1607, A YEAR OF (SOME) SIGNIFICANCE: TRANSLATION OF THE FIRST EUROPEAN TEXT IN MATHEMATICS — *ELEMENTS* — INTO CHINESE

Man-Keung SIU
The University of Hong Kong, Department of Mathematics, The University of Hong Kong, Pokfulam, Hong Kong
mathsiu@hkucc.hku.hk

ABSTRACT

The Italian Jesuit Matteo Ricci and the Chinese scholar-official XU Guang-qi of the Ming Dynasty collaborated to produce a translation of the first six books of *Elements* (more precisely, the fifteen-book-version *Euclidis Elementorum Libri XV* compiled by Christopher Clavius in the latter part of the fifteenth century) in Chinese in 1607, with the title *Ji He Yuan Ben* (Source of Quantity). This paper attempts to look at the historical context that made *Elements* the first European text in mathematics to be translated in China, and how the translated text was received at the time as well as what influence the translated text exerted in various domains in subsequent years, if any, up to the first part of the 20th century. This first European text in mathematics transmitted into China led the way of the first wave of transmission of European science into China, while a second wave and a third wave followed in the Qing Dynasty, but each in a rather different historical context. Besides comparing the styles and emphases of mathematical pursuit in the eastern and the western traditions the paper looks at the issue embedded in a wider intellectual and cultural context.

1 Introduction

The title of this paper (which is the text of a talk given at the ESU6 in July of 2010) is inspired by that of a well-received book by the historian Ray Huang [Huang, 1981]. Huang's book *1587, A Year of No Significance* was translated into Chinese soon after its publication and was given a more informative but perhaps less pithy title *Wanli Shiwu Nian* (In the Fifteenth Year of the Reign of Emperor Wanli). Huang begins his book with the passage:

> "Really, nothing of great significance happened
> in 1587, the year of the Pig. […] Let me begin
> my account with what happened on March 2,
> 1587, an ordinary working day."

His intention is to give an account of history from a "macrohistory" viewpoint, which he further exemplifies in a subsequent book titled *China: A Macrohistory* [Huang 1988/1997]. The purpose is to give an analysis of events that occurred in a long span in time, viewed from a long distance with a broad perspective. In this respect events, some of which might not reveal its true significance when it initially happened, cumulated in time to produce long-term effects. It is in a similar vein that this author tries to tell the story of the event that occurred in 1607 depicted in the title.

This paper attempts to look at the historical context that made *Elements* the first European text in mathematics to be translated in China, and how the translated text was received at the time as well as what influence the translated text exerted in various domains in subsequent years, if any, up to the first part of the 20th century.

This first European text in mathematics transmitted into China led the way of the first wave of transmission of European science into China, while a second wave and a third wave followed in the Qing Dynasty, but each in a rather different historical context. Section 4 of this paper (which is a record of an accompanying three-hour workshop conducted at the ESU6) deals with a comparison of the styles and emphases of mathematical pursuit in the Eastern and the Western traditions.

The readers may query whether it would be more appropriate to give such a talk in 2007, which coincided with the 400^{th} anniversary of the translation of *Elements* into Chinese. Indeed, several symposiums were held on this theme in 2007. In particular, on that occasion this author gave a talk that touches on the influence of *Elements* in Western culture and in China, as well as the pedagogical influence of *Elements*. The text of the 2007 talk (given at the Institute of Mathematics of Academia Sinica in Taipei) was published in a paper in Chinese in that same year [Siu, 2007]. The content and emphasis of that paper differ from those in this paper, but naturally are related to it. We do have a historical reason for giving this talk at the ESU6 held in 2010, for the year marks the 400 anniversary of the passing of Matteo Ricci, one of the two protagonists in this endeavour of enhancing understanding between Europe and China.

2 Translation of *Elements* into Chinese

The story started with the "era of exploration" when Europeans found a way to go to the East via sea route. Various groups took the path for various reasons, among whom were the missionaries. As a byproduct of the evangelical efforts of the missionaries an important page of intellectual and cultural encounter between two great civilizations unfolded in history.

From around 1570 to 1650 the most prominent group of missionaries that came to spread Christian faith in China were the Jesuits sent by the Society of Jesus, which was founded by Ignatius of Loyola in 1540. Of the many Jesuits this paper focuses attention on only one, Matteo Ricci (1552-1610), and of the many contributions of Ricci in the transmission of Western learning into China this paper focuses attention on only one, his collaboration with XU Guang-qi (1562-1633) in translating Euclid's *Elements* into Chinese.

The translation was based on the version of *Elements* compiled by Christopher Clavius (1538-1612) in 1574 (with subsequent editions), a fifteen-book edition titled *Euclidis Elementorum Libri XV*. Ricci learnt mathematics from Clavius at Collegio Romano where he studied from September 1572 to May 1578 before being sent to the East for missionary work.

On August 7, 1582 Ricci arrived in Macau, which was a trading colony in China set up by the Portugese with the consent of the Ming Court in 1557. Macau is the first as well as the last European colony in East Asia, being returned to Chinese sovereignty as a Special Administrative Region of China in 1999. Together with its neighbouring city of Hong Kong, which became a British colony in 1842 and returned to Chinese sovereignty in 1997, the two places played an important role in the history of the rise of modern China in a rather subtle way.

From Macau Ricci proceeded to move into mainland China and finally reach Peking (Beijing) in January of 1601. He became the most prominent Catholic missionary in China. When he passed away on May 11, 1610, he was the first non-Chinese that was granted the right to be buried on Chinese soil, an indication of the high esteem he was held in at the time. (Incidentally, Protestant missionary work also began in Macau with the arrival of Robert Morrison (1782-1834) of the London Missionary Society in 1807.)

Ricci left with us a very interesting and informative account of his life and missionary work in China in the form of a journal that was prepared for publication by a contemporary Jesuit Nicolas Trigault (1577-1628) in 1615. Let us quote a few passages from this journal of Ricci's [Ricci/Gallagher, 1942/1953, p.235, p.476].

