for the MNF facing an exogenous liquidity constraint, depending on whether the actual value of ρ is below, equal to, or above the critical value, $\rho^* > 0$, respectively. However, since ρ^* is a function of k while k^{**} is a function of ρ, we cannot directly apply the results of Proposition 2 to infer the MNF's optimal futures position in this case.

To characterize h^{**} when $\rho > 0$, we formulate the MNF's ex-ante decision problem as a two-stage optimization problem. In the first stage, we derive the MNF's optimal liquidation threshold, $k(h)$, for a given short futures position, h:

$$k(h) = \arg\max_{k>0} \int_{-\infty}^{k/h} \mathrm{E}_\delta\{u[(f_0+\varepsilon_1+\rho\varepsilon_1+\delta)x-(\varepsilon_1+\rho\varepsilon_1+\delta)h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1 + \int_{k/h}^{\infty} \mathrm{E}_\delta\{u[(f_0+\varepsilon_1+\rho\varepsilon_1+\delta)x-\varepsilon_1 h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1. \tag{10}$$

Using Leibniz's rule, the first-order condition for the right-hand side of Eq. (10) is given by

$$\mathrm{E}_\delta\{u[f_0x+(1+\rho)k(h)(x/h-1)+\delta(x-h)]\} - \mathrm{E}_\delta\{u[f_0x+\rho k(h)x/h+\delta x+k(h)(x/h-1)]\} = 0. \tag{11}$$

In the second stage, we solve the MNF's optimal short futures position, h^{**}, taking the liquidation threshold, $k(h)$, that solves Eq. (11) as given. The complete solution is thus h^{**} and $k^{**} = k(h^{**})$.

The second-stage optimization problem is given by

$$\max_{h>0}\ G(h) = \int_{-\infty}^{k(h)/h} \mathrm{E}_\delta\{u[(f_0+\varepsilon_1+\rho\varepsilon_1+\delta)x-(\varepsilon_1+\rho\varepsilon_1+\delta)h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1 + \int_{k(h)/h}^{\infty} \mathrm{E}_\delta\{u[(f_0+\varepsilon_1+\rho\varepsilon_1+\delta)x-\varepsilon_1 h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1. \tag{12}$$

Differentiating $G(h)$ with respect to h and evaluating the resulting derivative at $h = x$ yields

$$G'(x) = -(1+\rho)u'(f_0x)\int_{-\infty}^{k(x)/x} \varepsilon_1 g(\varepsilon_1)\ \mathrm{d}\varepsilon_1 - \int_{k(x)/x}^{\infty} \mathrm{E}_\delta\{u'[(f_0+\rho\varepsilon_1+\delta)x]\}\varepsilon_1 g(\varepsilon_1)\ \mathrm{d}\varepsilon_1, \tag{13}$$

where we have used Eq. (11) with $h = x$. We assume that $G(h)$ is strictly concave so that $h^{**} > (<)\ x$ if $G'(x) > (<)\ 0$.[8] The following proposition offers a sufficient condition under which $G'(x) > 0$.

[8] We numerically verify in Section 5 that this assumption is valid.

Proposition 4. *Suppose that the autocorrelation coefficient, ρ, is positive and that the risk-averse MNF can choose a threshold level, k, to commit to prematurely liquidating its futures position at $t = 1$. If*

$$\rho \geq \frac{\mathrm{E}_\delta\{u'[(f_0 + \delta)x]\}}{u'(f_0 x)} - 1, \tag{14}$$

the MNF's optimal short futures position, h^{**}, is an over-hedge.

Proof. See Appendix D. □

When the MNF's utility function is quadratic, i.e., $u'''(\pi) \equiv 0$, we have $\mathrm{E}_\delta\{u'[(f_0 + \delta)x]\} = u'(f_0 x)$ since $\mathrm{E}_\delta(\delta) = 0$. In this case, the right-hand side of condition (14) vanishes and thus condition (14) is satisfied for all $\rho > 0$. When the MNF is prudent, i.e., $u'''(\pi) \geq 0$, it follows from $\mathrm{E}_\delta(\delta) = 0$ and Jensen's inequality that $\mathrm{E}_\delta\{u'[(f_0 + \delta)x]\} \geq u'(f_0 x)$ and thus the right-hand side of condition (14) is strictly positive. Since condition (14) is sufficient but not necessary for the optimality of an over-hedge, it may still very well be the case that the prudent MNF would optimally opt for an over-hedge for all $\rho > 0$.

