Commodity Procurement Risk Management with Futures Contracts: A Dynamic Stack-and-Roll Approach

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Abstract: Procuring material from commodity spot markets can flexibly fulfil a forward production demand, but increase the risk of high procurement cost due to spot price volatility. In this paper, a dynamic stack-and-roll hedging approach using futures contracts is proposed. The approach aims at mitigating the procurement cost risk and optimising the terminal revenue received from the procurement and hedging activities. It separates the procurement planning horizon into multiple stages, along with varying hedging positions in the nearby futures contracts. Hedging positions are adjusted in response to commodity price behaviour and contemporary perceived information about forward production demand. Guided by the mean-variance criteria over the terminal revenue, dynamic programming is applied to derive a closed-form solution for optimal hedging positions in a discrete-time Markovian setting. Numerical experiments are carried out to assess the proposed approach with explicit solution in a realistic stochastic environment. The price processes are modelled by a fractal nonlinear regression model using real price data of China's commodity market, while demand information process is modelled by Bayesian formula. The results show that the proposed approach outperforms naive hedging strategy, and effectively mitigates the procurement cost risk.

Keywords: Dynamic stack-and-roll approach, Commodity procurement, Risk mitigation, Hedge

Ⅰ Introduction and Review of Related Literature

Procurement planning and optimisation is a vital issue in managing supply chains, aiming at matching demand with supply at the lowest cost [1]. Emerging B2B technologies facilitate manufacturers' procurement of raw material in the commodity spot market, which can fulfil the production demand with negligible lead time. On the one hand, procurement in spot market can closely match supply and demand, and then is an attractive procurement mode especially for small-sized manufacturers with bargaining power that is too limited to win a flexible contract. On the other hand, manufacturers relying on spot market sourcing are prone to suffer from a high procurement cost risk due to spot price volatility. The cost risk could further be exaggerated by realised production demand. In this research, a dynamic stack-and-roll hedging approach using futures contracts is integrated with spot procurement, in order to mitigate the cost risk caused by fluctuating spot price and uncertain production demand, and finally optimise the terminal revenue received from the procurement and hedging activities.

The stack-and-roll hedge refers to a strategy that rolls over a series of positions in short-maturity futures. Manufacturers with long-term procurement commitments may prefer this strategy to manage their procurement risk. Since in practice, the long-maturity futures that matches the long-term commitment tends to bear unreasonable price due to its lack of trading liquidity, while the short-maturity futures is frequently traded, and its price is closely correlated with the spot price. Especially when the long-maturity futures market is missing, a sequence of short-maturity rollover futures becomes a good substitution [2]. At the same time, strategically conducting the stack-and-roll hedge received increased academic scrutiny, after Metallgesellschaft AG incurred a heavy loss through the controversial using of a naive stack-and-roll strategy [3-7]. Most of the studies focus on minimising the variance of hedged return assuming a complete frictionless market. Meanwhile the proposed hedging approach tries to make trade-offs between maximising expected hedged return and minimising the variance for that return in an incomplete market framework. Empirical studies will demonstrate that the proposed hedging approach robustly outperforms the naive strategy in managing procurement risk.

The stack-and-roll approach is inherently a discrete multistage strategy, which renders perfect hedge infeasible, so an appropriate criterion as the hedging objective should be chosen to reflect a hedger's preference. There are increasing literatures on discrete multistage financial hedging, e.g. Schweizer [8], Gugushvili [9], Cerny [10], Basak and Chabakauri [11-12], among others. Considering a value-maximising manufacturer who wishes to grow the expected revenue as well as mitigate its variance risk, the mean-variance criteria over the terminal procurement revenue are selected in this research. Guided by mean-variance criteria, dynamic programming is applied to solve the procurement optimisation problem. The solving process is developed from the work of Basak and Chabakauri [11-12], and further elaborates their work mainly in two ways. First, this research derives a discrete closed-form presentation of optimal stack-and-roll hedging policy under mean-variance objective. Second, hedging quantity risk, i.e. uncertain production demand, is also accounted for in the optimisation problem. Moreover, minimum-variance hedge will be applied as a benchmark for comparing with the mean-variance stack-and-roll hedge.

