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On the neutrality of debt in investment intensity

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Abstract This paper examines the interaction between investment and financing decisions of a firm using a real options approach. The firm is endowed with a perpetual option to invest in a project at any time by incurring an irreversible investment cost at that instant. The amount of the irreversible investment cost is directly related to the intensity of investment that is endogenously chosen by the firm. At the investment instant, the firm can finance the project by issuing debt and equity, albeit subject to an exogenously given credit constraint that prohibits the firm's debt-to-asset ratio from exceeding a prespecified threshold. The optimal capital structure of the firm is determined by the trade-off between interest tax-shield benefits and bankruptcy costs of debt. Irrespective of whether the exogenously given credit constraint is binding or not, we show that leverage has no impact on the firm's optimal investment intensity, thereby rendering the neutrality of debt in investment intensity. Similar to earlier work, we show that debt is not neutral to investment timing in general, and the levered firm invests earlier than the unlevered firm in particular.

Keywords Capital structure · Investment intensity · Investment timing · Real options

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1 Introduction

Does debt financing affect a firm's investment decisions? Under perfect capital market assumptions, it is well-known from the seminal work of Modigliani and Miller (1958) that corporate financing and investment decisions are independent and thus can be made separately. However, when market imperfections such as corporate taxes, bankruptcy costs, and agency conflicts are introduced, the extant literature by and large establishes a linkage between corporate financing and investment decisions such that they are no longer separable (see, e.g., Dotan and Ravid 1985; Dammon and Senbet 1988; Mauer and Triantis 1994; Childs et al. 2005; Décamps and Djembissi 2007). In this paper, we use a real options approach to examine how debt financing affects a firm's investment decisions in general and its investment intensity in particular, where optimal leverage is determined within a standard trade-off model of capital structure *à la* Leland (1994) and Goldstein et al. (2001).

Our real options model features an owner-managed firm that operates in continuous time. The firm is initially endowed with a perpetual option to invest in a project at any time by incurring an irreversible investment cost at that instant. The amount of the irreversible investment cost determines the intensity of investment, which is a choice variable of the firm. The project generates a stream of stochastic cash flows that follow a lognormal diffusion process and increase with the intensity of investment, as in Capozza and Li (1994, 2002) and Bar-Ilan and Strange (1999). The firm makes three decisions regarding the undertaking of the project: the timing, intensity, and financing of investment.

The firm's investment timing decision is characterized by a threshold (the investment trigger) such that the project is undertaken at the first instant when the cash flow from the project reaches the investment trigger from below (see, e.g., McDonald and Siegel 1986; Dixit and Pindyck 1994). The firm's investment intensity decision affects the amount of the irreversible investment cost according to a known technology that exhibits decreasing returns to scale. At the investment instant, the firm makes its financing decision by issuing debt and equity, where the debt issued is perpetual with a constant coupon payment per unit time. There is an exogenously given credit constraint that prohibits the firm's debt-to-asset ratio from exceeding a prespecified threshold, which defines the firm's debt capacity. The firm chooses the optimal coupon payment so as to trade off the interest tax-shields against the bankruptcy costs of debt. The firm also chooses the optimal time to default on the debt obligation. Upon default, shareholders get nothing and debt holders receive the liquidation value.

Within our real options model, we show that debt financing has no impact on the firm's optimal investment intensity, irrespective of whether the exogenously given credit constraint is binding or not. In other words, the firm's decision on investment intensity is completely neutral to debt financing. To understand the intuition of this seemingly surprising result, we consider a benchmark case wherein the firm is unlevered. In this benchmark case, the unlevered firm chooses the optimal investment intensity that equates the value of the firm per unit intensity of investment to the marginal cost of investment at the investment instant. This is the usual optimality condition that the marginal return on investment is equal to the marginal cost of investment at the optimum. On the other hand, the unlevered firm chooses the optimal investment trigger taking into account the opportunity cost arising from killing the investment option when the project is undertaken, which is captured by the option value multiple (see Abel et al. 1996). The optimal investment trigger as such equates the value of the firm at the investment instant to the investment cost augmented by the option value multiple. Combining these two optimality conditions implies that the optimal investment intensity is the one at which the marginal cost of investment is equal to the average cost augmented by the option value multiple.[1]

When the firm is allowed to issue debt, the exogenously given credit constraint comes into play. The binding credit constraint makes the project less valuable and thus reduces the marginal return on investment. A higher level of investment intensity, on the other hand, relaxes the credit constraint and thus lowers the marginal cost of investment. The binding credit constraint also reduces the option value multiple, thereby making the net present value of the project at the investment instant less than that when the credit constraint does not bind. Since the adjustment to the option value multiple and that to the marginal

[1] Indeed, Wong (2009) shows that this optimality condition remains intact when the investment cost becomes partially reversible.

cost of investment are exactly the same (both of them come from the shadow price of the binding credit constraint), the firm's optimal investment intensity is ultimately determined by the optimality condition that is identical to that in the benchmark case. That is, the optimal investment intensity is the one that equates the marginal cost of investment to the average cost augmented by the option value multiple, thereby rendering the neutrality of debt in investment intensity within our real options model. We further show that debt is not neutral to investment timing in general, and the levered firm invests earlier than the unlevered firm in particular. The non-neutrality of debt in investment timing is due entirely to the interest tax-shield benefits of debt, net of bankruptcy costs, which induce the levered firm to accelerate the undertaking of the project as compared to the unlevered firm.

Our real options model is a direct extension of Belhaj and Djembissi (2007) by endogenizing the intensity of investment. In the absence of the exogenously given credit constraint, it is well-known from the real options literature on capital structure (see, e.g., Goldstein et al. 2001; Strebulaev 2007; Tserlukevich 2008) that there is a scaling property in that the optimal coupon payment as well as the values of debt and equity are all linear functions of the cash flow at the investment instant, which in turn is linearly related to the irreversible investment cost. The optimal investment intensity as such depends only on the characteristics of the project, and not on the factors that determine the optimal capital structure, thereby rendering the neutrality of debt in investment intensity. The binding credit constraint destroys the scaling property.[2] On the one hand, the binding credit constraint lowers the marginal cost of investment and thus tends to raise the investment intensity. On the other hand, it also lowers the value of the levered firm at the investment instant and thus tends to reduce the investment intensity. Our contribution is to show that these two opposing tendencies exactly offset each other as far as the investment intensity is concerned. The neutrality of debt in investment intensity as such is not just an obvious consequence of the scaling property.

Sabarwal (2005), like Belhaj and Djembissi (2007) and us, establishes the non-neutrality

[2] This can be easily seen from Eq. (37) that the net present value of the project at the investment instant depends not only on the irreversible investment cost but also on the shadow price of the binding credit constraint.

of debt in investment timing. His results, however, are driven by agency conflicts and not by corporate income taxes and bankruptcy costs. Specifically, Sabarwal (2005) assumes that debt holders, taking the firm's investment trigger as given, choose the coupon payment and determine the value of debt such that they break even for an exogenously given amount of debt financing. The firm, on the other hand, chooses the investment trigger so as to maximize the ex-ante equity value, taking the coupon payment selected by debt holders as given. This gives rise to the well-known risk-shifting problem of Jensen and Meckling (1976) that shareholders have incentives to increase the riskiness of the firm by investing earlier so as to transfer wealth from debt holders to themselves. In the rational expectations equilibrium, debt holders fully anticipate the risk shifting incentive of shareholders, and the optimal investment trigger is shown to be a strictly decreasing function of the debt-to-asset ratio due to the agency conflicts. In contrast, we show numerically that the optimal investment trigger in our real options model without any agency conflicts is non-monotonically related to the debt-to-asset ratio. While the interest tax-shield benefits of debt encourage early investment, the bankruptcy costs of debt deter the investment incentive. The former effect dominates (is dominated by) the latter effect for low (high) debt-to-asset ratios, thereby rendering a U-shaped pattern of the optimal investment trigger against the debt-to-asset ratio (see also Belhaj and Djembissi 2007).