5. Numerical examples

From the previous section, we know that we have to rely on numerical solutions to the model in order to characterize h^{**} for the prudent MNF. Doing so also allows us to quantify the optimal liquidation threshold, k^{**}.

To conduct the numerical analysis, we assume that the MNF has a negative exponential utility function: $u(\pi) = -e^{-\gamma\pi}$, where $\gamma > 0$ is the constant Arrow-Pratt measure of absolute risk aversion. We further assume that ε_1 and δ are normally distributed with means of zero and variances of 0.01. For normalization, we set $x = f_0 = 1$.

In Table 1, we set $\gamma = 2$ and report the optimal short futures position, h^*, and the critical autocorrelation coefficient, ρ^*, for different values of k and ρ. As is evident from Table 1, $h^* < (>)\ x = 1$ when $\rho < (>)\ \rho^*$, in accord with Proposition 2. Table 1 also reveals that h^* moves further away from a full-hedge as k decreases. That is, when the exogenous liquidity constraint becomes more severe, the MNF has to deviate more from full-hedging so as to better cope with the exchange rate risk and the liquidity risk simultaneously.

(Insert Table 1 here.)

Table 2 reports the optimal short futures position, h^{**}, and the optimal liquidation threshold, k^{**}, for different values of γ and ρ. Table 2 shows that a full-hedge is optimal if ρ is small, or else an over-hedge is optimal, implying that an under-hedge is never used.[9] This suggests that condition (14) in Proposition 4 is a rather weak condition. It is also evident from Table 2 that k^{**} decreases as either ρ increases or γ decreases. That is, the MNF is more willing to commit itself to prematurely liquidating its short futures position at $t = 1$ when either premature liquidation is indeed profitable at $t = 1$, or the MNF becomes less risk averse and thus does not mind to take on excessive risk.

(Insert Table 2 here.)

6. Optimal futures hedging under borrowing constraints

In this section, we follow Korn (2004) and Adam-Müller and Panaretou (2009) to consider an alternative setting in which the MNF is allowed to adjust its futures position at $t = 1$, albeit subject to a borrowing constraint. Let h_0 and h_1 be the numbers of the currency futures contracts sold (purchased if negative) by the MNF at $t = 0$ and $t = 1$, respectively, where all the contracts mature at $t = 2$. The MNF has no cash at $t = 1$ to pay any interim losses that may arise from its futures position, h_0, due to marking to market at that time. The MNF has to borrow at a rate that contains a constant mark-up, $r > 0$, over and above the riskless rate of interest. The MNF's random domestic currency income at $t = 2$ is therefore given by

$$\pi = e_2 x + (f_0 - f_2)h_0 + (f_1 - f_2)h_1 + r \min[(f_0 - f_1)h_0, 0]. \tag{15}$$

We solve the MNF's optimal futures positions by using backward induction.

Consider first that the decision problem of the MNF at $t = 1$. Taking the the futures position at $t = 0$, h_0, and the realization of the futures exchange rate at $t = 1$, f_1, as given,

[9] Indeed, Wong (2008) shows that $h^{**} \geq x$ for all $\rho > 0$ under constant absolute risk aversion.

the MNF chooses the futures position at $t = 1$, h_1, so as to maximize the expected utility of its domestic currency income at $t = 2$:

$$\max_{h_1} \mathrm{E}_\delta\{u[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x - \varepsilon_1 h_0 - (\rho\varepsilon_1 + \delta)(h_0 + h_1) + r\min(-\varepsilon_1 h_0, 0)]\}. \quad (16)$$

