Investors in Housing Market Search*

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Abstract

We add specialist investors—agents who attempt to profit from buying low and selling high—to a canonical housing market search model. These agents facilitate the turnover of mismatched houses on behalf of end-users and they may survive even if they face an arbitrarily large cost of financing vis-a-vis ordinary households. Multiple equilibrium may exist. In one equilibrium, the participation of investors is extensive, resulting in rapid turnover, a high vacancy rate, and high housing prices. In another equilibrium, few houses are bought and sold by investors. Turnover is sluggish, few houses are vacant, and prices are moderate. A decline in the rate at which investors finance investment, can rather paradoxically, lower investors participation and housing prices in equilibrium.

Key words: Search and matching, housing market, investors, liquidity, flippers

1 Introduction

In many housing markets, the purchases of owner-occupied houses by individuals who attempt to profit from buying low and selling high rather than for occupation are commonplace. For a long time, anecdotal evidence abounds as to how the participation of these specialist investors, who are popularly known as flippers in the U.S., in the housing market can be widespread.[1] More recently, empirical studies on

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[1]Out of the five transactions in a large developement in Hong Kong in August 2010, three were reported to involve investors who buy in anticipation of short-term gains (September 10 2010, *Hong*

the housing market have began to more systematically document the extent to which transactions in the housing market are motivated by a pure investment motive. Notable contributions include Depken et al. (2009) and Bayer et al. (2011). A related strand of investigation, which includes Rosen (2007), Shiller (2008), Wheaton and Nechayev (2008), among others, focuses on the role played by the investment motive in the housing market "bubble" in the U.S. in the late-1990s to the mid-2000s. A common theme in the discussion is that housing market flippings are destabilizing. Indeed, such sentiments are not uncommon among the public and policymakers in many places. In the U.S. in particular, interest deductibility applies to the first and second homes only and capital gains tax may only be exempted for properties sold after the first two years of purchase.[2]

In contrast, market markers in the financial markets, who buy and sell stocks for short-term gains and profit the bid-ask spreads in the interim, are often hailed for their role in providing liquidity to the market.[3] If the market makers in the financial market can help improve market liquidity, there should certainly be ample room for the flippers in the housing market to do just the same if not more, given that the housing market is far less liquid than the financial market.

The latter tendency, of course, is due to the fact that houses differ from one another along a large number of dimensions, and unlike many financial assets, are traded without the benefit of an organized exchange. Indeed, this recognition has prompted a sizeable literature on the application of the search and matching framework in Mortensen and Pissarides (1994) first developed for the labor market to study the determination of vacancy, turnover, sales volume, price, among other variables, in the housing market.[4] A common feature of the models in this literature is that the agents are exclusively end-users. A buyer is a household looking for a good match for occupation. A seller is a household which no longer finds its old house a desirable place of residence and is trying to sell it to someone who finds it as such. There are no specialist investors or flippers around.

Kong Economic Times). According to one industry insider, among all buyers of a new development in Hong Kong recently, only about 60% are buying for own occupation (November 20, 2010, *Wenweipo*).

[2]In recent times, the Chinese government has put in place severe restrictions on the ownership of a second home in a number of cities. In Beijing in particular, individuals who have not lived in the city for up to five years are barred from the purchase of a owner-occupied home altogether, measures meant to contain the speculative frenzy in the Chinese housing market in the last decade. See "Beijing's housing market bubbles," September 27 2007, *The New York Times*; "Wenzhou investors leave Beijing housing market," February 28 2011, *CNTV*. Similar measures were also in effect in Hong Kong, where since November 2010, housing units that are sold within two years of purchase are subjected to an extra 5-15% ad valorem transaction tax. See "Midland adds to jitters - over extra stamp duty," December 3, 2010, *The Hong Kong Standard.*

[3]See Coughenour and Deli (2002) and Coughenour and Saad (2004) for empirical studies on how market makers help provide liquidity in the stock market.

[4]Arnold (1989), Wheaton (1990), Williams (1995), Krainer (2001), Albrecht et. al. (2007), Diaz and Jerez (2009), Piazzesi and Schneider (2009), Ngai and Tenreyro (2010), to name just a few.

In this paper, we add specialist investors to a canonical housing market search model and study how investors participation affects price, turnover, vacancy, transaction volume, and welfare. In our model, the central role played by investors is to facilitate the turnover of mismatched houses on behalf of households. A crucial assumption is that ordinary households cannot hold more than one house at a time. The assumption, of course, can be justified by the usual liquidity constraint argument. In this case, a household which desires to move because the old house is no longer a good match must first sell it before the household can buy up a new house. In the usual housing market search model, the household must wait out a buyer who finds the old house a good match to arrive, which can be a lengthy process, especially in a buyer's market, one in which sellers outnumber buyers by a significant margin. This opens up profitable opportunities for specialist investors to just buy up the mismatched house at a discount in return for the time spent waiting for the eventual end-user buyer to arrive on behalf of the original owner. Meanwhile, transaction volume, vacancy, and housing price all increase with the extent of investors participation, whereas average Time-On-the-Market (TOM) declines in the interim.

According to Bayer et al. (2011), flippers in the housing market can be novice investors who buy en masse in an up market in the belief that the market may continue to go up and sell in panicky in a down market or can be sophisticated middlemen whose activities help provide liquidity in both up and down markets. Implicit in both popular discussions and previous studies on the investment motive in the housing market is that in reality the majority of flippers are more like novice investors than sophisticated middlemen, and any liquidity the flippers may provide is at best coincidental. If flippers as novice investors destabilize the market and flippers as sophisticated middlemen provide liquidity and help stabilize the market, on the whole, the pure investment motive in the housing market may only end up serving to facilitate the formation of speculative bubbles. Our analysis, however, suggests that the dichotomy may not be clear-cut. For tractability, we restrict attention to analyzing the steady state of a housing market and any kinds of timing market movements and speculative bubbles are ruled out by construction. Even so, wide swings in prices and transactions are distinct possibilities because multiple equilibrium is a natural outcome in a frictional housing market with specialist investors. In one equilibrium, the participation by investors is widespread, prices are high, and turnover is rapid. In another equilibrium, few houses are bought and sold by investors, prices are moderate, and turnover is sluggish. Thus, the participation of specialist investors in the housing market can be a double-edged sword—on the one hand, the investors help improve liquidity; on the other hand, when the extent of their participation can be fickle, the housing market can become more volatile as a result.

Our model has a number of readily testable implications. First, it trivially predicts a positive cross-section relation between transaction price and TOM—households can either sell to specialist investors at a discount or to wait for a better offer from an end-user buyer to arrive—which agrees with the evidence reported in Merlo and

Ortalo-Magne (2004), Leung et al. (2002) and Genesove and Mayer (1997), among others.[5]

Our model also predicts an aggregate positive relation between price and transaction volume on the one hand, and a negative relation between transaction volume and average TOM on the other hand—with greater investors participation, prices and sales both increase, whereas houses on average stay on the market for a shorter period of time. Indeed, these relationships have been important motivations behind much of the recent housing market search and matching literature.[6] Notable contributions include Krainer (2001), Diaz and Jerez (2009), and Ngai and Tenreyro (2010). In particular, Kranier (2001) shows, in various numerical analyzes, how a positive but temporary preference shock can give rise to higher prices and a greater volume of transaction, while Diaz and Jerez's (2009) analysis implies that an adverse shock to construction will shorten TOM, and may possibly lead to higher prices and a greater volume of transaction. The paper by Ngai and Tenreyro (2010) focuses on seasonal cycles in price and sales and they argue that increasing returns in the matching technology play a key role in generating such cycles. In all these papers, the increase in sales should be accompanied by a decline in vacancy—given that when a house is sold, it is sold to an end-user, who will immediately occupy it, vacancy must decline, or at least remains unchanged. In our model, however, when a house is sold, it may be sold to an investor, who will just leave it vacant for the time that it takes for an end-user buyer to arrive. Vacancy need not decline when sales increases, and in our model, it will increase instead.

Figure 1 depicts the familiar positive housing price-transaction volume correlation for the U.S. for the 1991-I to 2010-IV time period.[7] The usual housing market search model predicts that vacancy should decline in the housing market boom in the late-1990s to the mid-2000s and rise thereafter when the market collapses around 2007. Figures 2 and 3, however, show that any decline in vacancy is not apparent in the boom.[8] In fact, if there is any co-movement between vacancy on the one hand and price and transaction on the other hand in the run-up to the peak of the housing market boom in 2006, vacancy appears to have risen along with price and transac-

[5] Albrecht et al. (2007) emphasis another aspect of the results reported in Merlo and Ortalo-Magne (2004), which is that downward price revisions are increasingly likely when a house spends more and more time on the market.

[6] Stein (1995), who explains how the down-payment requirement plays a crucial role in amplifying shocks, is an early non-search-theoretic explanation for the positive relation between price and sales. Hort (2000), Leung et al. (2003), among others, provides recent evidence. Ho and Tse (2006) show that the same relation holds in the cross section.

[7] Housing Price is defined as the nominal house price divided by the CPI. The nominal house price is the transaction-based, seasonally-adjusted house price index from OFHEO. See http://www.fhfa.gov. The CPI is from the Federal Reserve Bank at St. Louis, seaonally-adjusted. We set Housing Price equal to 100 at 1991-I. Transation is from Moody.com. It is the sales in units of single-family homes and condominiums, seaonally-adjusted, and normed by the stock.

[8] Vacancy rate is for owner-occupied homes only and is from the Burean of Census CPS/HVS database.

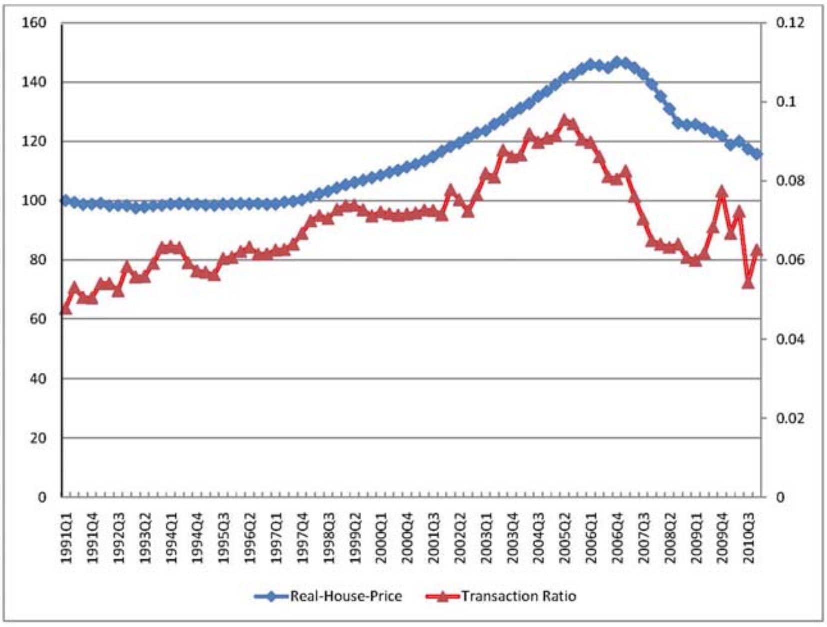

Figure 1: Price and Transaction

tion. While vacancy does not appear to have fallen to follow the market collapse, as predicted by our analysis, the tendency probably is a combined result of the massive amounts of bank foreclosures and unsold new constructions in the market bust—two forces absent in our analysis.

Insofar as the specialist investors in our model act as middlemen between the original homeowners and the eventual end-user buyers, this paper contributes to the literature on middlemen in search and matching pioneered by Rubinstein and Wolinsky (1987). Previously, it was argued that middlemen could survive by developing reputations as sellers of high quality goods (Li, 1998), by holding a large inventory of differentiated products to make shopping less costly for others (Johri and Leach, 2001; Shevchenko, 2004; Smith, 2004), by raising the matching rate in case matching is subject to increasing returns (Masters, 2007), and by lowering distance-related trade costs for others (Tse, 2011). This paper studies the role of middlemen in the provision of market liquidity. A simple model of housing market flippers as middlemen is also in Bayer et al. (2011). The model though is partial equilibrium in nature and cannot be used to answer many of the questions we ask in this paper. A recent paper by Wright and Wong (2011) shows that how, like our model, equilibrium in a search model with middlemen may exhibit bubble-like characters.

