The Provision of Reinforcement in Concrete Solids using the Generalized Genetic Algorithm

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Abstract: A generalized genetic algorithm has been developed to find the global optimal reinforcement contents in a concrete solid structure subjected to a general three-dimensional stress field. All feasible solutions are examined based on the genetic algorithm. The heterogeneous strategy used in the algorithm ensures that all of the local optimal regions are searched and the most optimal reinforcement content is found. The effectiveness of the proposed approach has been validated by comparing the steel contents evaluated using the present method with those obtained from other available methods. A more economic design is achieved by the proposed algorithm. The method developed provides the designer with a valuable tool for the determination of reinforcements in complicated solid concrete structures.

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Introduction

Stress distributions in reinforced concrete solid structures such as pile caps, transfer plates, dams, etc. are highly complicated. With advances in the use of computational methods in various structural analyses, it is not difficult to evaluate the internal stress distribution of solid structures using three-dimensional (3-D) finite element methods. However, after getting the internal stress fields from the finite element analysis, to date, the determination of the required reinforcement in a 3-D solid structure remains very complex and computationally tedious.

Clark (1976) developed a two-dimensional (2-D) approach for the provisions of tensile and compressive reinforcements to resist in-plane stresses at any point in a planar structure by considering the Mohr's stress circles of the applied stresses and concrete capacity. Hsu (1993) proposed a reinforcement design method with the consideration of strain compatibility and constitutive relations of concrete and reinforcements. Foster et al. (2003) and Law et al. (2007) extended Clark's approach to solve 3-D problems. The two proposed methods require finding an optimal solution among the feasible solutions by means of some optimization methods. The shortcoming of Law's approach is that the optimal solution is simply obtained from a trial-and-error process. This approach cannot guarantee that all possible solutions are exhausted; thereby, the solution obtained may not be the same as the global optimal solution.

It is reasonable to believe that the feasible solution region comprises several continuous spaces that are independent of each other. This means that (i) the number of feasible solutions is countless but Law's approach cannot identify all of the solutions and (ii) most optimization methods that rely on one continuous feasible solution space will not work for these problems.

In this paper, a genetic algorithm (GA) is proposed to perform the global optimal reinforcement design of a concrete solid structure. GA is an optimization module that simulates the biological evolutionary process (Holland, 1962, 1975). The method can find the global optimal solution with high probability because of the inherent implicit parallelism even if the optimization problem has a complicated feasible solution space (Goldberg and Segrest, 1987; Eiben et al., 1991; Whitely L.D., 1992). Furthermore, using the GA, the optimization process and the equation system solution process are relatively independent, and there are no data transferred from the two processes except design variables and the values of the objective function (Forrest, 1993). This feature makes GA particularly suitable for solving the present complicated optimization problems. GA has been widely used in different engineering fields and its fundamental theory is being developed and improved continuously. Parallel genetic algorithm (PGA) (Fogarty and Huang, 1991; Neuhause, 1991; Agrawal and Mathew, 2004) was proposed to improve the efficiency of GA. Niche technology devotes to increase the population diversity (Beasly et al., 1993; Goldberg et al., 1992). Hybrid genetic algorithm (HGA) (Espinoza and Minsker, 2006; Martin, 2009) combines some traditional searching algorithms in GA to enhance the ability of local search. Macro-evolutionary algorithm (MA) (Martin and Sole, 1999; Chen, 2003; Chen and Wang, 2010) is a concept of species evolution at the higher level which could improve the capability of searching global optima and avoid premature convergence during the selection process. Monti et al. (2010) discussed the efficiency and correctness of different real-coded genetic algorithms.

Formulation of a reinforced concrete design based on a 3-D stress field

Any point in a 3-D structure has three components of direct stresses and three components of shear stresses. Figure 1 shows a small tetrahedron of elements in concrete with direct stresses σ_x , σ_y , σ_z and shear stresses τ_{xy} , τ_{xz} , τ_{yz} in global coordinates (*X*, *Y*, *Z*).

By rotating the orientation of the axes to where the shear stresses diminish to zero, the direct stresses at such orientations will become the "principal stresses". Timoshenko and Goodier (1970) related the principal stresses and principal directions by using the following eigenvalue equation:

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \cdot \begin{bmatrix} \ell \\ m \\ n \end{bmatrix} = \sigma_i \begin{bmatrix} \ell \\ m \\ n \end{bmatrix} \qquad (i = 1,2,3)\,, \tag{1}$$

where σ_i is the principal stress (the eigenvalue) and $[\ell \quad m \quad n]^T$ is the direction cosine vector of the principal plane (the eigenvector) of Eq. (1). The principal stresses obtained by Eq. (1) were termed the "theoretical" principal stresses by Law et al. (2007), to distinguish them from the actual principal stresses in concrete after cracking and in the presence of reinforcements. The fundamental assumptions of Law's approach include that (i) the applied shear stress is resisted by the concrete only and (ii) the reinforcement is assumed to carry only the uniaxial stress in the direction of the bar. In the case that any of the "theoretical" principal stresses determined by Eq. (1) are tensile or exceed the allowable concrete strengths f_c , reinforcements should be added to resist the all the tensile force.