The first-order condition for program (16) is given by

$$\mathrm{E}_\delta\{u'[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x - \varepsilon_1 h_0 - (\rho\varepsilon_1 + \delta)(h_0 + h_1^*) + r\min(-\varepsilon_1 h_0, 0)]\}\rho\varepsilon_1$$

$$+\mathrm{Cov}_\delta\{u'[(f_0+\varepsilon_1+\rho\varepsilon_1+\delta)x-\varepsilon_1 h_0-(\rho\varepsilon_1+\delta)(h_0+h_1^*)+r\min(-\varepsilon_1 h_0, 0)], \delta\} = 0, \quad (17)$$

where h_1^* is the optimal futures position at $t = 1$, and we have used the property of the covariance operator, $\mathrm{Cov}_\delta(\cdot, \cdot)$, with respect to the probability density function of δ.[10] Since $u''(\pi) < 0$, it follows immediately from Eq. (17) that h_1^* is smaller than, equal to, or larger than $x - h_0$, depending on whether $\rho\varepsilon_1$ is positive, zero, or negative, respectively.

Anticipating that the optimal futures position at $t = 1$ is given by h_1^*, the MNF chooses h_0 at $t = 0$ so as to maximize the expected utility of its domestic currency income at $t = 2$:

$$\max_{h_0} \int_{-\infty}^{\infty} \mathrm{E}_\delta\{u[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x - \varepsilon_1 h_0$$

$$-(\rho\varepsilon_1 + \delta)(h_0 + h_1^*) + r\min(-\varepsilon_1 h_0, 0)]\}g(\varepsilon_1)\,\mathrm{d}\varepsilon_1. \quad (18)$$

Since h_1^* depends not only on h_0 but also on ε_1 when $\rho \neq 0$, program (18) can be solved analytically only in the case that $\rho = 0$. When $\rho = 0$, we have $h_1^* = x - h_0$ so that program (18) becomes

$$\max_{h_0} \int_{-\infty}^{\infty} u[f_0 x + \varepsilon_1(x - h_0) + r\min(-\varepsilon_1 h_0, 0)]g(\varepsilon_1)\,\mathrm{d}\varepsilon_1. \quad (19)$$

Differentiating the objective function in program (19) with respect to h_0, and evaluating the resulting derivative at $h_0 = x$ yields

$$\int_{-\infty}^{\infty} u'[f_0 x + r\min(-\varepsilon_1 x, 0)][-\varepsilon_1 + r\min(-\varepsilon_1, 0)]g(\varepsilon_1)\,\mathrm{d}\varepsilon_1$$

[10] For any two random variables, x and y, $\mathrm{Cov}_\delta(x, y) = \mathrm{E}_\delta(xy) - \mathrm{E}_\delta(x)\mathrm{E}_\delta(y)$.

$$= -\int_{-\infty}^{\infty} u'[f_0 x + r\min(-\varepsilon_1 x, 0)]\varepsilon_1 g(\varepsilon_1)\ \mathrm{d}\varepsilon_1$$

$$-r\int_{0}^{\infty} u'[f_0 x + r\min(-\varepsilon_1 x, 0)]\varepsilon_1 g(\varepsilon_1)\ \mathrm{d}\varepsilon_1. \tag{20}$$

The second term on the right-hand side of Eq. (20) is negative. The first term is the negative of the covariance between $u'[f_0 x + r\min(-\varepsilon_1 x, 0)]$ and ε_1, which is also negative because $u'[f_0 x + r\min(-\varepsilon_1 x, 0)]$ is weakly increasing ε_1 given that $u''(\pi) < 0$. Hence, by the strict concavity of the objective function of program (19), we conclude that the optimal futures position at $t = 0$, h_0^*, is less than x, when $\rho = 0$. This under-hedging result is consistent with the findings by Korn (2004) and Adam-Müller and Panaretou (2009) who restrict their attention to the case that $\rho = 0$.

To gain more insight into the characterization of h_0^* in the general case that $\rho \neq 0$, we have to rely on numerical analysis. As in Section 5, the MNF has the negative exponential utility function, $u(\pi) = -e^{-2\pi}$. We assume that ε_1 and δ are normally distributed with means of zero and variances of 0.01. For normalization, we set $x = f_0 = 1$. Table 3 reports h_0^* for different values of the mark-up rate, r, and the autocorrelation coefficient, ρ.

(Insert Table 3 here.)

