Novel Search Algorithms for Closed-Loop Transmit Diversity System with Limited Number of Feedback Bits

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Abstract:
We propose two simple closed-loop transmit diversity schemes with a fixed number of feedback bits being allocated for each transmit antenna. Under a low mobility condition and with perfect channel knowledge at the receiver, we show that our proposed schemes can achieve signal-to-noise ratio (*SNR)* values close to the optimal solution but with greatly reduced complexity. We also show that our methods outperform other suboptimal schemes such as the co-phasing method.

I. Introduction:
Antenna diversity is a key technology to improve the capacity and enhance the service quality of the wireless transmission systems. It could be applied to both uplink (Receive Diversity) and downlink (Transmit Diversity) transmissions. However, Receive Diversity is not feasible since most of the remote units only have single receive antenna. Also, due to the cost, size and power limitations, Receive Diversity with multiple antennas at the receiving end is generally not a desirable arrangement Thus, the multiple antenna technique is usually applied at the base station. When an additional antenna is introduced to the system, there is a general improvement in the reception quality of the remote mobile receiver within the system coverage area.

The transmit diversity scheme has already been included in the 3G wireless standard. The transmit diversity scheme in W-CDMA [1] could be classified into 2 types, namely open-loop and closed-loop. Open-loop scheme does not provide any feedback information to the transmit antennas while the closed-loop scheme does. Space-Time Transmit Diversity (STTD) using the technique described in [2] is an open-loop transmit diversity scheme being adopted by the Third Generation Partnership Project (3GPP) to maximize the diversity gain. The advantage of open-loop scheme is that the receiver complexity of the mobile station is kept low and no signaling overhead is required. However, the channel information is not utilized and may greatly affect the signal-to-noise ratio (*SNR*) performance at the receiving end. On the other hand, the closed-loop diversity technique makes full use of the changing transmission environment and adjusts the antenna gains adaptively. Gerlach and Paulraj first proposed the closed-loop transmit diversity scheme in 1994 [3], in which the receiving end calculates and feeds back to the transmitting side the optimal weight vector of each transmit antenna based on the received channel gain values. With the additional channel information, the closed-loop diversity scheme outperforms the open-loop scheme under low-mobility environments in which the delay of feedback signaling does not exceed the coherence time of the channel.

Partial channel feedback using a limited number of feedback bits under a limited bandwidth condition has been investigated in [4]. There should still exist an optimal quantized transmit weight vector which could achieve the maximum *SNR* value under different varying fading channel environments. However, the complexity would increase exponentially with the number of transmit antennas. In this paper, we study the difference between the *SNR* of the optimal scheme and our proposed methods under the limited quantized-feedback-bit condition and show that our proposed algorithms can yield a *SNR* value closed to the optimal scheme but require fewer computations. We also show by simulations that with different numbers of transmit antennas, the deviation of the *SNR* from the optimal scheme is small. This could greatly reduce the computation load of the receiving end in calculating the transmit weight vector but at the same time enjoy an excellent *SNR* performance.

The paper is organized as follows. We present the transmit diversity system model in section II. Our proposed transmit diversity methods

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and the computation complexities are described in section III. The simulation results and the comparison with other sub-optimal searching scheme are reported in section IV. Finally, we summarize our conclusion in section V.

II. System model:

We assume the system operates under a Rayleigh Fading Channel condition. The channel coefficients are assumed to be accurately estimated at the receiving end and there is no feedback delay between the transmitter and the receiver. The antennas are spaced far enough apart so that the signals can be assumed to suffer from independent fading. Using a model of M transmit antennas and one receive antenna, the transmitted signal could be expressed as:

$$X = \sqrt{P} dW$$

where P is the total transmit power, d is the data symbol, $X = [x_1, x_2, ...x_M]^T$ is the transmit signal vector and $W = [w_1, w_2, ...w_M]^T$ is the transmit weight vector assumed to be of unit norm. The channel gain is modeled by a $1xM$ complex matrix G with components g_i, where $i = 1, ..., M$, representing the independent complex gain of the signal from transmit antenna i to the receiver with zero mean and unity standard deviation. The received signal could thus be expressed as follows:

$$Y = GX + N$$

where N is a white Gaussian noise vector with independent zero mean complex Gaussian random variable elements and standard deviation σ_n. If maximal ratio combination is used at the receiver, the SNR is given as :

$$SNR = \frac{P}{\sigma_n^2} W^H G^H G W$$

Without any restriction on the number of feedback bits, the maximum SNR can be achieved by choosing a suitable W which is the eigenvector associated with the maximum eigenvalue of the Hermitian matrix $G^H G$. However, under the fixed number of feedback bits restriction, the transmit weight w has to be quantized. Assuming two feedback bits are used for each transmit antenna, the weight w can only be one of the elements of the quantization set specified in product form below:

$$w = \prod_{m=1}^{M} \left\{ e^{j2\pi(k-1)/4} / \sqrt{M}, k = 1,2,3,4 \right\}$$

To obtain an optimal quantized weight vector for maximizing the SNR, a complete search with a complexity of the order 4^M is required and the complexity will get very high with large M.