Effective hedging requires the commodity price and production demand to be accurately modelled. As commodity prices are discovered to possess fractal structures in many studies [13-14]. In this research, the commodity prices are supposed to be driven by a fractal nonlinear regression model. The nonlinear function is represented by a wavelet neural network trained by the extended Kalman Filter (EKF) algorithm. Monthly return and volatility are estimated by daily returns data in order to increase the prediction accuracy [15]. Since the production demand is a stochastic variable at the end of procurement horizon, Bayesian inference is appropriate to formulate the demand information updating process along the procurement horizon [16-19]. Under such a realistic stochastic environment, Monte Carlo simulation will be implemented to evaluate the procurement risk management performance.

This research is essentially concerned with supply chain operations management in the presence of online commodity market [20-24]. Chod, et al. [23] find out that operational flexibility and financial hedging tend to complement each other when both are used to mitigate demand risk. Ni, et al. [24] mitigate spot procurement risk using a multistage financial hedging approach. Our research enriches the literature by providing a discrete closed-form expression of a multistage procurement policy for online commodity procurement, aiming to maximise the mean-variance utility of a manufacturer who is faced by a long-term production demand.

Ⅱ Model Formulation for Spot Procurement with Stack-and-roll Hedging

General consideration

Consider a scenario in which a manufacturer plans to procure a certain kind of exchange-traded commodity as the raw material for forward production. The planning horizon is made up of T stages. Instead of entering into a contract with certain suppliers in advance, the manufacturer tries to enhance the sourcing flexibility using spot procurement at time T. In order to avoid suffering from procurement cost risk at time T, during the planning horizon [0, T], the manufacturer takes a stack-and-roll hedging strategy in the commodity future market. The manufacturer wishes to maximise the expected procurement revenue and meanwhile minimise the variance for that revenue.

Without loss of generality, the nearby futures contract will be adopted as the short-maturity contract [5], since it often attracts a sizable amount of trading activity and has good trading liquidity. Then it is assumed that each decision stage lasts for one month over the planning horizon. At the beginning of stage 0, the manufacturer initiates a position in the nearby futures contract based on the realised futures price, predicted commodity price behaviour and available information of production demand. At the end of stage 0, the futures position is liquidated before the outset of the next stage with updated information of production demand. At the end of the final stage, the futures positions are settled by cash while an actual procurement of commodities in the spot market is taken. The sequence of events in the hedging horizon is shown in Figure 1.

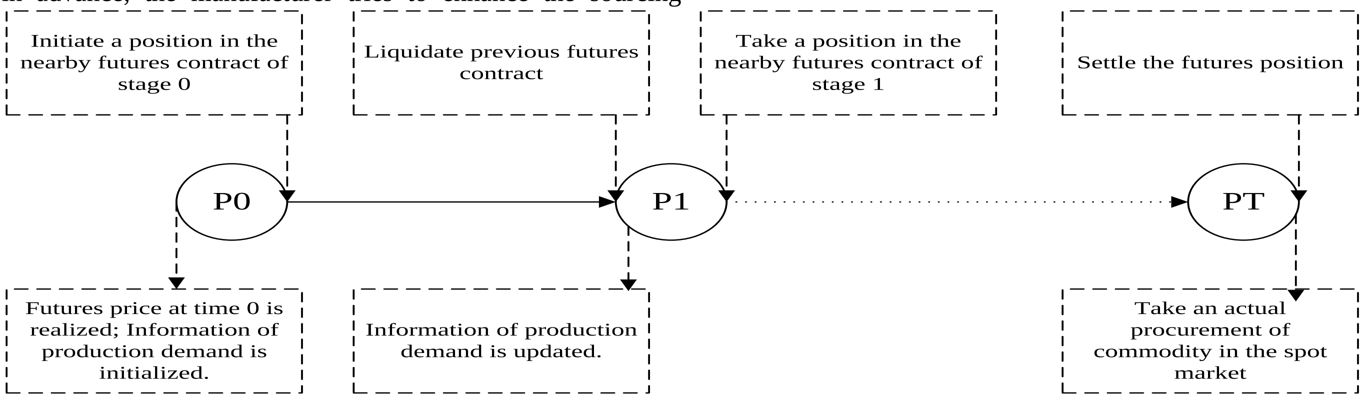


Figure 1 –Sequence of event in the spot procurement with stack-and-roll hedging

Variables and parameters

The mathematical notations and their definitions of the proposed model are listed below.