As is evident from Table 3, the optimal futures position at $t = 0$ is always an under-hedge, i.e., $0 < h_0^* < x = 1$, for all values of ρ such that h_0^* reaches the maximum at $\rho = 0$. The intuition of these results is as follows. The borrowing constraint makes hedging with the currency futures contracts at $t = 0$ costly. To limit the potential borrowing costs to be incurred at $t = 1$, the MNF is induced to sell short a smaller number of the currency futures contracts at $t = 0$. This is in line with the findings in Table 3 that h_0^* decreases as the mark-up rate, r, increases. The MNF finds the adjustment of its futures position at $t = 1$ more profitable if the futures exchange rates are more predictable, i.e., if ρ is either more positive or more negative. Hence, the MNF optimally reduces its futures position at $t = 0$ as ρ becomes either more positive or more negative, thereby rendering h_0^* to attain a maximum at $\rho = 0$.

In the setting wherein the MNF is prohibited from adjusting its futures position at $t = 1$, we know from Proposition 2 that an over-hedge at $t = 0$ is optimal should the autocorrelation coefficient, ρ, be sufficiently positive, which is in sharp contrast to the findings in Table 3. This suggests that the source of liquidity risk plays a pivotal role in determining the optimal futures hedging strategies for MNFs facing exchange rate uncertainty.

7. Conclusion

In this paper, we have examined the optimal design of a futures hedge program for a risk-averse multinational firm (MNF) under exchange rate uncertainty. To hedge its exchange rate risk exposure, the MNF trades unbiased currency futures contracts that are marked to market and require interim cash settlement of gains and losses. As in Lien and Wong (2005), the MNF is liquidity constrained in that it is obliged to prematurely liquidate its futures position on which the interim loss incurred exceeds a threshold level (i.e., the liquidation threshold).

When the liquidation threshold is exogenously given, we have shown that the MNF optimally opts for an under-hedge (an over-hedge) should the futures exchange rates be not too (sufficiently) positively autocorrelated. When the liquidation threshold is endogenously determined, we have shown that the MNF voluntarily chooses to prematurely liquidate its short futures position only if the futures exchange rates are positively autocorrelated. Since a positive autocorrelation implies that a loss from a futures position tends to be followed by another loss from the same position, the MNF finds premature liquidation of its futures position to be ex-post optimal. Thus, the optimal liquidation threshold is chosen by the MNF to strike a balance between ex-ante and ex-post efficient risk sharing. However, in the case that the futures exchange rates are uncorrelated or negatively autocorrelated, premature liquidation of the futures position is never ex-post optimal. The MNF as such prefers not to commit to any finite liquidation thresholds.

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Appendix A

The MNF's ex-ante decision problem is to choose a futures position, h, so as to maximize the expected utility of its random domestic currency income at $t = 2$, EU:

$$\int_{-\infty}^{k/h} \mathrm{E}_\delta\{u[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x - (\varepsilon_1 + \rho\varepsilon_1 + \delta)h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1$$

$$+ \int_{k/h}^{\infty} \mathrm{E}_\delta\{u[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x - \varepsilon_1 h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1 \tag{21}$$

if $h > 0$, and

$$\int_{-\infty}^{k/h} \mathrm{E}_\delta\{u[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x - \varepsilon_1 h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1$$

$$+ \int_{k/h}^{\infty} \mathrm{E}_\delta\{u[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x - (\varepsilon_1 + \rho\varepsilon_1 + \delta)h]\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1 \tag{22}$$

if $h < 0$. In order to solve the MNF's optimal futures position, h^*, we need to know which equation, Eq. (21) or Eq. (22), contains the solution.

Consider first the case that $h > 0$. Using Leibniz's rule to partially differentiate EU as defined in Eq. (21) with respect to h and evaluating the resulting derivative at $h \to 0^+$ yields

$$\lim_{h\to 0^+} \frac{\partial EU}{\partial h} = -\int_{-\infty}^{\infty} \mathrm{E}_\delta\{u'[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x](\varepsilon_1 + \rho\varepsilon_1 + \delta)\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1, \tag{23}$$

where we have used the fact that $\lim_{h\to 0^+} g(k/h) = 0$. Since ε_1 and δ has means of zero, the right-hand side of Eq. (23) is simply the negative of the covariance between $u'[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x]$ and $\varepsilon_1 + \rho\varepsilon_1 + \delta$ with respect to the joint probability density function of ε_1 and δ. Since $u''(\pi) < 0$, we have $\lim_{h\to 0^+} \partial EU/\partial h > 0$.