III. Proposed scheme:

Here we propose two simple transmit diversity schemes assumed under two feedback bits condition using simple and fast searching algorithms, yet producing a weight vector which could yield performance close to the optimal one. We model the channel gain of each transmit antenna as a vector in a complex plane, then the problem becomes to study how to rotate the complex vectors according to the weights so that the final combined vector has the maximum magnitude and thus maximum SNR. With two feedback bits, each vector is allowed to turn $z * \pi / 2$ where $z = 0, 1, 2, 3$ before it is combined with other vectors.

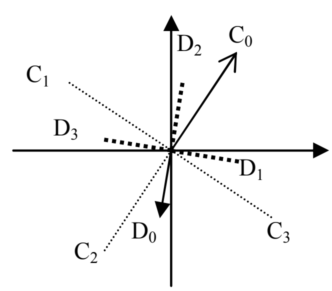


Figure 1. M=2 case

Figure 1 illustrates the M=2 case, where C_0 and D_0 are the original complex channel gain elements. The vectors are to be rotated so as to obtain the smallest angle difference. Under 2 feedback bits restriction, there should exist four groups of optimal vector combination such that the angle difference between the vectors is less than or equal to $\pi / 4$. Thus, there exist four groups of optimum vector combination: C_0 and D_2; C_3 and D_1; C_2 and D_0; C_1 and D_3. Any one of the above combinations would yield a final combined vector with maximum magnitude and hence maximum SNR. This approach can be extended to the search algorithms in the multi-antenna case as described below.

Method 1:

Firstly, the angles between any pair of the M channel vectors are compared. Thus there are

M(M-1)/2 possible pairs for comparison. The two vectors which form the smallest angle are combined. For the newly combined vector and the remaining *M-2* vectors, the comparing process is repeated to search for the next pair of vectors which forms the smallest angle . The procedure is iterated until only one vector is left.

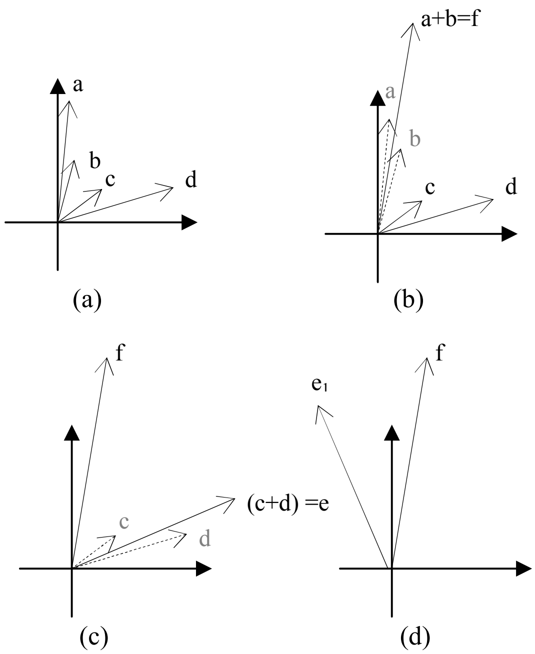
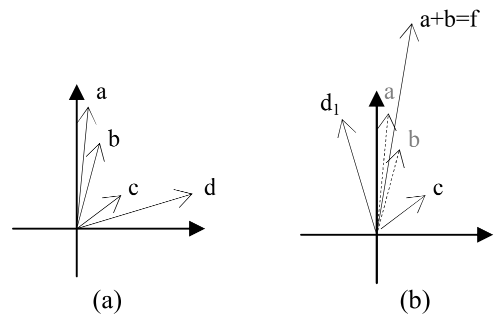


Figure 2. Steps of combining Complex Gaussian channel vectors using Method 1.

Figure2 illustrates the *M*=4 case. In Fig. 2a, vectors *a* and *b* forms a smallest angle and so are combined to form a new vector *f* as shown in Fig. 2b. Vectors *c, d, f* are then compared to search for the 2 vectors which forms a smallest angle. Here, vector *c* and *d* form a smallest angle, so they are combined to form vector *e* as shown in Fig. 2c. As the angle between vectors *e* and *f* are greater than $\pi/4$, vector *e* is rotated by $\pi/2$ anti-clockwise to give vector e_1 which is finally combined with *f* as shown in Fig. 2d.