Considering the equilibrium of the tetrahedrons, as shown in Figure 2, the following equations can be formulated:

$$\sigma_z - \rho_z f_s = \sigma_1 \ell_z^{\ 2} + \sigma_2 m_z^{\ 2} + \sigma_3 n_z^{\ 2} \tag{2}$$

$$\tau_{xz} = \sigma_1 \ell_x \ell_z + \sigma_2 m_x m_z + \sigma_3 n_x n_z \tag{3}$$

$$\tau_{yz} = \sigma_1 \ell_y \ell_z + \sigma_2 m_y m_z + \sigma_3 n_y n_z , \tag{4}$$

where σ_1, σ_2 and σ_3 are the principal stresses at the location considered, ρ_z, ρ_x, and ρ_y are steel ratios provided in the Z, X and Y directions, respectively, and f_s is the allowable stress of steel.

Likewise, similar equilibrium equations can be derived in the other directions.

Summing up the equilibrium equations, we obtain

$$\sigma_x - \rho_x f_s = \sigma_1 \ell_x^{\ 2} + \sigma_2 m_x^{\ 2} + \sigma_3 n_x^{\ 2} \tag{5}$$

$$\sigma_y - \rho_y f_s = \sigma_1 \ell_y^{\ 2} + \sigma_2 m_y^{\ 2} + \sigma_3 n_y^{\ 2} \tag{6}$$

$$\sigma_z - \rho_z f_s = \sigma_1 \ell_z^{\ 2} + \sigma_2 m_z^{\ 2} + \sigma_3 n_z^{\ 2} \tag{7}$$

$$\tau_{xy} = \sigma_1 \ell_x \ell_y + \sigma_2 m_x m_y + \sigma_3 n_x n_y \tag{8}$$

$$\tau_{xz} = \sigma_1 \ell_x \ell_z + \sigma_2 m_x m_z + \sigma_3 n_x n_z \tag{9}$$

$$\tau_{yz} = \sigma_1 \ell_y \ell_z + \sigma_2 m_y m_z + \sigma_3 n_y n_z . \tag{10}$$

By considering the fundamental property of direction cosines, one can get

$$\ell_x^{\ 2} + m_x^{\ 2} + n_x^{\ 2} = 1 \tag{11}$$

$$\ell_y^{\ 2} + m_y^{\ 2} + n_y^{\ 2} = 1 \tag{12}$$

$$\ell_z^{\,2} + m_z^{\,2} + n_z^{\,2} = 1\,. \tag{13}$$

The three principal planes are mutually perpendicular. Therefore,

$$\ell_x \ell_y + m_x m_y + n_x n_y = 0 \tag{14}$$

$$\ell_x \ell_z + m_x m_z + n_x n_z = 0 \tag{15}$$

$$\ell_y \ell_z + m_y m_z + n_y n_z = 0\,. \tag{16}$$

Eq.(11) to Eq.(16) are geometric equations.

The 12 equations (Eq.5 to Eq.16) will be used to work out the reinforcement ratios in the global directions under known values of applied stress σ_x, σ_y, σ_z, τ_{xy}, τ_{xz} and τ_{yz}. In general, there is no unique solution for this problem, as there are 15 unknowns and 12 equations. In view of the complexity of the solution of the system of equations, Law et al. (2007) suggested that, in the case that the "theoretical" principal stress exceeds f_c or is less than zero (i.e. in tension), the principal stress σ_3 is set to f_c or zero, leaving σ_1 and σ_2 to be determined. These assumptions are also adopted in the formulation in this paper. To find the optimal reinforced design, Law et al. (2007) further narrowed down the range of component of unknown direction cosines to search for feasible solutions. The optimal solution is chosen from a set of feasible solutions, which makes the total reinforcement ratio $\rho = |\rho_x| + |\rho_y| + |\rho_z|$ to a minimum.

In this paper, the set of equations (Eq.5 to Eq.16) is solved by the commercial software MatLab (MathWorks Inc, 2007a and 2007b). Unfeasible solutions are rejected by the abandon strategy (Gen and Cheng, 1997; Horst and Tuy, 1996), whereas numerical feasible solutions are transferred into the GA as a selecting individual. The optimal solution is obtained by the improved GA. The details of the GA will be presented in the following sections.