R_T	The wealth of manufacturer at the end of the planning horizon [0, T].
π_t	Long or short futures positions taken at the beginning of stage t , $t \in \{0,\ldots,T-1\}$
$R_{f,t}$	The tradable wealth in the futures market at the beginning of stage.
S_t	Commodity spot price at time t .
$F_{t,\tau}$	Commodity futures price at the beginning of stage t of a contract that matures at stage $t+\tau$.
D_T	Production demand realised at the end of the planning horizon.
K	Unit procurement cost for the manufacturer, if he chose to procure from spot market at the beginning of stage 0 and hold the commodity until time T .
c_t	Transaction cost in futures market.
r	Risk-free interest rate.
m	Futures margin per each futures contract.
λ	Risk-averse parameter

Mathematical model formulation

The procurement planning horizon can be regarded as a discrete-time incomplete-market Markovian setting with finite horizon [0, T]. The uncertainty of the setting is represented by a filtered probability space $(\Omega, \mathcal{F}, P)$ endowed with two Brownian motion in discrete time with a correlation ρ , denoted as $w_{s,t}$ and $w_{f,t}$. The evolution of commodity spot and nearby futures prices are described by two stochastic processes $\{ S_t, t = 0,1,...,T \}$ and $\{ F_t, t = 0,1,...,T \}$ on this probability space. All stochastic processes are assumed to be well defined and adapted to a filtration $\mathbb{F} = (\mathcal{F}_t)_{0 \le t \le T}$, which can be regarded as the information perceived at the beginning of each stage [19]. Let N be the number of trading period, the time increment can be denoted by $\Delta t = T/N$. In this stack-and-roll strategy, the increment Δt is taken to a month. Let $r_{s,t} = \ln(S_{t+\Delta t}/S_t)$ and $r_{f,t} = \ln(F_{t+\Delta t,0}/F_{t,\Delta t})$ respectively represents monthly return rate of spot price and nearby futures price. Their conditional mean and variance are defined in equations (1) and (2).

$$\mu_{s,t} = E\left[r_{s,t} \mid \mathcal{F}_t\right] \qquad \sigma_{s,t}^2 = E\left[{r_{s,t}}^2 \mid \mathcal{F}_t\right] - \mu_{s,t}^2, \quad 0 \le t < T \tag{1}$$

$$\mu_{f,t} = E\left[r_{f,t} \mid \mathcal{F}_t\right] \qquad \sigma_{f,t}^2 = E\left[{r_{f,t}}^2 \mid \mathcal{F}_t\right] - \mu_{f,t}^2, \quad 0 \le t < T \tag{2}$$

The spot and the nearby futures price follow the dynamics given by

$$\Delta S_t = \mu_{s,t} \cdot \Delta t + \sigma_{s,t} \cdot \Delta w_{s,t} \tag{3}$$

$$\Delta F_t = \mu_{f,t} \cdot \Delta t + \sigma_{f,t} \cdot \Delta w_{f,t} \tag{4}$$

The variation of spot procurement cost from time 0 to T is denoted by L_T, where

$$L_T = D_T \cdot \left(S_T - K\right) \tag{5}$$

In such an incomplete market setting, self-financing strategy using nearby futures is employed to hedge L_T, given an initial wealth $R_{f,0}$. The manufacturer chooses a hedging policy π, where π_t denotes the position in the nearby futures contract in stage t. Since π_t is decided based on the information perceived at the beginning of stage t, π_t should be $\mathcal{F}_t$ measurable. Let $M_t\left(\Omega, \mathcal{F}_t\right)$ be the function space that consists of all $\mathcal{F}_t$ measurable random variables, the hedging policy should fulfil the condition

$$\pi_t \in M_t\left(\Omega, \mathcal{F}_t\right) \tag{6}$$

The tradable wealth $R_{f,t}$ at the beginning of each stage is given by

$$R_{f,t} = e^{r\cdot\Delta t}(R_{f,t-1} - c_t) + \pi_{t-1}(e^{r_{f,t}} - m \cdot e^{r\cdot\Delta t}) \tag{7}$$

The wealth of the manufacturer at the end of the planning horizon is given by

$$R_T = R_{f,T} - L_T \tag{8}$$

The mean-variance criterion over the terminal wealth R_T is selected as the objective function. Due to this criterion, the manufacturer could maximise R_T for given level of its variance. The optimisation problem is formulated by

$$\max_{\pi\in\Pi} \; E[R_T^\pi] - \frac{\lambda}{2} Var[R_T^\pi] \tag{9}$$

where R_T^π is used to signify the dependence of the terminal wealth on π. Π is the set of admission policies conditioning on $M_t\left(\Omega, \mathcal{F}_t\right)$.