The next section presents the model. Section 3 begins with establishing two equilibrium relations between the extent of investors participation and market tightness. We next show the existence of equilibrium, followed by analyzes of the necessary and

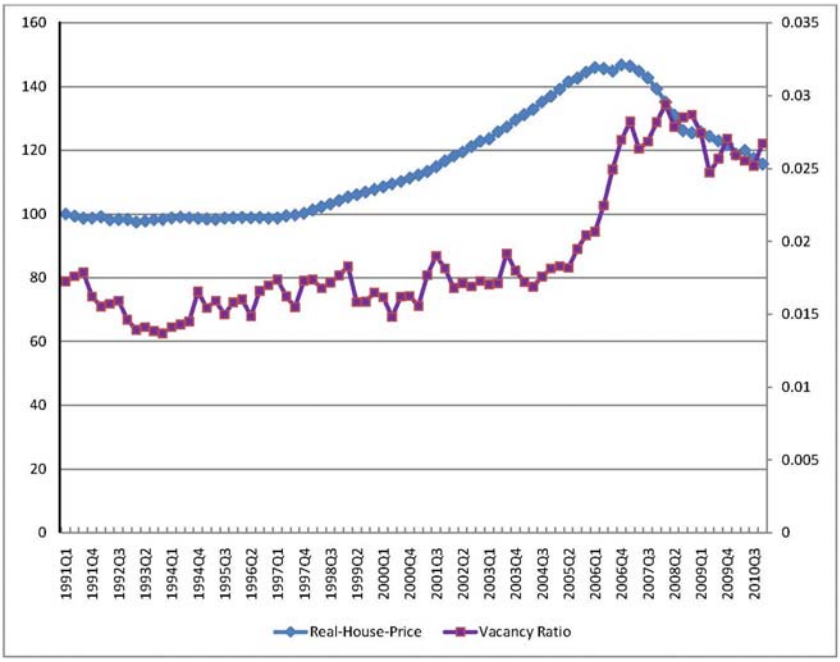

Figure 2: Price and Vacancy

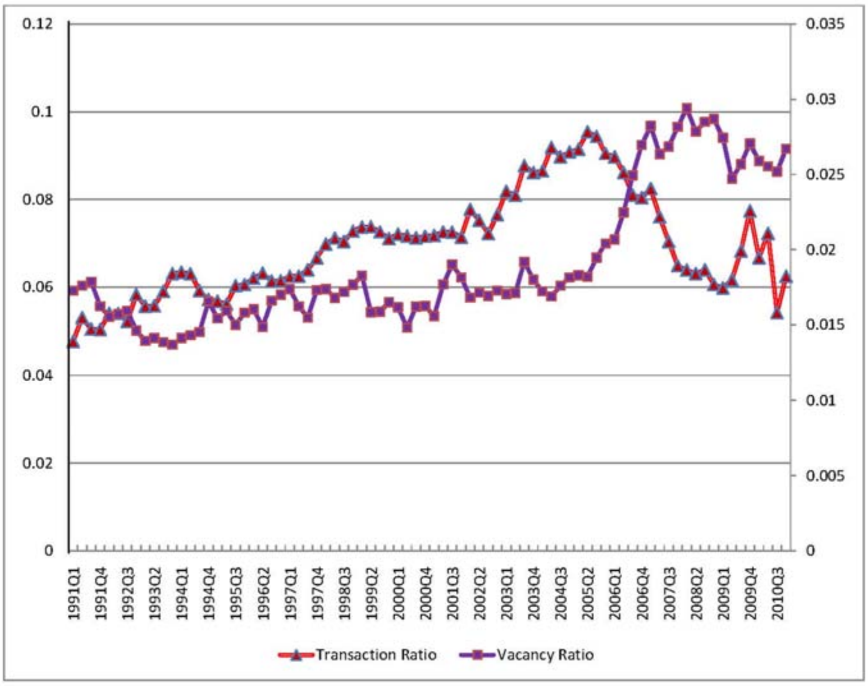

Figure 3: Transaction and Vacancy

sufficient conditions for multiplicity, how changes in investors' cost of financing influence the extent of their participation, housing price, and welfare. The results are then generalized to where the changes in the cost of financing affect both investors and ordinary owner-occupiers alike. Section 4 concludes. The longer and purely technical proofs are relegated to the Appendix.

2 Model

The city is populated by a continuum of measure one risk-neutral households, each of whom discounts the future at the same rate r_H. There are two types of housing in the city: owner-occupied, the supply of which is perfectly inelastic at $H < 1$ and rental, which is supplied perfectly elastically for a fixed rental of q. A household staying in a matched owner-occupied house derives a flow utility of $\upsilon > 0$, whereas a household either in a mismatched house or in rental housing none. A household-house match breaks up exogenously at a Poisson arrival rate δ, after which the household may continue to stay in the house but it no longer enjoys the flow utility υ. In the mean time, the household may choose to sell the old house and search out a new match. A crucial assumption is that a household cannot hold more than one house at a time. Then a mismatched homeowner must first sell the old house before she can buy a new one. The qualitative nature of our results should hold as long as there is a limit, not necessarily one, on the number of houses a household can own at a time. The one-house-limit assumption simplifies considerably.

The search market A seller meets a buyer in the search market at a Poisson arrival rate η, while a buyer finds a match in the same market at another Poisson arrival rate μ. The flow of matches in the search market is governed by a CRS matching function $M(B, S)$, where B and S denote, respectively, the measures of buyers and sellers in the market. Hence,

$$\mu = M(1, \theta), \tag{1}$$

$$\eta = \frac{\mu}{\theta}, \tag{2}$$

where $\theta = S/B$ denotes market tightness,[9] and that

$$\frac{\partial \mu}{\partial \theta} > 0, \quad \frac{\partial \eta}{\partial \theta} < 0.$$

Prices in the search market are determined by Nash bargaining.

[9] This definition of market tightness measures the intensity of competition among sellers. Equivalently, one can define market tightness as B/S to measure the intensity of competition among buyers. We find the first definition more convenient for this analysis.

The Walrasian investment market Instead of waiting out a buyer to arrive in the search market, a mismatched homeowner may sell her old house right away in a Walrasian market populated by specialist investors—agents who do not live in the houses they have bought but rather attempt to profit from buying low and selling high. Because homogeneous investors do not gain by selling and buying houses to and from one another, the risk-neutral investors may only sell in the end-user search market and will succeed in doing so at a rate equal to η, the same rate that any household-seller does in the market. We allow for investors to discount the future at a possibly different rate r_I than the households in the city.[10] In the competitive investment market, prices adjust to eliminate any excess returns in real estate investment.

Ours is not the first paper that assumes both a search market and a Walrasian market in the same model. A well-known paper is Lagos and Wright (2005) in the money search literature. Like Lagos and Wright (2005), our assumption of the coexistence of both kinds of market is more of a modeling device than is meant for realism.[11] Indeed, we recognize that the assumption of a Walrasian investment market seemingly completely contradicts the motivations for applying the search and matching framework to the study of the housing market. We believe that what we need, as a bare minimum, is not an investment market that is free of search frictions of any kind, but one in which the frictions are less severe than in the end-user market. If investors are entirely motivated by arbitrage considerations and do not care if the houses to be purchased are good matches for their own occupations, search should not a particularly serious problem. In reality, we imagine that households who intend to sell quickly and are willing to accept a lower price will convey their intentions to real estate agents, who in turn will play the role of market makers to alert investors the availability of such deals. The competition among investors should then drive prices up to just enough to eliminate any excess returns to investment. A Walrasian market assumption captures the favor of such arrangements in the simplest possible manner.

Asset values for investors Let V_I be the value of a vacant house to an investor and p_{IS} the price she expects to receive in the search market for selling the house. In the steady state,

$$r_I V_I = \eta \left(p_{IS} - V_I \right). \tag{3}$$

[10]While institutional investors may be able to finance investment at a lower interest rate, banks in many places charge higher mortgage interest rates for those who are buying a second home and for those who are not buying a house for occupation. We leave it as an open question as to whether $r_I \lessgtr r_H$ is the more empirically relevant case.

[11]On combining a search market and a Walrasian market also includes the framework introduced in Duffie et al. (2005) and extended in Lagos and Rocheteau (2007). In this framework, an ordinary investor seeks out a dealer through random search, who then trades on behalf of the investor in a competitive market.

Denote as p_{IB} the price the investor has paid for the house in the competitive investment market in the first place. In equilibrium,

$$p_{IB} = V_I. \tag{4}$$

Asset values for households There are three (mutually exclusive) states to which a household can belong:

1. in a matched house; value V_M,
2. in a mismatched house; value V_U,
3. in rental housing; value V_R.

The flow payoff for a matched owner-occupier first of all includes the utility she derives from staying in a matched house υ. The match will be broken, however, with a probability δ, after which the household may sell the house right away in the investment market at price p_{IB}. Alternatively, the household can continue to stay in the house while trying to sell it in the search market. In all then,

$$r_H V_M = \upsilon + \delta \left(\max \left\{ V_R + p_{IB}, V_U \right\} - V_M \right). \tag{5}$$

Let p_H denote the price a household-seller expects to receive in the search market. Then the flow payoff of a mismatched owner-occupier is equal to

$$r_H V_U = \eta \left(V_R + p_H - V_U \right). \tag{6}$$

Two comments are in order. First, in (6), the mismatched owner-occupier is entirely preoccupied with disposing the old house while she makes no attempt to search for a new match. This is due to the assumption that a household cannot hold more than one house at a time and the search process is memoryless. Second, under (5) and (6), the household has only one chance to sell the house in the investment market—at the moment the match is broken. Thereafter, the household must wait out a buyer in the search market to arrive. This restriction is without loss of generality in a steady-state equilibrium, in which the asset values and housing prices stay unchanging over time. No matter, after the house is disposed of, the household moves to rental housing to start searching for a new match. Hence,

$$r_H V_R = -q + \mu \left(V_M - \left(\alpha p_{IS} + (1 - \alpha) p_H \right) - V_R \right), \tag{7}$$

where α denotes the fraction of houses offered for sale in the search market held by investor-sellers and $1 - \alpha$ the fraction held by household-sellers.

Accounting identities Let n_M, n_U, and n_R denote the steady-state measures of matched households, mismatched homeowners, and renters, respectively. The population constraint reads

$$n_M + n_U + n_R = 1. \tag{8}$$

Each owner-occupied house in the city must be held either by a household in the city or by an investor. Hence,

$$n_M + n_U + n_I = H, \tag{9}$$

where n_I denotes both the measures of active investors and houses held by these individuals.

If each household can hold no more than one house at any moment, the only buyers in the search market are households in rental housing; i.e.,

$$B = n_R. \tag{10}$$

On the other hand, sellers in the search market include mismatched homeowners and investors, so that

$$S = n_U + n_I. \tag{11}$$

Housing market flows The flows into matched owner-occupied housing are households in rental housing, who just manage to buy up a house, whereas a fraction δ of matched households become mismatched in each time unit. In the steady state, with n_M stationary,

$$\mu n_R = \delta n_M. \tag{12}$$

Households' whose matches just break up may choose to sell their houses right away to investors or to wait out a buyer to arrive in the search market. Define

$$\Delta = V_R + p_{IB} - V_U \tag{13}$$

as the difference in payoff for a household between selling in the investment market $(V_R + p_{IB})$ and in the search market (V_U). In the steady state, the fraction of households who choose to sell in the investment market, given by

$$\alpha = \begin{cases} 0 & \Delta < 0 \\ [0,1] & \Delta = 0 \\ 1 & \Delta > 0 \end{cases}, \tag{14}$$

is the same fraction of all houses offered for sale in the search market which is held by investors. Hence, if the measure of mismatched homeowners (n_U) is to stay unchanging through time,

$$(1-\alpha)\,\delta n_M = \eta n_U, \tag{15}$$

where ηn_U gives the measure of mismatched homeowners who just manage to dispose of their properties in the search market. Next, those who enter rental housing include households who just sell their properties to investors and to end users, respectively. The exits are comprised of households who just find a match in owner-occupied housing, so that

$$\alpha\delta n_M + \eta n_U = \mu n_R, \tag{16}$$

for n_R to stay stationary. Finally, the measure of houses held by investors increase by the measure of houses recently mismatched households decide to dispose right away in the investment market and decline by the measure of houses investors manage to sell to end-users. Hence, in the steady state,

$$\alpha\delta n_M = \eta n_I. \tag{17}$$

Now, adding (12) and (15) yields exactly (16). Thus, only two of the three conditions constitute independent restrictions. That is, when the flows into and out of any two states for households are equal, the same must hold true for the remaining state. Besides, with

$$\eta = \frac{\mu}{\theta} = \mu\frac{B}{S} = \mu\frac{n_R}{n_U + n_I},$$

equation (15) becomes

$$\mu n_R = (1-\alpha)\,\delta n_M \frac{n_U + n_I}{n_U}.$$

Combine this equation with (12) and simplify,

$$n_U = \frac{1-\alpha}{\alpha} n_I. \tag{18}$$

Substituting the equation back into (15) yields just (17). Thus, indeed only two of the four steady-state flow equations are truly independent restrictions.[12]

Bargaining Prices in the search market fall out of Nash bargaining between a matched buyer-seller pair. There is only one buyer type in the search market—households in rental housing. The sellers, however, are either investors or mismatched homeowners. When a household-buyer is matched with an investor, the division of surplus in Nash Bargaining satisfies

$$V_M - p_{IS} - V_R = p_{IS} - V_I, \tag{19}$$

whereas when the household-buyer is matched with a household-seller, the division of surplus in Nash Bargaining satisfies[13]

$$V_M - p_H - V_R = V_R + p_H - V_U. \tag{20}$$

[12] To verify that the fraction of houses offered for sale in the search market which is held by investors is the same as the fraction of houses sold to investors out of all houses that are offered for sale in the first instance in the steady state, substitute (18) into $n_I/(n_U + n_I)$.

[13] With multiple types, the assumption of perfect information in bargaining is perhaps stretching a bit. We could have specified a bargaining game with imperfect information as in Harsanyi and Selten (1972), Chatterjee and Samuelson (1983), or Riddell (1981), for instance. It is not clear what may be the payoffs for the added complications.