The rest of this paper is organized as follows. Section 2 delineates our continuous-time model of an owner-managed firm that has a perpetual option to invest in a project under uncertainty. The firm has to make three decisions: the timing, intensity, and financing of its investment. Section 3 derives the values of debt and equity of the levered firm at the investment instant. Section 4 examines the optimal timing and intensity of investment in the benchmark case of all-equity financing. Section 5 characterizes the optimal investment and financing decisions of the firm in the absence of any exogenous credit constraints. Section 6 imposes onto the firm an exogenously given credit constraint that prohibits the firm from having a debt-to-asset ratio above a prespecified threshold. We analytically characterize the optimal investment and financing decisions of the firm, and numerically demonstrate the significance of the credit constraint on the behavior of the firm. The final section concludes.

2 The model

Consider a risk-neutral, owner-managed firm that has monopoly access to a perpetual option to invest in a project.[3] The firm is infinitely lived and operates in continuous time, where time is indexed by $t \in [0, \infty)$. The firm is subject to a symmetric corporate income tax system with full loss-offset provisions and a constant tax rate, $\tau \in (0, 1)$. The default-free term structure is flat with a known instantaneous rate of interest, $r > 0$.

The firm makes three decisions regarding the undertaking of the project: the timing, intensity, and financing of investment. The firm's investment intensity, $q \geq 0$, affects the stream of stochastic earnings before interest and taxes (EBIT), $\{qX_t : t \geq 0\}$, generated from the project, where $X_t > 0$ is a state variable specifying the project's EBIT at time t per unit intensity of investment. The stochastic process, $\{X_t : t \geq 0\}$, is governed by the following geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \; \mathrm{d}t + \sigma X_t \; \mathrm{d}Z_t, \tag{1}$$

where $\mu < r$ and $\sigma > 0$ are constant parameters, and $\mathrm{d}Z_t$ is the increment of a standard Wiener process under the risk-neutral probability space, $(\Omega, \mathcal{F}, \mathcal{Q})$.[4] Eq. (1) implies that the growth rate of X_t is normally distributed with a mean, $\mu\Delta t$, and a variance, $\sigma^2\Delta t$, over a time interval, Δt. The initial value of the state variable, $X_0 > 0$, is known at $t = 0$.

To undertake the project at endogenously chosen time, $t \geq 0$, and intensity, $q \geq 0$, the firm has to incur an irreversible investment cost, $I(q)$, at that instant, where $I(0) \geq 0$, $I'(0) = 0$, and $I'(q) > 0$ and $I''(q) > 0$ for all $q > 0$.[5] We further assume that the elasticity of the investment cost with respect to the intensity of investment, $qI'(q)/I(q)$, is strictly increasing in q.[6] It is well known that finding the optimal time to invest in the project is tantamount to finding a threshold value, X_I, of the state variable, X_t, such that the

[3] The assumption of risk neutrality is innocuous as long as there are arbitrage-free and complete financial markets in which assets can be traded to span the state variable that determines the value of the firm.

[4] The assumption that $\mu < r$ is needed to ensure that the value of the firm is finite.

[5] We allow for $I(0) > 0$ to account for some fixed set-up costs that are required to initiate the project. The strict convexity of $I(q)$ implies that the project exhibits decreasing returns to scale.

[6] I would like to thank an anonymous referee for pointing out this technical condition on $I(q)$, which ensures a unique maximum solution to the investment intensity.

firm optimally exercises the investment option at the first instant when X_t reaches X_I from below (see, e.g., McDonald and Siegel 1986; Dixit and Pindyck 1994). We refer to X_I as the investment trigger, which is a choice variable of the firm. Let $T_I = \inf\{t \geq 0 : X_t = X_I\}$ be the (random) first passage time of the state variable, X_t, to reach the investment trigger, X_I, from below, starting off at $t = 0$.

At the investment instant, T_I, the firm can issue debt and equity to finance the investment cost, $I(q)$, albeit subject to an exogenously given credit constraint. As in Belhaj and Djembissi (2007), the firm is prohibited from having a debt-to-asset ratio that exceeds a prespecified level, $\delta \in [0, 1]$. We refer to δ as the firm's debt capacity. In the extreme case that $\delta = 0$, the firm is restricted to be all-equity financed. The debt issued by the firm is perpetual in that debt holders receive a constant coupon payment, $C \geq 0$, per unit time until default occurs, where C is a choice variable of the firm. The coupon payments to debt holders are tax-deductible so that the interest tax-shield is τC per unit time.

Shareholders have limited liability and thus the option to default on their debt obligations. The optimal policy for shareholders is to default at the first instant when the value of equity vanishes, which is equivalent to solving the default trigger, X_D, at which the value of equity vanishes as the state variable, X_t, reaches X_D the first time from above (see, e.g., Leland 1994; Goldstein et al. 2001; Morellec 2001).[7] Let $T_D = \inf\{t \geq T_I : X_t = X_D\}$ be the (random) first passage time at which the default trigger, X_D, is reached from above, starting off at the investment instant, T_I.

At the default instant, T_D, the firm is immediately liquidated and absolute priority is enforced. Following Mello and Parsons (1992) and Morellec (2001), we assume that, after default, the new owners continue to employ the asset in its current use to yield the unlevered value, $V^U(q, X_D)$:

$$V^U(q, X_D) = \mathrm{E}_Q^{X_D}\left[\int_{T_D}^{\infty} e^{-r(t-T_D)}(1-\tau)qX_t \,\mathrm{d}t\right] = (1-\tau)\left(\frac{qX_D}{r-\mu}\right), \tag{2}$$

where $\mathrm{E}_Q^{X_D}(\cdot)$ is the expectation operator with respect to the risk-neutral probability mea-

[7] This stock-based definition of default implies that it is optimal for shareholders to inject capital in the firm as long as the firm has positive economic net worth, and that the firm is insolvent on a flow basis at the default instant.

sure, $\mathcal{Q}$, conditional on X_D. The liquidation value of the firm at the default instant, T_D, is then given by $(1-b)V^U(q, X_D)$, where $b \in [0, 1]$ is a parameter gauging the severity of bankruptcy costs.[8] Since absolute priority is enforced, shareholders get nothing and debt holders receive the liquidation value upon default.

We summarize the firm's investment and financing decisions by a triple, (q, X_I, C), that specifies the investment intensity, q, the investment trigger, X_I, and the coupon payment, C. The firm chooses the triple, (q, X_I, C), so as to maximize the ex-ante value of equity prior to the debt issuance, subject to the exogenously given credit constraint. Specifically, at the investment instant, T_I, the firm issues perpetual debt to raise $D(q, X_I, C)$ from debt holders, where $D(q, X_I, C) \leq \delta I(q)$. The difference, $I(q) - D(q, X_I, C) \geq (1-\delta)I(q)$, is raised from shareholders whose claim right after the debt issuance is worth $E(q, X_I, C)$. The ex-ante value of equity is therefore given by $E(q, X_I, C) - [I(q) - D(q, X_I, C)] = V(q, X_I, C) - I(q)$, where $V(q, X_I, C) = D(q, X_I, C) + E(q, X_I, C)$ is the value of the firm at the investment instant, T_I. Hence, maximizing the ex-ante value of equity is tantamount to maximizing the net present value of the project.

3 Valuation of corporate securities

In this section, we derive the values of debt and equity at the investment instant, T_I, taking the firm's investment and financing decisions, (q, X_I, C), as given. That is, we derive $D(q, X_I, C)$ and $E(q, X_I, C)$, where $q > 0$, $X_I > X_0$, and $C \geq 0$ are all fixed.