Based on the above discussion, the optimal stack-and-roll hedging policies can be obtained by solving the optimisation problem in equation (9), subject to constraints in equations (6), (7) and (8).

Ⅲ Determination of Optimal Stack-and-roll Hedging Policy

The problem formulated in Section 3 will be solved using dynamic programming to obtain optimal stack-and-roll hedging policies. For the dynamic hedging problems over terminal wealth R_T, dynamic programming is readily applicable to solve the problem with objective function in the form $E\left[u\left(R_T\right)\right]$, since the value function has the iterated-expectation property $E_t\left[u\left(R_T\right)\right] = E_t\left[E_{t+\Delta t}\left[u\left(R_T\right)\right]\right]$ based on the law of total expectation. However, the mean-variance objective function in equation (5) does not have such a property. According to the law of total variance, the value function is given by

$$\begin{aligned} &E_t\left[R_T\right] + Var_t\left[R_T\right] \\ &= E_t\left[E_{t+\Delta t}\left[R_T\right] + Var_{t+\Delta t}\left[R_T\right]\right] + Var_t\left[E_{t+\Delta t}\left[R_T\right]\right] \end{aligned} \tag{10}$$

Due to the presence of the term $Var_t\left[E_{t+\Delta t}\left[R_T\right]\right]$, direct application of the classical dynamic programming solution procedure is not feasible because Bellman's principle of optimality will be violated. To resolve this difficulty, Basak and Chabakauri [12] first derive a tractable recursive formulation for the mean-variance objective in equation (9), and obtain time-consistent analytical optimal policies in continuous time using dynamic programming. Based on their work, a discrete-time closed-form expression of optimal stack-and-roll hedging policy is developed. Moreover, in our approach, the optimal policy can be adjusted to the evolution of nonfinancial operating information, i.e. production demand.

The solution procedure is as follows. First, a recursive formulation for value function in dynamic programming will be derived. Given the objective function in equation (9) with optimal hedging policy π_s^*, $s \in \{t, \ldots, T-1\}$, the value function V, is defined as

$$V\left(R_{f,t}, F_t, L_t, t\right) \equiv E_t\left[R_T^*\right] - \frac{\lambda}{2} \cdot Var_t\left[R_T^*\right] \tag{11}$$

According to equation (10), the recursive presentation of value function is as follows

$$V_t = \max_{\pi_s^*, s\in\{t,\ldots,T-1\}} \left\{ E_t\left[V_{t+1}\right] - \frac{\lambda}{2} \cdot Var_t\left[E_{t+1}\left[R_T\right]\right] \right\} \tag{12}$$

where V_t is shorthand for $V\left(R_{f,t}, F_t, L_t, t\right)$. Since the transaction cost is proportional to futures position, let $\varepsilon = m \cdot e^{r\cdot\tau} + c_t \cdot e^{-r\cdot\tau} / \pi_t$, equation (7) can be rewritten as

$$R_{f,t} = e^{r\cdot\Delta t} R_{f,t-1} + \pi_{t-1}(e^{r_{f,t}} - \varepsilon) \tag{13}$$

Then the tradable wealth at the end of the planning horizon is given by

$$R_{f,T} = e^{r(T-t)\Delta t} R_{f,t} + \sum_{s=t}^{T-1} e^{r(T-s)\Delta t} \pi_s (e^{r_{f,t}} - \varepsilon) \tag{14}$$

Substituting equation (14) into (12) and letting

$$H_t = L_t - \sum_{s=t}^{T-1} e^{r(t-s)\Delta t} \pi_s (e^{r_{f,t}} - \varepsilon) \tag{15}$$

Then the evolution process of V_t is as follows

$$\max_{\pi_s^*, s\in\{t,\ldots,T-1\}} \left\{ E_t\left[\Delta V_t\right] - \frac{\lambda}{2} \cdot e^{2r(T-t-1)\Delta t} Var_t\left[R_{f,t+1} - H_{t+1}\right] \right\} = 0 \tag{16}$$