Equilibrium A steady-state equilibrium is made up of (1) matching probabilities and market tightness $\{\mu, \eta, \theta\}$, (2) asset values $\{V_I, V_M, V_U, V_R\}$, (3) measures of households in the various states and investors $\{n_M, n_U, n_R, n_I\}$, (4) measures of buyers and sellers in the search market $\{B, S\}$, (5) the fraction of mismatched households selling houses in the investment market $\{\alpha\}$, and (6) housing prices $\{p_{IB}, p_{IS}, p_H\}$ that satisfy (1)-(20) .

3 Analysis

To proceed analyzing the equilibrium of the model, we divide the equations of the model into two blocks and derive one independent relation between α and θ from each.

3.1 Investors participation, market tightness, and turnovers

The first equation block is made up of the accounting identities and the flow equations: (1), (2), (8)-(12), and (15)-(18), from which we can derive the first relation between α and θ.

Lemma 1 From the accounting identities and the housing market flow equations is an implicit function for θ of α: $\widetilde{\theta} : [0, 1] \to \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$, where

$$\frac{\widetilde{\theta}\,(1-H)}{1-\widetilde{\theta}\alpha} - \frac{\delta H - \mu\left(\widetilde{\theta}\right)(1-H)}{\delta + \mu\left(\widetilde{\theta}\right)\alpha} = 0, \tag{21}$$

and that $\partial\widetilde{\theta}/\partial\alpha < 0$. The lower and upper bounds $\widetilde{\theta}_L$ and $\widetilde{\theta}_U$, both of which are strictly positive and finite, are given by, respectively, the solutions of (21) at $\alpha = 1$ and $\alpha = 0$. Specifically, $\widetilde{\theta}_L < 1$.

Proof. In the Appendix. ■

Given α, once θ is determined by (21), the steady-state measures of households in the various states and investors are uniquely determined. The next Lemma summarizes the results.

Lemma 2

a. At $\alpha = 0$,

$$n_I = 0, \quad n_R = 1 - H, \quad n_M = H - n_U,$$

whereas n_U is given by the solution to

$$\frac{n_U}{1-H} - \mu^{-1}\left(\delta\frac{H - n_U}{1 - H + n_U}\right) = 0.$$

b. As α increases from 0,

$$\frac{\partial n_I}{\partial \alpha} > 0, \ \frac{\partial n_R}{\partial \alpha} > 0, \ \frac{\partial n_M}{\partial \alpha} > 0, \ \text{whereas} \ \frac{\partial n_U}{\partial \alpha} < 0.$$

c. At $\alpha = 1$,

$$n_U = 0, \quad n_R = 1 - H + n_I, \quad n_M = H - n_I,$$

whereas n_I is given by the solution to

$$\frac{n_I}{1 - H + n_I} - \mu^{-1}\left(\delta \frac{H - n_I}{1 - H + n_I}\right) = 0,$$

Proof. In the Appendix. ■

When no houses are sold to investors ($\alpha = 0$), trivially $n_I = 0$ in the steady state, whereas when all mismatched houses are sold to investors in the first instance, it is only reasonable that no household will stay in a mismatched house in the steady state, so that $n_U = 0$. And then as α increases from 0 towards 1, n_I should only increase and n_U decline. Lemma 2 confirms these intuitions. What is less obvious in the Lemma is that both n_R and n_M increase along with the increase in α. The first tendency follows from the fact that if both the city's population and the housing stock are given, a unit increase in the measure of houses held by investors must be matched by a unit decline in the measure of houses occupied by the households in the city. To follow then is the same unit increase in the city's households in rental housing. For the second tendency, with an increase in α, fewer households spend any time at all selling their old houses in the search market before initiating search for a new match. In the mean time, the decrease in θ (Lemma 1), through lowering μ, lengthens the time a household spends on average in rental housing before a new match can be found. By Lemma 2, the first effect dominates, so that more households are in matched owner-occupied housing in the steady state.

By Lemma 1, an increase in α lowers market tightness, given by

$$\theta = \frac{S}{B} = \frac{n_U + n_I}{n_R}.$$

Indeed, the relation also follows from the comparative statics results in Lemma 2. First, where $\partial n_R / \partial \alpha > 0$, there will be more buyers to follow an increase in α. Second, given that by (9),

$$n_U + n_I = H - n_M,$$

$$\frac{\partial S}{\partial \alpha} = \frac{\partial \left[n_U + n_I\right]}{\partial \alpha} = -\frac{\partial n_M}{\partial \alpha} < 0.$$

That is, when more households are matched, there must only be fewer houses on the market. The two tendencies—more buyers (n_R) and fewer sellers ($n_U + n_I$)—reinforce each other to result in a smaller θ.

In the model economy, the entire stock of vacant house comprises of houses held by investors. With a given housing stock, the vacancy rate is simply equal to n_I/H. A direct corollary of Lemma 2b is that:

Lemma 3 In the steady state, the vacancy rate for owner-occupied houses is increasing in α.

Housing market transactions per time unit in the model are comprised of (i) $\alpha\delta n_M$ houses sold from households to investors, (ii) ηn_I houses investors sell to households, and (iii) ηn_U houses sold by one household to another, adding up to an aggregate transaction volume,

$$T = \alpha\delta n_M + \eta n_I + \eta n_U.$$

Lemma 4 In the steady state, transaction volume is increasing in α.

Proof. In the Appendix. ■

The usual measure of turnover in the housing market is the time it takes for a house to be sold, what is known as Time-On-the-Market (TOM). Given that houses sold in the investment market are on the market for a vanishingly small time interval and houses sold in the search market for a length of time equal to $1/\eta$ on average, we may define the model's average TOM as

$$\frac{\alpha\delta n_M}{T} \times 0 + \frac{\eta n_I + \eta n_U}{T} \times \frac{1}{\eta}. \tag{22}$$

Lemma 5 In the steady state, on average TOM is decreasing in α.

Proof. In the Appendix. ■

There is a composition effect (more houses sold to investors for which TOM is equal to 0) and a "structural" effect (a larger η for houses sold in the search market given that $\partial\eta/\partial\theta < 0$ and $\partial\theta/\partial\alpha < 0$) reinforcing one another. In sum, Lemmas 4 and 5 confirm that with greater investors participation, sales increase and houses are sold more quickly on average.

TOM is a measure of the turnover of houses for sale, and as such does not carry any direct welfare implications. A more household-centric measure of turnover is the length of time a household (rather than a house) has to stay unmatched. We define what we call Time-Between-Matches (TBM) as the sum of two spells: (1) the time it takes for a household to sell the old house, and (2) the time it takes to find a new match. While the first spell (TOM) on average is shorter with an increase in α, the second is longer where the decline in θ to accompany the increase in α causes μ to fall. A priori then it is not clear what happens to the average length of the whole spell. The old house is sold more quickly. But it also takes longer on average to find a new match in a less tight market—a market with fewer sellers and more buyers. To examine which effect dominates, write the economy's average TBM as

$$\alpha\frac{1}{\mu}+(1-\alpha)\left(\frac{1}{\eta}+\frac{1}{\mu}\right)=\frac{1}{\mu}+\frac{1-\alpha}{\eta}.$$

where $1/\mu$ is the average TBM for households who sell in the investment market[14] and $1/\eta+1/\mu$ for households who sell in the search market.[15]

Lemma 6 In the steady state, on average TBM is decreasing in α.

Proof. Substituting from (12), (15), and then (8),

$$\frac{1}{\mu}+\frac{1-\alpha}{\eta}=\frac{1-n_M}{\delta n_M},$$

a decreasing function of n_M. But where $\partial n_M/\partial\alpha>0$, there must be a smaller average TBM. ■

Lemma 6 perhaps may be taken as the dual of Lemma 2a ($\partial n_M/\partial\alpha>0$). When matched households are more numerous in the steady state, on average people must be spending less time between matches.

In the present model, given α, equilibrium market tightness θ is completely isomorphic of the determination of asset values and housing prices. The same conclusion of course carries over to the determination of vacancy, turnover, and transaction volume. If not for the inclusion of investors in the model economy, α is identically equal to 0 and Lemma 1 would have completed the analysis of everything that seems to be of any interest at all. With the inclusion of investors and their extent of participation measured by α, Lemmas 5 and 6 show how changes in α affect the turnovers of houses and households, the latter of which can have important consequences on welfare—a question we shall address in the following . But first α is endogenous and in particular given by (14), and hence a function of asset values and housing prices in equilibrium. A complete analysis with investors participation must then also include an analysis of the determination of asset values and housing prices, to which we next turn.

3.2 Which market to sell?

The second equation block of the model is made up of the equations for asset values, households' optimization, and housing prices, given by (3)-(7), (13), (14), (19), and

[14] The household sells the old house instantaneously. Given a house-finding rate μ, the average TBM is then $1/\mu$.

[15] Let t_1 denote the time it takes the household to sell the old house in the search market and t_2-t_1 the time it takes the household to find a new match after the old house is sold. Then the household's TBM is just t_2. On average, $E[t_2]=\int_0^\infty \eta e^{-\eta t_1}\left(\int_{t_1}^\infty t_2\mu e^{-\mu(t_2-t_1)}dt_2\right)dt_1=1/\eta+1/\mu$.

(20), from which we shall derive a correspondence for α of θ that characterizes households' choices between selling in the investment market and in the search market. To this end, we first solve (3)-(7), (19), and (20) for the three housing prices and four asset values. For $\Delta \leq 0$, so that $\alpha \in [0, 1]$,

$$p_H = \frac{((\eta + r_H)(\eta + 2r_I) - ((1-\alpha)(\eta + r_I) + r_I)\mu)\upsilon + (2\delta + \eta + 2r_H)(\eta + 2r_I)q}{(2\delta + \eta + 2r_H)(\alpha\mu r_I + \eta r_H + 2r_I r_H)}, \tag{23}$$

$$p_{IS} = \frac{(\eta + r_I)((\eta + 2r_H - (1-\alpha)\mu)\upsilon + (2\delta + \eta + 2r_H)q)}{(2\delta + \eta + 2r_H)(\alpha\mu r_I + \eta r_H + 2r_I r_H)}, \tag{24}$$

$$p_{IB} = V_I = \frac{\eta((\eta + 2r_H - (1-\alpha)\mu)\upsilon + (2\delta + \eta + 2r_H)q)}{(2\delta + \eta + 2r_H)(\alpha\mu r_I + \eta r_H + 2r_I r_H)}, \tag{25}$$

$$V_M = \frac{(\eta + 2r_H)\upsilon}{r_H(2\delta + \eta + 2r_H)}, \tag{26}$$

$$V_U = \frac{\eta\upsilon}{r_H(2\delta + 2r_H + \eta)}, \tag{27}$$

$$V_R = \frac{((\eta + 2r_H)r_I + (1-\alpha)\eta(r_H - r_I))\mu\upsilon - r_H(\eta + 2r_I)(2\delta + \eta + 2r_H)q}{r_H(2\delta + \eta + 2r_H)(r_H(\eta + 2r_I) + \alpha\mu r_I)}. \tag{28}$$

On the other hand, for $\Delta > 0$, so that $\alpha = 1$,[16]

$$p_H = \frac{(\eta r_H - \mu r_I + 2\eta r_I + 2r_H r_I + \eta^2)\upsilon + (2\eta r_H + 2\delta r_I + 2\eta r_I + 4r_H r_I + \eta^2)q}{(\eta + 2r_H)(\eta r_H + \mu r_I + 2\delta r_I + 2r_H r_I)}, \tag{29}$$

$$p_{IS} = \frac{(r_I + \eta)(\upsilon + q)}{r_H(\eta + 2r_I) + r_I(\mu + 2\delta)}, \tag{30}$$

$$p_{IB} = V_I = \frac{\eta(\upsilon + q)}{r_H(\eta + 2r_I) + r_I(\mu + 2\delta)}, \tag{31}$$

$$V_M = \frac{(r_H(\eta + 2r_I) + \mu r_I)\upsilon - 2\delta r_I q}{r_H(r_H(\eta + 2r_I) + r_I(\mu + 2\delta))}, \tag{32}$$

$$V_U = \frac{\eta}{\eta + 2r_H}\frac{(r_H(\eta + 2r_I) + \mu r_I)\upsilon - 2\delta r_I q}{r_H(r_H(\eta + 2r_I) + r_I(\mu + 2\delta))}, \tag{33}$$

$$V_R = \frac{\mu r_I \upsilon - (r_H(\eta + 2r_I) + 2\delta r_I)q}{r_H(r_H(\eta + 2r_I) + r_I(\mu + 2\delta))}. \tag{34}$$

[16] With $\Delta > 0$ and $\alpha = 1$, no houses will be sold at price p_H and there will be no mismatched homeowners around earning a flow payoff $r_H V_U$. Equations (29) and (33), respectively, denote the price at which a deviating household can sell its house in the search market and the asset value for the household. The two equations are needed to derive the condition for which $\Delta > 0$ indeed holds.