The value of equity at the investment instant, T_I, is given by

$$E(q, X_I, C) = \mathrm{E}_{\mathcal{Q}}^{X_I}\left[\int_{T_I}^{T_D} e^{-r(t-T_I)}(1-\tau)(qX_t - C)\,\mathrm{d}t\right], \tag{3}$$

where T_D is the default instant at which the state variable, X_t, reaches the optimal default trigger, $X_D(q, C)$, from above, and $\mathrm{E}_{\mathcal{Q}}^{X_I}(\cdot)$ is the expectation operator with respect to the

[8] Even when $b = 0$, the firm will not entirely finance by issuing debt. Too much leverage risks bankruptcy with the concomitant losses of the tax deductibility of coupon payments, thereby imposing limits on the usage of debt in the absence of bankruptcy costs (see Brennan and Schwartz 1978; Leland 1994).

risk-neutral probability measure, $\mathcal{Q}$, conditional on X_I. Rewrite Eq. (3) as

$$E(q, X_I, C) = (1-\tau)\left(\frac{qX_I}{r-\mu} - \frac{C}{r}\right) + \mathrm{E}_{\mathcal{Q}}^{X_I}\left[\int_{T_D}^{\infty} e^{-r(t-T_I)}(1-\tau)(C - qX_t)\ \mathrm{d}t\right]. \tag{4}$$

Using the strong Markov property of Ito diffusions, we can write the second term on the right-hand side of Eq. (4) as

$$\mathrm{E}_{\mathcal{Q}}^{X_I}\left[e^{-r(T_D-T_I)}\right]\mathrm{E}_{\mathcal{Q}}^{X_D(q,C)}\left[\int_{T_D}^{\infty} e^{-r(t-T_D)}(1-\tau)(C - qX_t)\ \mathrm{d}t\right], \tag{5}$$

where $\mathrm{E}_{\mathcal{Q}}^{X_D(q,C)}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on $X_D(q, C)$. It is well-known (see, e.g., Karatzas and Shreve 1988; Dixit and Pindyck 1994) that

$$\mathrm{E}_{\mathcal{Q}}^{X_I}\left[e^{-r(T_D-T_I)}\right] = \left[\frac{X_D(q,C)}{X_I}\right]^{\alpha}, \tag{6}$$

if $X_I > X_D(q, C)$, where $\alpha = \mu/\sigma^2 - 1/2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2} > 0$. Substituting expression (5) and Eq. (6) into Eq. (4) yields

$$\begin{aligned} E(q, X_I, C) &= V^U(q, X_I) - (1-\tau)\frac{C}{r} \\ &\quad + \left\{(1-\tau)\frac{C}{r} - V^U[q, X_D(q,C)]\right\}\left[\frac{X_D(q,C)}{X_I}\right]^{\alpha}, \end{aligned} \tag{7}$$

where $V^U(q, X_I)$ is defined in Eq. (2) with X_D replaced by X_I and T_D replaced by T_I. The last term on the right-hand side of Eq. (7) is the value of the default option. Differentiating Eq. (7) with respect to $X_D(q, C)$ and solving the first-order condition yields the optimal default trigger:

$$X_D(q, C) = \left(\frac{r-\mu}{q}\right)\left(\frac{\alpha}{\alpha+1}\right)\frac{C}{r}. \tag{8}$$

It is evident from Eq. (8) that $0 < qX_D(q, C) < C$ if $C > 0$, i.e., the firm is insolvent on a flow basis at the default instant (see also Leland 1994; Goldstein et al. 2001; Morellec 2001). If $C = 0$, it follows from Eqs. (7) and (8) that $X_D(q, 0) = 0$ and $E(q, X_I, 0) = V^U(q, X_I)$.

The value of debt at the investment instant, T_I, is given by

$$D(q, X_I, C) = \mathrm{E}_Q^{X_I}\left\{\int_{T_I}^{T_D} e^{-r(t-T_I)}C\ \mathrm{d}t + e^{-r(T_D-T_I)}(1-b)V^U[q, X_D(q, C)]\right\}, \tag{9}$$

where $V^U[q, X_D(q, C)]$ and $X_D(q, C)$ are given by Eqs. (2) and (8), respectively. Rewrite Eq. (9) as

$$D(q, X_I, C) = \frac{C}{r} - \mathrm{E}_Q^{X_I}\left[e^{-r(T_D-T_I)}\right]\left\{\frac{C}{r} - (1-b)V^U[q, X_D(q, C)]\right\}, \tag{10}$$

where the second term on the right-hand side of Eq. (10) follows from the strong Markov property of Ito diffusions. Substituting Eq. (6) into Eq. (10) and rearranging terms yields

$$\begin{aligned} D(q, X_I, C) = {} & \frac{C}{r}\left\{1 - \tau\left[\frac{X_D(q, C)}{X_I}\right]^\alpha\right\} \\ & -\left\{(1-\tau)\frac{C}{r} - V^U[q, X_D(q, C)]\right\}\left[\frac{X_D(q, C)}{X_I}\right]^\alpha \\ & -bV^U[q, X_D(q, C)]\left[\frac{X_D(q, C)}{X_I}\right]^\alpha. \end{aligned} \tag{11}$$

The first term on the right-hand side of Eq. (11) is the value of the coupon payments net of the forgone interest tax-shield benefits due to default. The second term is the value of the default option that is given to shareholders. The last term is the value of bankruptcy costs. If $C = 0$, it follows from Eqs. (8) and (11) that $D(q, X_I, 0) = 0$.

For all $C > 0$, the value of the levered firm at the investment instant, $V(q, X_I, C)$, is given by the sum of the value of debt, $D(q, X_I, C)$, and the value of equity, $E(q, X_I, C)$. Using Eqs. (7) and (11), we have

$$\begin{aligned} V(q, X_I, C) = {} & V^U(q, X_I) + \frac{\tau C}{r}\left\{1 - \left[\frac{X_D(q, C)}{X_I}\right]^\alpha\right\} \\ & -bV^U[q, X_D(q, C)]\left[\frac{X_D(q, C)}{X_I}\right]^\alpha, \end{aligned} \tag{12}$$

where $V^U(q, X_I)$ is defined in Eq. (2) with X_D replaced by X_I and T_D replaced by T_I, and $X_D(q, C)$ is given by Eq. (8). The first term on the right-hand side of Eq. (12) is the value of the firm should it be unlevered. The second term is the value of the interest tax-shield benefits of debt. The last term is the value of bankruptcy costs.

4 Benchmark case of all-equity financing

In this section, we consider a benchmark wherein the firm is restricted to finance the project solely with equity. This is the case when the firm has zero debt capacity, i.e., $\delta = 0$. This is also the case studied by Capozza and Li (1994, 2002) and Bar-Ilan and Strange (1999).

The value of the unlevered firm at $t = 0$ is given by

$$F^U(X_0) = \max_{q>0, X_I>X_0} \mathrm{E}_{\mathcal{Q}}^{X_0}\Big\{e^{-rT_I}[V^U(q, X_I) - I(q)]\Big\}, \tag{13}$$

where T_I is the investment instant, $\mathrm{E}_{\mathcal{Q}}^{X_0}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_0, and $V^U(q, X_I)$ is defined in Eq. (2) with X_D replaced by X_I and T_D replaced by T_I. It is well-known (see, e.g., Karatzas and Shreve 1988; Dixit and Pindyck 1994) that

$$\mathrm{E}_{\mathcal{Q}}^{X_0}\Big(e^{-rT_I}\Big) = \Big(\frac{X_0}{X_I}\Big)^{\beta}, \tag{14}$$

if $X_I > X_0$, where $\beta = 1/2 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2} > 1$. Substituting Eqs. (2) and (14) into Eq. (13) yields

$$F^U(X_0) = \max_{q>0, X_I>X_0} \Big[(1-\tau)\Big(\frac{qX_I}{r-\mu}\Big) - I(q)\Big]\Big(\frac{X_0}{X_I}\Big)^{\beta}. \tag{15}$$

The first-order conditions for the optimization problem on the right-hand side of Eq. (15) are given by

$$(1-\tau)\Big(\frac{X_I^U}{r-\mu}\Big) = I'(q^U), \tag{16}$$

and

$$(1-\tau)\Big(\frac{q^U X_I^U}{r-\mu}\Big) = \Big(\frac{\beta}{\beta-1}\Big)I(q^U), \tag{17}$$

where q^U and X_I^U are the optimal investment intensity and trigger of the unlevered firm, respectively.