Substituting equation (14) into (11), the value function is shown to be linear in tradable wealth $R_{f,t}$, then the value function can be represented as

$$V\left(R_{f,t}, F_t, L_t, t\right) = e^{r(T-t)\Delta t} R_{f,t} + \hat{V}\left(F_t, L_t, t\right) \tag{17}$$

Substituting equation (17) into (16) and computing the variance term, we get

$$\max_{\pi_s^*, s\in\{t,\dots,T-1\}} \{\Delta\hat{V}_t + e^{r(T-t-1)}\pi_t\left(e^{r_{f,t}} - \varepsilon\right)$$
$$-\frac{\lambda}{2}\cdot e^{2r(T-t-1)\Delta t}\left(Var_t(H_{t+1}) - 2\pi_t Cov_t\left(H_{t+1}, e^{r_{f,t}}\right) + \pi_t^2 Var_t(e^{r_{f,t}})\right)\} = 0 \quad (18)$$

Then the optimal hedging policy is given by

$$\pi_t^* = \frac{Cov_t\left[H_{t+1}, e^{r_{f,t}}\right]}{\operatorname{var}_t(e^{r_{f,t}})} + \frac{1}{\lambda}\cdot\frac{E_t\left[e^{r_{f,t}} - \varepsilon\right]}{\operatorname{var}_t(e^{r_{f,t}})}\cdot e^{-r\cdot(T-t-1)\Delta t} \quad (19)$$

According to the definition of H_t and the tradable wealth, we can get

$$H_t = E_t\left[L_T e^{-r(T-t)\Delta t}\right] - E_t\left[R_{f,T}^* e^{-r(T-t)\Delta t} - R_{f,t}\right] \quad (20)$$

where

$$E_t\left[L_T e^{-r(T-t)\Delta t}\right] = E_t\left[E_{t+1}\left[L_T e^{-r(T-t-1)\Delta t}\right]e^{-r\Delta t}\right]$$
$$= e^{-r\Delta t}E_t\left[H_{t+1} + E_{t+1}\left[R_{f,T}^* e^{-r(T-t-1)\Delta t} - R_{f,t+1}\right]\right]$$
$$= e^{-r\Delta t}E_t\left[H_{t+1}\right] + E_t\left[R_{f,T}^* e^{-r(T-t)\Delta t} - e^{-r\Delta t}\cdot R_{f,t+1}\right] \quad (21)$$

The recursive expression of H_t is given by

$$H_t = e^{-r\Delta t}E_t\left[H_{t+1}\right] - E_t\left[e^{-r\Delta t}\cdot R_{f,t+1} - R_{f,t}\right]$$
$$= e^{-r\Delta t}E_t\left[H_{t+1}\right] - E_t\left[e^{-r\Delta t}\cdot\left(e^{r\Delta t}\cdot R_{f,t} + \pi_t^*\cdot\left(E_t\left[e^{r_{f,t}}\right] - \varepsilon\right)\right) - R_{f,t}\right]$$
$$= e^{-r\Delta t}E_t\left[H_{t+1}\right] - \pi_t^* E_t\left[\left(e^{r_{f,t}} - \varepsilon\right)\right]\cdot e^{-r\Delta t} \quad (22)$$

The optimal hedging policy is given by

$$\pi_t^* = \frac{E_t\left[H_{t+\Delta t}\right] - e^{r\Delta t}H_t}{E_t\left[\left(e^{r_{f,t}} - \varepsilon\right)\right]} \quad (23)$$

Substituting (19) into (22), the recursive expression of H_t is as follows

$$H_t = e^{-r\Delta t}E_t\left[\left\{1 + \left(E_t\left[e^{r_{f,t}}\right] - e^{r_{f,t}}\right)\cdot\frac{E_t\left[e^{r_{f,t}} - \varepsilon\right]}{\operatorname{var}_t\left[e^{r_{f,t}}\right]}\right\}H_{t+\Delta t}\right] - \frac{E_t^2\left[e^{r_{f,t}} - \varepsilon\right]}{\lambda\operatorname{var}_t\left[e^{r_{f,t}}\right]}\cdot e^{-r(T-t)\Delta t}$$

(24)