Now, consider the case that $h < 0$. Using Leibniz's rule to partially differentiate EU as defined in Eq. (22) with respect to h and evaluating the resulting derivative at $h \to 0^-$ yields

$$\lim_{h\to 0^-} \frac{\partial EU}{\partial h} = -\int_{-\infty}^{\infty} \mathrm{E}_\delta\{u'[(f_0 + \varepsilon_1 + \rho\varepsilon_1 + \delta)x](\varepsilon_1 + \rho\varepsilon_1 + \delta)\}g(\varepsilon_1)\ \mathrm{d}\varepsilon_1, \tag{24}$$

where we have used the fact that $\lim_{h\to 0^-} g(k/h) = 0$. Inspection of Eqs. (23) and (24) reveals that $\lim_{h\to 0^+} \partial EU/\partial h = \lim_{h\to 0^-} \partial EU/\partial h > 0$. Since EU as defined in either Eq. (21) or Eq. (22) is strictly concave, the MNF's optimal futures position, h^*, must be a short position, i.e., $h^* > 0$.

Appendix B

Differentiating $L(\rho)$ with respect to ρ yields

$$L'(\rho) = -u'(f_0x)\int_{-\infty}^{k/x} \varepsilon_1 g(\varepsilon_1)\,\mathrm{d}\varepsilon_1 - \int_{k/x}^{\infty} \mathrm{E}_\delta\{u''[(f_0+\rho\varepsilon_1+\delta)x]\}\varepsilon_1^2 x g(\varepsilon_1)\,\mathrm{d}\varepsilon_1$$

$$+\mathrm{E}_\delta\{u'[(f_0+\delta)x+\rho k]\}g(k/x)k^2/x^2. \tag{25}$$

Since ε_1 has a mean of zero, the first term on the right-hand side of Eq. (25) is positive. The other two terms are also positive because $u'(\pi) > 0$ and $u''(\pi) < 0$. Thus, we have $L'(\rho) > 0$ for all ρ.

Using the fact that ε_1 has a mean of zero, we can write Eq. (7) as

$$L(\rho) = \int_{k/x}^{\infty} \Big\{(1+\rho)u'(f_0x) - \mathrm{E}_\delta\{u'[(f_0+\rho\varepsilon_1+\delta)x]\}\Big\}\varepsilon_1 g(\varepsilon_1)\,\mathrm{d}\varepsilon_1$$

$$+\Big\{\mathrm{E}_\delta\{u[(f_0+\delta)x+\rho k]\} - u(f_0x)\Big\}g(k/x)k/x^2. \tag{26}$$

Evaluating Eq. (26) at $\rho = 0$ yields

$$L(0) = \Big\{u'(f_0x) - \mathrm{E}_\delta\{u'[(f_0+\delta)x]\}\Big\}\int_{k/x}^{\infty} \varepsilon_1 g(\varepsilon_1)\,\mathrm{d}\varepsilon_1$$

$$+\Big\{\mathrm{E}_\delta\{u[(f_0+\delta)x]\} - u(f_0x)\Big\}g(k/x)k/x^2. \tag{27}$$

Since $u''(\pi) < 0$ and $\mathrm{E}_\delta(\delta) = 0$, Jensen's inequality implies that $\mathrm{E}_\delta\{u[(f_0+\delta)x]\} < u(f_0x)$. The second term on the right-hand side of Eq. (27) is negative. Since $u'''(\pi) \geq 0$, it follows from $\mathrm{E}_\delta(\delta) = 0$ and Jensen's inequality that $\mathrm{E}_\delta\{u'[(f_0+\delta)x]\} \geq u'(f_0x)$. The first term on the right-hand side of Eq. (27) is non-positive and thus $L(0) < 0$. Now, consider the case that ρ is sufficiently large such that $(1+\rho)u'(f_0x) > \mathrm{E}_\delta\{u'[(f_0+\rho\varepsilon_1+\delta)x]\}$ for all $\varepsilon_1 > 0$ and $u(f_0x) < \mathrm{E}_\delta\{u[(f_0+\delta)x+\rho k]\}$. Thus, for ρ sufficiently large, it follows from Eq. (26) that $L(\rho) > 0$.