Optimization module and generalized genetic algorithm

In contrast to the binary code of Standard Genetic Algorithms (SGAs), General Genetic Algorithms (GGAs) employ real code. Real code and corresponding genetic operators improve the fitness of the optimal population and optimization efficiency (Gerald et al., 2009; Martin, 2009; Pei et al., 2009). A GGA is adopted in this paper, and the details will be discussed in the following sections.

Optimization module

As an optimization problem, one can construct an optimization module for the reinforcement design method based on 3-D stress fields. First, the objective function can be described as

$$\min \quad \rho = |\rho_x| + |\rho_y| + |\rho_z| . \tag{17}$$

Second, the number of design variables is 2 because there are 14 unknowns and 12 equations (σ_3 has been assumed to reach f_c or zero). Two design variables can be selected in 9 direction cosines. For simplicity, l_x and m_x are selected as design variables in the following optimization process. In the GA, the design variables l_x and m_x are called chromosomes, denoted by x_1 and x_2. A vector assembled by the design variables is called an individual, denoted as X_i^T:

$$X_i^T = \left(x_{i1}^T \quad x_{i2}^T\right). \tag{18}$$

The superscript T represents the current generation ordinal. The subscript i represents the population number of the individual.

The range of x_1 and x_2 is

$$-1 \le x_1 \le 1 \tag{19}$$

$$-1 \le x_2 \le 1. \tag{20}$$

Third, the principal stresses σ_1 and σ_2 of concrete should not be negative, as concrete tensile strength is ignored and all of the applied tensile stress is resisted by the steel reinforcement. Furthermore, σ_1 and σ_2 should not exceed f_c. Thus, the constraints are

$$0 \le \sigma_1 \le f_c \tag{21}$$

$$0 \le \sigma_2 \le f_c. \tag{22}$$

We then obtain the optimization module of the reinforcement design method based on 3-D stress fields.

Macroscopic genetic strategy

The population isolation mechanism (De Jong, 1975; Goldberg, 1987), which can quickly converge and effectively avoid pre-maturity, is adopted in this paper. Before optimization, the parameter space is divided into several subspaces. Seed selection and genetic operations are performed in each subspace independently in the following genetic processes. Finally, the global optimal solution is acquired by comparing the local optimal solutions. The essence of the population isolation mechanism is an artificial intervention method in the genetic optimization process, which can advance the computation parallelism of the GA. The efficiency of the population isolation mechanism has been proven elsewhere in the literature.

The whole optimization process was divided into two phases, i.e., an asymptotic phase and a cataclysmic phase. In the asymptotic phase, the GA searches for feasible solutions in any possible region. In the cataclysmic phase, the GA searches for the local optimal solution precisely in the local optimal region. The two phases are transformed gradually by the inherent property of the arithmetic crossover operator and the adaptive zoom of the mutation operator, which will be elaborated in the next section.

Genetic operators

There are 3 fundamental genetic operators in GAs, i.e., selection, crossover and mutation. The selection operator, also called the reproduction operator, is used to determine if an individual should be reproduced or eliminated from the population based on its fitness value. In general, the selection operator ensures that an individual with a high fitness value survives with higher probability, and, as a result, the better gene can be reserved and reproduced in the next generation (Goldberg, 1989; Davis, 1991). In SGAs, the selection of the operator is performed based only on probability, i.e., the parent generation is forbidden to compete with the progeny generation. Therefore, the best individual may not be preserved in the next generation. To improve the convergence probability and find the global optimal solution effectively, the optimum reserved strategy, which allows the parent generation to compete with the progeny generation, is adopted in this paper. The validity of the optimum reserved strategy is justified in theory and practice. According to the schema theorems (Whitley, 1992), SGAs with the optimum reserved strategy can converge to the global optimal solution by probability; whereas SGAs without the optimum reserved strategy cannot converge to the global optimal solution by probability. This finding shows the importance of the optimum reserved strategy.

