PERFORMANCES OF TURBO CODES WITH FADING COMPENSATION IN MULTIPATH CHANNELS

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Abstract – This paper studies the performance of turbo-coded system in the frequency non-selective correlated Rayleigh fading channels. The turbo-coded system employs a pilot-symbol-aided (PSA) technique for fading compensation and interleaving for spreading the error bursts to reduce the error rate. The PSA technique is also used to provide the decoder with channel side information (CSI) for better performance in fading channel. Two PSA techniques, three normalized Doppler spreads and different interleaving sizes have been investigated. Results of computer simulations have shown that, an improved first-order fading prediction technique can improve the BER performance by a factor of about 62, relative to those obtained using the pilot symbols only. Results have also shown that an interleaving depth equal to one-quarter the reciprocal of normalized Doppler spread can achieve a good BER performance.

I. INTRODUCTION

Turbo codes have been shown to be close to the Shannon's capacity limit in the additive white Gaussian noise (AWGN) channels [1]. In order to apply this powerful coding technique in wireless channels, more works have to be carried out. Some researchers have already investigated the applications of turbo codes over the fading channels [2-5]. However, in these works, one of the important assumptions made is perfect phase tracking of the fading process. This assumption is not realistic because, in the fading environments both phase and amplitude distortions are introduced into the signals. Thus the signals will suffer significant degradation in the error-rate performances.

There are different ways to counter fading. This paper proposes to use pilot-symbol-aided (PSA) transmission to counter fading. This is because the PSA process can easily be incorporated with the turbo coding process. In a turbo-coded system, the transmitted data-symbol sequence is turbo coded into frames of data symbols and so the pilot symbols can easily be inserted into these frames without any addition framing process [6]. At the receiver, the fading effects of the data symbols can be estimated and corrected before feeding into the turbo decoder for decoding.

In section 2, the system model used in the computer simulations is introduced. Section 3 describes two PSA techniques for fading compensation and CSI extraction. Discussion of simulation results is given in section 4. Finally, section 5 summarizes our investigation.

II. SYSTEM MODEL

The turbo-coded system studied here is shown in Figure 1. The turbo encoder is half rated and has the generator matrix $(1,5/7)_8$ and a block size of 1000. The turbo encoder encodes the binary data $\{u_k\}$ (where k is the integer symbol index), into the sequence $\{x_k^s, x_k^p\}$, where x_k^s, $x_k^p \in \{-1, 1\}$ and the sequences $\{x_k^s\}$ and $\{x_k^p\}$ represent the systematic sequence and parity-check sequences respectively. The coded symbols are fed to a block interleaver with a buffer of size M rows (depth) by N columns (span) to produce $\{d_k\}$. The encoded symbols are written in successive M rows and transmitted over the channel in N columns. For every (L-1)-interleaved symbols, a known pilot symbol p_k is inserted to form an L-symbol frame as shown in Figure 2. The resultant sequence $\{s_k\}$ are filtered by a premodulation filter and then QPSK modulated. QPSK is used here because it can increase the bandwidth efficiency without increasing the complexity of system, relative to BPSK used in [4,5]. However, in QPSK,

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because the systematic information is transmitted together with parity information in the same QPSK symbol(s), they are not corrupted by independent distortion; hence it can be expected that the error rate performance of QPSK is worse. The QPSK signal is then subjected to a time-varying frequency-nonselective Rayleigh fading channel where the signal s_k is multiplied by a Rayleigh variable $c_k = A_k + j B_k = |c_k| e^{j\phi}$. The A_k and B_k are modeled as a statistically independent, real-valued and zero-mean Gaussian low-pass process whose bandwidth equals the Doppler shift. Without loss of generality we normalize the power of $\{c_k\}$ to unity. The AWGN process's variables n_k's are added to the faded symbol, where n_k's are independent and identically distributed zero-mean complex Gaussian random variables with variance σ_n^2.