Now, with either (23)-(28) or (29)-(34), one can show that, by means of straightforward calculations, Δ has just the same sign as

$$\mathcal{S}_{\Delta} \equiv \left(\frac{r_H}{r_I} - 1 - z\right)\eta + \mu - 2\left(\delta + r_H\right)z, \tag{35}$$

where $z = q/v$. The expression $\mathcal{S}_{\Delta}$ may be interpreted as the incentives for mismatched homeowners to sell in the investment market. Such incentives are unambiguously weakened at larger r_I and q; investors may only offer a lower price when they face a higher cost of financing and it becomes less attractive for households to switch to rental housing sooner by quickly selling the mismatched house in the investment market if rental housing is more costly. On the other hand, the incentives are strengthened at a larger v; it becomes more attractive to shorten Time-Between-Matches by quickly selling in the investment market if there is a higher reward for staying in a matched owner-occupied house.

In any case, by virtue of (14), we can define a correspondence $\widehat{\alpha} : \mathcal{R}_{+} \rightrightarrows [0,1]$ that gives the fraction of households selling in the investment market in equilibrium,

$$\widehat{\alpha}(\theta) = \begin{cases} 0 & \mathcal{S}_{\Delta} < 0 \\ [0,1] & \mathcal{S}_{\Delta} = 0 \\ 1 & \mathcal{S}_{\Delta} > 0 \end{cases}. \tag{36}$$

Notice that since $\mathcal{S}_{\Delta}$ does not depend on α, for each θ, $\mathcal{S}_{\Delta}$ is either negative, equal to 0, or positive for all α. Hence in case $\mathcal{S}_{\Delta} = 0$ holds at some θ, any α on the unit interval is potentially equilibrium. Which particular α actually qualifies as equilibrium will depend on which value satisfies the $\theta = \widetilde{\theta}(\alpha)$ relation as defined in Lemma 1.

3.3 Equilibrium

We now have two relations between α and θ: the $\widetilde{\theta}(\alpha)$ function in (21) and the $\widehat{\alpha}(\theta)$ correspondence in (36). A steady-state equilibrium is any $\{\alpha, \theta\}$ pair that simultaneously satisfies the two relations.

3.3.1 Existence

Formally, define $F \equiv \widehat{\alpha}\left(\widetilde{\theta}(\alpha)\right)$, a correspondence mapping $[0,1]$ into itself. Equilibrium is any fixed point of F.

Proposition 1 Equilibrium exists for all positive $\{r_H, r_I, v, q, \delta, H\}$ tuple.

Proof. To establish existence, we apply Kakutani's fixed point theorem for correspondence to show that F has a fixed point. First, the unit interval is clearly a compact, convex, and nonempty subset of the one-dimensional Euclidean space. Second, since $\widetilde{\theta}(\alpha)$ is defined for all $\alpha \in [0,1]$ and is positive-valued, and that $\widehat{\alpha}$ is

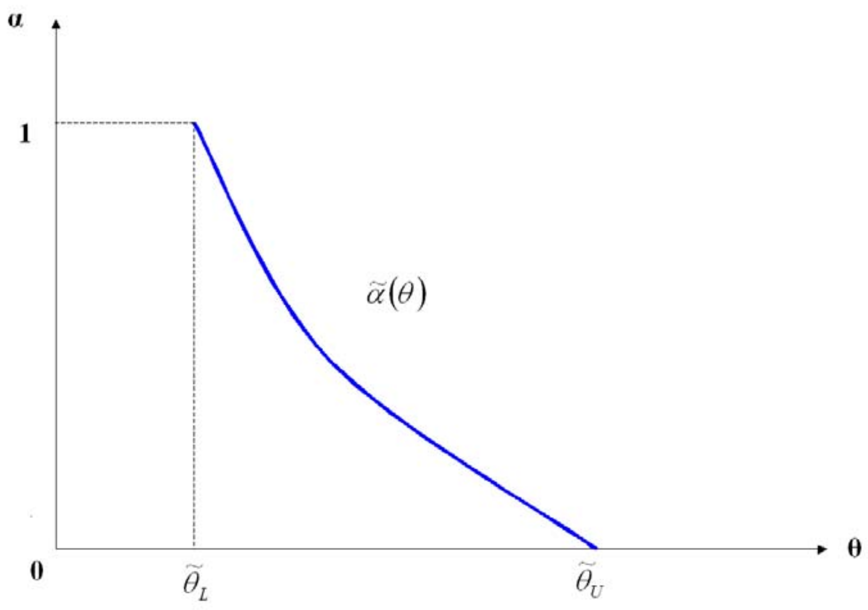

Figure 4: The $\widetilde{\alpha}$ function

nonempty for all $\theta > 0$, F must be positive-valued and nonempty for all $\alpha \in [0,1]$. Whenever F is multi-valued, F is the entire unit interval. Then it must be convex. Finally, with $\widetilde{\theta}$ continuous and $\widehat{\alpha}$ possessing a closed graph by virtue of the continuity of $\mathcal{S}_\Delta$, F must have a closed graph as well. Then by Kakutani's fixed point theorem, F has a fixed point. ■

3.3.2 Multiplicity

To check for uniqueness and multiplicity, we begin with inverting the $\widetilde{\theta}$ function to define $\widetilde{\alpha} \equiv \widetilde{\theta}^{-1}$, where $\widetilde{\alpha} : \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right] \to [0,1]$. Given that $\partial\widetilde{\theta}/\partial\alpha < 0$, likewise, $\partial\widetilde{\alpha}/\partial\theta < 0$. That is, $\widetilde{\alpha}(\theta)$ decreases continuously from 1 at $\theta = \widetilde{\theta}_L$ to 0 at $\theta = \widetilde{\theta}_U$. Figure 4 depicts one possible $\widetilde{\alpha}(\theta)$ schedule.

To characterize the $\widehat{\alpha}$ correspondence, we ask how the sign of $\mathcal{S}_\Delta$ may change with θ.

Lemma 7 Assume that $-\theta\frac{\partial^2\mu}{\partial\theta^2}\mu + 2\frac{\partial\mu}{\partial\theta}\left(\theta\frac{\partial\mu}{\partial\theta} - \mu\right) < 0$.[17] Then, as a function of θ,

a. if $r_I < \widehat{r}_I$, where

$$\widehat{r}_I \equiv \frac{r_H}{1+z}, \tag{37}$$

[17] The condition holds always if μ is isoelastic.

$\mathcal{S}_\Delta$ is U-shaped, starting out and ending up equal to infinity, in which case either $\mathcal{S}_\Delta$ stays positive for all θ or $\mathcal{S}_\Delta = 0$ holds at two values for θ.

b. if $r_I \geq \widehat{r}_I$, $\mathcal{S}_\Delta$ is upward-sloping throughout, starting out negative and ending up positive, in which case there is a unique solution of θ to $\mathcal{S}_\Delta = 0$.

Proof. In the Appendix. ■

Now, by virtue of Lemma 7, for $r_I < \widehat{r}_I$, and if $\mathcal{S}_\Delta$ stays positive for all θ, then $\widehat{\alpha}(\theta) = 1$ for all $\theta \geq 0$. Panel A of Figure 5 depicts the graph of $\widehat{\alpha}(\theta)$ in this case. On the other hand, if there are two solutions to $\mathcal{S}_\Delta = 0$, say $\widehat{\theta}_1$ and $\widehat{\theta}_2$, where $\widehat{\theta}_1 < \widehat{\theta}_2$,

$$\widehat{\alpha}(\theta) = \begin{cases} 0 & \theta \in \left(\widehat{\theta}_1, \widehat{\theta}_2\right) \\ [0,1] & \theta = \widehat{\theta}_1 \text{ and } \widehat{\theta}_2 \\ 1 & \theta \in \left[0, \widehat{\theta}_1\right) \cup \left(\widehat{\theta}_2, \infty\right) \end{cases}.$$

Panels C and D of Figure 5 depict two possible graphs of $\widehat{\alpha}(\theta)$ in this case. And then for $r_I \geq \widehat{r}_I$,

$$\widehat{\alpha}(\theta) = \begin{cases} 0 & \theta < \widehat{\theta} \\ [0,1] & \theta = \widehat{\theta} \\ 1 & \theta > \widehat{\theta} \end{cases},$$

where $\widehat{\theta}$ is the unique root of $\mathcal{S}_\Delta = 0$. Panels E and F of Figure 5 depict two possible graphs of $\widehat{\alpha}(\theta)$ in this case.

To summarize, for large $\theta = S/B$, $\widehat{\alpha}(\theta) = 1$ must hold, whether or not $r_I \lesseqgtr \widehat{r}_I$—whereas it can take a long time to sell in a tight search market where sellers outnumber buyers by a significant margin, mismatched homeowners are better off to just sell in the investment market at a discount. For small θ, sellers possess considerable bargaining power and houses are sold at high prices as a result, as can be verified by differentiating (23)-(25). In such a market, households can similarly prefer to just sell right away in the investment market to capitalize on the high price at which houses are sold. This tendency is particularly pronounced for smaller r_I under which investors can afford to pay relatively high prices. For intermediate θ, neither of the two incentives to sell in the investment market is strong enough to cause $\mathcal{S}_\Delta \geq 0$.

Given the $\widetilde{\alpha}(\theta)$ schedule in Figure 4 and the $\widehat{\alpha}(\theta)$ graphs in Figure 5, equilibrium is any $\{\alpha, \theta\}$ pair at which the two intersect. To proceed, we first define the graph of the $\widehat{\alpha}(\theta)$ correspondence as being:

1. *Nondecreasing* over a given interval of θ if for any two θ' and θ'' in the interval, where $\theta' < \theta''$, no element of $\widehat{\alpha}(\theta')$ is strictly greater than any element of $\widehat{\alpha}(\theta'')$.

2. *Decreasing* at some θ' if there exists an nonempty interval $(\theta', \theta'']$ where at least one element of $\widehat{\alpha}(\theta')$ is greater than all elements of $\widehat{\alpha}(\theta)$ for some $\theta \in (\theta', \theta'']$.

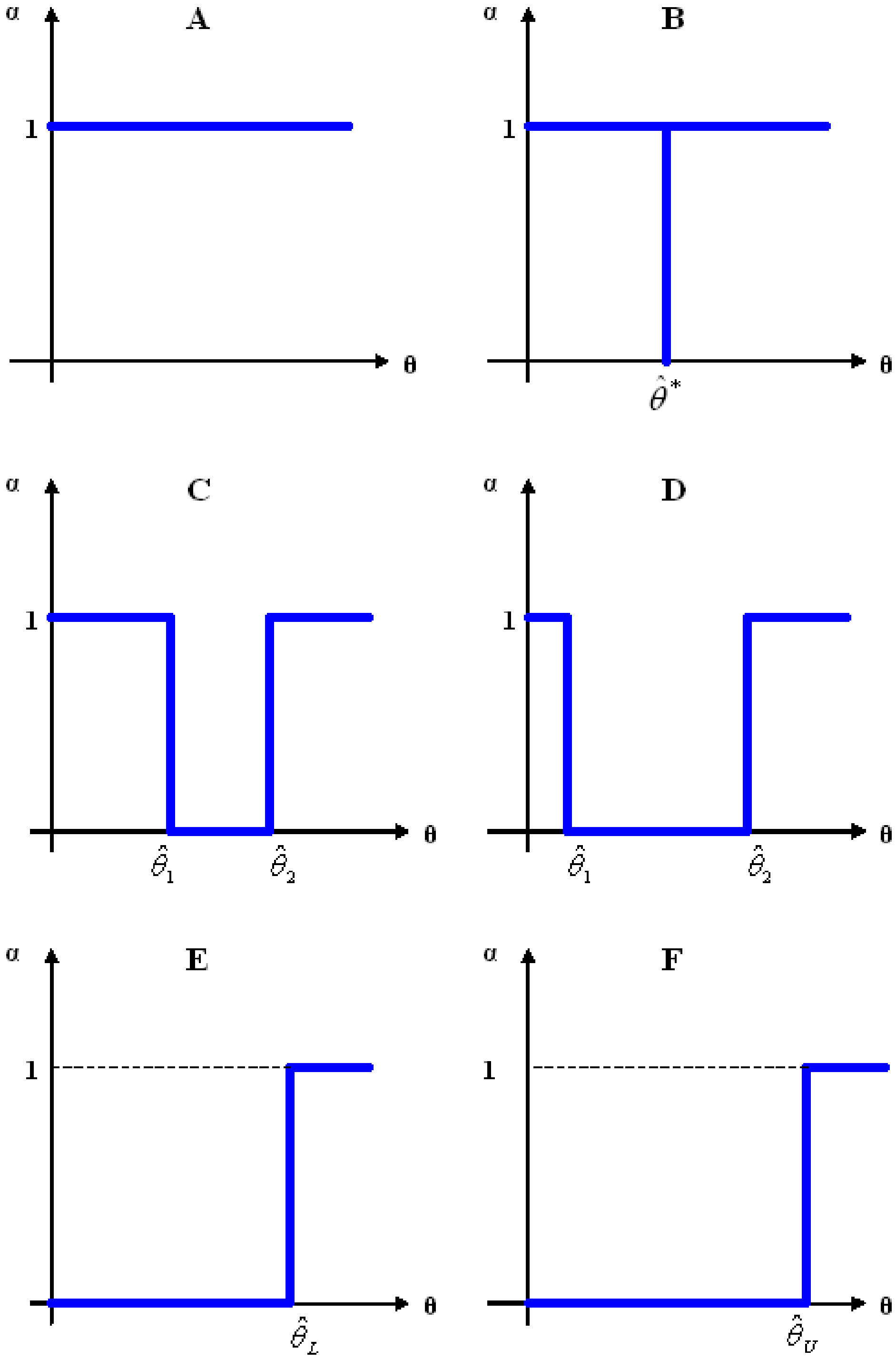

Figure 5: The $\widehat{\alpha}(\theta)$ correspondence

Proposition 2 For $r_I \geq \widehat{r}_I$ and for arbitrarily small r_I, the graph of $\widehat{\alpha}(\theta)$ is nondecreasing throughout, in which case equilibrium is unique. Otherwise, where there are two solutions of θ to $\mathcal{S}_\Delta = 0$, so that the graphs of $\widehat{\alpha}(\theta)$ is decreasing at $\widehat{\theta}_1$, there exist multiple equilibrium if and only if $\widehat{\theta}_1 \in \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$.