The crossover operator simulates the reconstructive process of sexual reproduction in biology. Genes are exchanged among individuals by the crossover operator, and then new individuals, including more complex genes, are reconstructed (De Jong, 1975). The crossover operator is always selected by the special problem under study. The Arithmetic crossover operator, a widely used crossover operator in real code GGAs, is adopted in this study. Assuming X_i^T and X_j^T are two individuals that will cross in T generation population, the progeny generation, X_i^{T+1} and X_j^{T+1}, created by the arithmetic crossover operator are

$$\begin{cases} X_i^{T+1} = X_i^T + \tau_1 (X_i^T - X_j^T) \\ X_j^{T+1} = X_j^T + \tau_2 (X_i^T - X_j^T) \end{cases}, \tag{23}$$

where τ_1 and τ_2 are two random numbers that are uniformly distributed in the range of [-1, 1]. The arithmetic crossover operation ensures that all neighborhoods of X_i^T and X_j^T are searched and the area between X_i^T and X_j^T is well considered. If there is only one local optimal solution in the solution space or one of the local solutions is obviously better than others (which occurs with high probability in many optimization problems), then the arithmetic crossover operation moves individuals to the global optimal solution gradually. The mean distance of individuals is then shortened, and the GGA is transferred from the asymptotic phase to the cataclysmic phase.

The mutation operator simulates gene mutation in biology. In SGAs, the mutation operator randomly switches several bits of the bit string from 0 to 1 or from 1 to 0. This mutation operator is not suitable for GGAs because GGAs are coded with real numbers. Therefore, many mutation operators are created to adapt GGAs. Adaptive random mutation, as a practicable real code mutation operator, is adopted in this paper. Assuming that the code of the mutation individual X_i^T is

$$X_i^T = \left(x_{i1}^T \quad x_{i2}^T \right), \tag{24}$$

a chromosome is selected as the mutation bit arbitrarily, e.g., x_{i1}^T. The mutation result is

$$x_{i1}^{T+1} = x_{i1}^T + \tau b, \tag{25}$$

where τ is a random numbers which is uniformly distributed in the range of [-1, 1], and b is the radius of the value range. This mutation ensures that x_{i1}^{T+1} is acquired in the neighborhood of x_{i1}^T that is represented by $U\left(x_{i1}^T, b\right)$ and the radius b is a variable that is varied by the generation and determined by

$$b = \begin{cases} \frac{1}{2^T} b_s & \text{when} \quad \frac{1}{2^T} b_s > b_l \\ b_l & \text{else} \end{cases}, \tag{26}$$

where b_l and b_s are, respectively, the lower and upper bounds of the mutation range. Eq. (26) ensures the adaptive zoom ability of the mutation operator. The mutation range constantly decreases with evolution processing. In the asymptotic phase, the range is big and the mutation operator participates in the global search. In the cataclysmic phase, the range is small and the mutation operator is used to search the local optimal area precisely to find the local optimal solutions. The lower bound b_l is used to ensure the functionality of the mutation operator, because it is meaningless if the range becomes too small.

Inbreeding criterion and heterogeneous strategy

Generally speaking, GAs are good at global searching, whereas GGAs are accomplished at local searching. Although the population isolation mechanism decreases the probability of pre-maturity, it is not guaranteed that the global optimal solution will be found by a GGA. In fact, the convergence efficiency obtained by arithmetic crossover and adaptive random mutation occurs at the expense of random searching. To avoid inbreeding and pre-maturity, a heterogeneity strategy is proposed to improve the GGA.

Mechanism of the heterogeneous strategy

The heterogeneous strategy is a new genetic strategy that selects seeds out of the current search region with low probability in the course of evolution and sends them into the population to obtain population diversity. The heterogeneous strategy simulates the manual intervention operator in biology, and the aim is to obtain more optimal species. The procedural property of the GA ensures the achievement of the desired strategy. The mechanism of the heterogeneous strategy is to

(i) monitor the optimization procession and startup the heterogeneous strategy if inbreeding occurs in the present population;

(ii) select several heterogeneous seeds out of the present searching region and substitute the inbreeding individuals that have low fitness;

(iii) reproduce the progeny generation by the population including heterogeneous seeds and start the next evolution circulation.

Inbreeding criterion

Inbreeding can be judged by calculating the mean distance of the population. Assuming X_i^T and X_j^T are two individuals of the current population:

$$X_i^T = (x_{i1}^T \quad x_{i2}^T) \tag{27}$$

$$X_j^T = (x_{j1}^T \quad x_{j2}^T) \tag{28}$$

The distance between X_i^T and X_j^T can be calculated by

$$d(X_i^T, X_j^T) = \sqrt{(x_{i1}^T - x_{j1}^T)^2 + (x_{i2}^T - x_{j2}^T)^2} \tag{29}$$

Given a critical value d_r, the probability of the distance of 2 random different individuals in the T generation population less than d_r can be examined by Eq. (30).

$$P(T) = P\left[d(X_i^T, X_j^T)\right] < d_r \tag{30}$$

If $P(T)$ exceeds the given inbreeding criterion p_b, one can judge that inbreeding occurs in the T generation population. In other words, the inbreeding criterion is

$$P\left[d\left(X_i^T, X_j^T\right) < d_r\right] > p_b \tag{31}$$

It can be seen that a minority of individuals in the population inevitably approach each other. However, if many individuals assemble in a small feasible solution region, a manual intervention strategy, such as the heterogeneous strategy, should be adopted to ensure the optimization efficiency of the GGA. This is consistent with the rule of manually controlled species optimization in biology.