At the receiver, the received sequence $\{r_k'\}$ is filtered by a postdemodulation filter which is taken to have the same characteristics as the premodulation at the transmitter. The resultant frequency response of the premodulation filter and postdemodulation filter in cascaded has a raised cosine roll-off of 100%. Assume that perfect carrier and clock recovery has been achieved. Thus, the signal at time $t = kT$ sec can be written as

$$r_k = c_k s_k + n_k \tag{1}$$

where c_k is the effects of fading introduced in the fading channel. By using the pilot symbols, the fading effects can be estimated and then used to compensate the fading effects on the coded symbols within the frame. Two prediction techniques are described in the following section. In Fig. 1, after fading compensation, the received coded symbols are deinterleaved to produce $\{y_k^s, y_k^p\}$, where $\{y_k^s\}$ and $\{y_k^p\}$ are the systematic and parity-check sequences fed to the turbo decoder implemented using the log-MAP algorithm. Eight iterations are used in the algorithm to obtain the binary data $\{\hat{u}_k\}$ in the studies. In the study, it has been assumed that the receiver has a perfect knowledge of the variance σ_n^2.

III. PILOT-SYMBOL-AIDED TECHNIQUES

First-Order Prediction [6]

The sample signal in the i-th position of the k-th received pilot frame can be written as

$$r_{k,i} = c_{k,i} s_{k,i} + n_{k,i} \tag{2}$$

where $s_{k,i}$ is either a pilot symbol or a data symbol, $c_{k,i}$ is the change in the i-th symbol of the k-frame, $n_{k,i}$ is the noise sample in the i-th symbol of the k-frame.

For $i = 0$, the sample signal is

$$r_{k,0} = p_{k,0} c_{k,0} + n_{k,0} \tag{3}$$

where $p_{k,0}$ is the pilot symbol in the k-th frame.

Since the pilot symbol $p_{k,0}$ is known at the receiver, $c_{k,0}$ can be obtained using Eqn. (3) as

$$c_{k,0} = \frac{r_{k,0}}{p_{k,0}} - \frac{n_{k,0}}{p_{k,0}} \tag{4}$$

The estimates of the fading effects for the rest of the data symbols $\{\hat{c}_{k,i}\}$ can be obtained by first-order prediction, i.e.

$$\hat{c}_{k,i} = \left(1 - \frac{i}{L}\right)\hat{c}_{k,0} + \left(\frac{i}{L}\right)\hat{c}_{k+1,0} \tag{5}$$

for $i = 1, 2, ..., L-1$.

The estimated signal $\hat{c}_{k,i}$ is used to correct the fading effects in the received signal sample, $r_{k,i}$.

Improved First-Order Prediction [6]

For the frame lengths of 4, 8, 16, ..., 2^n, a better BER performance can be obtained by the proposed first-order prediction using the pilot and data symbols. The changes on the pilot symbols in the k-th frame, $\hat{c}_{k,0}$, and in the k+1-th frame, $\hat{c}_{k+1,0}$, can be estimated from $r_{k,0}, p_{k,0}$, $r_{k+1,0}$, and $p_{k+1,0}$, using Eqn. (4). The improved estimate of $c_{k,L/2}$ in the L/2-th symbol of the k-th frame is given by

$$\hat{c}_{k,L/2} = \frac{\hat{c}_{k,0} + \hat{c}_{k+1,0}}{2} \tag{6a}$$

which is then used to correct the fading effect in $r_{k,L/2}$ to give an estimate of the data signal

$$\hat{r}_{k,L/2} = \frac{r_{k,L/2}}{\hat{c}_{k,L/2}} \tag{6b}$$

The signal $\hat{r}_{k,L/2}$ is fed to the threshold detector to produce the data signal $\hat{d}_{k,L/2}$ which is used as a known symbol to correct the data symbols $r_{k,L/4}$ and $r_{k,3L/4}$. Since $\hat{d}_{k,L/2}$ is a possible signal vector on the constellation and $r_{k,L/2}/\hat{d}_{k,L/2}$ is closer to $c_{k,L/4}$ and $c_{k,3L/4}$ in a slowly-faded channel, the better estimates of $c_{k,L/4}$ and $c_{k,3L/4}$ can be obtained as, respectively,

$$\tilde{c}_{k,L/4} = \frac{1}{2}\left(\hat{c}_{k,0} + \frac{r_{k,L/2}}{\hat{d}_{k,L/2}}\right) \tag{7a}$$

$$\tilde{c}_{k,3L/4} = \frac{1}{2}\left(\frac{r_{k,L/2}}{\hat{d}_{k,L/2}} + \hat{c}_{k+1,0}\right) \tag{7b}$$