Solving Eqs. (16) and (17) yields our first proposition.

Proposition 1 *The unlevered firm's optimal investment intensity, q^U, is the unique solution to*

$$I'(q^U) = \left(\frac{\beta}{\beta - 1}\right)\frac{I(q^U)}{q^U}, \tag{18}$$

and the unlevered firm's optimal investment trigger, X_I^U, is given by

$$X_I^U = \left(\frac{r - \mu}{1 - \tau}\right)\left(\frac{\beta}{\beta - 1}\right)\frac{I(q^U)}{q^U}. \tag{19}$$

Proof Dividing Eq. (17) by q^U and substituting the resulting equation into Eq. (16) yields Eq. (18). The uniqueness of q^U follows from the fact that $qI'(q)/I(q)$ is strictly increasing in q. Rearranging terms of Eq. (17) yields Eq. (19). □

To see the intuition of Proposition 1, we use Eq. (2) to write Eqs. (16) and (17) as

$$\frac{V^U(q^U, X_I^U)}{q^U} = I'(q^U), \tag{20}$$

and

$$V^U(q^U, X_I^U) - I(q^U) = \left(\frac{\beta}{\beta - 1} - 1\right)I(q^U), \tag{21}$$

respectively. Eq. (20) states that the optimal investment intensity, q^U, equates the value of the unlevered firm per unit intensity of investment, $V^U(q^U, X_I^U)/q^U$, to the marginal cost of investment, $I'(q^U)$, at the investment instant. The literature on irreversible investment under uncertainty refers to the expression, $\beta/(\beta - 1) > 1$, as the option value multiple (see Abel et al. 1996). It measures the wedge between the value of the project at the investment instant, $V^U(q^U, X_I^U)$, and the investment cost, $I(q^U)$, which captures the opportunity cost arising from killing the investment option when the project is undertaken, as is evident from Eq. (21). Combining Eqs. (20) and (21) yields Eq. (18). That is, at the optimal investment intensity, q^U, the marginal cost of investment, $I'(q^U)$, is equal to the average cost, $I(q^U)/q^U$, augmented by the option value multiple, $\beta/(\beta-1)$.

5 The case of no exogenous credit constraints

In this section, we consider the case that the exogenously given credit constraint does not exist. In this case, the firm issues perpetual debt to raise $D(q, X_I, C)$ from debt holders at the investment instant, T_I. The difference, $I(q) - D(q, X_I, C)$, is raised from (paid to if negative) shareholders. The ex-ante value of equity prior to the debt issuance is therefore given by $E(q, X_I, C) - [I(q) - D(q, X_I, C)] = V(q, X_I, C) - I(q)$.

The value of the levered firm at $t = 0$ is given by

$$F(X_0) = \max_{q>0, X_I>X_0, C\geq 0} \mathrm{E}_{\mathcal{Q}}^{X_0}\Big\{e^{-rT_I}[V(q, X_I, C) - I(q)]\Big\}, \tag{22}$$

where $\mathrm{E}_{\mathcal{Q}}^{X_0}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_0, and $V(q, X_I, C)$ is defined in Eq. (12). Substituting Eqs. (2), (8), (12), and (14) into Eq. (22) yields

$$F(X_0) = \max_{q>0, X_I>X_0, C\geq 0} \Bigg\{(1-\tau)\left(\frac{qX_I}{r-\mu}\right) + \frac{\tau C}{r} - \left[\tau + b(1-\tau)\left(\frac{\alpha}{\alpha+1}\right)\right] \times\left(\frac{\alpha}{\alpha+1}\right)^{\alpha}\left(\frac{r-\mu}{qX_I}\right)^{\alpha}\left(\frac{C}{r}\right)^{\alpha+1} - I(q)\Bigg\}\left(\frac{X_0}{X_I}\right)^{\beta}. \tag{23}$$

We solve the optimization problem on the right-hand side of Eq. (23) in two steps. First, we derive the optimal coupon payment, $C(q, X_I)$, for a given pair of $q > 0$ and $X_I > X_0$. Then, we derive the optimal investment intensity and trigger, q^L and X_I^L, taking the schedule of the optimal coupon payments, $C(q, X_I)$, as given. The solution to the optimization problem on the right-hand side of Eq. (23) is therefore given by q^L, X_I^L, and $C^L = C(q^L, X_I^L)$.

For a given pair of $q > 0$ and $X_I > X_0$, the first-order condition for the optimization problem on the right-hand side of Eq. (23) is given by

$$\tau - [\tau(\alpha+1) + b(1-\tau)\alpha]\left(\frac{\alpha}{\alpha+1}\right)^{\alpha}\left(\frac{r-\mu}{qX_I}\right)^{\alpha}\left[\frac{C(qX_I)}{r}\right]^{\alpha} = 0, \tag{24}$$

where $C(qX_I)$ is the optimal coupon payment, which, from Eq. (24), is a function of the EBIT at the investment instant, qX_I. Solving Eq. (24) for $C(qX_I)$ yields

$$C(qX_I) = r\phi\left(\frac{\alpha+1}{\alpha}\right)\left(\frac{qX_I}{r-\mu}\right). \tag{25}$$

where $\phi = \{\tau/[\tau(\alpha+1) + b(1-\tau)\alpha]\}^{1/\alpha} \in (0,1)$. Substituting Eqs. (24) and (25) into the right-hand side of Eq. (23) yields

$$F(X_0) = \max_{q>0, X_I>X_0} \left\{[1-\tau(1-\phi)]\left(\frac{qX_I}{r-\mu}\right) - I(q)\right\}\left(\frac{X_0}{X_I}\right)^{\beta}. \tag{26}$$

Inspection of Eqs. (15) and (26) reveals that the effect of the optimal leverage on firm value, i.e., $F(X_0) - F^U(X_0)$, is equivalent to that of a reduction in the corporate income tax rate from τ to $\tau(1-\phi)$ on the value of the unlevered firm. The following proposition is thus an immediate consequence of Proposition 1.

Proposition 2 *If the exogenously given credit constraint does not exist, the optimal investment intensity of the levered firm, q^L, is identical to that of the unlevered firm, q^U. Furthermore, the optimal investment trigger of the levered firm, X_I^L, is given by*

$$X_I^L = \left[\frac{r-\mu}{1-\tau(1-\phi)}\right]\left(\frac{\beta}{\beta-1}\right)\frac{I(q^L)}{q^L}, \tag{27}$$

which is strictly less than that of the unlevered firm, X_I^U, the optimal coupon payment of the levered firm, C^L, is given by

$$C^L = \left[\frac{r\phi}{1-\tau(1-\phi)}\right]\left(\frac{\alpha+1}{\alpha}\right)\left(\frac{\beta}{\beta-1}\right)I(q^L), \tag{28}$$

and the optimal debt-to-asset ratio of the levered firm, δ^L, is given by

$$\delta^L = \left[\frac{\phi}{1-\tau(1-\phi)}\right]\left[\frac{\tau(\alpha+2-\tau) + b(1-\tau)(\alpha+1-\tau)}{\tau(\alpha+1) + b(1-\tau)\alpha}\right]\left(\frac{\beta}{\beta-1}\right), \tag{29}$$

where $\phi = \{\tau/[\tau(\alpha+1)+b(1-\tau)\alpha]\}^{1/\alpha} \in (0,1)$.