"[…] Whoever may think that ethics, physics and mathematics are not important in the work of the Church, is unacquainted with the taste of the Chinese, who are slow to take a salutary spiritual potion, unless it be seasoned with an intellectual flavouring. […] All this, what we have recounted relative to a knowledge of science, served as seed for a future harvest, and also as a foundation for the nascent Church in China. […] but nothing pleased the Chinese as much as the volume on the *Elements* of Euclid. This perhaps was due to the fact that no people esteem mathematics as highly as the Chinese, despite their method of teaching, in which they propose all kinds of propositions but without demonstrations. […] The result of such a system is that anyone is free to exercise his wildest imagination relative to mathematics, without offering a definite proof of anything. In Euclid, on the contrary, they recognized something different, namely, propositions presented in order and so definitely proven that even the most obstinate could not deny them."

Is it really true that the notion of a mathematical proof was completely absent from ancient Chinese mathematics as Ricci remarked? This is a debatable issue [Siu, 1993, pp.345-346]. In Section 4 we will see one example (**Problem 2**) that would have made Ricci think otherwise, had he the opportunity of having access to the commentaries of LIU Hui of the 3rd century.

To Ricci, who studied mathematics under Clavius, the treatise *Elements*, compiled by Euclid (c.325-265 B.C.E.) in the early third century B.C.E., was the basis of any mathematical study. He therefore suggested to his Chinese friend XU Guang-qi that *Elements* should be the first mathematical text to be translated. XU Guang-qi set himself to work very hard on this project. He went to listen to Ricci's exposition of *Elements* every day in the afternoon (since he could not read Latin, while Ricci was well versed in Chinese) and studied laboriously, and at night he wrote out in Chinese everything he had learnt by day. We are told according to an account by Ricci: "When he [XU Guang-qi] began to understand the subtlety and solidity of the book, he took such a liking to it that he could not speak of any other subject with his fellow scholars, and he worked day and night to translate it in a clear, firm and elegant style. […] Thus he succeeded in reaching the end of the first six books which are the most necessary and, whilst studying them, he mingled with them other questions in mathematics." We are further told that "He [XU Guang-qi] would have wished to continue to the end of the Geometry; but the Father [Matteo Ricci] being desirous of devoting his time to more properly religious matters and to rein him in a bit told him to wait until they had seen from experience how the Chinese scholars received these first books, before translating the others." [Bernard, 1935, pp.67-68]

The translated text was published in 1607 and was given the title *Ji He Yuan Ben* (Source of Quantity). In the preface Ricci said:

> "[……] but I said: "No, let us first circulate this in order that those with an interest make themselves familiar with it. If, indeed, it proves of some value, then we can always translate the rest." Thereupon he [XU Guang-qi] said, "Alright. If this book indeed is of use, it does not necessary have to be completed by us." Thus, we stopped our translation and published it, […]".

But in his heart XU Guang-qi wanted very much to continue the translation. In a preface to a revised edition of *Ji He Yuan Ben* in 1611 he lamented, "It is hard to know when and by whom this project will be completed." This deep regret of XU Guang-qi was resolved only two and a half centuries later when the Qing mathematician LI Shan-lan (1811-1882) in collaboration with the English missionary Alexander Wylie (1815-1887) translated Book VII to Book XV in 1857 (based on the English translation of *Elements* by Henry Billingsley published in 1570) [Xu, 2005].

XU Guang-qi was a Chinese scholar brought up in the Confucian tradition, upholding the basic tenet of self-improvement and social responsibility, leading to an aspiration for public service and an inclination to pragmatism. He first got to know Catholic missionaries by an incidental encounter with the Jesuit Lazzaro Cattaneo (1560-1640) in the southern province of Guangdong, who probably introduced him to Ricci. XU Guang-qi was baptized (under the Christian name Paul) in 1603. He saw in Western religion and Western science and mathematics an excellent way to cultivate the mind and a supplement to Confucian studies. He also saw in Western science and technology the significant role it would play in improving the well-being of his countrymen. This eagerness on his part to study Western learning was very much welcomed by Ricci as it was in line with the tactics adopted by the Jesuit missionaries in making use of Western science and mathematics to attract and convert the Chinese literati class who usually occupied important positions in the Imperial Court. Matteo Ricci impressed the Chinese intellectuals as an erudite man of learning, thereby commanding their trust and respect [Siu, 1995/1996, p.148].

This is a good point to insert an explanation of the term "*ji he*" in the title of the translated text. This term has become the modern Chinese terminology for geometry. Some people suggests that it is a transliteration of the Western word "geometria (geometry)", Two reason can be raised against this view: (1) The word "geometria" does not appear in the title of Clavius' fifteen-book version of *Elements*. (In fact, nowhere in Euclid's *Elements* does the word "geometria" appear.) (2) Jesuit missionaries in those days were rather cautious about employing anything "un-Chinese" and transliteration was considered to be one such. A reading of the translated definitions in Book V, which is on Eudoxus' theory of proportion, will reveal that "*ji he*" is the technical term for "magnitude". In traditional Chinese mathematical classics the term "*ji he* (how much, how many)" frequently appears to begin a problem. We may conjecture that XU Guang-qi, who was familiar with this term because of his knowledge on traditional Chinese mathematics (the part that he had access to), thought of borrowing it to translate the technical term "magnitude" in *Elements*. By putting the term as a keyword in the title XU Guang-qi probably noticed the significance of the notion of "magnitude" in *Elements*. With the passing of time the original technical meaning of "*ji he*" (as "magnitude") was forgotten. Instead, because *Ji He Yuan Ben* (comprising the first six books of *Elements*) deals with properties of geometric figures such as triangles, squares, parallelograms and circles, the term acquires meaning as the name of the subject, replacing the term *xing xue* (study of figures) employed in the nineteenth century [Siu, 1995/1996, pp.160-161].

3 View of XU Guang-qi about *Elements*

By the Ming Dynasty the mathematical legacy in China was no longer preserved and nurtured in the way it should be. Quite a number of important mathematical classics were either completely lost or left in an incomplete form. As a scholar brought up in the Confucian tradition XU Guang-qi was aware that mathematics had once occupied a significant part of education and statecraft in China and should be restored to its former position of importance. He ascribed the unsatisfactory state of the subject at his time to two factors, which he expressed in 1614 in the preface to another translated European mathematical text (*Epitome Arithmeticae Practicae* compiled by Christopher Clavius in 1583, translated by LI Zhi-zao (1565–1630) also in collaboration with Matteo Ricci):

> "There are two main causes for negligence and dilapidation of mathematics that set in only during several past centuries. Firstly, scholars in pursuit of speculative philosophical studies despise matters of practical concern. Secondly, sorcery encroaches upon mathematics to turn it into a study filled with mysticism."