The inbreeding criterion should be different in the asymptotic phase and the cataclysmic phase. In the asymptotic phase, the GA needs to search all of the solution space. As we know, even slight inbreeding may lead to searching of the dead ends in this phase. For this reason, the inbreeding criterion should be more critical in this phase. In the cataclysmic phase, individuals assemble in several optimal regions and the final optimal solution is likely found. Accordingly, the declaration of inbreeding should be performed more carefully. The transformation of the inbreeding criterion is realized by the modification of the critical value d_r:

$$d_r = \begin{cases} \frac{1}{2^T} d_{rs} & when \quad \frac{1}{2^T} d_{rs} > d_{rl} \\ d_{rl} & else \end{cases}, \tag{32}$$

where d_{rl} and d_{rs} are the lower and upper bounds of d_r, respectively. This setup ensures that the heterogeneous strategy works effectively but does not interrupt the evolution excessively.

Heterogeneous seed selection

Assuming X_i^T and X_j^T are two inbreeding individuals and the corresponding fitness values are $f(X_i^T)$ and $f(X_j^T)$, without loss of generality, consider

$$f(X_i^T) < f(X_j^T) \tag{33}$$

The fitness value of X_i^T is smaller and the individual i is substituted by a heterogeneous seed.

There are two methods to select heterogeneous seeds. In the first method, one can select heterogeneous seeds randomly in the neighborhood of the inbreeding individuals. This method is similar to the aforementioned mutation operator. If

$$X_i^T = (x_{i1}^T \quad x_{i2}^T)\,, \tag{34}$$

assuming x_{i1}^T is the design variable to be operated (which is selected then randomly):

$$x_{i1}^T * = x_{i1}^T + \tau b_h\,, \tag{35}$$

where τ is a random number that is uniformly distributed in the range of [-1, 1], and b_h is the radius of the range. b_h is also varied by the generation ordinal T:

$$b_h = \begin{cases} \frac{1}{2^T} b_{hs} & when \quad \frac{1}{2^T} b_{hs} > b_{hl} \\ b_{hl} & else \end{cases}\,,$$

where b_{hl} and b_{hs} are the lower and upper bounds of b_h, respectively. The selected heterogeneous seed is then

$$X_i^T * = (x_{i1}^T * \quad x_{i2}^T)\,. \tag{36}$$

By substituting X_i^T with $X_i^T *$ in the present population, the heterogeneous operator is achieved.

In the asymptotic phase, the random selection method improves the diversity of the population and avoids inbreeding. In the cataclysmic phase, inbreeding individuals are usually alike, and the difference always occurs in several specific design variables because of the imbalance of the optimization process. Except for these several specific design variables, the other variables are identical. This feature can be used to yield more efficient heterogeneous seeds by considering the evolution trend. Assuming X_i^T and X_j^T are two inbreeding individuals and only the 2nd variable is different

$$\begin{cases} X_i^T = (x_{i1}^T \quad x_{i2}^T) \\ X_j^T = (x_{i1}^T \quad x_{j2}^T) \end{cases} \tag{37}$$

considering $f(X_i^T) < f(X_j^T)$, the 2nd variable of the heterogeneous seed can be determined by

$$x_{i2}^T * = \begin{cases} x_{j2}^T + \tau b_h & when \quad x_{i2}^T < x_{j2}^T \\ x_{j2}^T - \tau b_h & when \quad x_{i2}^T > x_{j2}^T \end{cases}. \tag{38}$$

The heterogeneous seed is then

$$X_i^T * = (x_{i1}^T \quad x_{ik}^T *). \tag{39}$$

It can be seen from Eq. (39) that the principles of the latter method of heterogeneous seed yielding are

(i) to consider the relationship between the fitness function and the k^{th} design variable while ignoring the other identical variables;

(ii) as the identical value of variables is determined by X_i^T or X_j^T, assume that the fitness function is monotonic or that there is only one local optimal solution in the neighborhood of x_{jk}^T;

(iii) to determine the evolution trend of the population and to find the better design variable $x_{ik}^T *$ so as to obtain the more valuable individual $X_i^T *$.