Method 2:

This method is similar to method 1 but with less computational complexity. Again, this method first combines the vectors which form the smallest angle to form a new vector. However, further comparisons on the smallest angles formed are on vectors pair formed by the newly combined vector and any one of the remaining vectors. The remaining vectors can be rotated to within $\pi/4$ of the new vector, thus, the newly formed vector will then combine with its closest neighbor to form another new vector. The process continues until the final single vector is obtained. Figure 3 shows an example of our second proposed method:



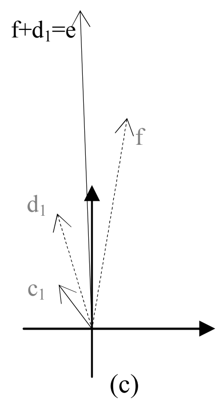


Figure 3. Steps of combining Complex Gaussian Channel vectors using Method 2

In Fig. 3a, there are 4 vectors. Vectors *a* and *b* form the smallest angle and so are combined to produce a new vector *f* as shown in Fig. 3b. Vector *f* is then compared with the remaining vectors *c* and *d* which are allowed to rotate to within $\pi/4$ of vector *f*. Here, vector *d* is rotated $\pi/2$ anticlockwise to give d_1 so as to fall within $\pi/4$ of vector f. Thus, vectors *f* and d_1 are combined to form e. Finally, the remaining vector *c* also has to be rotated by $\pi/2$ anticlockwise to form c_1 in order to fall within the $\pi/4$ region of vector *e*. The final vector is then formed by combining vectors *e* and c_1.

Complexity of the proposed scheme:

Method 1:

With only 2 feedback bits for each antenna, the weight vector is allowed to rotate $z*\pi/2$ where *z=0, 1, 2, 3.* Initially, comparisons are made among all *M* vectors in order to find a smallest angle pair. This amounts to *M(M-1)/2* possible pairs for comparison. After the first combination, there are *(M-1)* vectors left and the number of comparisons becomes *(M-1)(M-2)/2.* Repeating the process, the total number of comparisons is *M(M-1)(M+1)/6.* As the vectors are allowed to rotate, the maximum complexity for this proposed scheme is thus *2M(M-1)(M+1)/3*, having a computational complexity of the order of M^3 which is much lower than that of the optimal searching technique as *M* increases.

Method 2:

Same as method 1, initially there are $M(M-1)/2$ possible pairs for comparison. As vector pairs are formed between the newly combined vector and any one of the remaining vectors, there are only $(M-2)$ comparisons. The process continues and the final number of comparisons is $(M-1)^2$. After accounting for the rotations, the maximum complexity of method 2 is $4(M-1)^2$ which is of the order of M^2 only.

IV. Simulation results:

We simulate the *SNR* performance of methods 1 and 2 and compare the results with another sub-optimal searching algorithm, the co-phasing algorithm [5]. Simulations are performed with different numbers of transmit antenna cases. The number of feedback bits is restricted to two for each transmit antenna in all cases. Tables I and II show the difference in $SNR(S_i)$ and the average distance (D_i) between the proposed methods and the optimal solution. Results show that the *SNR* values achieved by both method 1 and method 2 are very close to that of the optimal one especially when the number of antennas is small. When compared to the co-phasing method shown in Table 3, the two proposed methods outperform the co-phasing scheme *SNR* difference and also show superior performance in terms of the average distance.

	M =3	M=4	M=5	M=6
Si <0.01	99.41%	97.44%	94.96%	92.16%
Di	2.4385e-004	0.0014	0.0032	0.0056

Table 1. Result from Method 1

	M =3	M=4	M=5	M=6
Si <0.01	99.41%	98.09%	96.66%	95.42%
Di	2.4385e-004	0.0010	0.0020	0.0031

Table 2. Result from Method 2

	M =3	M=4	M=5	M=6
Si <0.01	84.58%	72.35%	63.39%	56.12%
Di	0.0331	0.0737	0.1067	0.1410

Table 3. Result from Co-Phasing Algorithm

V. Conclusion:

In order to maximize the *SNR*, an optimal weight vector has to be determined. Optimal searching method is seldom used due to the high complexity. This paper has introduced two sub-optimal searching algorithms to obtain the weight vectors of the closed-loop transmit diversity scheme under a limited number of feedback bits condition.

It has been shown that the proposed methods can achieve the *SNR* performance close to the optimal solution and are superior to the other sub-optimal method, co-phasing scheme, especially in a higher number of antenna cases. Furthermore, compared with the optimal search, the computational complexity of the proposed methods is greatly reduced.

VI. References:

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