He saw in the introduction of Western mathematics, which was novel to him, a way to revive the indigenous mathematical tradition. He had a wider vision of mathematics, not just as an intellectual pursuit but as a subject of universal applications as well. In an official memorial submitted to the Emperor in 1629, he said, "Furthermore, if the study of measure and number [mathematics] is understood, then it can be applied to many problems [other than astronomy] as a by-product." Such problems were labelled by him in ten categories: (1) weather forecast, (2) irrigation, (3) musical system, (4) military equipment, (5) accounting, (6) building, (7) machine, (8)topography, (9) medical practice, (10) timepieces.

Despite XU Guang-qi's emphasis on utility of mathematics, he was sufficiently perceptive to notice the essential feature about *Elements*. Commenting on the merits of the book in the preface to *Ji He Yuan Ben*, he said:

> "As one proceeds from things obvious to things subtle, doubt is turned to conviction. Things that seem useless at the beginning are actually very

> useful, for upon them useful applications are based. It can be truly described as the envelopment of all myriad forms and phenomena, and as the erudite ocean of a hundred school of thought and study."

He stressed this point in another translated text (also in collaboration with Matteo Ricci) in 1608, that of parts of *Geometria practica* compiled by Christopher Clavius in 1606, retitled as *Ce Liang Fa Yi* (Methods and Principles in Surveying):

> "It has already been ten years since Master *Xitai* [Matteo Ricci] translated the methods in surveying. However, only started from 1607 onwards the methods can be related to their principles. Why do we have to wait? It is because at that time the six books of *Ji He Yuan Ben* were just completed so that the principles could be transmitted. As far as the methods are concerned, are they different from that of measurement at a distance in *Jiu Zhang* [*Suan Shu*] and *Zhou Bi* [*Suan Jing*]? They are not different. If that is so, why then should they be valued? They are valued for their principles."

In the same year XU Guang-qi published *Ce Liang Yi Tong* (Similarities and Differences in Surveying) in which he tried to explicate traditional Chinese surveying methods by the Western mathematics he had just learnt from *Elements*. In the introduction to the book he said:

> "In the chapter on *gou gu* (study of right-angled triangles) of *Jiu Zhang Suan Shu* there are several problems on surveying using gnomon and the trysquare, the methods of which are more or less similar to those in the recently translated *Ce Liang Fa Yi* (Methods and Principles in Surveying). The *yi* (principles) are completely lacking. Anyone who studies them cannot understand where they are derived from. I have therefore provided new *lun* (proofs) so that examination of the old text becomes as easy as looking at the palm of your hand."

A more explicit explanation can be found in an official memorial he submitted to the Emperor in 1629:

> "[… not knowing that] there are *li* (theory), *yi* (principle), *fa* (method) and *shu* (calculation) in it. Without understanding the theory we cannot derive the method; without grasping the principle we cannot do the calculation. It may require hard work to understand the theory and

to grasp the principle, but it takes routine work
to derive the method and to do the calculation."

As a scholar brought up in the Confucian tradition XU Guang-qi even saw in *Elements* the derived benefit in moral education. In an essay titled *Ji He Yuan Ben Za Yi* (Various Reflections on *Ji He Yuan Ben*) written in 1607 he said, "Five categories of personality will not learn from this book: those who are impetuous, those who are thoughtless, those who are complacent, those who are envious, and those who are arrogant. Thus to learn from this book one not only strengthens one's intellectual capacity but also builds a moral base."

For an in-depth analysis of the translation of *Ji He Yuan Ben* readers are strongly recommended to consult the book by Peter Engelfriet, which is a revised and expanded version of his 1996 doctoral dissertation at Leiden University [Engelfriet, 1998]. For an analysis of the work of XU Guang-qi in synthesizing Western mathematics and ancient Chinese mathematics in the context of mathematics in the Ming period readers may consult a paper of Peter Engelfriet and this author [Engelfriet & Siu, 2001]. For a general discussion on the contribution of XU Guang-qi in fostering development in science in 17th century China, readers may consult a paper of this author [Siu, 1995/1996].

It may be of interest, if just for the sake of speculation, to raise a few hypothetical questions:

(1) How much would XU Guang-qi have achieved in mathematics if he had concentrated his effort on this one discipline?

(2) What would have happened if he had known about the various commentaries on the controversial Fifth Postulate?

(3) What would have happened if he had mastered Latin just as Ricci had mastered Chinese?

(4) What would have happened if he had the chance and the inclination to actually pay a visit to Europe at the time and to return to China with what he experienced and observed over there?

Nothing of that sort happened in history. Besides cultural obstacle there were at the time adverse social and political factors that did not work in favour of the first dissemination of Western learning in China. "Ironically, the ready acceptance of Western science by this small circle of open-minded scholar-officials, as exemplified by XU Guang-qi, also turned out to be a reason for their failure, for in the eyes of the conservative ministers and the general populace, this small group of converts were over-enthusiastic about the alien culture. They lacked the support of the host culture, so to speak."[Siu, 1995/1996, p.171].

4 An inscribed square in a right-angled triangle

Through working out a series of problems built around one specific question, that of an inscribed square in a right-angled triangle, we will compare the styles and emphases of mathematical pursuit in the Eastern and the Western traditions. In the following problems the labelling in the figure refers to that specified in the corresponding passage, sometimes with an accompanying figure. (This exercise was actually carried out in a three-hour workshop in the ESU6.)

Problem 1: Given a right-angled triangle *ABC* with *AC* as its hypotenuse, how would you inscribe a square in it, i.e., construct a square *BFED* with *D* on *AB*, *E* on *AC*, and *F* on *BC* (Figure 1)?