Figure 3 illustrates the heterogeneous seed selection based on the trend of evolution. This figure is based on a hypothesis that the design space and fitness function are all continuous in a little neighborhood of x_{jk}^T. This hypothesis is valid for most engineering optimization problem. In this hypothesis， $x_{ik}^T *$ can be found in the range of $(x_{jk}^T - b_h, x_{jk}^T)$ as $x_{ik}^T > x_{jk}^T$. Besides, if $x_{jk}^T - b_h$ is to the right of the local optimal solution, the function in the range $(x_{jk}^T - b_h, x_{jk}^T)$ is monotonous. $x_{ik}^T *$ can be selected using a similar process.

The probability fitness function that has only one peak value in the neighborhood of x_{jk}^{T} is very high. Consequently, heterogeneous seeds selected based on the evolution trend are always better than the inbreeding individuals; therefore, the convergence speed is increased, especially in the cataclysmic phase.

It should be noted that if most individuals are inbreeding, only several heterogeneous seeds will be selected to enter the population because the intensity of manual interference should not be so strong that it destroys the stability of the population structure, or else the inherent virtue of the GA will be lost.

In this paper, the improved GGA adopting the heterogeneous strategy will be used to find the best provision of reinforcements in concrete solids.

Implementation of GGA with the heterogeneous strategy

The computational procedure of GGA with the heterogeneous strategy is listed below.

(i) Divide the solution space into several subspaces. The calculation steps (ii to ix) are performed in each subspace independently and simultaneously;

(ii) Initialize all of the variables involved in the GGA;

(iii) Select the initial population randomly;

(iv) Place half of the individuals of the population randomly into the crossover pool and perform arithmetic crossover;

(v) Perform the selection operator on the population including the crossover parent generation and the generating progeny generation. Reserve the

(vi) Select half of the individuals of the population randomly and perform adaptive random mutation;

(vii) Perform the selection operator on the population including the mutation parent generation and the generating progeny generation. Reserve the better half and join them into the un-mutated parent generation to constitute the next population

(viii) Start up the heterogeneous strategy if inbreeding occurred in the present population, or else move to the next step;

(ix) Compute the mean fitness of the population. If the optimal individual is unchanged in p evolution circulation, go to the next step (considering that the algorithm has converged and the termination conditions have been met), otherwise go back to step (iv);

(x) Compare the local optimal populations in the subspaces and select the best individuals to constitute the final optimal population;

(xi) End the program and terminate the genetic optimization.

The crossover probability and mutation probability are different for different SGAs because of the intermediate population.

GGAs divide the optimization into an asymptotic phase and a cataclysmic phase. Cooperation of rough searching in the cataclysmic phase and precise searching in the asymptotic phase ensures that the global optimal solution can be found with high probability.

Examples

Twelve examples were computed to verify the accuracy and effectiveness of the proposed method. The design parameters of the former two examples are the same as described by Foster et al. (2003). The concrete cube strength is $f_{cu} = 65$ MPa and $f_c = 1.8 \times \sqrt{f_{cu}} = 14.51$ MPa, and the steel strength is $f_s = 400$ MPa. The design parameters of the last ten examples are the same as those given in Law et al. (2007). The design was carried out in accordance with BS 8110 (1997), including (i) $f_c = 0.4 f_{cu} = 14$ MPa (grade 35 concrete with f_{cu}=35MPa) for the case of no cracking; (ii) $f_c = 1.8 \times \sqrt{f_{cu}} = 10.65$ MPa with concrete cracking; and (iii) steel strength as $f_s = 0.87 f_y = 0.87 \times 460 = 400.2$ MPa. The "theoretical" principal stresses determined by Eq. (1) are denoted by σ_1, σ_2 and σ_3. The applied stresses for the twelve cases are tabulated in Table 1.

To perform the heterogeneous strategy, two critical parameters, d_r and b_h, need to be selected beforehand. These parameters are decided by the characters of the design space and population size N. In this paper, as the design space is a square, the upper bound of d_r (denoted as d_{rs}) can be computed by

$$d_{rs} = \frac{1}{4} d_t = \frac{x_{\max} - x_{\min}}{\sqrt{N}} = 0.2582 \tag{40}$$

where d_t is the "deserved distance" of individuals based on the hypothesis that all individuals are evenly distributed in the design space. By the same hypothesis, one can let $b_{hs} = 4d_t$, where b_{hs} is the upper bound of b_h.

The present calculations revealed that the average startup time of the heterogeneous strategy in these 12 examples is 7.58, and most inbreeding occurred in the last 5 generations.