Proof Using Eqs. (15) and (26), we apply the results in Proposition 1 to get $q^L = q^U$ and Eq. (27). Substituting Eq. (27) into Eq. (25) yields Eq. (28). Substituting Eqs. (2), (8), (27), and (28) into Eq. (11) yields $D(q^L, X_I^L, C^L) = \delta^L I(q^L)$, where δ^L is defined in Eq. (29). □

The intuition of Proposition 2 is as follows. It is evident from Eq. (24) that the schedule of the optimal coupon payments, $C(qX_I)$, is linear in the EBIT at the investment instant, qX_I. The values of equity and debt, evaluated at $C(qX_I)$, are thus also linear in qX_I, as is evident from Eqs. (7) and (11). This is referred to as the scaling property in the real options literature on capital structure (see, e.g., Goldstein et al. 2001; Strebulaev 2007; Tserlukevich 2008). As is shown on the right-hand side of Eq. (26), the value of the interest tax-shield benefits of debt, net of bankruptcy costs, is equal to $\tau\phi qX_I/(r-\mu)$, which is tantamount to the case of a reduction in the corporate income tax rate from τ to $\tau(1-\phi)$ faced by the unlevered firm (see also Belhaj and Djembissi 2007). From Proposition 1, we know that reducing the corporate income tax rate has no effect on the unlevered firm's optimal investment intensity, q^U, but decreases its optimal investment trigger, X_I^U, thereby invoking Proposition 2.

Substituting Eq. (27) into the expression inside the curly brackets on the right-hand side of Eq. (26) yields

$$V(q^L, X_I^L, C^L) - I(q^L) = \left(\frac{\beta}{\beta-1} - 1\right) I(q^L). \tag{30}$$

Since $q^U = q^L$, it follows from Eqs. (21) and (30) that the net present value of the unlevered project at the investment instant, $T_I^U = \inf\{t \geq 0 : X_t = X_I^U\}$, is exactly equal to that of the levered project at the investment instant, $T_I^L = \inf\{t \geq 0 : X_t = X_I^L\}$. Using Eqs. (15) and (26) and Propositions 1 and 2, we have

$$F(X_0) - F^U(X_0) = \left\{\left[\frac{1-\tau(1-\phi)}{1-\tau}\right]^{\beta} - 1\right\} F^U(X_0) > 0. \tag{31}$$

That is, the value of the levered firm at $t = 0$, $F(X_0)$, exceeds that of the unlevered firm, $F^U(X_0)$, by the value of the interest tax-shield benefits of debt (net of bankruptcy costs), which is given by the right-hand side of Eq. (31). These results are the same as the findings in Belhaj and Djembissi (2007).

While debt is neutral to the intensity of investment, it is not neutral to the timing of investment, as is shown in Proposition 2.[9] Such non-neutrality of debt in investment timing is due entirely to the interest tax-shield benefits of debt, net of bankruptcy costs, which induce the levered firm to accelerate the undertaking of the project as compared to the unlevered firm. Indeed, in the absence of corporate income taxes and bankruptcy costs, i.e., $\tau = b = 0$, it follows from Eqs. (19) and (27) that $X_I^U = X_I^L$, thereby rendering the Modigliani-Miller theorem.

Unlike us, Sabarwal (2005) establishes the non-neutrality of debt in investment timing without relying on corporate income taxes and bankruptcy costs.[10] His results are driven by agency conflicts. Specifically, Sabarwal (2005) assumes that debt holders, taking the firm's investment trigger as given, choose the coupon payment and determine the value of debt such that they break even in their investment. The firm, on the other hand, chooses the investment trigger so as to maximize the ex-ante equity value, taking the coupon payment selected by debt holders as given. For an exogenously given amount of debt financing, the perpetual debt contract is correctly priced by rational expectations in equilibrium. To see how Sabarwal's (2005) assumptions affect our results, we fix the investment intensity at q^L, and suppose that debt holders naively believe that the firm would choose the investment trigger X_I^L, given by Eq. (27). Debt holders as such demand the coupon payment, C^L, given by Eq. (28), and pay the amount, $D(q^L, X_I^L, C^L) = \delta^L I(q^L)$, to the firm, where δ^L is defined in Eq. (29). The firm then chooses the investment trigger, X_I, so as to maximize

[9] It is well known (see, e.g., Sarkar 2000; Shackleton and Wojakowski 2002; Wong 2007, 2008; Thijssen 2009) that the expected time to exercise the investment option (the investment timing) is given by $\mathrm{E}_Q^{X_0}(T_I) = \ln(X_I/X_0)/(\mu - \sigma^2/2)$, whenever $\mu > \sigma^2/2$. Hence, the investment trigger and the investment timing are positively related.

[10] The real options model of Sabarwal (2005) is a straightforward extension of Pindyck (1988, 1991).

the ex-ante value of equity:

$$\max_{X_I > X_0} \; \mathrm{E}_Q^{X_0}\Big\{e^{-rT_I}\{E(q^L, X_I, C^L) - [I(q^L) - D(q^L, X_I^L, C^L)]\}\Big\}$$

$$= \max_{X_I > X_0} \; [V(q^L, X_I, C^L) - I(q^L)]\left(\frac{X_0}{X_I}\right)^{\beta}$$

$$+[D(q^L, X_I^L, C^L) - D(q^L, X_I, C^L)]\left(\frac{X_0}{X_I}\right)^{\beta}, \tag{32}$$

where we have used Eq. (14) and $V(q^L, X_I, C^L) = D(q^L, X_I, C^L) + E(q^L, X_I, C^L)$. Differentiating the right-hand side of Eq. (32) with respect to X_I, and evaluating the resulting derivative at $X_I = X_I^L$ yields

$$-\frac{\partial D(q^L, X_I, C^L)}{\partial X_I}\bigg|_{X_I = X_I^L}\left(\frac{X_0}{X_I^L}\right)^{\beta} = -\frac{\alpha}{X_I^L}\left[\frac{C^L}{r} - D(q^L, X_I^L, C^L)\right]\left(\frac{X_0}{X_I^L}\right)^{\beta} < 0, \tag{33}$$

where we have used the fact that X_I^L is the investment trigger that maximizes the value of the levered firm, i.e., the first term on the right-hand side of Eq. (32). Eq. (33) implies that the firm has incentives to lower the investment trigger from X_I^L so as to reduce the value of debt from $D(q^L, X_I^L, C^L)$. This is the well-known risk-shifting problem of Jensen and Meckling (1976) that shareholders have incentives to increase the riskiness of the firm (in our case to invest in the project earlier) so as to transfer wealth from debt holders to themselves. Debt holders are, of course, rational and fully anticipate the risk-shifting problem. In the rational expectations equilibrium, debt holders break even and the firm chooses the optimal investment trigger that is less than X_I^L.[11]

6 The case of exogenous credit constraints

In this section, we resume the original case that the exogenously given credit constraint, $D(q, X_I, C) \leq \delta I(q)$, prevails, where $\delta \in (0, 1]$ is the maximum debt-to-asset ratio. In this

[11] See Mauer and Sarkar (2005) for the complete analysis that incorporates both the agency and trade-off considerations.

case, the value of the firm at $t = 0$ is given by

$$F(X_0) = \max_{q>0, X_I > X_0, C \geq 0} \mathrm{E}_{\mathcal{Q}}^{X_0}\Big\{e^{-rT_I}[V(q, X_I, C) - I(q)]\Big\}$$
$$\text{s.t.} \qquad D(q, X_I, C) \leq \delta I(q), \tag{34}$$

where T_I is the (random) investment instant, $\mathrm{E}_{\mathcal{Q}}^{X_0}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_0, and $D(q, X_I, C)$ and $V(q, X_I, C)$ are defined in Eqs. (11) and (12), respectively.