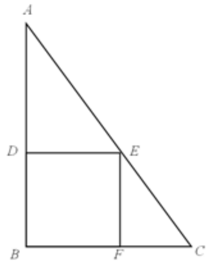

Figure 1

Remarks: There are various ways to solve this problem. One way that is close to the style of Euclid would be to bisect $\angle ABC$ by *BE* (with *E* on *AC*) [justified by I.9], then drop perpendiculars *ED*, *EF* (with *D* on *AB* and *F* on *BC*) [justified by I.12]. It can be proved that *BFED* is the inscribed square we want. (Throughout this section, I.9 means Proposition 9 in Book I of Euclid's *Elements*, etc.)

Another way is to first construct a square *ABB′A′* with *AB* as one side. Join *BA′*, which intersects *AC* at *E*. Drop perpendiculars *ED*, *EF* (with *D* on *AB* and *F* on *BC*). It can be proved that *BFED* is the inscribed square we want. The second way may look just like the first way, but the second way can be generalized readily to construct an inscribed square in an arbitrary triangle *ABC*, which is not necessarily right-angled. To do this, drop a perpendicular *AH* to *BC* (with *H* on *BC*). Construct the square *AHB′A′* with *B′* on *BC* and on the other side of *AH* as *B*. Join *BA′* to intersect *AC* at *E*. Draw *EDG* parallel to *BC* (with *G* on *AB* and *D* on *AH*). Drop perpendiculars *EF*, *GI* (with *F*, *I* on *BC*). It can be proved that *IFEG* is the inscribed square we want.

There are yet other ways to construct an inscribed square in an arbitrary triangle *ABC*. For instance, erect a square *WZYX* inside the triangle *ABC* (with *X* on *AB* and *WZ* on and inside *BC*). Join *BY* and produce to intersect *AC* at *E*. Draw *EG* parallel to *BC* (with *G* on *AB*) and drop perpendiculars *GI* and *EF* (with *I*, *F* on *BC*). It can be proved that *IFEG* is an inscribed square in triangle *ABC*. Or one can carry out a similar procedure by starting with a square on BC that lies outside the triangle *ABC*.

It is interesting to note a construction by the English mathematician John Speidell in his book *A geometrical extraction*, *or* , *A compendious collection of the chiefest and choisest problems* (1616), which somehow combines the feature of the problem for a right-angled triangle and an arbitrary triangle (Figure 2).

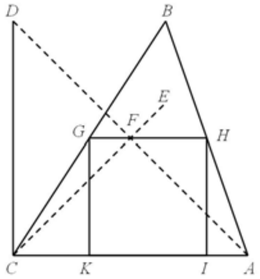

Figure 2

Erect a perpendicular *CD* to the base *CA* with *CD* equal to the height of *B* above *CA*, then bisect $\angle ACD$ (= a right angle) by *CE*. Let *CE* intersect *AD* at *F*. Draw *GFH* (with G on BC and H on AB) parallel to CA. Drop perpendiculars GK, HI (with K, I on CA). It can be proved that $KIHG$ is the inscribed square we want.

There is a common feature in all of the different methods exhibited above that is characteristic of the style of Greek geometry expounded in Euclid's *Elements*. In Euclid's exposition of geometry a definition (for instance, an inscribed square in a given triangle) does not guarantee existence. Existence is justified by a construction. Each one of these methods actually constructs such an inscribed square in a given triangle. Before one is not even certain whether such an inscribed square exists or not, one would not go ahead to calculate the length of its side.

Now that we know such an inscribed square exists we can ask what the length of its side is. It can be shown from each construction that the side x of the inscribed square in a right-angled triangle with sides of length a, b containing the right angle is given by $x = \dfrac{ab}{a+b}$ (Exercise). More generally, for an arbitrary triangle ABC with base $BC = b$ and altitude $AH = h$, the side x of the inscribed square $IFEG$ (with I, F on BC, G on AB and E on AC) is given by $x = \dfrac{hb}{h+b}$ (Exercise).

Problem 2: **Problem 1** appears as Problem 15 of Chapter 9 in *Jiu Zhang Suan Shu* (Nine Chapters on the Mathematical Art) compiled between 100 B.C. and A.D. 100. Study the original text (English translation in **Appendix 1**) and explain the formula by **two** different proofs given in the commentary by LIU Hui in the mid 3rd century (Figure 3).

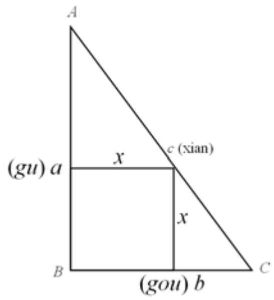

Figure 3

Remarks: The first method is a "visual proof" of the formula $x = \dfrac{ab}{a+b}$ by dissecting and re-assembling coloured pieces (Figure 4). A similar but more interesting computation was devised by LIU Hui for the next problem in the book, of an inscribed circle of a right-angled triangle [Siu, 1993, pp.349-352].

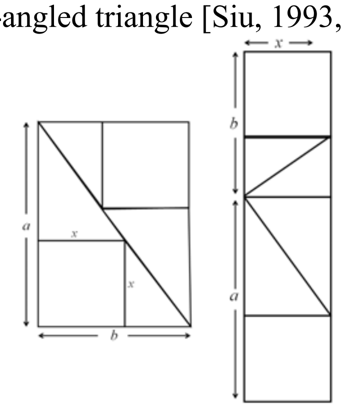

Figure 4

The second method is based on the theory of proportion, making use of the so-called *Jinyou method* (known as the *Rule of Three* in the western world) and the *principle of invariant ratio* (which is basically the same as the content of I.43). Although there was no theory of similar triangles developed in ancient Chinese mathematics, a special case of it in the situation of right-angled triangles was frequently employed with dexterity and proved to be rather adequate for most purposes.

Problem 3: **Problem 1** does not appear in Euclid's *Elements* but appears as a particular case of Added Proposition 15 of Book VI in *Euclidis Elementorum Libri XV* compiled by Christopher Clavius in 1574, which was translated into Chinese in *Ji He Yuan Ben* of 1607. Study the original text (English translation in **Appendix 2**) and compare this explanation with that of LIU Hui's, or that of your own (in **Problem 1**). Is there a different emphasis in these explanations?