These signals $\tilde{c}_{k,L/4}$ and $\tilde{c}_{k,3L/4}$ are used to correct $r_{k,L/4}$ and $r_{k,3L/4}$, respectively, to obtain the signals

$$\hat{r}_{k,L/4} = \frac{r_{k,L/4}}{\tilde{c}_{k,L/4}} \tag{8c}$$

$$\hat{r}_{k,3L/4} = \frac{r_{k,3L/4}}{\tilde{c}_{k,3L/4}} \tag{8d}$$

The process continues until all the coded symbols are done.

The improved first-order prediction makes use of the pilot and data symbols for fading compensation and so has a better accuracy. However it has the constraint that the frame length must be 4, 8, 16 ... or 2^n, where n is an integer. Here, a modification is used to remove this constraint as follows. The middle position of data symbols can be re-estimated using the neighbor pilot symbols as described previously. If the number of data symbols is odd, only one data symbol is required to estimate. Otherwise, the two data symbols in the middle positions are re-estimated. Then, the data frame is equally subdivided (e.g. 1000 data symbols divided into two 500 data symbols) and the newly estimated data symbols are used as the new references. The middle positions of divided frames are estimated again. The process is repeated until all data symbols are done.

It has been shown that the modification of the channel reliability $L_c = (2/\sigma_n^2)|c_k|$ in the BCJR-MAP algorithm, where $|c_k| = \sqrt{A_k^2 + B_k^2}$ is known as CSI, could greatly improve the BER performance [2]. Here. CSI for each symbol is used in the decoding process in this study.

IV. RESULTS AND DISCUSSIONS

Intensive Monte Carlo simulations have been carried out to evaluate the BER performance of the system of Figure 1. The signal-to-noise ratio is taken as

$$SNR = 10\left[\log\left(\frac{E_b}{N_0}\right)\right] dB \tag{9}$$

where E_b is the average transmitted energy per bit (after taking into account the pilot symbols) at the transmitter output of Figure 1 and N_0 is the one-sided power spectral density of the additive white Gaussian noise measured at the same point.

The BER performances of the turbo coded signal, at SNR = 15 dB using $50(\text{depth}) \times 40(\text{span})$ block interleaver, with different pilot frame lengths L and normalized Doppler spreads of $f_dT = 0.01$, 0.005 and 0.001 are shown in Fig. 3. It can be seen that with $f_dT = 0.001$, the performances are about the same with L = 10 to 100. However, with $f_dT = 0.005$ and 0.01, degradation occurs from at L = 20 and 50, respectively. This is because in fast fading, the correction of fading has to be made more often than in slow fading.

To assess the effective of the two prediction techniques, we define an improvement factor as follows:

$$\text{Improvement factor} = \frac{\text{BER of system using pilot symbols only}}{\text{BER of system using pilot and data symbols}} \tag{10}$$

The improvement factors with $f_dT = 0.01$, 0.005 and 0.001, for different pilot frame lengths using 50×40 block interleaver, are shown in Figs. 4a, 4b and 4c, respectively. At low SNRs, none of the pilot symbol-aided technique is recommended because the presence of Gaussian noise reduces the accuracy of the estimates $\tilde{c}_{k,i}, \hat{c}_{k,i}, \hat{r}_{k,i}$. With $f_dT = 0.01$, a SNR of 15dB and a pilot frame length L of 11, the BER performance can be improved by a factor of about 62. Whereas with $f_dT = 0.005$, a SNR of 15 dB and a pilot frame length L of 11, the improved factor reduces to about 32. With $f_dT = 0.001$, a SNR at 20dB and a frame length of 101, the improvement factor reduces to 1.6. Figure 4 shows that, if the pilot frame length L is too long, the improvement factor is less than 1 which means that the improved first-order prediction is worse than the first-order prediction.