Proof. First, if $r_I \geq \widehat{r}_I$, the $\widehat{\alpha}(\theta)$ graph, as depicted in either Panel E or F of Figure 5, is nondecreasing throughout. Then there can be one and only one $\{\alpha, \theta\}$ pair at which the downward-sloping $\widetilde{\alpha}(\theta)$ in Figure 4 can meet a nondecreasing $\widehat{\alpha}(\theta)$. Next, by (35), $\lim_{r_I \to 0} \mathcal{S}_\Delta = \infty$. Hence for arbitrarily small r_I, $\mathcal{S}_\Delta$ stays positive for all θ and the $\widehat{\alpha}(\theta)$ graph is as depicted in Panel A of Figure 5. Clearly, the unique equilibrium is $\alpha = 1$and $\theta = \widetilde{\theta}_L$.

Otherwise, the $\widehat{\alpha}(\theta)$ graph must be like the two depicted in Panels C and D in Figure 5. In this case, by construction, for all $\theta \leq \widehat{\theta}_1$, $\mathcal{S}_\Delta \geq 0$, so that $1 \subset \widehat{\alpha}(\theta)$ for all $\theta \leq \widehat{\theta}_1$. Thus if $\widetilde{\theta}_L \leq \widehat{\theta}_1$, $\alpha = 1$ and $\theta = \widetilde{\theta}_L$ is equilibrium. Given that $\widetilde{\theta}_U \geq \widehat{\theta}_1$, either that $\widetilde{\theta}_U \in \left[\widehat{\theta}_1, \widehat{\theta}_2\right]$ or that $\widetilde{\theta}_U > \widehat{\theta}_2$. In the first case, since $0 \subset \widehat{\alpha}(\theta)$ for all $\theta \in \left[\widehat{\theta}_1, \widehat{\theta}_2\right]$ where $\mathcal{S}_\Delta \leq 0$, $\alpha = 0$ and $\theta = \widetilde{\theta}_U$ is equilibrium. Next, consider the case of $\widetilde{\theta}_U > \widehat{\theta}_2$. But if $\widetilde{\theta}_L \leq \widehat{\theta}_1$, $\widetilde{\theta}_U > \widehat{\theta}_2 > \widehat{\theta}_1 \geq \widetilde{\theta}_L$; i.e., $\widehat{\theta}_2 \in \left(\widetilde{\theta}_L, \widetilde{\theta}_U\right)$. Then there exists an $\alpha \in (0,1)$ that satisfies $\alpha = \widetilde{\alpha}\left(\widehat{\theta}_2\right)$. At $\theta = \widehat{\theta}_2$, any $\alpha \in [0,1] \subset \widehat{\alpha}\left(\widehat{\theta}_2\right)$. Thus, $\alpha = \widetilde{\alpha}\left(\widehat{\theta}_2\right)$ and $\theta = \widehat{\theta}_2$ is equilibrium. This proves that $\widehat{\theta}_1 \in \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$ is sufficient for multiplicity given an $\widehat{\alpha}(\theta)$ graph as depicted in either Panel C or D in Figure 5. To show that it is also necessary, notice that if $\widehat{\theta}_1 \notin \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$, the point at which $\widehat{\alpha}(\theta)$ is decreasing, $\widehat{\alpha}(\theta)$ is nondecreasing throughout $\left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$, and as in where $r_I \geq \widehat{r}_I$, there can be just one point at which $\widehat{\alpha}(\theta)$ and $\widetilde{\alpha}(\theta)$ intersect. ■

The graph of the $\widehat{\alpha}(\theta)$ correspondence must be nondecreasing for large θ. Proposition 2 says that if it possesses a point $\left(\widehat{\theta}_1\right)$ at which it is decreasing and if the point lies within $\left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$, there can be multiple equilibrium. Figures 6 and 7 illustrate the situations covered by the condition. In both figures, there are three equilibria, at (i) $\alpha = 1$ and $\theta = \widetilde{\theta}_L$, (ii) $\alpha = \widetilde{\alpha}\left(\widehat{\theta}_1\right)$ and $\theta = \widehat{\theta}_1$, and for Figure 6, (iiia) $\alpha = \widetilde{\alpha}\left(\widehat{\theta}_2\right)$ and $\theta = \widehat{\theta}_2$, whereas for Figure 7, (iiib) $\alpha = 0$ and $\theta = \widetilde{\theta}_U$. Intuitively, for small θ, under which houses are sold at relatively high prices, households find it advantageous to sell their old houses quickly in the investment market. And then if all mismatched houses are sold in the investment market in the first instance, there will be more rapid turnover and fewer houses are for sale in the search market to cause a small θ. Thus, $\alpha = 1$ and $\theta = \widetilde{\theta}_L$ is equilibrium in the situations depicted in Figures 6 and 7. For moderate values for θ, households' incentives to sell in the investment market are weakened. On the other hand, if fewer or none mismatched houses are sold in

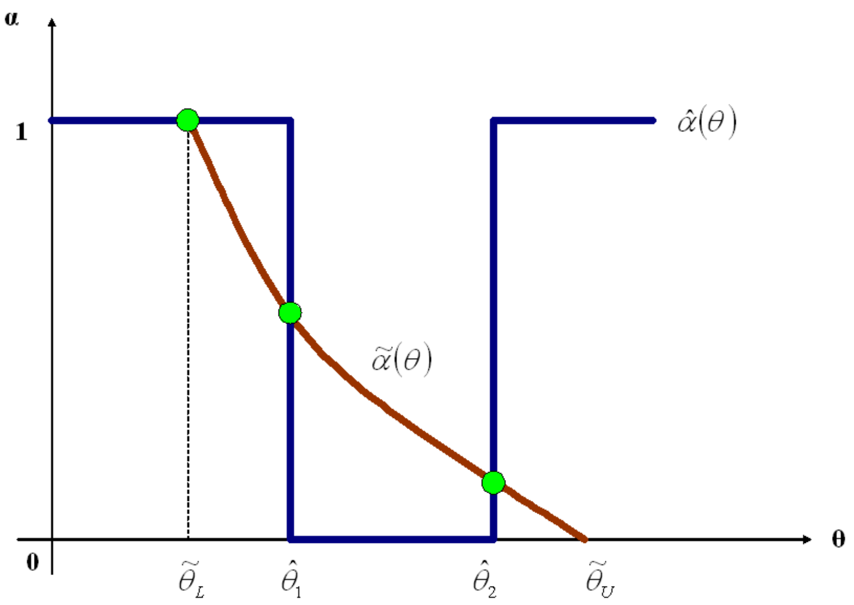

Figure 6: Multiple equilibrium – $\widehat{\theta}_2 \leq \widetilde{\theta}_U$

the investment market at all, turnover slows down and more houses are for sale in the search market to cause a larger θ. As a result, a smaller α and a larger θ is also equilibrium in Figures 6 and 7.

The $\widehat{\theta}_1$ equilibrium in Figures 6 and 7 is unstable, however, in the sense that any small perturbation should cause the economy to move to either the $\widetilde{\theta}_L$ equilibrium or to the $\widehat{\theta}_2$ equilibrium in Figure 6 and the $\widetilde{\theta}_U$ equilibrium in Figure 7. Consider for instance, a small increase in θ from where $\theta = \widehat{\theta}_1$. Then $\mathcal{S}_\Delta$ becomes negative; no mismatched homeowners will sell in the investment market immediately thereafter. To follow the decrease in α is a further increase in θ. Eventually, the economy should settle on the $\widetilde{\theta}_2$ steady-state equilibrium in the environment depicted in Figure 6 and on the $\widetilde{\theta}_U$ equilibrium in the environment depicted in Figure 7. There can be catastrophic declines in the extent of investors participation then when the economy is hit by some seemingly unimportant shocks to θ. With the existence of multiple steady-state equilibrium, the extent of investors participation can be fickle, especially when the economy happens to be at an unstable equilibrium initially. In the following, we shall show that to accompany such discrete declines in the extent of investors participation are discrete declines in housing prices. The economy is thus prone to apparent periodic boom-bust cycles—episodes usually attributed to the rise and bursting of speculative bubbles.

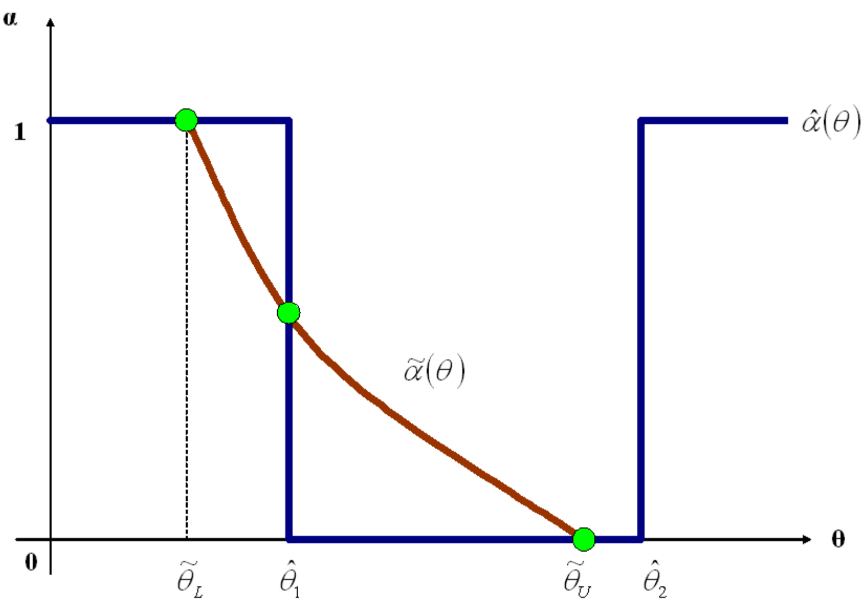

Figure 7: Multiple equilibrium – $\widehat{\theta}_2 > \widetilde{\theta}_U$

3.4 Cost of financing and investors participation

A natural question to ask is how the extent of investors participation depends on the rate at which investors finance investment. A not unreasonable conjecture is that when investors face a lower cost of financing, they can offer higher prices to mismatched homeowners, and then in equilibrium more houses should be sold in the investment market. But such a conclusion must be qualified given that equilibrium may not be unique. For instance, a small decline in r_I may possibly result in agents coordinating to a low α equilibrium from a high α equilibrium. Furthermore, the equilibrium effects of the incentives to sell in the investment market could turn out to be rather perverse. In particular, in Figures 6 and 7, for $\theta \in \left[0, \widehat{\theta}_1\right]$, $\widehat{\alpha}(\theta) = 1$—housing prices are high with a low θ and as such just selling right away in the investment market may dominate waiting out in the search market. Now, think of a decline in r_I; housing prices rise at each θ, which may well cause an expansion of the $\left[0, \widehat{\theta}_1\right]$ interval. If equilibrium is at $\widehat{\theta}_1$, the increase in $\widehat{\theta}_1$ is a movement down $\widetilde{\alpha}(\theta)$, giving rise to a smaller α. To more systematically investigate the effects of r_I on α, we begin with:

Lemma 8

a. For $r_I < \widehat{r}_I$, $\mathcal{S}_\Delta$ is U-shaped, with a well-defined minimum. Write $\mathcal{S}^*_\Delta = \min_\theta \mathcal{S}_\Delta$.

i. For small r_I, $\mathcal{S}_\Delta^* > 0$.

ii. $\partial \mathcal{S}_\Delta^* / \partial r_I < 0$. As r_I increases, before r_I reaches $\widehat{r_I}$, $\mathcal{S}_\Delta^* = 0$ at some $\theta = \widehat{\theta}^*$.

iii. Thereafter, as r_I continues to increase, $\mathcal{S}_\Delta^*$ falls below 0, and that the two roots of $\mathcal{S}_\Delta = 0$, $\widehat{\theta}_1$ and $\widehat{\theta}_2$ diverge as $\partial\widehat{\theta}_1/\partial r_I < 0$ but $\partial\widehat{\theta}_2/\partial r_I > 0$.

iv. As $r_I \to \widehat{r_I}$, $\widehat{\theta}_1 \to 0$ and $\widehat{\theta}_2 \to \widehat{\theta}_L$ for some limiting value $\widehat{\theta}_L$.

b. For $r_I \geq \widehat{r_I}$, $\mathcal{S}_\Delta$ becomes upward-sloping throughout.

i. At $r_I = \widehat{r_I}$, the unique root of $\mathcal{S}_\Delta = 0$, $\widehat{\theta} = \widehat{\theta}_L$, the limiting value of $\widehat{\theta}_2$.

ii. Thereafter, $\partial\widehat{\theta}/\partial r_I > 0$; while as r_I becomes arbitrarily large, $\widehat{\theta} \to \widehat{\theta}_U$ for some finite $\widehat{\theta}_U > 1$.