Remarks: The construction is effected by dropping the perpendicular AD on BC (with D on BC), and divide AD at E such that $AE : ED = AD : BC$. In the original text this construction was justified by Added Proposition 1 of Book VI of *Ji He Yuan Ben*, which may be wrongly ascribed; the more likely justification seems to be Proposition 10 of Book VI within the same book (Figure 5).

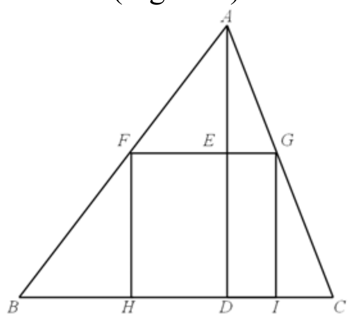

Figure 5

There is a supplemented method that is just a specialization in the case when ABC is a right-angled triangle with $\angle ABC$ equal to a right angle (Figure 1). An appended remark at the end says that the side of the inscribed square $BFED$ in a right-angled triangle ABC must be a mean proportion of AD, FC, thus affording a motivation of the construction.

Problem 4: In 1609 XU Guang-qi wrote *Gou Gu Yi* (Principle of the Right-angled Triangle) in which he attempted to synthesize knowledge about a right-angled triangle contained in *Jiu Zhang Suan Shu* and in Euclid's *Elements* (or more precisely, the version by Clavius from which he learn Euclidean geometry). In particular, Problem 4 is about an inscribed square in a right-angled triangle. Study the original text (English translation in **Appendix 3**). In your opinion to what extent did XU Guang-qi succeed in accomplishing the synthesis (Figure 6)?

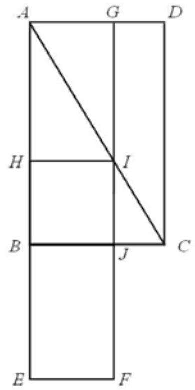

Figure 6

Remarks: Xu Guang-qi started with the formula

$$HB = BJ = JI = IH = \frac{AB \times BC}{AB + BC}$$

and tried to reduce back to Added Proposition 15 in Book VI of *Ji He Yuan Ben*, that is, prove that H divided AB such that $AH : HB = AB : BC$. He made use of a number of results of reciprocally related figures in Book VI of *Ji He Yuan Ben*. His proof may sound rather round-about and awkward, an indication of an "unnatural" attempt to combine two different styles that may not be as compatible! But we should admire the intention of XU Guang-qi in this effort of what he described as "*hui tong* (to understand and to synthesize)".

Problem 5: Added Proposition 15 of Book VI in *Euclidis Elementorum Libri XV* actually gives the answer to a more general problem, which specializes to the formula for the case of a right-angled triangle. **(a)** Devise a proof by dissection along the line of thinking of LIU Hui. **(b)** In the case of a right-angled triangle the answer to the general problem would give two different ways to "inscribe a square in a right-angled triangle". Compare these two ways.

Remarks: Not to spoil the fun of the reader, a solution to **(a)** will be left as an exercise. The two ways in **(b)** give different answers.

Problem 6: In what way is the result in **Problem 2** a special case of the formula offered by the Indian mathematician Bhaskara (also known as Bhaskara II or Bhaskaracharya) in Problem 161 of Chapter 5 of *Lilavati* (12^{th} century)? Problem 161 is about two vertical poles, the top of each being connected by a string to the bottom of the other. One is asked to compute the height of the intersecting point of the strings from the ground.

Remarks: If the two poles of height a, b are at a distance ℓ apart, then it can be seen that the height x of the intersecting point above ground is given by $x = \dfrac{ab}{a+b}$ (Exercise). In other words, x is the harmonic mean of a and b, independent of ℓ ! (Explain this independence geometrically.) The form of the relationship rings a bell. When ℓ is made equal to a, it becomes apparent that x is nothing but the side of the inscribed square in a right-angled triangle.

A comparison of the methods in **Problem 1** and **Problem 2** will show a general difference in approach between ancient Chinese mathematics and Greek mathematics. Roughly speaking we can borrow the terms "algorithmic mathematics" and "dialectic mathematics" coined by Peter Henrici [Henrici, 1974, p.80] to describe the two approaches. Ideally speaking these two approaches should complement and supplement each other with one containing some part of the other like *yin* and *yang* in Chinese philosophy. Further discussion on these two approaches and cognitive thinking in the West and East revealed in the activity of proof and proving may be found in another two papers of this author on mathematical proofs [Siu, 2009a; Siu, 2011].

5 Influence exerted by *Ji He Yuan Ben* in China

In his essay, *Ji He Yuan Ben Za Yi* of 1607 XU Guang-qi commented:

> "The benefit derived from studying this book is many. It can dispel shallowness of those who learn the theory and improve their concentration. It can supply fixed methods for those who apply to practice and kindle their creative thinking. Therefore everyone in this world should study this book."

But realizing the actual situation he also commented in the same essay:

> "This book has wide applications and is particularly needed at this point in time. […] In the preface Mister Ricci also expressed his wish to promulgate this book so that it can be made known to everybody who will then study it. Few people study it. I surmise everybody will study it a hundred years from now, at which time they will regret that they study it too late. They would wrongly attribute to me the foresight [in introducing this book], but what foresight have I really?"

However, near to a hundred years later, the situation was still far from what he would like to see. In the preface to *Shu Xue Yao* (The Key to Mathematics) written by DU Zhi-geng (second part of 17th century) in 1681, LI Zi-jin (1622-1701) said, "Even those gentlemen in the capital who regard themselves to be erudite scholars keep away from the book [*Elements*], or close it and do not discuss its content at all, or discuss it with incomprehension and perplexity."

The Chinese in the 17th and 18th centuries did not seem to feel the impact of the essential feature of Western mathematics exemplified in Euclid's *Elements* as strongly as XU Guang-qi. Thus, the influence of the newly introduced Western mathematics on mathematical thinking in China was not as extensive and as directly as XU Guang-qi had imagined. The effect was gradual and became apparent only much later. However, the fruit was brought forth elsewhere, not in mathematics but perhaps of an even higher historical importance.