With the terminal condition $H_T = L_T$, the discrete-time closed-form expression of optimal hedging policy is as follows

$$\pi_t^* = \frac{E_t\left[\gamma\cdot L_T\cdot e^{-r(T-t-1)\Delta t}\right]}{E_t\left[\left(e^{r_{f,t}} - \varepsilon\right)\right]} - e^{-r(T-t-1)\Delta t}\sum_{s=t+1}^{T-1}\frac{E_s^2\left[e^{r_{f,s}} - \varepsilon\right]}{\lambda E_t\left[\left(e^{r_{f,t}} - \varepsilon\right)\right]\operatorname{var}_s\left[e^{r_{f,s}}\right]} + \frac{E_t\left[e^{r_{f,t}} - \varepsilon\right]}{\lambda\operatorname{var}_t\left[e^{r_{f,t}}\right]}e^{-r(T-t-1)\Delta t}$$

(25)

where

$$\gamma = \left\{\left(e^{r_{f,t}} - E_t\left[e^{r_{f,t}}\right]\right)\cdot\left(E_t\left[e^{r_{f,t}} - \varepsilon\right]/\operatorname{var}_t\left[e^{r_{f,t}}\right]\right)\right\}\cdot$$
$$\prod_{s=t+1}^{T-1}\left\{1 + \left(E_s\left[e^{r_{f,s}}\right] - e^{r_{f,s}}\right)\left(E_s\left[e^{r_{f,s}} - \varepsilon\right]/\operatorname{var}_s\left[e^{r_{f,s}}\right]\right)\right\} \quad (26)$$

ⅣModel Evaluation and Monte Carlo Simulation

In this study, Monte Carlo simulation is carried out to evaluate the performance of the proposed model with optimal policy derived in Section 4. Another major research issue concerns the modelling of the commodity price and production demand, which will also be discussed in this section.

Stochastic processes of copper prices at China's commodity market

Copper is very widely used in industrial production, and yet its volatile price movements over the past several years have been a major concern for manufacturers. Without loss of generality, the proposed model is applied to help a manufacturer procure copper from China's commodity market. The proposed model requires an accurate prediction of monthly return and volatility of copper prices, as indicated in equation (25) and (26). In order to increase the accuracy, daily returns of copper prices are first modelled and then used to estimate the monthly return and volatility [15].

The most prevailing commodity price models are the term structure model [25] and its extensions. However these models are inappropriate to be incorporated in the stack-and-roll strategy, because the models assume all futures are fairly priced relative to each other, and hence assume away the trading liquidity risk [6]. Since recent studies on testing fractal structure of commodity price confirm the nonlinear dependence of the prices series, nonlinear regression models are developed to model daily return rates of copper spot price and nearby futures price. A wavelet neural network is adopted to represent the nonlinear function, which can capture the fractal property of copper prices [26]. At the same time, copper spot and futures price are frequently find to be co-integrated [27]. To accommodate the fractal structure and the possibility of a long-run equilibrium relationship, spot and nearby futures return rates are modelled as follows

$$r_{s,t}^d = \alpha_{s0} + f(r_{s,t-1}^d \dots r_{s,t-i}^d, r_{f,t-1}^d) + \beta B_{t-1} + \varepsilon_{s,t} \quad (27)$$

$$r_{f,t}^d = \alpha_{f0} + f(r_{f,t-1}^d \dots r_{f,t-i}^d, r_{s,t-1}^d) + \beta B_{t-1} + \varepsilon_{f,t} \quad (28)$$

where $B_{t-1} = \ln\left(p_{s,t-1}\right) - \theta\cdot\ln\left(p_{f,t-1}\right)$ represents the long-run equilibrium relationship; $r_{s,t}^d$ is the daily return rate of copper spot price; $r_{f,t}^d$ is the daily return rate of the nearby copper futures price; f is the function of wavelet neural network. The Akaike information criterion (AIC) is applied to select the self-correlation lagged order i. EKF algorithm is employed to determine the model parameters in (27) and (28), using daily closing copper spot price of Shanghai Changjiang Nonferrous Metals Market and nearby copper futures price of Shanghai Futures Exchange. The in-sample training data covers from January 4th, 2005 to February 15th, 2011, while the out-of-sample testing data covers from Feb 16th, 2011 to September 26, 2011. A forecasting performance comparison between wavelet neural network (WNN) model and linear autoregressive model is carried out. The results shown in Table 1 indicate that WNN model can improve the prediction accuracy of copper prices.