Proof. In the Appendix. ■

By characterizing how $\mathcal{S}_\Delta$ behaves as a function of r_I, the results of Lemma 8 help trace out the evolution of the $\widehat{\alpha}(\theta)$ correspondence as r_I increases. In particular, by a(i), for small r_I, $\widehat{\alpha}(\theta) = 1$ as depicted in Panel A of Figure 5. And then by a(ii), when r_I has reached a certain level, $\mathcal{S}_\Delta^*$ falls just equal to 0, at $\theta = \widehat{\theta}^* \equiv \arg\min \mathcal{S}_\Delta$ for the given r_I. At $\theta = \widehat{\theta}^*$, $\widehat{\alpha}(\theta) = [0, 1]$, whereas $\widehat{\alpha}(\theta)$ remains equal to 1 for all other θ, as depicted in Panel B of Figure 5. Thereafter, as r_I continues to increase, by a(iii), the $\widehat{\alpha}(\theta)$ correspondence becomes like the one in Panel C. Any further increase in r_I lowers $\widehat{\theta}_1$ and raises $\widehat{\theta}_2$; the $\widehat{\alpha}(\theta)$ correspondence in Panel C turns into the one in Panel D. As $r_I \to \widehat{r_I}$, by (a.iv), $\widehat{\theta}_1 \to 0$ and $\widehat{\theta}_2$ tends to some limiting value $\widehat{\theta}_L$. By (b.i), this limiting value is the unique root of $\mathcal{S}_\Delta = 0$ when $r_I \geq \widehat{r_I}$ first holds; Panel D turns into Panel E. Eventually, as r_I rises to an arbitrarily large value, by (b.ii), $\widehat{\theta}$ tends to some finite limiting value $\widehat{\theta}_U$; Panel E gradually turns into Panel F.

Granted that $\widetilde{\alpha}(\theta)$ is independent of r_I, the effects of an increase in r_I on equilibrium α and θ can then be read off by superimposing the same given $\widetilde{\alpha}(\theta)$ successively into Panels A to F of Figure 5. We can conclude from this exercise the following.

Proposition 3

a. For small r_I, equilibrium is $\alpha = 1$ and $\theta = \widetilde{\theta}_L$. For large r_I, in equilibrium, $\alpha < 1$ and $\theta > \widetilde{\theta}_L$. Specifically, if $\widehat{\theta}_U \geq \widetilde{\theta}_U$, as r_I becomes large, $\alpha = 0$ and $\theta = \widetilde{\theta}_U$. Otherwise, α stays positive for arbitrarily large r_I.

b. If $\alpha = 1$ and $\theta = \widetilde{\theta}_L$ is not equilibrium at r_I, the pair is not equilibrium at $r_I' > r_I$. If $\alpha = 0$ and $\theta = \widetilde{\theta}_U$ is equilibrium at r_I, the pair is equilibrium at $r_I' > r_I$.

c. Any $\widehat{\theta}_2$ or $\widehat{\theta}$ equilibrium is increasing in r_I, and therefore the accompanying α is decreasing in r_I.

d. Any $\widehat{\theta}_1$ equilibrium is decreasing in r_I, and therefore the accompanying α is increasing in r_I.

Proof. Given that for small r_I, the $\widehat{\alpha}(\theta)$ correspondence is given by the one in Panel A of Figure 5, the first part of (a) follows immediately. For large r_I, the $\widehat{\alpha}(\theta)$ correspondence tends to the one in Panel F. In this case, θ can remain equal to $\widetilde{\theta}_L$ only if the entire $\widetilde{\alpha}(\theta)$ schedule lies to the right of $\widehat{\theta}_U$; i.e., $\widetilde{\theta}_L \geq \widehat{\theta}_U$. Given that $\widetilde{\theta}_L < 1$ (Lemma 1) and $\widehat{\theta}_U > 1$ (Lemma 8), the condition cannot hold. Hence, for large r_I, in equilibrium, $\alpha < 1$ and $\theta > \widetilde{\theta}_L$. Next, if $\widehat{\theta}_U \geq \widetilde{\theta}_U$, in the limit when r_I becomes arbitrarily large, the downward-sloping $\widetilde{\alpha}$ must meet the horizontal axis at where $\widehat{\alpha}(\theta) = 0$, in which case equilibrium is $\alpha = 0$ and $\theta = \widetilde{\theta}_U$. Otherwise, the downward-sloping $\widetilde{\alpha}$ must meet the vertical segment of $\widehat{\alpha}$, yielding a $\widehat{\theta}$ equilibrium, where $\alpha > 0$. For (b), note that as r_I increases, the set of θ over which $1 \subset \widehat{\alpha}(\theta)$ shrinks and the set of θ over which $0 \subset \widehat{\alpha}(\theta)$ expands. Parts (c) and (d) are direct corollaries of (a.iii) and (b.ii) of Lemma 8, given that $\widetilde{\alpha}$ is downward-sloping. ■

Parts (a)-(c) conform to the intuitive notion that an increase (decrease) in r_I should have a negative (positive) effect on investors participation. Still, it is surprising that α can remain strictly positive even for arbitrarily large r_I, for which investors can only finance investment at a huge disadvantage vis-a-vis households. In the model economy, the central role investors play is to facilitate the rapid disposal of mismatched houses on behalf of households. By (a), investors can carry on playing the role even at a very high cost of financing. The condition for this to be the case, $\widehat{\theta}_U < \widetilde{\theta}_U$, holds for large $\widetilde{\theta}_U$. By (50) in the proof of Lemma 1, $\widetilde{\theta}_U$ is increasing in H and becomes arbitrarily large when $H \to 1$. In general, we can show that $\widetilde{\theta} = S/B$ is increasing in H for each α. Intuitively, with a larger stock of owner-occupied houses, fewer households would stay in rental housing, causing a decline in $B = n_R$, whereas more houses would be for sale in the search market, giving rise to a larger S. If $\widetilde{\theta}_U$ becomes large enough, a role for investors can remain no matter what, if mismatched homeowners find it too difficult to sell in the end-user search market.

Notice that Part (d) of the Proposition corresponds to the discussion preceding Lemma 8—a decline in r_I causes an expansion of the $\left[0, \widehat{\theta}_1\right]$ interval through the positive effect of a lower r_I on housing prices. Then any $\widehat{\theta}_1$ equilibrium will involve a smaller α. On the contrary, by Part (c), a decline in r_I will result in an increase in α in any $\widehat{\theta}_2$ or $\widehat{\theta}$ equilibrium. Recall that $\widehat{\theta}_2$ and $\widehat{\theta}$ are the smallest θ in the respective intervals $\left[\widehat{\theta}_2, \infty\right)$ and $\left[\widehat{\theta}, \infty\right)$, over which mismatched homeowners find it advantageous to sell in the investment market because it takes too long to sell in the search market. A decline in r_I, by raising the price investors are able to offer to mismatched homeowners, enlarge these intervals. Where the declines in $\widehat{\theta}_2$ and $\widehat{\theta}$ are movements up the downward-sloping $\widetilde{\alpha}$, in equilibrium α increases.

On the whole, one can conclude that a given decline in r_I does give rise to a greater level of investors participation if one is willing to dismiss any $\widehat{\theta}_1$ equilibrium on stability grounds and the possibility that agents may coordinate to a smaller α equilibrium in case there exist multiple equilibrium. However, we are of the opinion that the analysis does not, strictly speaking, allow us to reach any such unambiguous conclusion and we cannot rule out occasions, admittedly rare, in which a decline in r_I can, rather perversely, cause a lower level of investors participation.

3.5 Housing prices and asset values

3.5.1 $\mathcal{S}_\Delta < 0$

In a steady-state equilibrium in which $\mathcal{S}_\Delta < 0$, $\alpha = 0$, and $\theta = \widetilde{\theta}_U$, houses will be sold at just one price p_H, as given by (23) evaluated at $\alpha = 0$,

$$p_H = \frac{(\eta + r_H - \mu)\,\upsilon + (2\delta + \eta + 2r_H)\,q}{(2\delta + \eta + 2r_H)\,r_H}. \tag{38}$$

As long as $\mathcal{S}_\Delta$ is not strictly positive, asset values V_M and V_U remain given by (26) and (27), respectively, whereas V_R in (28) simplifies to

$$V_R = \frac{\mu\upsilon - (2\delta + \eta + 2r_H)\,q}{r_H\,(2\delta + \eta + 2r_H)}.$$

In this case, of course, prices and asset values are independent of r_I in the complete absence of investors participation. A decline in r_H will lead to higher housing prices but market tightness, vacancy, turnover, and transaction volume will just stay at the given levels entirely determined by the level of the housing stock, the rate matched households becomes mismatched, and the matching technology in the search market, as described in Lemmas 1-6 with $\alpha = 0$.

3.5.2 $\mathcal{S}_\Delta > 0$

In a steady-state equilibrium in which $\mathcal{S}_\Delta > 0$, $\alpha = 1$, and $\theta = \widetilde{\theta}_L$, houses will be sold at two prices, from households to investors at p_{IB}, given by (31), in the investment market and at p_{IS}, given by (30), from investors to households in the search market, where $p_{IB} < p_{IS}$. In the model economy, houses sold from households to investors stay on the market for a vanishingly small time interval, whereas houses sold from investors to households in the search market stay on the market for on average $1/\eta > 0$ units of time. A positive relation between price and TOM in the cross section then emerges, a relation that matches an important stylized fact in the real estate literature. Besides, with $p_{IB} < p_{IS}$, the model trivially implies that houses bought by flippers are at lower prices than are houses bought by non-flippers. Depken et al. (2009) indeed find the tendency to hold in their hedonic price regressions.

Not surprisingly, both p_{IB} and p_{IS} are decreasing in r_I, as can be readily verified by differentiating (31) and (30). The same negative effects are felt on V_M and V_I as well—when housing prices are lower, there can only be lower asset values for matched homeowners and investors. The effect on V_R, however, is positive as would-be buyers benefit from the lower housing prices. On the whole, aggregate welfare, defined as

$$W = n_M V_M + n_R V_R + n_I V_I,$$

is decreasing in r_I, where the negative effect on $n_M V_M$ alone suffices to dominate the positive effect on $n_R V_R$.[18] Hence, any decline in r_I, while causing housing prices to increase and making would-be home buyers worse off as a result, has an overall positive effect on welfare, whether or not investors' interests are taken into account.

3.5.3 $\mathcal{S}_\Delta = 0$

In a steady-state equilibrium in which $\mathcal{S}_\Delta = 0$, $\alpha \in (0,1)$, and $\theta \in \left(\widetilde{\theta}_L, \widetilde{\theta}_U\right)$, houses can be sold for three prices. In addition to the two prices p_{IB} and p_{IS} for transactions between an investor and a household, with $\alpha \in (0,1)$, there will also be transactions between two households, carried out at price p_H. Now, solving $\mathcal{S}_\Delta = 0$ for μ and substituting the result into (23), (24), and (25), respectively, yield

$$p_H = p_{IS} = \frac{(\eta + r_I)\,\upsilon}{(2\delta + \eta + 2r_H)\,r_I}, \tag{39}$$

$$p_{IB} = \frac{\eta\upsilon}{(2\delta + \eta + 2r_H)\,r_I}. \tag{40}$$

With $p_H = p_{IS}$ in equilibrium, there will, however, be just two distinct prices at which houses are sold—at p_{IB} in the investment market and at $p_H = p_{IS}$ in the search market. Just as in the case of $\mathcal{S}_\Delta > 0$, the model implies a positive relation between TOM and price and that houses bought by flippers are at lower prices. Also, just as in the case of $\mathcal{S}_\Delta > 0$, p_{IS} $(= p_H)$, as well as p_{IB}, are decreasing in r_I.

If α and θ were not already settled at their respective boundary values, housing prices, as given by (39) and (40), can also change in response to movements in θ associated with any change in α; any decrease in θ caused by an increase in α (Lemma 1) will result in higher housing prices. Investors participation, by itself, has a positive impact on housing prices. If there exist multiple equilibrium, α can increase just by itself when the economy, for whatever reason, moves to a high α equilibrium from an initial low α equilibrium or vice versa, without any change in parameter values. Alternatively, a seemingly insignificant shock to α can result in catastrophic changes in housing prices—tendencies that usually are attributed to the formation and bursting of speculative bubbles.