Three leading figures responsible for the so-called "Hundred-day Reform" of 1898 — KANG You-wei (1858-1927), LIANG Qi-chao (1873-1929), TAN Si-tong (1865-1898) — were strongly influenced by their interest in acquiring Western learning. In 1888 KANG You-wei wrote a book titled *Shi Li Gong Fa Quan Shu*

(Complete Book on Concrete Principles and Postulates [of Human Relationship]), later incorporated into his masterpiece *Da Tong Shu* (Book of Great Unity) of 1913. It carries a shade of the format of *Elements*, as the title suggests. The book *Ren Xue* (On Moral Philosophy) written by TAN Si-tong and published posthumously in 1899, carries an even stronger shade of the format of *Elements*, reminding one of the book *Ethics* by Baruch Spinoza (1632-1677) of 1675 that began with definitions and postulates. To educate his countrymen in modern thinking TAN Si-tong established in 1897 a private academy known as the Liuyang College of Mathematics in his hometown, stating clearly in a message on the mission of the college that mathematics is the foundation of science, and yet the study starts with mathematics but does not end with it. Apparently, he was regarding mathematics as assuming a higher position than just a technical tool in the growth of a whole-person in liberal education. In his famous book *Qing Dai Xue Shu Gai Lun* (Intellectual Trends of the Qing Period), originally published in *Reform Magazine* in 1920/1921, LIANG Qi-chao remarked (English translation by Immanuel C.Y. Hsü [Liang, 1959]):

> "Since the last phase of the Ming, when Matteo Ricci and others introduced into China what was then known as Western learning (*xi xue*), the methods of scholarly research had changed from without. At first only astronomers and mathematicians credited [the new methods], but later on they were gradually applied to other subjects."

The "Hundred-day Reform" ended in failure despite the initiation and support of Emperor Guangxu (reigned 1875-1908) because of the political situation of the time. TAN Si-tong met with the tragic fate of being arrested and executed in that same year, while KANG You-wei and LIANG Qi-chao had to flee the country and went to Japan. This was one important step in a whole series of events that culminated in the overthrow of Imperial Qing and the establishment of the Chinese Republic in 1911.

Within mathematics itself, *Ji He Yuan Ben* did have some influence, gradual as it was. For a more detailed discussion readers are recommended to consult the book by Peter Engelfriet [Engelfriet, 1998]. By the first part of the 20th century the Chinese began to appreciate the deeper meaning of *Elements*. An illuminating remark came from an eminent historian CHEN Yin-ke (1890-1969) who said in an epilogue to the Manchurian translation of *Ji He Yuan Ben* in 1931 (translated into English by this author):

> "The systematic and logical structure of Euclid's book is of unparalleled preciseness. It is not just a book on number and form but is a realization of the Greek spirit. The translated text in the Manchurian language and the version in *Shu Li Jing Yun* (Collected Basic Principles of Mathematics) are edited to lend emphasis on utility of the subject, not realizing that, by so doing, the original essence has been lost."

6 The three waves of transmission of European science into China

The translation of *Elements* by XU Guang-qi and Matteo Ricci led the way of the first wave of transmission of European science into China, while a second wave and a third wave followed in the Qing Dynasty, but each in a rather different historical context.

The gain of this first wave seemed momentary and passed with the downfall of the Ming Dynasty. "Looking back we can see its long-term influence, but at the time this small window which opened onto an amazing outside world was soon closed again, only to be forced open as a wider door two hundred years later by Western gunboats that inflicted upon the ancient nation a century of exploitation and humiliation, thus generating an urgency to know more about the Western world." [Siu, 1995/1996, pp.170-171].

The second wave came in the wake of the first wave and lasted from the mid 17th century to the mid 18th century. Instead of Chinese scholar-officials the chief promoter was Emperor Kangxi of Qing Dynasty (reigned 1654-1722). Instead of Italian and Portugese Jesuits the western partners were mainly French Jesuits, the so-called "King's Mathematicians" sent by Louis XIV, the "Sun King" of France (reigned 1643-1715), in 1685 [Du & Han, 1992].

This group of Jesuits, led by Jean de Fontaney (1643-1710) reached Peking in 1688. An interesting account of their lives and duties in the Imperial Court was recorded in the journal written by one of the group, Joachim Bouvet (1656-1730) [Bouvet, 1697]. By imperial decree an intensive course of study on Western science and mathematics was organized to take place in the Imperial Palace, with the French Jesuits as tutors, for Emperor Kangxi and some of the princes. The happenings of this second wave form an interesting and intricate story that cannot be discussed in detail in this paper for want of space. It reflects an attitude of learning when the student (Emperor Kangxi) regards himself in a much more superior position than his teachers! A main conclusion is the compilation of a monumental one-hundred-volume treatise *Lu Li Yuan Yuan* (Origins of Mathematical Harmonics and Astronomy) commissioned by Emperor Kangxi, worked on by a large group of Jesuits, Chinese scholars and official astronomers. The project started in 1713 and the treatise was published in 1722/1723, comprising three parts: *Li Xiang Kao Cheng* (Compendium of Observational Computational Astronomy), *Shu Li Jing Yun* (Collected Basic Principles of Mathematics), *Lu Lu Zheng Yi* (Exact Meaning of Pitchpipes). Interested readers will find a more in-depth discussion of this second wave in a paper of Catherine Jami [Jami, 2002].

The third wave came in the last forty years of the 19th century in the form of the so-called "Self-strengthening Movement" after the country suffered from foreign exploitation during the First Opium War (1839-1842) and the Second Opium War (1856-1860). This time the initiators were officials led by Prince Gong (1833-1898) with contribution from Chinese scholars and Protestant missionaries coming from England or America, among whom were LI Shan-lan and Alexander Wylie who completed the translation of *Elements*. In 1862 *Tong Wen Guan* (College of Foreign Languages) was established by decree, at first serving as a school for studying foreign languages to train interpreters but gradually expanded into an institute of learning Western science. The slogan of the day, which was "learn the strong techniques of the

“[Western] barbarians” in order to control them”, reflected the purpose and mentality during that period. In 1866 a mathematics and astronomy section was added to *Tong Wen Guan*, with LI Shan-lan as its head of department. In 1902 *Tong Wen Guan* became part of Peking Imperial University, which later became what is now Beijing University [Siu, 2009b, pp.203-204]. For a general discussion on the history of the rise of modern China readers may consult some standard texts [Fairbank & Reischauer, 1973; Hsü, 1970/2000].

