

# Short-Circuited Series Stubs for Application in Uniplanar Low-Pass Filters

Rui Li<sup>1</sup>, Sheng Sun<sup>2</sup>, Lei Zhu<sup>3</sup>

<sup>1</sup>*Institute of Microelectronics, A\*STAR (Agency for Science, Technology and Research), Singapore  
email: lir@ime.a-star.edu.sg*

<sup>2</sup>*Department of Electrical & Electronic Engineering, The University of Hong Kong, Hong Kong  
email: sunsheng@ieee.org;*

<sup>3</sup>*School of Electrical & Electronic Engineering Nanyang Technological University, Singapore  
email: ezhu@ntu.edu.sg*

**Abstract** — An alternative uniplanar low-pass filter with series stubs is presented in this paper and its upper stopband is extended up to five times of its cutoff frequency. The basic filter structure primarily consists of short-circuited series stubs on slot line and connecting lines on coplanar waveguide. Under the target of Chebyshev function low-pass frequency responses, all the element values involved in this filter topology is determined via efficient synthesis design approach rather try-to-cut optimization approach. Later on, the connecting line between adjacent stubs is replaced by its equivalent T-shaped transmission line to suppress the higher order harmonic bands. As design examples, two five-pole uniplanar filters with and without harmonic-suppressed elements embedded are designed, fabricated and measured.

**Index Terms** — Low pass filter, short-circuited series stub, synthesis design, harmonic suppression.

## I. INTRODUCTION

As a key building block, microwave low-pass filters have been widely used in modern wireless communication systems. High performance low-pass filters are always in great demand. Extensive studies have been carried out for decades and numerous low-pass filters based on various design topologies have been reported in the literature [1]-[7]. A typical class of microwave low-pass filters was constituted using open-circuited shunt stubs and connecting lines of quarter-/half-wavelength in [3] and [4]. Due to the frequency-distributed property, this kind of filters suffers from relatively narrow upper stopband. Another category of low-pass filter composed of hairpin or C-section resonators exhibits elliptic function response based on the original idea in [5] became quite popular recently, because of its compact size and extended upper stopband [6]. Since the attenuation poles cannot be located near the passband, the rejection skirt near the cutoff was not increased so much. A semi-lumped low-pass filter with attractive features of compactness and sharp rolloff was then proposed in [7], however, the lump components increase the fabrication difficulty in practice.

An alternative class of uniplanar low-pass filters with Chebyshev function responses is firstly proposed by using

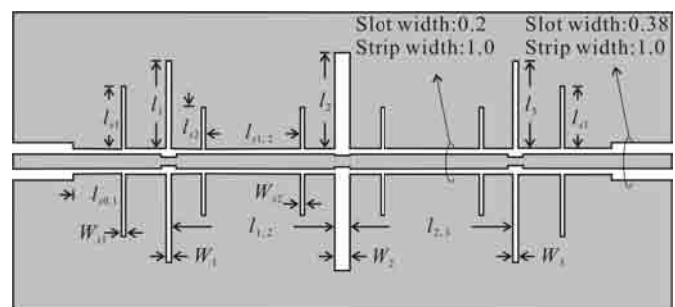


Fig. 1. The proposed Chebyshev low-pass filter with extended upper stopband. Dimensions:  $l_1 = l_5 = 6.0$ ,  $l_2 = 6.60$ ,  $l_{1,2} = l_{2,3} = 9.31$ ,  $l_{s1} = 3.72$ ,  $l_{s2} = 2.30$ ,  $l_{s0,1} = 2.23$ ,  $l_{s1,2} = 4.89$ ,  $W_1 = W_3 = 0.34$ ,  $W_2 = 1.0$ ,  $W_{s1} = 0.10$ ,  $W_{s2} = 0.10$ . All units are in mm.

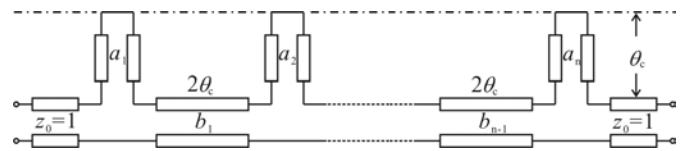


Fig. 2. Transmission-line model of the Chebyshev lowpass filter with series stubs ( $n = 3$ ) [9].