Along any $\widehat{\theta}_2$ or $\widehat{\theta}$ equilibrium, a decline in r_I will lead to an increase in α and a decrease in θ (Proposition 3c). In this case, by (39) and (40), the direct and indirect effects of an interest rate decrease reinforce each other to raise housing prices. A more

[18] Substituting from (32), (34), (31), (53), (54), and (56) and differentiating verify the claim.

intriguing scenario is when a decline in r_I shall cause α to decrease instead either because the economy moves to a low α equilibrium from an initial high α equilibrium in case there exist multiple equilibrium or because the economy is moving along a $\widehat{\theta}_1$ equilibrium (Proposition 3d). In what direction housing prices will move cannot be unambiguously read off from (39) and (40) as the direct effect of the interest rate decline and the indirect effect through a supposedly lower α and therefore a higher θ affect housing prices differently. To proceed, we solve $\mathcal{S}_\Delta = 0$ for r_I and substitute the result into (39) and (40), respectively,

$$p_{IS} = p_H = \frac{(\eta + r_H - \mu)\, \upsilon + (2\delta + \eta + 2r_H)\, q}{r_H\, (2\delta + \eta + 2r_H)}, \tag{41}$$

$$p_{IB} = \frac{(\eta - \mu)\, \upsilon + (2\delta + \eta + 2r_H)\, q}{r_H\, (2\delta + \eta + 2r_H)}. \tag{42}$$

The two equations are independent of r_I; whatever effects a given change in r_I will have on housing prices are subsumed through the effects of changes in θ that follow the change in r_I obtained from holding $\mathcal{S}_\Delta = 0$.

Proposition 4 Holding $\mathcal{S}_\Delta = 0$, changes in r_I, whether positive or negative, will cause housing price to increase (decrease), as long as to follow the interest rate change is an increase (decrease) in α.

Proof. By straightforward differentiation of (41) and (42) with respect to θ and noting that $\partial\widetilde{\theta}/\partial\alpha < 0$. ■

By Proposition 4 then, the indirect effect of a decline in r_I on housing prices through changes in α and θ always dominates the direct effect shall the two effects be of opposite tendencies. A surprising implication then is that a given decline in investors' cost of financing can, rather paradoxically, lead to decreases in housing prices, if to follow the interest rate decrease is also a decrease in investors participation.

A direct corollary of Lemmas 3-5 and Proposition 4 is that:

Proposition 5 Holding $\mathcal{S}_\Delta = 0$, changes in r_I will cause a positive relation between housing price and transaction volume. In the mean time, average TOM and TBM decline, whereas vacancy rate increases.

Now, if the greater level of investors participation has the effects of raising transaction volume and housing price, while lowering average TOM, our model then provides a plausible explanation for the well-known positive time series relation between the first two variables and the negative relation of the two with TOM. Besides, the present model also predicts a rising vacancy rate amidst the increase in price and sales and the decrease in average TOM, a prediction which is unique among housing market search models. Figures 1-3 in the Introduction show that the predictions are at the very least not obviously inconsistent with the data. A prediction of a negative correlation between vacancy on the one hand and price and transaction on the other hand, an implication that follows naturally from housing market search models in which the agents are exclusively end-users, appears to be considerably more difficult to reconcile with the data.

Asset values V_M and V_U, as given by (26) and (27), do not depend on r_I as such, as long as $\mathcal{S}_\Delta \leq 0$, while $V_I = p_{IB}$, given by (40), is decreasing in r_I. As to V_R, first substitute the solution for μ from $\mathcal{S}_\Delta = 0$ to yield,

$$V_R = \frac{\eta (r_I - r_H) \upsilon}{(2\delta + \eta + 2r_H) r_I r_H}, \tag{43}$$

which is increasing in r_I. In sum, investors are worse off and would-be home buyers benefit from the lower housing prices that result from any increase in r_I at each θ. Just as for housing prices, to fully account for the effects of a given change in r_I on asset values, we should also check how asset values change in response to the changes in α and θ triggered by the change in r_I. To this end, note that V_M and V_U, given by (26) and (27), respectively, as well as V_I, given by (40), are decreasing in θ; household-homeowners and investors are better off when a less tight housing market gives rise to higher housing prices. Asset values for households in rental housing, V_R, as given by (43), can increase or decrease with a given decline in θ, depending on whether $r_I \lessgtr r_H$. Furthermore, changes in α will also result in changes in the measures of investors and households in the various states as described in Lemma 2. Aggregate welfare,

$$\begin{aligned} W &= n_M V_M + n_U V_U + n_R V_R + n_I V_I \\ &= n_M \frac{(\eta + 2r_H) \upsilon}{r_H (2\delta + \eta + 2r_H)} + n_U \frac{\eta \upsilon}{r_H (2\delta + 2r_H + \eta)} + n_R \frac{\eta (r_I - r_H) \upsilon}{(2\delta + \eta + 2r_H) r_I r_H} \\ &\quad + n_I \frac{\eta \upsilon}{(2\delta + \eta + 2r_H) r_I}, \end{aligned}$$

can then depend on r_I in a complicated manner.

To proceed, we substitute for n_M, n_U, n_R, and n_I, respectively from (53)-(56) in the Appendix and the solution for r_I from $\mathcal{S}_\Delta = 0$,

$$W = \left(\frac{\eta (2r_H H + \eta + \delta)}{(2\delta + \eta + 2r_H)(\eta + \delta)} - (1 - H) \frac{(1 + z - \theta) \eta + 2(\delta + r_H) z}{(2\delta + \eta + 2r_H)} \right) \frac{\upsilon}{r_H}. \tag{44}$$

Just as in (39) and (40) for housing prices, this expression for W is independent of r_I, where any effect r_I will have on W is subsumed in how changes in θ to follow the interest rate change affect W.

Proposition 6 Holding $\mathcal{S}_\Delta = 0$, changes in r_I, whether positive or negative, will cause aggregate asset values to rise (fall), as long as to follow the interest rate change is an increase (decrease) in α.

Proof. Differentiating (44) with respect to θ and noting that $\partial\widetilde{\theta}/\partial\alpha < 0$. ■

Together, Propositions 4 and 6 say that whenever housing prices rise in response to a given change in r_I, negative or otherwise, aggregate asset values will rise, even if households in rental housing can be worse off and that they must be more numerous in a housing market in which more houses are held by investors.

3.6 A general change in interest rate

So far, we have restricted attention to analyzing the effects of an increase in r_I alone on investors participation and housing prices. Many of the implications, however, survive for a general change in interest rate that affects both investors and ordinary households alike. Specifically, write R for r_H/r_I in (35),

$$\mathcal{S}_\Delta = (R - 1 - z)\,\eta + \mu - 2\,(\delta + r_H)\,z. \tag{45}$$

Then equiproportionate increases in r_H and r_I, while leaving R unchanged, lower $\mathcal{S}_\Delta$. A general increase in interest rate thus weakens mismatched homeowners' incentives to sell in the investment market, just as an increase in r_I, holding fixed r_H, does. Analogous to Proposition 3 is that:

Proposition 7 Holding constant R at some given level,

a. for sufficiently large r_H, in equilibrium, $\alpha < 1$ and $\theta > \widetilde{\theta}_L$. Eventually, as r_H rises above a certain level, $\alpha = 0$ and $\theta = \widetilde{\theta}_U$ must obtain.

b. if $\alpha = 1$ and $\theta = \widetilde{\theta}_L$ is not equilibrium at r_H, the pair is not equilibrium at $r'_H > r_H$. If $\alpha = 0$ and $\theta = \widetilde{\theta}_U$ is equilibrium at r_H, the pair is equilibrium at $r'_H > r_H$.

c. any $\widehat{\theta}_2$ or $\widehat{\theta}$ equilibrium is increasing in r_H, and therefore the accompanying α is decreasing in r_H.

d. any $\widehat{\theta}_1$ equilibrium is decreasing in r_H, and therefore the accompanying α is increasing in r_H.

Proof. Hold constant R and allow r_H to increase; by (45),

$$\frac{\partial \mathcal{S}^*_\Delta}{\partial r_H} = 2z < 0.$$

For large r_H then, the $\widehat{\alpha}(\theta)$ graph cannot be like the ones in Panels A and B of Figure 5. For $r_I \geq \widehat{r}_I$ (i.e., $R \leq 1 + z$), so that there exists a unique root $\widehat{\theta}$ to $\mathcal{S}_\Delta = 0$, $\partial\widehat{\theta}/\partial r_H > 0$, given that $\partial\mathcal{S}_\Delta/\partial r_H < 0$. By (45) and with $R - 1 - z < 0$, $\lim_{r_H\to\infty}\widehat{\theta} = \infty$. For $r_I < \widehat{r}_I$ (i.e., $R > 1 + z$), there are two roots $\widehat{\theta}_1$ and $\widehat{\theta}_2$ to

$\mathcal{S}_{\Delta} = 0$. Again, given that $\partial \mathcal{S}_{\Delta}/\partial r_H < 0$, $\partial \widehat{\theta}_1/\partial r_H < 0$ and $\partial \widehat{\theta}_2/\partial r_H > 0$. By (45) and with $R - 1 - z > 0$, as $r_H \to \infty$, there can just be two θ that solves $\mathcal{S}_{\Delta} = 0$, equal to zero and infinity; thus $\lim_{r_H \to \infty} \widehat{\theta}_1 = 0$ and $\lim_{r_H \to \infty} \widehat{\theta}_2 = \infty$. Parts (a)-(d) of the Propositions then follow immediately. ■

Intuitively, at a higher cost of financing in general, housing prices fall, and the reward to quickly selling in the investment market diminishes. But just as in the case of an increase in r_I alone, it is not possible to conclude unambiguously that investors participation must fall, where there exist multiple equilibrium, and that any $\widehat{\theta}_1$ equilibrium is actually decreasing in r_H.

The effects of the general increase in the cost of financing on housing prices are similar to those of an increase in r_I by itself. The following proposition summarizes the results.

Proposition 8

a. If in equilibrium, $\mathcal{S}_{\Delta} < 0$, $\alpha = 0$, and $\theta = \widetilde{\theta}_U$, houses will be sold at just one price, p_H, as given by (38), which is independent of r_I but decreasing in r_H. Equiproportionate increases in r_H and r_I then lower housing prices.

b. If in equilibrium, $\mathcal{S}_{\Delta} > 0$, $\alpha = 1$, and $\theta = \widetilde{\theta}_L$, houses will be sold at two prices, p_{IS} and p_{IB}, as given by (30) and (31), respectively. Equiproportionate increases in r_H and r_I lower both prices.

c. If in equilibrium, $\mathcal{S}_{\Delta} = 0$, $\alpha \in (0, 1)$, and $\theta \in \left(\widetilde{\theta}_L, \widetilde{\theta}_U\right)$, houses will be sold at two prices, $p_{IS} = p_H$ and p_{IB}, as given by (39) and (40), respectively.

   i. Equiproportionate increases in r_H and r_I, holding θ fixed, lower both prices and so does an increase in θ that follows from a decline in α.

   ii. Holding $\mathcal{S}_{\Delta} = 0$, equiproportionate changes in r_H and r_I, whether positive or otherwise, raise p_{IB} as long as to follow the interest rate changes is an increase in α and for $R \in [0, 1 + z]$. The same positive effect is felt on $p_{IS} = p_H$ for R in neighborhoods of $R = 0$, 1, and $1 + z$.

Proof. In the Appendix. ■

Notice that by (c.i), if to follow the equiproportionate increases in r_H and r_I is a decline in investors participation α, housing prices must unambiguously decline, just as when an increase in r_I alone causes α to fall will lower housing prices for sure. More generally, (c.ii) is concerned with how prices may change when the increases in interest rate may be followed by either an increase or a decline in α, as in the situations covered in Proposition 4. Also as in Proposition 4, here prices will increase if α happens to rise to follow the interest rate changes, positive or otherwise, if the values of R are appropriately chosen. The last restrictions are sufficient, but not necessary, conditions, and that the conclusions should hold under weaker conditions.

4 Concluding remarks

By allowing for the participation of specialist investors, without any assumed or acquired heterogeneity and endogenous search efforts, our model predicts a positive relation between transaction price and TOM in the cross section, a relation found in numerous empirical studies. Our predictions of the relations among price, transaction volume, and average TOM in the aggregate also mimic the pattern found in the data. Unlike previous studies, these relations emerge from our model through interest rate shocks, arguably a more plausible channel than shocks to preference and construction—channels that previous studies rely upon.

Mismatched households will only be able to find new matches in a market where vacant houses are abundant. On the other hand, few houses will be vacant if mismatched households cannot quickly sell. Flippers in the housing market help mitigate such coordination problems. In this manner, the change in the vacancy rate and the change in the number of houses bought by flippers are (almost) two sides of the same coin. Where investors participation raises demand and prices, our analysis suggests vacancy tends to increase in an up market. This implication, as shown in Figures 1-3, appears to be borne out in the run-up to the peak of the housing market boom of late-1990s to mid-2000s.

Meanwhile, the existence of multiple equilibrium appears a natural outcome of the investment motive in a frictional housing market. When the extent of investors participation can be fickle, prices can change discretely in response to a discrete change in the former. The model economy can then exhibit bubble-like characteristics with prices fluctuating widely without any apparent changes in "market fundamentals". Undoubtedly, our analysis cannot be the complete analysis of "speculative bubbles" in the housing market. Credit market conditions, market psychology, and the dynamics of price movements must also feature prominently. Nevertheless, we show that even in the absence of such factors, the interaction of the strength of the incentives to sell quickly to flippers and the extent of their activities suffice to imply an intrinsically volatile housing market.