We form the Lagrangian for the optimization problem on the right-hand side of Eq. (34):

$$\mathcal{L} = \Big\{(1-\tau)\Big(\frac{qX_I}{r-\mu}\Big) + \frac{\tau C}{r} - \Big[\tau + b(1-\tau)\Big(\frac{\alpha}{\alpha+1}\Big)\Big]$$
$$\times\Big(\frac{\alpha}{\alpha+1}\Big)^{\alpha}\Big(\frac{r-\mu}{qX_I}\Big)^{\alpha}\Big(\frac{C}{r}\Big)^{\alpha+1} - I(q)\Big\}\Big(\frac{X_0}{X_I}\Big)^{\beta} + \lambda\Big\{\delta I(q) - \frac{C}{r}$$
$$+\Big[1-(1-b)(1-\tau)\Big(\frac{\alpha}{\alpha+1}\Big)\Big]\Big(\frac{\alpha}{\alpha+1}\Big)^{\alpha}\Big(\frac{r-\mu}{qX_I}\Big)^{\alpha}\Big(\frac{C}{r}\Big)^{\alpha+1}\Big\}, \tag{35}$$

where we have substituted Eqs. (2), (8), (11), (12), and (14) into Eq. (34), and $\lambda \geq 0$ is the Lagrange multiplier to the exogenous credit constraint, $D(q, X_I, C) \leq \delta I(q)$. Let (q^L, X_I^L, C^L) be the solution to the optimization problem on the right-hand side of Eq. (34), and $\lambda^L \geq 0$ be the optimal Lagrange multiplier.

Solving the Kuhn-Tucker conditions for the Lagrangian in Eq. (35) yields the following proposition.

Proposition 3 *Irrespective of whether the exogenously given credit constraint is binding or not, the optimal investment intensity of the levered firm, q^L, is identical to that of the unlevered firm, q^U. Furthermore, the optimal investment trigger of the levered firm, X_I^L, is strictly less than that of the unlevered firm, X_I^U.*

Proof See Appendix A. □

The intuition of Proposition 3 is as follows. The Kuhn-Tucker conditions for the Lagrangian in Eq. (35) imply the following pair of optimality conditions (see Appendix A for the derivation):

$$\frac{V(q^L, X_I^L, C^L)}{q^L} - \lambda^L \delta \left(\frac{X_I^L}{X_0} \right)^\beta \frac{I(q^L)}{q^L} = \left[1 - \lambda^L \delta \left(\frac{X_I^L}{X_0} \right)^\beta \right] I'(q^L), \tag{36}$$

and

$$V(q^L, X_I^L, C^L) - I(q^L) = \left[1 - \lambda^L \delta \left(\frac{X_I^L}{X_0} \right)^\beta \right] \left(\frac{\beta}{\beta - 1} - 1 \right) I(q^L). \tag{37}$$

When $\lambda^L = 0$, the exogenously given credit constraint does not bind at the optimum so that Proposition 2 holds. Indeed, Eqs. (36) and (37) with $\lambda^L = 0$ reduce to Eqs. (20) and (21) in the benchmark case of all-equity financing. The more interesting case is the one in which the exogenously given credit constraint is binding at the optimum so that $\lambda^L > 0$. This is the case when $0 < \delta < \delta^L$, where δ^L is defined in Eq. (29). Inspection of Eqs. (36) and (37) reveals that we need to take into account the shadow price of the binding credit constraint, i.e., $\lambda^L \delta (X_I^L / X_0)^\beta > 0$. The binding credit constraint makes the project less valuable and thus reduces the marginal return on investment by an amount, $\lambda^L \delta (X_I^L/X_0)^\beta I(q^L)/q^L$, as is evident from the left-hand side of Eq. (36). On the other hand, a higher level of investment intensity relaxes the credit constraint and thus lowers the marginal cost of investment by the fraction, $\lambda^L \delta (X_I^L/X_0)^\beta$, as is shown on the right-hand side of Eq. (36). Eq. (37) implies that the binding credit constraint reduces the option value multiple by the same fraction, $\lambda^L \delta (X_I^L/X_0)^\beta$. The net present value of the project at the investment instant, $V(q^L, X_I^L, C^L) - I(q^L)$, is therefore less than that under the non-binding credit constraint, which is equal to that with all-equity financing.[12] Since the adjustment to the option value multiple and that to the marginal cost of investment are exactly the same and given by the fraction, $\lambda^L \delta (X_I^L/X_0)^\beta$, the firm's optimal investment intensity is ultimately determined by the optimality condition identical to that in the benchmark case.[13] That is, at the optimal

[12] Indeed, Belhaj and Djembissi (2007) offer a numerical example that shows a U-shaped pattern of the value of the levered firm at the investment instant against the debt capacity, δ, which is consistent with our results. See also Table 1.

[13] Of course, if the the adjustment to the option value multiple and that to the marginal cost of investment are not the same, which may be the case for some variant models of ours (see, e.g., Sarkar and Zapatero 2003; Sarkar 2008; Wong and Wu 2009), debt is no longer neutral to investment intensity.

investment intensity, q^L, the marginal cost of investment, $I'(q^L)$, is equal to the average cost, $I(q^L)/q^L$, augmented by the option value multiple, $\beta/(\beta-1)$. Proposition 3 thus extends the neutrality of debt in investment intensity, as is shown in Proposition 2, to the more general case of exogenous credit constraints within our real options model.

Using Eqs. (15), (34), and (37) and Propositions 1 and 3, we have

$$F(X_0) - F^U(X_0) = \left[\left(\frac{X_I^U}{X_I^L}\right)^\beta - 1 - \lambda^L\delta\left(\frac{X_I^U}{X_0}\right)^\beta\right]F^U(X_0), \tag{38}$$

where the right-hand side of Eq. (38) is the value of the interest tax-shield benefits of debt net of bankruptcy costs. When the exogenously given credit constraint does not bind at the optimum, i.e., $\delta \geq \delta^L$, we have $\lambda^L = 0$ so that Eq. (38) reduces to Eq. (31). In the case that $0 < \delta < \delta^L$, it is clear from the fact that the optimal choice of the unlevered firm is feasible to, but is not chosen by, the levered firm ($X_I^L < X_I^U$ from Proposition 3) so that $F(X_0) > F^U(X_0)$.

To gain more insight into how the severity of the exogenously given credit constraint affects the levered firm, we conduct the following numerical analysis with respect to different values of the debt capacity, $\delta \in [0, 1]$. We set the investment cost function, $I(q) = 10 + q^4$, the annualized riskless rate of interest, $r = 8\%$, the corporate income tax rate, $\tau = 15\%$, and the bankruptcy cost parameter, $b = 30\%$. The state variable, X_t, takes on the initial value, $X_0 = 1$, with the annualized growth rate, $\mu = 2\%$, and the annualized standard deviation, $\sigma = 30\%$. Table 1 reports our numerical results.