Since $L(0) < 0$, $L(\rho) > 0$ for ρ sufficiently large, and $L'(\rho) > 0$, there must exist a unique point, ρ^*, that solves $L(\rho^*) = 0$ and $\rho^* > 0$. Thus, for all $\rho < (>)\ \rho^*$, we have $L(\rho) < (>)\ 0$. It then follows from Eq. (3) and the second-order condition for program (2) that $h^* < (>)\ x$ for all $\rho < (>)\ \rho^*$. This completes the proof of Proposition 2.

Appendix C

To facilitate the exposition, we fix $h = x$ and let the MNF choose k to solve the following problem:

$$\max_{k\geq 0}\ F(k) = u(f_0x)\int_{-\infty}^{k/x} g(\varepsilon_1)\,\mathrm{d}\varepsilon_1 + \int_{k/x}^{\infty} \mathrm{E}_\delta\{u[(f_0+\rho\varepsilon_1+\delta)x]\}g(\varepsilon_1)\,\mathrm{d}\varepsilon_1. \tag{28}$$

Differentiating $F(k)$ with respect to k yields

$$F'(k) = \Big\{u(f_0x) - \mathrm{E}_\delta\{u[(f_0+\delta)x+\rho k]\}\Big\}g(k/x)/x. \tag{29}$$

Denote k^* as the solution to program (28).

Consider first the case that $\rho \leq 0$. In this case, we have $u(f_0x) \geq u(f_0x+\rho k) > \mathrm{E}_\delta\{u[(f_0+\delta)x+\rho k]\}$, where the second inequality follows from $u''(\pi) < 0$, $\mathrm{E}_\delta(\delta) = 0$, and Jensen's inequality. Eq. (29) then implies that $F'(k) > 0$ for all $k > 0$ and thus $k^* = \infty$. From Proposition 1, we know that $h^* = x$ if $k = \infty$. It thus follows immediately from $k^* = \infty$ that $k^{**} = \infty$ and $h^{**} = x$ when $\rho \leq 0$.

Now consider the case that $\rho > 0$. Note that $\mathrm{E}_\delta\{u[(f_0+\delta)x+\rho k]\}$ is increasing in k since $u'(\pi) > 0$ and $\rho > 0$. When $k \to 0^+$, it follows from $u''(\pi) < 0$, $\mathrm{E}_\delta(\delta) = 0$, and Jensen's inequality that $\mathrm{E}_\delta\{u[(f_0+\delta)x+\rho k]\} < u(f_0x)$. On the other hand, when $k \to \infty$, we have $\mathrm{E}_\delta\{u[(f_0+\delta)x+\rho k]\} > u(f_0x)$. Thus, there exists a unique point, $k^* \in (0,\infty)$, such that $F'(k^*) = 0$, as is evident from Eq. (29). In this case, it must be true that $k^{**} < \infty$. To see this, suppose the contrary in that $k^{**} = \infty$. Then, from Proposition 1, we have $h^{**} = x$, which would imply $k^* = k^{**} = \infty$, a contradiction to $k^* < \infty$. Hence, we have $k^{**} < \infty$ when $\rho > 0$. This completes the proof of Proposition 3.

Appendix D

Using the fact that ε_1 has a mean of zero, we can write Eq. (13) as

$$G'(x) = \int_{k^*/x}^{\infty} \Big\{(1+\rho)u'(f_0 x) - \mathrm{E}_\delta\{u'[(f_0 + \rho\varepsilon_1 + \delta)x]\}\Big\}\varepsilon_1 g(\varepsilon_1)\ \mathrm{d}\varepsilon_1. \tag{30}$$

For all $\varepsilon_1 > k^*/x$, risk aversion implies that $\mathrm{E}_\delta\{u'[(f_0+\rho\varepsilon_1+\delta)x]\} < \mathrm{E}_\delta\{u'[(f_0+\delta)x]\}$ since $\rho > 0$. It then follows from condition (14) that $(1+\rho)u'(f_0 x) > \mathrm{E}_\delta\{u'[(f_0+\rho\varepsilon_1+\delta)x]\}$ for all $\varepsilon_1 > k^*/x$. From Eq. (30), $G'(x) > 0$ and thus $h^{**} > x$.