Table 1: Performance comparison when forecasting out-of-sample data

spot/futures	NMSE	MAE	DS
WNN	1.3993/1.3548	0.0115/0.0144	56/55
AR	1.7542/1.7154	0.0187/0.0179	51/50

After we have obtained the daily return data, the monthly return and volatility can be calculated using equation (29). It is assumed that there are 22 trading days in a month.

$$\operatorname{var}\left(r_{x,t}\right) = \sum_{k=1}^{22}(r_{x,t+k}^d - \overline{r}_{x,t}^d)^2 + 2\sum_{k=i}^{22}\left(\operatorname{cov}\left(r_{x,t+k}^d, r_{x,t+k-1}^d\right) + \dots + \operatorname{cov}\left(r_{x,t+k}^d, r_{x,t+k-i}^d\right)\right)$$

$$r_{x,t} = \sum_{k=1}^{22} r_{x,t+k}^{d}, x \in \{f, s\} \tag{29}$$

Random demand and information updating

Following the demand uncertainty structure as that of Iyer and Bergen [17], production demands assumed to follow a normal distribution as follows

$$f(D \mid \upsilon) \sim N(\upsilon, \delta^2) \tag{30}$$

where υ is the mean of D and is also normally distributed as follows

$$g(\upsilon) \sim N(\mu, \tau^2) \tag{31}$$

At the beginning of stage 0, the production demand is a normal distribution $m(D) \sim N(\mu, \tau^2 + \delta^2)$. As more information about demand will be received along the planning horizon, the knowledge of the demand distribution will be improved by applying Bayesian inference. Assuming that information collected during each stage is converted by the manufacturer into an estimation of the production demand, i.e. d_t, the demand distribution conditioning on d_t is $g(\upsilon \mid d_t) \sim N(\mu(d_t), \tau^2(d_t))$, where $\frac{1}{\tau^2(d_t)} = \frac{1}{\delta^2} + \frac{1}{\tau^2(d_{t-1})}$ and $\mu(d_t) = \frac{\delta^2 \mu(d_{t-1}) + \tau^2(d_{t-1}) d_t}{\delta^2 + \tau^2(d_{t-1})}$, which implies that $m(D \mid d_t) \sim N(\mu(d_t), \tau^2(d_t) + \delta^2)$. Then we can see that with information gathered along the planning horizon, demand uncertainty will be reduced as time goes on.

Ⅴ Simulation results and discussion

To simulate the monthly return and volatility of copper spot price and nearby futures price, the initial spot price is set to be 60,000 Yuan/ton while the nearby futures price is set to be 60,100 Yuan/ton. The unit procurement cost at stage 0 is set to be 63000 Yuan/ton. Besides, at stage 0, we assume that $f(D \mid \upsilon) \sim N(\upsilon, 0.1)$ and $g(\upsilon) \sim N(1, 0.1)$. For simplicity, we assume that $d_t = \mu = 1$, where $t \in \{0, \ldots, T-1\}$, which means that information received along the planning horizon enhance the probability of expected demand μ, and thus gradually reduce the variance of demand estimation. A total 5000 pairs of sample paths are generated by Monte Carlo simulation for copper price returns and production demand.

First, the proposed stack-and-roll hedging strategy is assessed by comparing the hedged with unhedged procurement revenue. The result is shown in Figure 2. From Figure 2 we can see that the hedged revenue is much less volatile than the unhedged one and the expected value of the hedged revenue is much larger than the unhedged one.

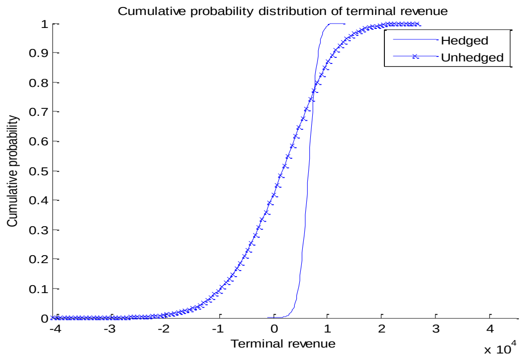
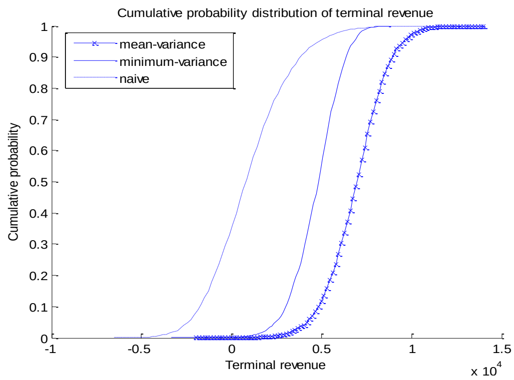