Ideally, the depth of block interleaver should be infinite to achieve independent fading. However, in a practical system, the interleaving depth M is finite, this implies that, the received symbols remain correlated after deinterleaving. Paul and Dominic [7] proved that an interleaving depth equivalent to one-fifth to one-quarter the duration of a fade cycle (defined as the reciprocal of the maximum f_dT) is almost as good as full interleaving using Trellis Coded PSK modulations. On the other hand, the interleaving span is related to the memory of encoder. Thus, a larger span is required if the memory size increases. In this paper, we try to find out the optimum depth and span by computer simulations.

Figures 5 and 6 show the BER performances using simple first-order interpolation prediction with $L = 11$ with $f_dT = 0.005$. Figure 5 shows that an increase of interleaving depth from 25 to 50 has about 1.5 dB improvement at BER = 1E-3. An increase of interleaving depth from 50 to 100 has no significant on the BER performance. Figure 6 shows that improvements can be obtained by increasing the interleaving span from 20 to 100.

V. CONCLUSION

The performances of a turbo-coded signal incorporated with PSA techniques and interleaving have been studied using computer simulations. Results of computer simulation have shown that, at high SNRs, and in the fast faded channels, the improved first-order prediction has substantial improvements in BER performance. The BER performance can be improved by a factor about 62 by using improved first-order prediction. Results have also shown that an interleaving depth equal to one-quarter the reciprocal of normalized Doppler spread should be enough to produce a good BER performance.

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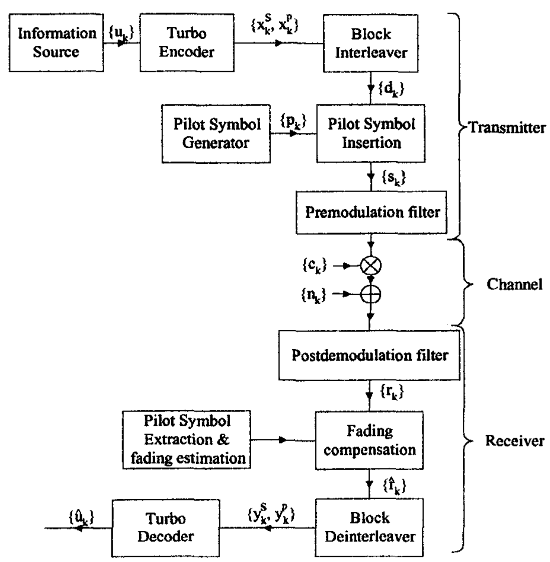

Figure 1: System model

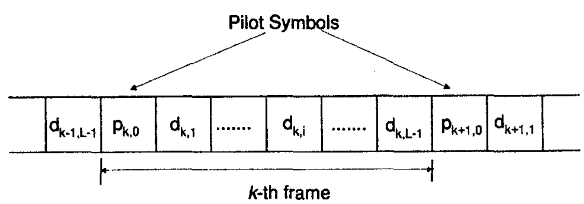

Figure 2: Frame structure of transmitted symbol sequence

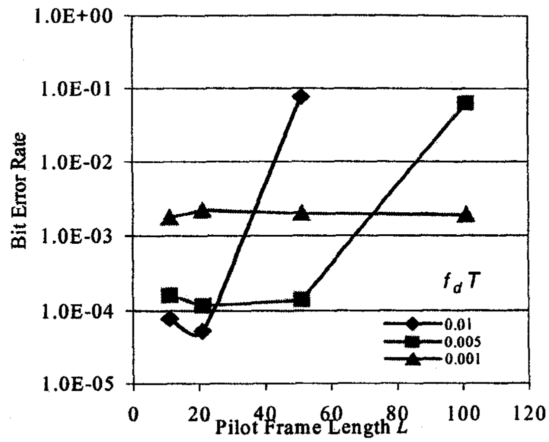

Figure 3: BER performance with different frame lengths L and normalized Doppler spreads at SNR = 15dB.