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Table 1
Optimal futures positions

	$k = 0.05$		$k = 0.1$		$k = 0.15$		$k = 0.2$	
	ρ^*	h^*	ρ^*	h^*	ρ^*	h^*	ρ^*	h^*
$\rho = 0.01$		0.9381		0.9337		0.9566		0.9787
$\rho = 0.03$		0.9728		0.9659		0.9801		0.9906
	0.0460	1.0000	0.0514	1.0000	0.0462	1.0000	0.0395	1.0000
$\rho = 0.07$		1.0391		1.0286		1.0287		1.0225
$\rho = 0.09$		1.0707		1.0592		1.0538		1.0383

Notes: The multinational firm has a negative exponential utility function: $u(\pi) = -e^{-2\pi}$. The underlying random variables, ε_1 and δ, are normally distributed with means of zero and variances of 0.01. Both the foreign currency cash flow, x, and the initial futures exchange rate, f_0, are normalized to unity. This table reports the optimal short futures position, h^*, and the critical autocorrelation coefficient, ρ^*, for different values of the liquidation threshold, k, and the autocorrelation coefficient, ρ.

Table 2
Optimal futures positions and liquidation thresholds

	$\gamma = 1$		$\gamma = 2$		$\gamma = 3$		$\gamma = 4$	
	h^{**}	k^{**}	h^{**}	k^{**}	h^{**}	k^{**}	h^{**}	k^{**}
$\rho = 0.01$	1.0000	0.5000	1.0000	0.9884	1.0000	1.4852	1.0000	1.4998
$\rho = 0.02$	1.0000	0.2494	1.0000	0.5000	1.0000	0.7481	1.0000	0.9901
$\rho = 0.05$	1.0726	0.1000	1.0070	0.1999	1.0000	0.2970	1.0000	0.3990
$\rho = 0.1$	1.2313	0.0473	1.0738	0.0997	1.0188	0.1485	1.0073	0.2000
$\rho = 0.2$	1.5103	0.0185	1.2385	0.0472	1.1349	0.0737	1.1031	0.0990
$\rho = 0.5$	2.1670	0.0036	1.6292	0.0121	1.4354	0.0243	1.3294	0.0357

Notes: The multinational firm has a negative exponential utility function: $u(\pi) = -e^{-\gamma\pi}$, where γ is a positive constant. The underlying random variables, ε_1 and δ, are normally distributed with means of zero and variances of 0.01. Both the foreign currency cash flow, x, and the initial futures exchange rate, f_0, are normalized to unity. This table reports the optimal short futures position, h^{**}, and the optimal liquidation threshold, k^{**}, for different values of the risk aversion coefficient, γ, and the autocorrelation coefficient, ρ.

Table 3
Optimal futures positions

	$r = 0.04$	$r = 0.06$	$r = 0.08$	$r = 0.1$
	h_0^*	h_0^*	h_0^*	h_0^*
$\rho = -0.5$	0.9408	0.9126	0.8854	0.1954
$\rho = -0.2$	0.9540	0.9321	0.9109	0.3553
$\rho = -0.1$	0.9555	0.9343	0.9137	0.3726
$\rho = 0$	0.9560	0.9350	0.9146	0.3780
$\rho = 0.1$	0.9555	0.9343	0.9137	0.3726
$\rho = 0.2$	0.9540	0.9321	0.9109	0.3553
$\rho = 0.5$	0.9408	0.9126	0.8854	0.1954

Notes: The multinational firm has a negative exponential utility function: $u(\pi) = -e^{-2\pi}$. The underlying random variables, ε_1 and δ, are normally distributed with means of zero and variances of 0.01. Both the foreign currency cash flow, x, and the initial futures exchange rate, f_0, are normalized to unity. This table reports the optimal short futures position at $t = 0$, h_1^*, for different values of the mark-up rate, r, and the autocorrelation coefficient, ρ.