(Insert Table 1 here)

The first row in Table 1 gives the optimal investment intensity and investment trigger of the unlevered firm, $q^U = 2.0551$ and $X_I^U = 2.4508$, respectively. The value of the unlevered firm at $t = 0$ is $F^U(X_0) = 10.006$, and the net present value of the unlevered project at the investment instant is $V^U(q^U, X_I^U) - I(q^U) = 43.517$. When the firm is allowed to have positive debt capacity, i.e., $\delta \in (0, 1]$, the optimal investment intensity of the levered firm remains unchanged and equal to $q^L = 2.0551$. The exogenously given credit constraint is always binding at the optimum for all $\delta \in (0,1]$ such that the optimal coupon payment, C^L, and the value of the levered firm, $F(X_0)$, are both strictly increasing in δ.[14] The last row in Table 1 reports the numerical solution to the case of no exogenous credit constraints. In this unconstrained scenario, the optimal debt-to-asset ratio is $\delta^L = 1.2885$ and the value of the levered firm is the highest, $F(X_0) = 10.839$. Table 1 shows that both the optimal investment trigger, X_I^L, and the net present value of the project at the investment instant, $V(q^L, X_I^L, C^L) - I(q^L)$, exhibit U-shaped patterns against the debt capacity, δ, such that the former reaches a minimum at $\delta = 0.9$ while the latter reaches a minimum at $\delta = 0.7$. Table 1 also shows that $X_I^L < X_I^U = 2.4508$, which is consistent with the results in Propositions 2 and 3. Relaxing the severity of the exogenously given credit constraint (i.e., increasing the debt capacity, δ) does not always hasten investment by lowering the investment trigger, particularly when δ is sufficiently high. While the interest tax-shield benefits of debt encourage early investment, the bankruptcy costs of debt deter the investment incentive. The former effect on the investment trigger, X_I^L, dominates (is dominated by) the latter effect for low (high) values of δ, thereby rendering the U-shaped pattern of X_I^L against δ. These results are consistent with the findings in Belhaj and Djembissi (2007).

7 Conclusion

In this paper, we have examined the interaction between investment and financing decisions of an owner-managed firm using a real options approach. The firm is endowed with a perpetual option to invest in a project at any time by incurring an irreversible investment cost at that instant. The amount of the irreversible investment cost determines the intensity of investment with decreasing returns to scale. The project generates a stream of cash flows that is stochastic over time and increases with the intensity of investment. The firm can finance the project by issuing debt and equity, albeit subject to an exogenously given

[14] Belhaj and Djembissi (2007) use a much smaller value of the annualized standard deviation, $\sigma = 5\%$. In their numerical example, the optimal debt-to-asset ratio in the unconstrained case is $\delta^L = 0.96$ so that the exogenously given credit constraint does not bind at the optimum for all $\delta \in [0.96, 1]$.

credit constraint that prohibits the firm's debt-to-asset ratio from exceeding a prespecified threshold. The optimal capital structure of the firm is determined by the trade-off between interest tax-shield benefits and bankruptcy costs of debt.

Within our real options model, we have established the neutrality of debt in investment intensity. Irrespective of whether the exogenously given credit constraint is binding or not, we have shown that debt financing does not affect the firm's optimal investment intensity. Debt financing, however, has real effect on investment timing. Specifically, the optimal investment trigger of the levered firm is strictly smaller than that of the unlevered firm, thereby implying that the former undertakes the project earlier than the latter.

In this paper, we have employed the standard real options model in which the state variable follows a geometric Brownian motion. It is of great interest to see whether the neutrality of debt in investment intensity is robust to alternative stochastic processes. For example, Sarkar and Zapatero (2003) develop a trade-off model of capital structure with mean reverting earnings (see also Sarkar 2003). They show that the scaling property no longer holds even in the absence of any exogenous credit constraints. It is thus an interesting extension to examine how debt financing affects a firm's investment decisions in the context of Sarkar and Zapatero (2003). We leave this challenge for future research.

Appendix A

Proof of Proposition 3 The Kuhn-Tucker conditions for the Lagrangian in Eq. (35) are given by

$$\Big\{(1-\tau)\Big(\frac{q^L X_I^L}{r-\mu}\Big) - I'(q^L)q^L$$

$$+[\tau(\alpha+1)+b(1-\tau)\alpha]\Big(\frac{\alpha}{\alpha+1}\Big)^{\alpha+1}\Big(\frac{r-\mu}{q^L X_I^L}\Big)^{\alpha}\Big(\frac{C^L}{r}\Big)^{\alpha+1}\Big\}\Big(\frac{X_0}{X_I^L}\Big)^{\beta}$$

$$+\lambda^L\Big\{\delta I'(q^L)q^L - [1+\tau\alpha+b(1-\tau)\alpha]\Big(\frac{\alpha}{\alpha+1}\Big)^{\alpha+1}\Big(\frac{r-\mu}{q^L X_I^L}\Big)^{\alpha}\Big(\frac{C^L}{r}\Big)^{\alpha+1}\Big\} = 0, \quad \text{(A.1)}$$

$$\left\{(1-\tau)\left(\frac{q^L X_I^L}{r-\mu}\right)-\left(\frac{\beta}{\beta-1}\right)\left[I(q^L)-\frac{\tau C^L}{r}\right]\right.$$

$$\left.-\left(\frac{\alpha+\beta}{\alpha}\right)\left[\frac{\tau(\alpha+1)+b(1-\tau)\alpha}{\beta-1}\right]\left(\frac{\alpha}{\alpha+1}\right)^{\alpha+1}\left(\frac{r-\mu}{q^L X_I^L}\right)^{\alpha}\left(\frac{C^L}{r}\right)^{\alpha+1}\right\}\left(\frac{X_0}{X_I^L}\right)^{\beta}$$

$$+\lambda^L\left[\frac{1+\tau\alpha+b(1-\tau)\alpha}{\beta-1}\right]\left(\frac{\alpha}{\alpha+1}\right)^{\alpha+1}\left(\frac{r-\mu}{q^L X_I^L}\right)^{\alpha}\left(\frac{C^L}{r}\right)^{\alpha+1}=0, \tag{A.2}$$

$$\left\{\tau-[\tau(\alpha+1)+b(1-\tau)\alpha]\left(\frac{\alpha}{\alpha+1}\right)^{\alpha}\left(\frac{r-\mu}{q^L X_I^L}\right)^{\alpha}\left(\frac{C^L}{r}\right)^{\alpha}\right\}\left(\frac{X_0}{X_I^L}\right)^{\beta}$$

$$-\lambda^L\left\{1-[1+\tau\alpha+b(1-\tau)\alpha]\left(\frac{\alpha}{\alpha+1}\right)^{\alpha}\left(\frac{r-\mu}{q^L X_I^L}\right)^{\alpha}\left(\frac{C^L}{r}\right)^{\alpha}\right\}=0, \tag{A.3}$$

and

$$\lambda^L\left\{\delta I(q^L)-\frac{C^L}{r}+\left[\frac{1+\tau\alpha+b(1-\tau)\alpha}{\alpha+1}\right]\left(\frac{\alpha}{\alpha+1}\right)^{\alpha}\left(\frac{r-\mu}{q^L X_I^L}\right)^{\alpha}\left(\frac{C^L}{r}\right)^{\alpha+1}\right\}=0, \tag{A.4}$$

where (q^L, X_I^L, C^L) is the solution to the optimization problem on the right-hand side of Eq. (34), and $\lambda^L \geq 0$ is the optimal Lagrange multiplier. If $\lambda^L > 0$, the exogenously given credit constraint must be binding, as is evident from Eq. (A.4).