short-circuited series stubs [8] instead of open-circuited shunt stubs in the classical low-pass filters [3] and [4]. As a dual-case of the filters with shunt stubs in [3]-[4], its transmission line model only has series stubs and it is utilized herein to make up uniplanar low-pass filters on hybrid coplanar waveguide and slot line structures without needing via hole in the formulation of short-circuited ends. In analog to the classical filter theory in [3]-[4], an efficient synthesis design approach is established [8]. In this paper, the low-pass filter with series short-ended stubs is further discussed, emphasizing the performance of upper stopband. Afterwards, a lowpass filter with extended upper stopband is presented as shown in Fig. 1, where an equivalent T-shaped line section with a short-circuited series stub is newly developed and synthesized for harmonic suppression. The overall filter layout is simulated via commercial design

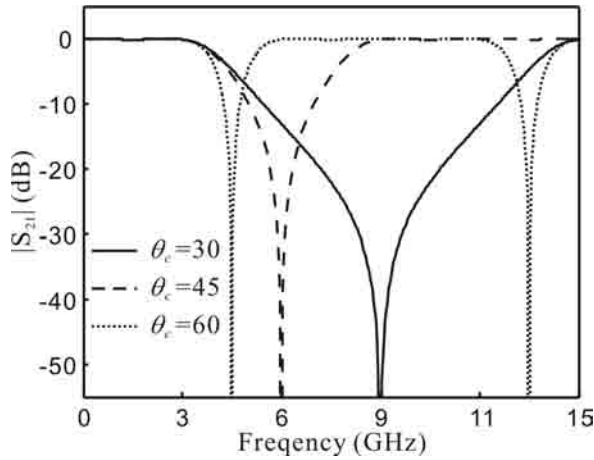


Fig. 3. Transmission line model frequency responses of low-pass filter ( $n = 2$ ) under varied cutoff electrical length  $\theta_c$ .

software, i.e., Agilent ADS software [9], for theoretical validation. Finally, two low-pass filters are designed and fabricated for comparison. The measured results are found to agree with those obtained from transmission line model and electromagnetic simulation and show good low-pass filtering responses up to five times ( $5f_c$ ) of the cutoff frequency ( $f_c$ ).

## II. LOW-PASS FILTER WITH SERIES STUBS

Fig. 2 shows the generalized transmission line model of the proposed low-pass filter with short-circuited series stubs. It is composed of a cascade of  $n$  series short-circuited stubs with normalized impedance  $a_i$  and electrical length  $\theta_c$  ( $\theta_c$ : electrical length at cutoff frequency  $f_c$ ) through  $(n-1)$  uniform connecting lines with normalized impedance  $b_i$  and electrical length  $2\theta_c$ . Following the direct synthesis procedures in [3]-[4], the insertion loss function  $P_L$  of the above network can be obtained as the following,

$$P_L = 1 + \left| \frac{P_{2n-1}(\sin \theta)}{\cos \theta} \right|^2. \quad (1)$$

where  $P_{2n-1}$  is a polynomial of degree  $(2n-1)$ . By matching the derived insertion loss function with the targeted insertion loss function (2) that exhibits desired Chebyshev equal-ripple responses [4], the element values of each section can be accordingly determined.

$$P_L = 1 + h^2 \left\{ \frac{(1 + \sqrt{1-x_c^2})T_{2n-1}\left(\frac{x}{x_c}\right) - (1 - \sqrt{1-x_c^2})T_{2n-3}\left(\frac{x}{x_c}\right)}{2\sqrt{1-x^2}} \right\}^2 \quad (2)$$

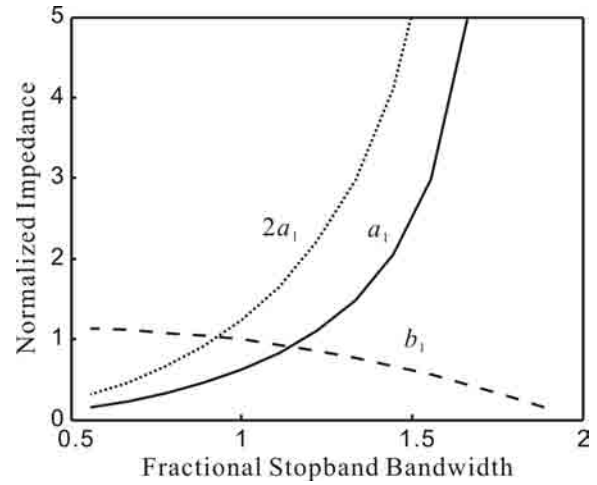


Fig. 4. Normalized impedances ( $a_1$  and  $b_1$ ) as a function of the cutoff electrical length ( $\theta_c$ ) for a low-pass prototype with  $n = 2$  and  $h = 0.1$  dB.

where  $x = \sin \theta$ ,  $x_c = \sin \theta_c$ ,  $T_n(x)$  is the Chebyshev polynomial function of the first kind of degree  $n$  and  $h$  is the specified equal-ripple level in the concerned frequency band.