Figure 2. Comparison of hedged and unhedged procurement revenue

Second, the performance of the proposed mean-variance hedging strategy, minimum-variance and naive hedging strategy are assessed through comparing the cumulative probability distribution of the three, as shown in Figure 3, from which we can see that the proposed stack-and-roll strategy has the best hedging performance.

Figure 3 Performance comparisons of different hedging strategies

Third, the effect of risk-averse parameter on the stack-and-roll strategy is shown in Figure 4, from which we can see that hedging strategy with high risk-averse level can reduce more revenue volatility risk only at the expense of lower expected revenue.



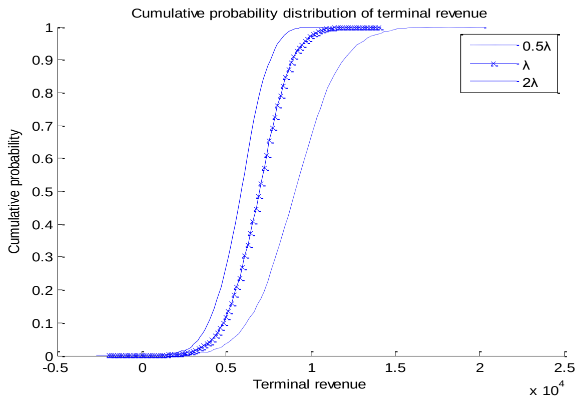


Figure 4. Effective of risk-averse parameter on the stack-and-roll strategy

Fourth, experiments are implemented to evaluate the effect of updating demand information on stack-and-roll strategy. From Figure 5, we can see that ignoring the updating information can deteriorate the hedging performance, especially when the information is more reliable.

Ⅵ Conclusions

A dynamic stack-and-roll hedging strategy is developed in this study to manage the risk of spot procurement and optimise the terminal revenue received from procurement and hedging activities. A discrete-time closed-form expression of optimal hedging policies is derived by dynamic programming, which is determined by commodity price behaviour and timely perceived production demand. The performance of proposed model is evaluated by Monte Carlo simulation, when it is applied to procure copper in China's commodity market. In order to simulate a more real stochastic environment, Bayesian inference is adopted to model the demand information updating process, while a nonlinear regression model is developed to model the copper price behaviour. Since monthly data are weakly correlated and contains less market information, daily returns are used to compute monthly return and volatility of copper prices, in order to increase the estimation accuracy. From the simulation results, the proposed procurement risk management model can perform robustly better than unhedged spot procurement and other existing hedging strategies.

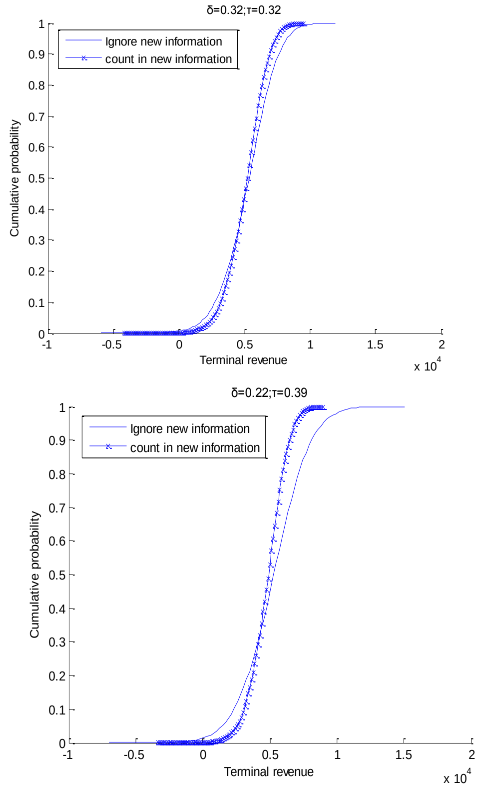


Figure 5. Effect of demand information updating on stack-and-roll strategy

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