Multiplying C^L/r to Eq. (A.3) and adding the resulting equation to Eq. (A.1) yields

$$[V(q^L, X_I^L, C^L)-I'(q^L)q^L]\left(\frac{X_0}{X_I^L}\right)^{\beta}+\lambda^L\left\{\delta I'(q^L)q^L-\frac{C^L}{r}\right.$$

$$\left.+\left[\frac{1+\tau\alpha+b(1-\tau)\alpha}{\alpha+1}\right]\left(\frac{\alpha}{\alpha+1}\right)^{\alpha}\left(\frac{r-\mu}{q^L X_I^L}\right)^{\alpha}\left(\frac{C^L}{r}\right)^{\alpha+1}\right\}=0, \tag{A.5}$$

where we have used Eq. (35) for $V(q^L, X_I^L, C^L)$. Substituting Eq. (A.4) into Eq. (A.5) yields Eq. (36). Multiplying $C^L/r(\beta-1)$ to Eq. (A.3) and subtracting the resulting equation from Eq. (A.2) yields

$$\left[V(q^L, X_I^L, C^L)-\left(\frac{\beta}{\beta-1}\right)I(q^L)\right]\left(\frac{X_0}{X_I^L}\right)^{\beta}$$

$$+\frac{\lambda^L}{\beta-1}\left\{\frac{C^L}{r}-\left[\frac{1+\tau\alpha+b(1-\tau)\alpha}{\alpha+1}\right]\left(\frac{\alpha}{\alpha+1}\right)^{\alpha}\left(\frac{r-\mu}{q^L X_I^L}\right)^{\alpha}\left(\frac{C^L}{r}\right)^{\alpha+1}\right\}=0, \tag{A.6}$$

where we have used Eq. (35) for $V(q^L, X_I^L, C^L)$. Substituting Eq. (A.4) into Eq. (A.6) yields Eq. (37). Subtracting Eq. (36) from Eq. (37) yields

$$\left[1-\lambda^L\delta\left(\frac{X_I^L}{X_0}\right)^\beta\right]\left[I'(q^L)-\left(\frac{\beta}{\beta-1}\right)\frac{I(q^L)}{q^L}\right]=0. \tag{A.7}$$

Define the following coupon payment:

$$\hat{C}=r\left(\frac{\alpha+1}{\alpha}\right)\left[\frac{\tau}{\tau(\alpha+1)+b(1-\tau)\alpha}\right]^{1/\alpha}\left(\frac{q^LX_I^L}{r-\mu}\right). \tag{A.8}$$

If we evaluate the left-hand side of Eq. (A.3) at $C^L=\hat{C}$, the left-hand side of Eq. (A.3) becomes

$$-\frac{\lambda^L(1-\tau)[\tau+b(1-\tau)]\alpha}{\tau(\alpha+1)+b(1-\tau)\alpha}\le 0, \tag{A.9}$$

where the equality holds only when $\lambda^L=0$, i.e., only when the credit constraint is not binding. By the concavity of the optimization problem on the right-hand side of Eq. (34), Eqs. (A.3) and (A.9) imply that $C^L=\hat{C}$ if $\lambda^L=0$ and $C^L<\hat{C}$ if $\lambda^L>0$. Hence, if $\lambda^L>0$, it follows from either Eq. (A.3) or Eq. (A.8) that

$$1-[1+\tau\alpha+b(1-\tau)\alpha]\left(\frac{\alpha}{\alpha+1}\right)^\alpha\left(\frac{r-\mu}{q^LX_I^L}\right)^\alpha\left(\frac{C^L}{r}\right)^\alpha>0. \tag{A.10}$$

Eq. (A.3) implies that

$$\left[\tau\left(\frac{X_0}{X_I^L}\right)^\beta-\lambda^L\right]\left\{1-[1+\tau\alpha+b(1-\tau)\alpha]\left(\frac{\alpha}{\alpha+1}\right)^\alpha\left(\frac{r-\mu}{q^LX_I^L}\right)^\alpha\left(\frac{C^L}{r}\right)^\alpha\right\}>0. \tag{A.11}$$

It follows from Eqs. (A.10) and (A.11) that $\lambda^L<\tau(X_0/X_I^L)^\beta$ if $\lambda^L>0$. Since $0\le\lambda^L<\tau(X_0/X_I^L)^\beta$, we have $\lambda^L\delta<(X_0/X_I^L)^\beta$. Eq. (A.7) as such reduces to

$$I'(q^L)=\left(\frac{\beta}{\beta-1}\right)\frac{I(q^L)}{q^L}. \tag{A.12}$$

It then follows from Eqs. (18) and (A.12) that $q^L=q^U$.

Multiplying C^L/r to Eq. (A.3) and adding the resulting Substituting Eqs. (A.3) and (A.4) into Eq. (A.2) yields

$$\left[(1-\tau)\left(\frac{q^LX_I^L}{r-\mu}\right)-\left(\frac{\beta}{\beta-1}\right)I(q^L)+\left(\frac{\alpha}{\alpha+1}\right)\frac{\tau C^L}{r}\right]\left(\frac{X_0}{X_I^L}\right)^\beta$$

$$+\lambda^L\left[\left(\frac{\beta}{\beta-1}\right)\delta I(q^L)-\left(\frac{\alpha}{\alpha+1}\right)\frac{C^L}{r}\right]=0. \tag{A.13}$$

If $\lambda^L > 0$, Eqs. (A.4) and (A.10) imply that

$$\delta I(q^L)-\left(\frac{\alpha}{\alpha+1}\right)\frac{C^L}{r}>0. \tag{A.14}$$

Eq. (A.13) implies that

$$\begin{aligned} X_I^L &\leq \left(\frac{r-\mu}{1-\tau}\right)\left[\left(\frac{\beta}{\beta-1}\right)\frac{I(q^L)}{q^L}-\left(\frac{\alpha}{\alpha+1}\right)\frac{\tau C^L}{rq^L}\right] \\ &< \left(\frac{r-\mu}{1-\tau}\right)\left(\frac{\beta}{\beta-1}\right)\frac{I(q^U)}{q^U}=X_I^U, \end{aligned} \tag{A.15}$$

where the first inequality follows from the fact that $\lambda^L \geq 0$ and Eq. (A.14) holds if $\lambda^L > 0$, the second inequality follows from Eq. (A.3) that $C^L > 0$ and $q^L = q^U$, and the equality follows from Eq. (19). □

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Table 1 Behavior of the firm facing the exogenously given credit constraint for different values of the debt capacity

Debt capacity	Investment intensity	Investment trigger	Default trigger	Coupon payment	Project NPV	Firm value
0	2.0551	2.4508	0	0	43.517	10.006
0.1	2.0551	2.4150	0.0426	0.2247	42.885	10.102
0.2	2.0551	2.3814	0.0862	0.4540	42.304	10.197
0.3	2.0551	2.3506	0.1308	0.6889	41.784	10.289
0.4	2.0551	2.3231	0.1766	0.9301	41.342	10.378
0.5	2.0551	2.2995	0.2238	1.1786	40.987	10.463
0.6	2.0551	2.2803	0.2725	1.4353	40.738	10.543
0.7	2.0551	2.2665	0.3229	1.7010	40.612	10.616
0.8	2.0551	2.2587	0.3752	1.9766	40.631	10.681
0.9	2.0551	2.2578	0.4296	2.2629	40.817	10.737
1	2.0551	2.2647	0.4861	2.5604	41.192	10.782
1.2885	2.0551	2.3342	0.6609	3.4816	43.517	10.839

The risk-neutral owner-managed firm has an option to invest in a project. The firm's investment decisions are characterized by the investment trigger, X_I, at which the investment option is exercised, and by the investment intensity, q, according to the investment cost function, $I(q) = 10 + q^4$. At the investment instant, the firm has to choose the coupon payment, C, to raise the amount, $D(q, X_I, C)$, from debt holders subject to the exogenously given credit constraint, $D(q, X_I, C) \leq \delta I(q)$, where $\delta \in [0, 1]$ is the debt capacity (i.e., the maximum debt-to-asset ratio). The net present value of the project is $V(q, X_I, C) - I(q)$ at the investment instant. The value of the firm is equal to the value of the investment option at $t = 0$. The parameter values are as follows: the riskless rate of interest, r, is 8%; the corporate income tax rate, τ, is 15%; the bankruptcy cost parameter, b, is 30%; and the state variable, X_t, takes on the initial value, $X_0 = 1$, with the annualized growth rate, $\mu = 2\%$, and the annualized standard deviation, $\sigma = 30\%$.