Fig. 3 plots the frequency responses of the Chebyshev low-pass filter with  $n = 2$ ,  $h = 0.1$  dB,  $\theta_c = 30^\circ$ ,  $45^\circ$  and  $60^\circ$  at cutoff frequency 3 GHz. The upper cutoff phases are entirely determined by  $(180^\circ - \theta_c)$  and they are equal to  $150^\circ$ ,  $135^\circ$  and  $120^\circ$ , respectively. Hence, the fractional bandwidth of the upper stopband of the lowpass filter is defined as  $\Delta = 2 - \theta_c/45^\circ$ . By setting the same cutoff frequency, the low-pass filter with the smallest  $\theta_c$  has the largest upper stopband. However, as a tradeoff, the roll-off at the cutoff becomes slower as shown in Fig. 3.

The low-pass filter in Fig. 2 can be implemented on hybrid coplanar-waveguide and slotline structures [8]. Due to uniplanar feature of this filter, the short-circuited series stubs can be easily realized on slot lines while the uniform connecting lines between two adjacent stubs can be formed on coplanar-waveguide. Now, the relationship between the element values of a low-pass filter with  $n = 2$ , which are the normalized impedance for both two stubs  $a_1$  and middle connecting line  $b_1$ , and the fractional upper stopband bandwidth  $\Delta$  is investigated as shown in Fig. 4. The normalized impedance  $a_1$  becomes extremely large as wider upper stopband is required. On the other hand, the value of  $b_1$  does not change significantly and gradually goes down from unity ( $z_0 = 1$ ) with the increasing  $\Delta$ . In other words, the upper stopband bandwidth is limited by the impedance range of the series stub. In this case, the implementation of the stubs with slotline will have the largest upper stopband around 122% and 137% for balanced and non-balanced topology.

To further improve the upper stopband performance, the uniform connecting line between two adjacent stubs, even the feeding lines can be replaced by its equivalent T-shaped transmission lines. As a dual case of the T-shaped line with an open-circuited shunt stub in [10], a T-shaped line with a short-

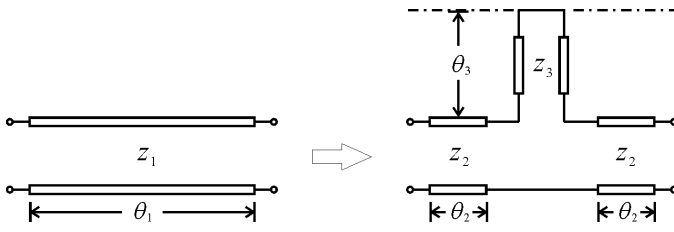


Fig. 5. Topologies of a uniform transmission line and its equivalent T-shaped line with a short-circuited series stub.

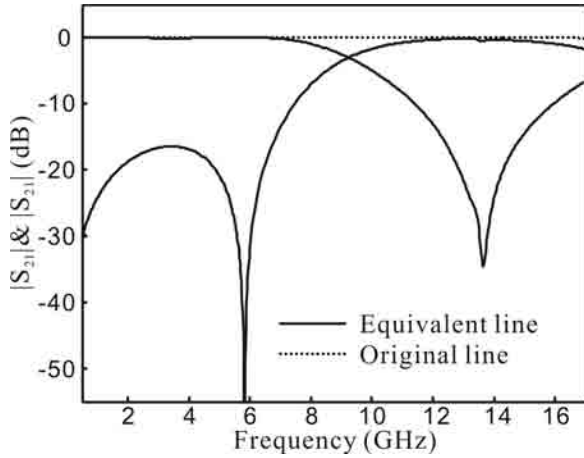


Fig. 6. Simulated frequency responses of the original connecting line and the T-shaped equivalent line.

circuited series stub is newly developed here, as shown in Fig. 5. For the original uniform transmission line with length of  $\theta_1$ , the  $ABCD$  matrix is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{original} = \begin{bmatrix} \cos \theta_1 & jz_1 \sin \theta_1 \\ jy_1 \sin \theta_1 & \cos \theta_1 \end{bmatrix} \quad (3)$$