The theme and mood of the three waves of transmission of European science into China were reflected in the respective slogans prevalent in each period. In the first part of the 17th century the idea was: “In order to surpass we must try to understand and to synthesize.” In the first part of the 18th century it became: “Western learning has its root in Chinese Learning.” In the latter part of the 19th century the slogan took on a very different tone: “Learn the strong techniques of the ‘[Western] barbarians’ in order to control them.”

In a paper on European science in China Catherine Jami says:

> “[…] the cross-cultural transmission of scientific learning cannot be read in a single way, as the transmission of immutable objects between two monolithic cultural entities. Quite the contrary: the stakes in this transmission, and the continuous reshaping of what was transmitted, can be brought to light only by situating the actors within the society in which they lived, by retrieving their motivations, strategies, and rationales within this context.” [Jami, 1999, p.430]

In a paper on the life and work of XU Guang-qi this author once suggested:

> “It will be a meaningful task to try to trace the “mental struggle” of China in the long process of learning Western science, from the endeavour of XU Guang-qi, to the resistance best portrayed by the vehement opposition of YANG Guang-xin, to the promulgation of the theory that “Western science had roots in ancient China”, to the self-strengthening movement, and finally to the “naturalization” of western science in China. It is a complicated story embedded in a complicated cultural-socio-political context.” [Siu, 1995/1996, p.171]

In the words of the historian Immanuel Hsü, this “mental struggle” is “an extremely hard struggle against the weight of pride and disdain for things foreign, and the inveterate belief that the bountiful Middle Kingdom had nothing to learn from the outlandish barbarians and little to gain from their association.” [Hsü, 1970/2000, p.10] Viewed in this light the attempt and foresight of XU Guang-qi stand out all the more unusual, visionary and admirable.

Appendix 1

Now given a right-angled triangle whose *gou* is 5 *bu* and whose *gu* is 12 *bu*. What is the side of an inscribed square? The answer is 3 and 9/17 *bu*.

Method (See Figure 3): Let the sum of the *gou* and the *gu* be the divisor; let the product of the *gou* and the *gu* be the dividend. Divide to obtain the side of the square.

Commentary of Liu Hui: The product of the *gou* and the *gu* is the area of a rectangle comprising crimson triangles, indigo triangles and yellow squares, each in two. Place the two yellow squares at the two ends; place the crimson triangles and indigo triangles, with figures of the same type combined together, in between so that their respective *gu* and *gou* coincide with the side of the yellow square. These pieces form a rectangle. Its width is the side of the yellow square; its length is the sum of the *gou* and *gu*. Hence the sum of the *gou* and the *gu* becomes the divisor. In the figure of the right-angled triangle with its inscribed square, on the two sides of the square there are smaller right-angled triangles, for which the relation between their sides retains the same ratio as that of the original right-angled triangle. The respective sums of the smaller *gou* and *gu* of the right-angled triangle on the *gou* [which is equal to the *gou*] and that of the smaller *gou* and *gu* of the right-angled triangle on the *gu* [which is equal to the *gu*] become the mean proportion. Let the *gu* be the mean proportion and the sum of the *gou* and the *gu* be the other term of the ratio. Apply Rule of Three to obtain the side of the inscribed square with the *gou* being 5 *bu*. Let the *gou* be the mean proportion and the sum of the *gou* and the *gu* be the other term of the ratio. Apply Rule of Three to obtain the side of the inscribed square with the *gu* being 12 *bu*. This [second] method does not follow the method explained at the beginning, but it produces the dividend and the divisor. In the next problem on the inscribed circle of a right-angled triangle when we utilize Rule of Three and Rule of Proportional Distribution, this method again becomes apparent.

Appendix 2

Added Proposition 15 of Book VI: Given a triangle, it is required to produce its inscribed square.

Method (See Figure 5): If *ABC* is an acute-angled triangle and it is required to produce its inscribed square, through *A* construct *AD* [*D* on *BC*] which is perpendicular to *BC*. Divide *AD* at *E* such that *AE* : *ED* = *AD* : *BC* (Book VI, Proposition 1, Added Proposition (?)) . Through *E* construct *FG* [*F* on *AB* and *G* on *AC*] parallel to *BC*. From *F* and *G* respectively construct *FH* [*H* on *BC*] and *GI* [*I* on *BC*] parallel to *ED*. The figure *FHIG* is what is required to produce. If the triangle is right-angled or obtuse-angled, then drop the perpendicular from the right angle or the obtuse angle respectively and proceed as before.

Proof: FEG is parallel to *BC*, so *BD* : *DC* = *FE* : *EG* (Book VI, Proposition 4, Added Proposition). By ratio componendo *BC* : *DC* = *FG*: *EG*. We also have *DC* : *AD* = *EG* : *AE* (Book VI, Proposition 4, Corollary). By ratio ex aequali *BC* : *AD* = *FG* : *AE*. We also have *AD* : *BC* = *AE* : *ED* [and *BC* : *AD* = *FG* : *AE*]. By ratio ex aequali *BC* : *BC* = *FG* : *ED*. Since *BC* and *BC* are equal, we have *FG* and *ED* are equal. *FG* is equal to *HI* (Book I, Proposition 34). *ED*, *FH* and *GI* are all equal, so the four sides *FG*, *GI*, *IH*, *HF* are all equal. *EDH* is a right angle, so *FHD* is a right angle

(Book I, Proposition 29). The other angles are also right angles. Hence *FHIG* is a square.

Supplemented method (See Figure 1): If in a right-angled triangle *ABC* it is required to produce its inscribed square with *ABC* as one of its right angle, then divide the perpendicular *AB* at *D* such that *AD* : *DB* = *AB* : *BC* (Book VI, Proposition 10). Through *D* construct *DE* [*E* on *AC*] parallel to *BC*. Through *E* construct *EF* [*F* on *BC*] parallel to *AB*. The figure *DBFE* is what is required to produce.

Proof: *BC* : *AB* = *DE* : *AD* (Book VI, Proposition 4, Corollary) and *AB* : *BC* = *AD* : *DB*. By ratio ex aequali *BC* : *BC* = *DE* : *D*B. Since *BC* and *BC* are equal, we have *DE* and *DB* are equal. Hence *DBFE* is a square.