5 Appendix

Proof of Lemma 1 The first step is to use (8) and (9) to write

$$n_R = 1 - H + n_I. \tag{46}$$

Then, by (10), (11), (18), and (46),

$$\theta = \frac{S}{B} = \frac{n_U + n_I}{n_R} = \frac{\frac{1-\alpha}{\alpha} n_I + n_I}{1 - H + n_I} = \frac{n_I}{\alpha \left(1 - H + n_I\right)}. \tag{47}$$

Solve the equation for n_I,

$$n_I = \frac{\theta\alpha \left(1 - H\right)}{1 - \theta\alpha}. \tag{48}$$

Next, by (12) and (8),

$$\mu n_R = \delta\left(1 - n_U - n_R\right).$$

Rearrange and then substitute from (18) and (46),

$$(\mu + \delta)\left(1 - H + n_I\right) = \delta\left(1 - \frac{1-\alpha}{\alpha} n_I\right).$$

Solve the equation for n_I,

$$n_I = \alpha \frac{\delta H - \mu(\theta)(1 - H)}{\delta + \mu(\theta)\alpha}. \tag{49}$$

Setting the LHSs of (48) and (49) equal yields (21). Implicitly differentiating yields a negative partial derivative. From (21), $\widetilde{\theta}_U$ is given by

$$\delta\widetilde{\theta}_U + \mu\left(\widetilde{\theta}_U\right) = \frac{\delta H}{1 - H}, \tag{50}$$

while $\widetilde{\theta}_L$ solves

$$\frac{\delta\widetilde{\theta}_L + \mu\left(\widetilde{\theta}_L\right)}{1 - \widetilde{\theta}_L} = \frac{\delta H}{1 - H}. \tag{51}$$

Given that the LHSs of both conditions are positive and finite for $H < 1$, $\widetilde{\theta}_L$ and $\widetilde{\theta}_U$ are strictly positive and finite, and that $\widetilde{\theta}_L < 1$.

Proof of Lemma 2

Comparative statics Solve (21) for

$$\alpha = \frac{\delta H - (1 - H)(\mu + \delta\theta)}{\theta\delta H}. \tag{52}$$

Substitute (52) into (48) yields

$$n_I = \frac{\delta H - (1 - H)(\mu + \delta\theta)}{\mu + \delta\theta}. \tag{53}$$

Substitute (53) into (46) yields

$$n_R = \frac{\delta H}{\mu + \delta\theta}. \tag{54}$$

Substitute (53) into (18) yields

$$n_U = \frac{\delta H(\theta - 1) - (1 - H)(\mu + \delta\theta)}{\mu + \delta\theta}. \tag{55}$$

Finally, by (8), (54), (55),

$$n_M = \frac{\mu H}{\mu + \delta\theta}. \tag{56}$$

The comparative statics in the Lemma can be obtained by differentiating (53)-(56), respectively, with respect to θ, and then noting that $\partial\widetilde{\theta}/\partial\alpha < 0$.

Boundary values At $\alpha = 0$, by (17), $n_I = 0$. Then, by (46), $n_R = 1 - H$ and by (9), $n_M = H - n_U$. To obtain the equation for n_U, substitute (50) into (55) and simplify. For $\alpha = 1$, by (15), $n_U = 0$. And then by (46), $n_R = 1 - H + n_I$. Thus, with (8), $n_M = H - n_I$. The equation for n_I is obtained by substituting (51) into (53) and simplify.

Proof of Lemma 4 First substituting from (9), and then from (52) and (56),

$$T = \alpha\delta n_M + (H - n_M)\,\eta = \delta\frac{\eta H}{\eta+\delta}\frac{1+\theta}{\theta} - \eta\,(1-H)\,. \tag{57}$$

Differentiating and simplifying,

$$\frac{\partial T}{\partial\theta} = \frac{\delta H}{\theta}\left[\theta\frac{\partial\eta}{\partial\theta}\left(\left(\frac{1}{\eta+\delta}\right)^2\delta\frac{1+\theta}{\theta} - \frac{1-H}{\delta H}\right) - \frac{\eta}{\eta+\delta}\frac{1}{\theta}\right].$$

From (50),

$$\frac{1-H}{\delta H} \leq \frac{1}{\theta\,(\eta+\delta)},$$

since $\theta \leq \widetilde{\theta}_U$. Thus

$$\begin{aligned}
\frac{\partial T}{\partial\theta} &\leq \frac{\delta H}{\theta}\left[\theta\frac{\partial\eta}{\partial\theta}\left(\left(\frac{1}{\eta+\delta}\right)^2\delta\frac{1+\theta}{\theta} - \frac{1}{\theta\,(\eta+\delta)}\right) - \frac{\eta}{\eta+\delta}\frac{1}{\theta}\right] \\
&= \frac{\delta H}{\theta^2\,(\eta+\delta)}\left[\left(\frac{\delta\theta-\eta}{\eta+\delta}\right)\frac{\partial\eta}{\partial\theta}\theta - \eta\right] \\
&\leq \frac{\delta H}{\theta^2\,(\eta+\delta)}\left[\left(\frac{\delta\theta-\eta}{\eta+\delta}\right)\frac{\partial\eta}{\partial\theta}\theta - \frac{\partial\eta}{\partial\theta}\theta\right] \\
&= \frac{\delta H}{\theta^2\,(\eta+\delta)}\left[\frac{\delta\theta+\delta}{\eta+\delta}\right]\frac{\partial\eta}{\partial\theta}\theta < 0,
\end{aligned}$$

where the second inequality follows from the fact that $\partial\mu/\partial\theta > 0$ and that $\eta = \mu/\theta$. But then $\partial\widetilde{\theta}/\partial\alpha < 0$; hence $\partial T/\partial\alpha > 0$.

Proof of Lemma 5 Substituting from (9) and (57), (22) becomes

$$\frac{H - n_M}{\alpha\delta n_M + (H - n_M)\,\eta}.$$

Differentiating with respect to α yields

$$\frac{\delta\,(n_M - H)\,n_M - \delta H\alpha\frac{\partial n_M}{\partial\alpha} - (H - n_M)^2\,\frac{\partial\eta}{\partial\theta}\frac{\partial\theta}{\partial\alpha}}{\left[\alpha\delta n_M + (H - n_M)\,\eta\right]^2} < 0,$$

since $\partial n_M/\partial\alpha > 0$, $\partial\eta/\partial\theta < 0$, and $\partial\theta/\partial\alpha < 0$.

Proof of Lemma 7 Starting with $\theta = 0$, $\mu = 0$, while $\eta \to \infty$, so that $\mathcal{S}_\Delta$ becomes

$$\lim_{\theta \to 0} \left(\frac{r_H}{r_I} - 1 - z \right) \eta - 2 \left(\delta + r_H \right) z,$$

which is equal to positive infinity/a finite negative number/negative infinity if $r_I \lesseqgtr \widehat{r}_I$. On the other hand, as $\theta \to \infty$, $\eta = 0$ and $\mu \to \infty$, so that $\mathcal{S}_\Delta$ becomes

$$\lim_{\theta \to \infty} \mu - 2 \left(\delta + r_H \right) z, \tag{58}$$

an expression that tends to positive infinity. Differentiating,

$$\frac{\partial \mathcal{S}_\Delta}{\partial \theta} = \left(\frac{r_H}{r_I} - 1 - z \right) \frac{\partial \eta}{\partial \theta} + \frac{\partial \mu}{\partial \theta}, \tag{59}$$

which is guaranteed positive if $r_I \geq \widehat{r}_I$. In this case, $\mathcal{S}_\Delta$ starts out at either negative infinity or a finite negative number, is increasing throughout and eventually tends to positive infinity. A unique θ then solves $\mathcal{S}_\Delta = 0$. On the other hand, if $r_I < \widehat{r}_I$, $\mathcal{S}_\Delta$ starts out and ends up equal to positive infinity. It must therefore be initially decreasing but eventually increasing. If the condition in the Lemma holds, (59) changes sign just once. This can be shown by differentiating (59) and evaluating at where it is equal to zero, which leads to an expression which is positive if the condition holds. Then, at where (59) vanishes, $\mathcal{S}_\Delta$ is convex.

Proof of Lemma 8 By (35), $\lim_{r_I \to 0} \mathcal{S}_\Delta = \infty$. Thus, for arbitrarily small r_I, $\mathcal{S}_\Delta^* > 0$. This proves (a.i). Differentiating (35) and by the Envelope Theorem,

$$\frac{\partial \mathcal{S}_\Delta^*}{\partial r_I} = -\frac{r_H}{r_I^2} \eta < 0.$$

This proves the first part of (a.ii). As to the second part, notice that

$$\lim_{r_I \to \widehat{r}_I} \mathcal{S}_\Delta = \mu - 2 \left(\delta + r_H \right) z, \tag{60}$$

which is minimized at $\theta = 0$, yielding a negative $\mathcal{S}_\Delta^*$ in the limit. Given that $\mathcal{S}_\Delta^*$ is continuous in r_I, $\mathcal{S}_\Delta^* = 0$ must hold before r_I has reached $\widehat{r}_I$. For (a.iii), differentiating $\mathcal{S}_\Delta = 0$ and for $i = 1, 2$,

$$\frac{\partial \widehat{\theta}_i}{\partial r_I} = \frac{\frac{r_H}{r_I^2} \eta}{\frac{\partial \mathcal{S}_\Delta}{\partial \theta}}.$$

Where $\mathcal{S}_\Delta$ is decreasing at $\widehat{\theta}_1$ and increasing at $\widehat{\theta}_2$, $\partial \widehat{\theta}_1 / \partial r_I < 0$ and $\partial \widehat{\theta}_2 / \partial r_I > 0$. For (a.iv), notice that $\partial \mathcal{S}_\Delta / \partial \theta$, as given by (59), can only be negative as $r_I \to \widehat{r}_I$ at $\theta \to 0$. This proves $\widehat{\theta}_1 \to 0$ as $r_I \to \widehat{r}_I$. The limiting value for $\widehat{\theta}_2$ is given by the solution to

$$\mu \left(\widehat{\theta}_2 \right) = 2 \left(\delta + r_H \right) z,$$

which of course is simply the definition of $\widehat{\theta}_L$. The positivity of $\partial\widehat{\theta}/\partial r_I$ is due to the same reason for the positivity of $\partial\widehat{\theta}_2/\partial r_I$. The limiting value of $\widehat{\theta}$ as r_I becomes arbitrarily large is given by the solution to

$$\mu\left(\widehat{\theta}_U\right)\left(1-\frac{1+z}{\widehat{\theta}_U}\right)=2\left(\delta+r_H\right)z.$$

Given that the RHS is positive and finite, $\widehat{\theta}_U$ is finite and satisfies $\widehat{\theta}_U > 1+z > 1$. This completes the proof of (b).

Proof of Proposition 8 Part (a) merely repeats the discussions in Section 3.5.1. For (b), substitute $r_I = r_H R^{-1}$ into (30) and (31) and differentiate. For (c), substituting $r_I = r_H R^{-1}$ into (39) and (40), respectively, yields,

$$p_{IS}=p_H=\frac{\left(\eta+r_H R^{-1}\right)\upsilon}{\left(2\delta+\eta+2r_H\right)r_H R^{-1}}, \tag{61}$$

$$p_{IB}=\frac{\eta\upsilon}{\left(2\delta+\eta+2r_H\right)r_H R^{-1}}, \tag{62}$$

both of which are decreasing in r_H and θ. Solve $\mathcal{S}^*_\Delta = 0$ from (45) for r_H,

$$r_H=\frac{\left(R-1-z\right)\eta+\mu}{2z}-\delta, \tag{63}$$

and substituting the result into (61) and (62), respectively, gives

$$p_{IS}=p_H=\frac{\left(2\eta zR+\eta R-\eta-\eta z+\mu-2\delta z\right)q}{\left(\eta R-\eta+\mu\right)\left(\eta R-\eta-\eta z+\mu-2\delta z\right)}, \tag{64}$$

$$p_{IB}=\frac{2\upsilon z^2 R\eta}{\left(\eta R-\eta+\mu\right)\left(\eta R-\eta-\eta z+\mu-2\delta z\right)}. \tag{65}$$

Differentiating (65) with respect to θ and then substituting for η and $\partial\eta/\partial\theta$ using the identity $\eta = \mu/\theta$ yield an expression whose sign is given by that of

$$\left(\mu-\theta\mu'\right)\left(R-1\right)\left(R-z-1\right)-\left(\left(2R-2-z+\theta\right)\mu'+\mu-2z\delta\right)\theta^2.$$

The expression is strictly negative at $R = 0$ and $R = 1+z$ if the RHS of (63) is positive. And then differentiating twice with respect to R yields

$$2\left(\mu-\theta\mu'\right)>0,$$

given the concavity of μ. This establishes that p_{IB} in (65) must be decreasing in θ for $R \in [0, 1+z]$. For $p_{IS} = p_H$, differentiating (64) with respect to θ and evaluating at $R = 0$, 1, and $1+z$, respectively, all yield a strictly negative expression as long as the RHS of (63) is positive. This establishes that $p_{IS} = p_H$ in (64) must be decreasing in θ for R in neighborhoods of 0, 1, and $1+z$.

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