The  $ABCD$  matrix of its equivalent series-type T-shaped line is given by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{new} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{line} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{stub} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{line} \quad (4)$$

where

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{line} = \begin{bmatrix} \cos \theta_2 & jz_2 \sin \theta_2 \\ jy_2 \sin \theta_2 & \cos \theta_2 \end{bmatrix} \quad (4a)$$

and

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{stub} = \begin{bmatrix} 1 & jz_3 \tan \theta_3 \\ 0 & 1 \end{bmatrix} \quad (4b)$$

By equaling (3) and (4) when  $\theta_1 = \pi/2$ , we have

$$z_2 = \frac{z_1}{\sin 2\theta_2 - \cot 2\theta_2 \cos^2 \theta_2} \quad (5a)$$

and

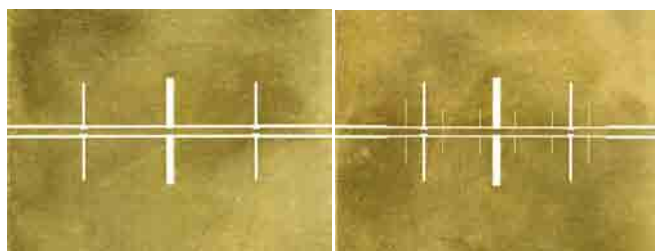
$$z_3 = \frac{z_1 \cot 2\theta_2}{(\sin 2\theta_2 - \cot 2\theta_2 \cos^2 \theta_2) \tan \theta_3} \quad (5b)$$

where  $\theta_2$  should be less than  $\pi/4$ , and  $\theta_3$  is determined by the position of the harmonic frequency. Thus, for a given uniform transmission line with an impedance of  $z_1$ , its equivalent T-shaped line with a short-circuited series stub can be synthesized through (5). Fig. 6 plots the simulated results of the original connecting line between two adjacent series stubs and its T-shaped equivalent line. The impedance of the original line is  $47 \Omega$ . It is observed that the frequency response of the equivalent line is close to the original line from DC to 6.5 GHz, and it generates one transmission zero at 13.6 GHz. The higher order harmonic band is therefore suppressed and the upper stopband will be widened. Similarly, the  $50 \Omega$  feeding line can also be replaced by such an equivalent line, which has similar electrical properties in the concerned frequency band as the original line.

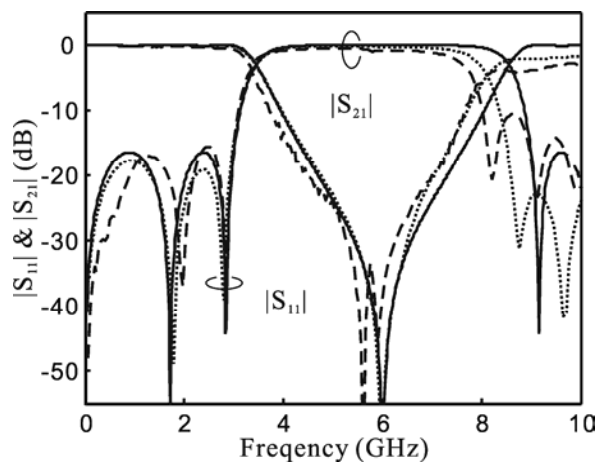
### III. RESULTS AND DISCUSSION

Final optimization procedure is executed via full-wave simulator that can take all the discontinuities into account, two prototype low-pass filters ( $n = 3$ ,  $h = 0.1$  dB and  $\theta_c = 45^\circ @ 3$ GHz) with/without T-shaped transmission line are designed and fabricated on a substrate with relative dielectric constant of 10.8 and thickness of 0.635 mm. Fig. 7(a) shows the photographs of these two fabricated filters. Firstly, for the direct synthesis designed low-pass filter, the measured frequency responses are depicted together with the ones obtained from transmission line model and full-wave simulation. It can be seen from Fig. 7(b) that the three sets of curves agree with each other very well. They all exhibit three consistent transmission poles in the low passband less than about 3.4 GHz and an attenuation pole with deep rejection around 5.8 to 6.0 GHz. These results have evidently validated the proposed transmission line model and predicted performances of these filters implemented on hybrid structure. Table I gives the detailed comparisons of the electrical parameters among these three sets of results from the model-based synthesis approach, full-wave simulation and microwave measurement.