The results obtained in the present study together with those described by Forster et al. (2003) and Law et al. (2007) are presented in Tables 2 and 3 for comparison. To demonstrate the efficiency of GGA and SGA, the first two examples were studied using SGA and GGA approaches. As shown in Table 2, lesser amount of total steel contents were determined by the GGA for all of the cases, which demonstrated that GGA can converge to the global optimum solution in a higher probability than SGA. Furthermore, by comparing the amount of steel determined by using Law's method and GGA, lesser amount of total steel contents were found by the GGA. This finding means that a more economic design can be achieved by the proposed method. As GAs have been integrated in many popular commercial packages, this method can be easily applied to reinforcement design in general practice.

Conclusions

As an improvement of the reinforcement design approach based on 3-D stress field proposed by Law et al. (2007), this paper is devoted to finding the optimal reinforcement design via an improved GGA. Since the feasible solutions are solved by MatLab and the global optimal solution is acquired by the GA, the optimal reinforcement design is robust.

An improved GGA is proposed in this paper based on the inbreeding criterion and heterogeneous strategy. The inbreeding criterion is used to judge the occurrence of inbreeding whereas the heterogeneous strategy is used to eliminate the inbreeding individuals. Many effective techniques such as the population isolation mechanism, optimum reserved strategy and arithmetic crossover were also adopted in the computation. It has been demonstrated from the solution of the optimization problem that the improved GGA is suitable for solving complicated optimization problems. The reinforcement design method based on the improved GGA can find the global optimal design solution of a general reinforced concrete element. The method provides the designer with a valuable tool for the dimensioning of reinforcements in concrete solid structures.

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Table.1. – Applied stresses of the 12 examples

	1	2	3	4	5	6	7	8	9	10	11	12
σ_x (kPa)	3000	-3000	-3000	-3000	-3000	-3000	17000	17000	-2000	3000	17000	5000
σ_y (kPa)	15000	15000	15000	5000	10000	-5000	15000	15000	2000	7000	15000	15000
σ_z (kPa)	4000	16000	-4000	-4000	-4000	-4000	16000	0	-5000	0	16000	16000
τ_{xy} (kPa)	2500	2500	2500	2500	2500	2500	2500	2500	-6000	-6000	2500	2500
τ_{xz} (kPa)	2000	2000	2000	2000	2000	2000	0	0	4000	4000	2000	2000
τ_{yz} (kPa)	3000	3000	3000	3000	3000	3000	0	0	-2000	-2000	3000	3000
σ_1 (kPa)	1437	-3472	-2272	-3163	-2576	-7664	18693	18693	8280	3280	12336	4263
σ_2 (kPa)	4157	12481	-5601	-5656	-5618	-5358	13307	13307	4320	-680	14642	12500
σ_3 (kPa)	16406	18992	15873	6819	11194	1023	16000	0	-7600	-1260	21023	19238

Table 2- The steel contents of Examples 1 and 2

	Example 1					Example 2				
	ρ_x (%)	ρ_y (%)	ρ_z (%)	ρ_{total} (%)	Difference in ρ_{total} (%)	ρ_x (%)	ρ_y (%)	ρ_z (%)	ρ_{total} (%)	Difference in ρ_{total} (%)
Foster et al. approach	2.302	0.970	2.301	5.573	N/A	0.942	0.000	0.942	1.884	N/A
SGA approach	2.456	0.964	1.986	5.406	-3.00	0.961	0.023	0.927	1.912	1.49
GGA approach	2.469	0.890	2.031	5.390	-3.27	0.974	0.000	0.886	1.860	-1.25

Table 3- Summary of the steel contents of Example 3 to Example 12

Example	Law et al. approach				GGA approach				Difference in ρ_{total} (%)
	ρ_x (%)	ρ_y (%)	ρ_z (%)	ρ_{total} (%)	ρ_x (%)	ρ_y (%)	ρ_z (%)	ρ_{total} (%)	
3	1.859	1.612	1.777	5.248	2.094	1.494	1.637	5.224	-0.45
4	0.004	1.664	1.251	2.910	0.009	1.611	1.268	2.888	-0.77
5	0.003	0.701	0.000	0.705	0.002	0.698	0.004	0.703	-0.26
6	1.020	2.065	2.399	5.485	1.015	2.045	2.395	5.455	-0.55
7	1.337	1.666	1.458	4.462	1.336	1.633	1.449	4.418	-0.99
8	1.277	0.000	1.855	2.799	1.255	0.000	1.521	2.777	-0.80
9	1.344	0.417	1.455	3.216	1.356	0.416	1.441	3.213	-0.11
10	1.165	2.191	1.601	4.957	1.158	2.194	1.600	4.951	-0.12
11	1.374	0.874	0.500	2.749	1.368	0.871	0.498	2.737	-0.45
12	1.374	0.874	0.000	2.249	1.368	0.871	0.000	2.239	-0.45