Appended: In the right-angled triangle *ABC* it is required to produce its inscribed square with *ABC* as one of its right angle. The side of this inscribed square must be a mean proportion of *AD* and *FC*. This is because *AD* : *DE* = *EF* : *FC* (Book VI, Proposition 4, Corollary).

Appendix 3

Method (See Figure 6): *Gu AB* is 36, *gou BC* is 27. It is required to produce its inscribed square. Let the product of *gou* and *gu* be the dividend. Let the sum of *gou* and *gu* be the divisor, which is *AE* equal to 63. Divide and obtain each side of the inscribed square, *HB* and *BJ*, to be 15.428.

Proof: *AB* = 36, *BC* = 27. Let their product 972 be the dividend. This is the [area of the] rectangle *ABCD*. Let the sum 63 be the divisor. This is the straight line *AE*. Divide to obtain the side *EF* to be 15.428. This makes the rectangle *AEFG* equal in area to the rectangle *ABCD* (Book VI, Proposition 16). Let *FG* intersect *BC* at *J* and *AC* at *I*, then the figure *BHIJ* is an inscribed square of the right-angled triangle *ABC*.

Why? *ABCD* and *AEFG* are reciprocally related figures, that is, *AB* : *AE* = *BJ* : *BC* (Book VI, Proposition 15 (14?)). By ratio dividendo *AB* : *BE* = *BJ* : *JC*, that is, *AB* : *BC* = *BJ* : *JC* (because *BC* = *BE*). We also have *AH* : *HI* = *IJ* : *JC* (Book VI, Proposition 4). By ratio alternando *AH* : *IJ* = *HI* : *JC*. But *HB* = *IJ*, *BJ* = *HI*. Therefore *AH* : *HB* = *BJ* : *JC*. Since *AB* : *BC* = *BJ* : *JC* and *AH* : *HB* = *BJ* : *JC*, we have *AB* : *BC* = *AH* : *HB*. Hence *BHIJ* is an inscribed square (Book VI, Added Proposition 15).

REFERENCES

– Bernard, H., 1935, *Matteo Ricci's Scientific Contribution to China*, Peiping (Beijing): Henri Vetch.
– Bouvet, J., 1697, *Histoire de l'empereur de la Chine*: *presentée au Roy*, Paris: Etienne Michallet.
– Du, S.R., Han, Q., 1992, "The contribution of French Jesuits to Chinese science in the seventeenth and eighteenth centuries", *Impact of Science on Society* **167**, 265-275.
– Engelfriet, P., 1998, *Euclid in China: The Genesis of the First Chinese Translation of Euclid's Elements Book I-VI* (*Ji He Yuan Ben; Beijing, 1607) and its Reception Up to 1723*, Leiden: Brill.
– Engelfriet, P., Siu, M.K., 2001, "XU Guang-qi's attempts to integrate western and Chinese mathematics", in *Statecraft & Intellectual Renewal in Late Ming China*: *The Cross-cultural Synthesis of XU Guang-qi* (*1562-1633*), C. Jami, P. Engelfriet & G. Blue (eds.), Leiden: Brill, pp.279-310.
– Fairbank, J.K., Reischauer, E.O., 1973, *China*: *Tradition and Transformation*, Boston: Houghton Mifflin.
– Henrici, P., 1974, "Computational complex analysis", *Proceedings of Symposia in Applied Mathematics* **20**, 79-86.
– Hsü, C.Y., 1970/2000, *The Rise of Modern China*, Oxford: Oxford University Press.
– Huang, R., 1981, *1587*, *A Year of No Significance*, New Haven: Yale University Press.

– Huang, R., 1988/1997, *China*: *A Macro History*, Armonk: M.E. Sharpe.
– Jami, C., 1999,““European science in China” or “Western Learning”? Representations of cross-cultural transmission, 1600-1800”, *Science in Context* **12(3)**, 413-434.
– Jami, C., 2002, “Imperial control and western learning: The Kangxi Emperor’s performance”, *Late Imperial China* **23(1)**, 28-49.
– Liang, Q.C., 1959, *Intellectual Trends of the Qing Period* (Translated by Immanuel C.Y. Hsü), Cambridge: Harvard University Press.
– Ricci, M., Gallagher, I.J. (trans.), 1942/1953, *China in the Sixteenth Century*: *The Journals of Mathew Ricci, 1583-1610*, New York: Random House.
– Siu, M.K., 1993, “Proof and pedagogy in ancient China: Examples from Liu Hui’s commentary on Jiu Zhang Suan Shu”, *Educational Studies in Mathematics* **24**, 345-357.
– Siu, M.K., 1995/1996, “Success and failure of XU Guang-qi: Response to the first dissemination of European science in Ming China”, *Studies in History of Medicine and Science*, *New Series* **14(1-2)**, 137-178.
– Siu, M.K., 2007, ““Mr Ou [Euclid]” in China for four hundred years” [in Chinese], *Science & Culture Review* **4(6)**, 97-114.
– Siu, M.K., 2009a, “The algorithmic and dialectic aspects in proof and proving”, in *Proceedings of the ICMI Study 19 on Proof and Proving*, *Volume 2*, F.L.Lin, F.J.Hsieh, G.Hanna & M. de Villiers (eds.), Taipei: National Taiwan Normal University, pp.160-165.
– Siu, M.K., 2009b, “Mathematics education in East Asia from antiquity to modern times”, in *Dig Where You Stand: Proceedings of a Conference on On-going Research in the History of Mathematics Education, Gardabaer*, *June 20-24*, *2009*, K. Bjarnadottir, F. Furinghetti & G. Schubring (eds.), Reykjavik: University of Iceland, pp.197-208.
– Siu, M.K., 2011, Proof within the Western and the Eastern cultural traditions: Implications for mathematics education, in *19th ICMI Study Volume, May 2009, Taipei,* G. Hanna, M. de Villiers (eds.), to appear.
– Xu, Y.B., 2005, “The first Chinese translation of the last nine books of Euclid’s Elements and its source”, *Historia Mathematica* **32 (1)**, 4-32.