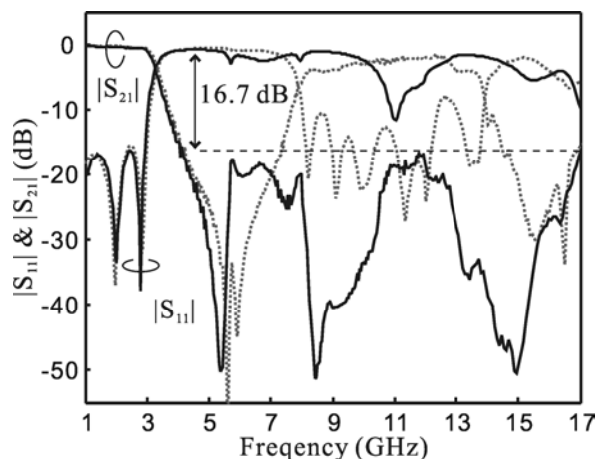
For the filter with embedded equivalent T-shaped transmission line as shown in the right side in Fig. 7(a), its measured frequency responses are plotted together in Fig. 7(c) with its original one shown in left-side in Fig. 7(a). It can be seen that the performance in the desired low passband has been almost unchanged, while the upper stopband has been greatly extended to 15 GHz with a suppression level below 16.7 dB.



(a)



(b)



(c)

Fig. 7. Five-pole low-pass filters. (a) Photographs of Filter I (left) & Filter II (right). (b) S-parameters of Filter I, theory (solid line), simulated (dotted line), and measured (dashed line). (c) Measured S-parameters of Filter I (dotted line) & Filter II (solid line).

#### IV. CONCLUSION

A class of uniplanar Chebyshev low-pass filters with short-circuited series stubs and its upper stopband performance are investigated. On a basis of a transmission line network composed of series stubs cascading through uniform connecting lines, this low-pass filter is efficiently synthesized in a simple step and its prototype is then formed on hybrid coplanar-waveguide and slotline structures. By replacing the uniform connecting lines with equivalent T-shaped lines, the

TABLE I  
COMPARISONS FOR LOW-PASS PROTOTYPE FILTER

	Theory	EM simulation	Measurement
3 dB cut-off (GHz)	3.44	3.45	3.34
Maximum in-band insertion loss (dB)	0.1	0.12	0.54
Minimum in-band return loss (dB)	16.43	17.72	15.51
3 dB upper cut-off (GHz)	8.56	8.12	7.89

upper stopband has been improved. The measured results of the fabricated filters show good agreement with the theoretical predictions and thus provide experimental verifications.

#### REFERENCES

- [1] G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance Matching Networks and Coupling Structure*. New York: McGraw-Hill, 1980
- [2] D. M. Pozar, *Microwave Engineering*. Reading, MA: Addison-Wesley, 1990.
- [3] O.P. Gupta and R.J. Wenzel, "Design Tables for a Class of Optimum Microwave Bandstop Filters," *IEEE Trans. Microw. Theory and Tech.*, vol. 18, pp.402-404, Jul. 1970.
- [4] R. Levy, "A new class of distributed prototype filters with applications to mixed lumped/distributed component design," *IEEE Trans. Microw. Theory and Tech.*, vol. 18, no.12, pp.1064-1071, Dec. 1970.
- [5] R. J. Wenzel, "Small elliptic-function low-pass filters and other applications of microwave c sections," *IEEE Trans. Microw. Theory Tech.*, vol. 18, no. 12, pp. 1150-1158, Dec. 1970.
- [6] L.-H. Hsieh and K. Chang, "Compact elliptic-function low-pass filters using microstrip stepped-impedance hairpin resonators," *IEEE Trans. Microw. Theory Tech.*, vol. 51, no. 1, pp. 193-199, Jan. 2003.
- [7] J.-W. Shen, "A compact semi-lumped low-pass filter for harmonics and spurious suppression," *IEEE Microw. Wireless Compon. Lett.*, vol. 10, no. 3, pp. 92-93, Mar. 2000.
- [8] R. Li, S. Sun, L. Zhu, "Direct synthesis of transmission line low-pass/high-pass filters with series stubs," *IET Microwaves, Antennas & Propagation*, vol. 3, no. 4, pp. 654-662, Jun. 2009.
- [9] Advanced Design System (ADS) 2005a, Agilent Technol., Palo Alto, CA, 2005.
- [10] W. -H. Tu and K. Chang, "Compact second harmonic-suppressed bandstop and bandpass filters using open stubs," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 6, pp. 2497-2502, Jun. 2006.