A List of Figure Captions

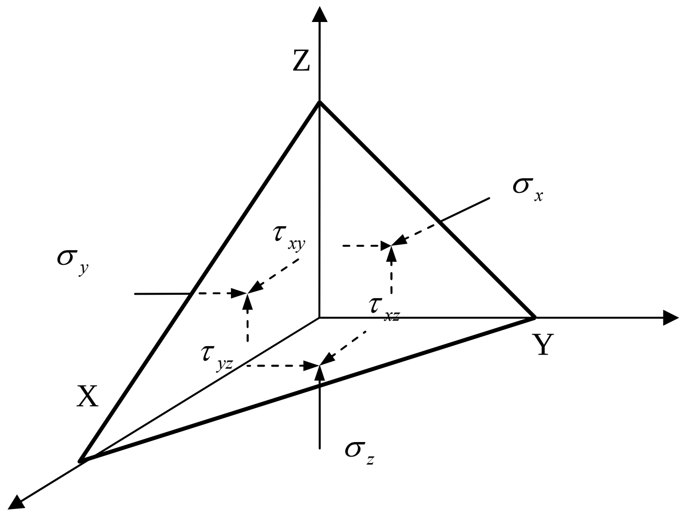

Fig. 1. Three-dimensional stress field

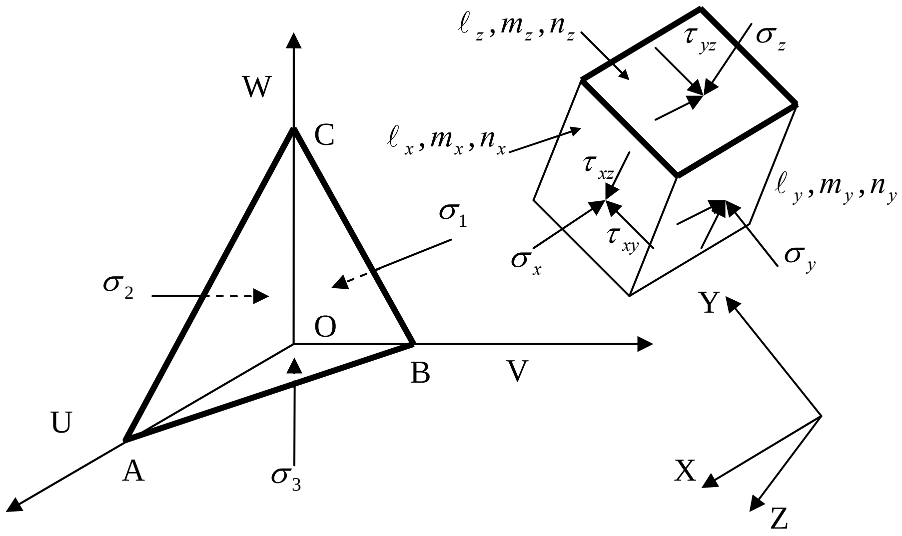

Fig. 2. Equilibrium of forces on a 3-D brick element

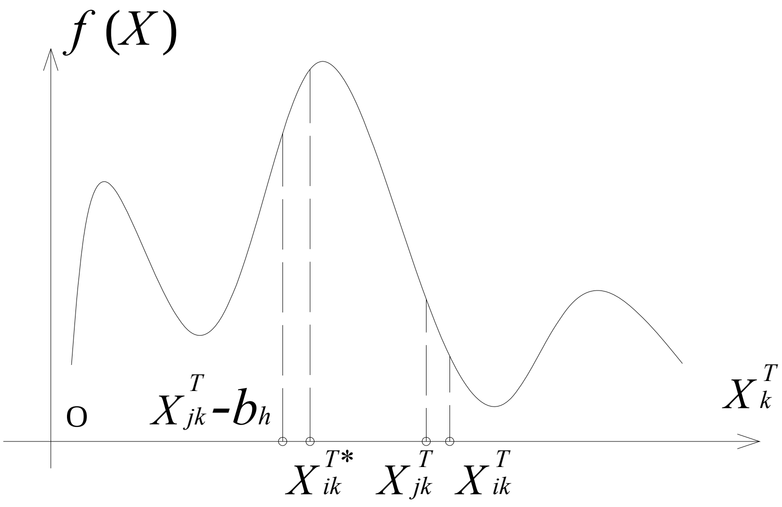

Fig. 3. Selection of heterogeneous seeds based on the evolution trend