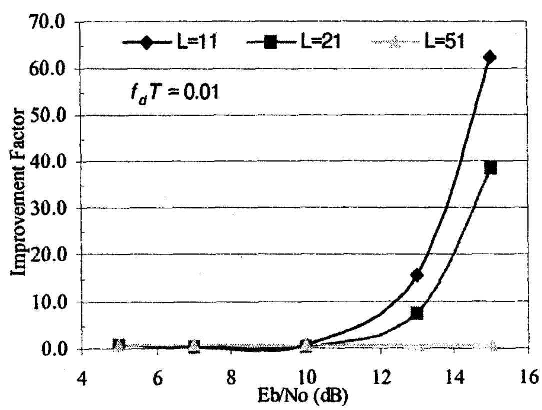

Figure 4a

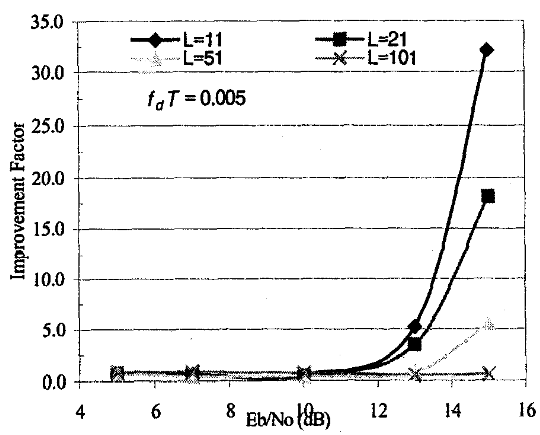

Figure 4b

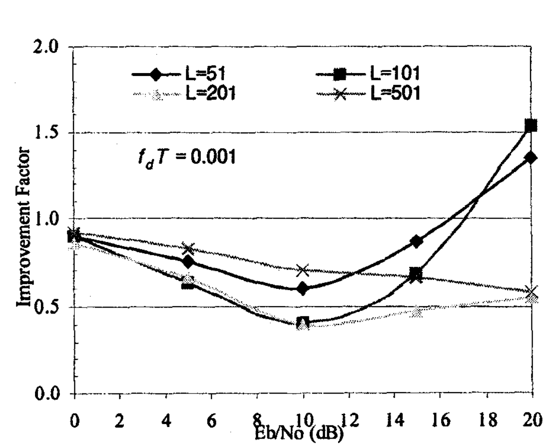

Figure 4c

Figure 4: Improvement Factor with a) $f_dT = 0.01$, b) $f_dT = 0.005$ and c) $f_dT = 0.001$

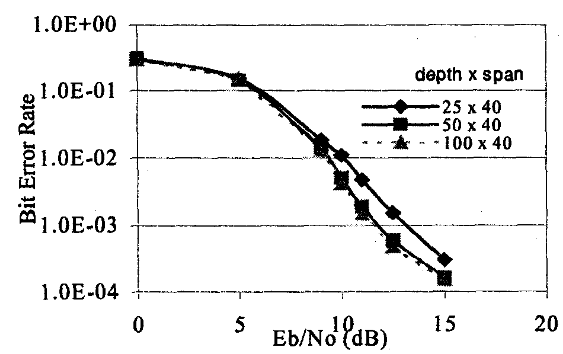

Figure 5: BER performance with improved first-order prediction and different block interleaver sizes $M\times40$

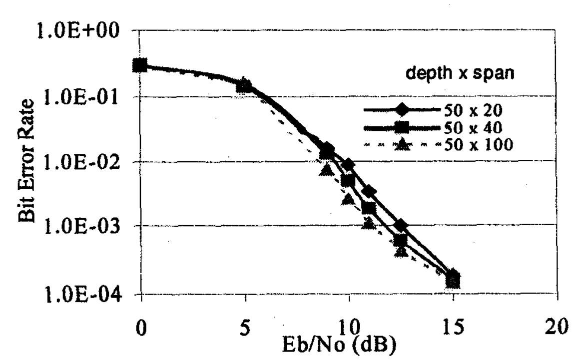

Figure 6: BER performance with improved first-order prediction and block interleaver size of 50 $\times N$