Non-orthogonal Transmission in Multi-user Systems With Grassmannian Beamforming

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***Abstract*—Aiming to achieve the sum-rate capacity in multi-user multi-input multi-output (MIMO) channels with N_t antennas implemented at the transmitter, opportunistic beamforming (OBF) generates N_t orthonormal beams and serves N_t users during each transmission, which results in high scheduling delay over the users, especially in densely populated wireless networks. Non-orthogonal OBF with more than N_t transmit beams can be exploited to serve more users simultaneously and further decreases scheduling delay. However, the inter-beam interference will inevitably deteriorate the sum-rate. Therefore, there is a tradeoff between the sum-rate and the increasing number of transmit beams. In this context, the sum-rate of non-orthogonal OBF with $N > N_t$ beams are studied, where the transmitter is based on the Grassmannian beamforming. Our results show that non-orthogonal OBF is an interference-limited system. Moreover, when the inter-beam interference reaches its minimum for fixed N_t and N, the sum-rate scales as $N \ln\left(\frac{N}{N-N_t}\right)$ and it decreases monotonically with N for fixed N_t. Numerical results corroborate the accuracy of our analyses.**

I. Introduction

For different scheduling strategies in the downlink of multi-user communication systems, there are two conflicting goals [1], [2]. One is to satisfy the quality of service (QoS) requirements of different users, such as scheduling delay. The other aims to maximize system throughput. Round-robin scheduling falls into the former case and it follows a strict order to serve each user once in each round such that it guarantees minimum scheduling delay over the users. On the contrary, opportunistic scheduling exploits multi-user diversity gain and achieves the sum-rate capacity as the number of users K approaches infinity [3], [4]. When the base station (BS) is equipped with $N_t > 1$ antennas, opportunistic scheduling can be implemented by opportunistic beamforming (OBF), which generates $N = N_t$ beams and serves up to N_t users for each channel use [5], [6]. While it achieves the sum-rate capacity, OBF results in significant scheduling delay over the users in the densely populated wireless networks. In practice, shorter scheduling delay is much more desirable for the delay-sensitive traffic such as audio/video streaming, and for urgent data with deadline such as alarm applications [7], [8].

Conventionally, opportunistic and round-robin scheduling can be combined to keep a balance between scheduling delay and sum-rate [9], [10]. On the other hand, for the OBF, if $N > N_t$ beams are generated for each channel use, more users can be simultaneously served and scheduling delay will be further reduced. Unfortunately, in this case, the inter-beam interference will inevitably deteriorate the sum-rate. Therefore, there is a tradeoff between the sum-rate and the increasing number of transmit beams. This motivates us to investigate the relationship between the sum-rate and the number of beams for the OBF with $N > N_t$ beams. Hereafter, the conventional OBF with $N = N_t$ beams is denoted by "orthogonal OBF" while the scheme with $N > N_t$ beams is referred to as "non-orthogonal OBF".

In this paper, we focus on the general non-orthogonal OBF with $N > N_t$ transmit beams. Increasing N is essentially equivalent to increasing spatial multiplexing gain. On the contrary, the inter-beam interference among non-orthogonal beams will inevitably deteriorate the data-rate on each beam. Therefore, a key question is: how does the number of transmit beams N and inter-beam interference affect the final sum-rate? In this paper, the distribution function of the received signal-to-interference plus noise ratio (SINR) is first developed. Based on the distribution function, the achievable sum-rate and the sum-rate scaling law are established. In particular, the sum-rate scaling law reveals that non-orthogonal OBF is an interference-limited system. Moreover, when the inter-beam interference is minimized for fixed number of transmit antennas N_t and fixed number of transmit beams N, the sum-rate monotonically decreases with the number of transmit beams N.

The rest of this paper is organized as follows. Section II describes the system model and the scheduling strategy. In Section III, the distribution function of the received SINR is developed. The sum-rate is analyzed in Section IV. Simulation results and discussions are presented in Section V and, finally, Section VI concludes the paper.

II. System Model And Scheduling Strategy

A. System Model

We consider the downlink transmission from a BS equipped with N_t antennas to K single-antenna users. The number of users K is assumed to be larger than N_t and all users are scattered geographically and do not cooperate. The channels

The work was supported in part by the HKU Seed Funding Programme, Project No. 200911159084.

978-1-61284-231-8/11/$26.00 ©2011 IEEE

from the BS to each user are assumed to be constant during one time slot and vary independently from one time slot to another. Different channels among users are mutually independent and identically distributed. Also, the average SNRs for all users are assumed to be identical.

At each time slot, N different symbols in the vector $\boldsymbol{x} = [x_1, \cdots, x_N]^H$, where the superscript H denotes the Hermitian operation, are simultaneously transmitted onto N different beams. Prior to transmission, $\boldsymbol{x}$ is multiplied by a $N_t \times N$ beamforming matrix $\boldsymbol{B} = [\boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_N]$, where $\boldsymbol{b}_n$ denotes the beamforming vector for beam n, $n = 1, \cdots, N$. Therefore, the received symbol of user k is given by

$$y_k = \sqrt{\frac{\rho}{N}} \sum_{n=1}^{N} \boldsymbol{h}_k^H \boldsymbol{b}_n x_n + z_k, \tag{1}$$

where ρ stands for the transmit power for each user, $\boldsymbol{h}_k = [h_1^k, h_2^k, \cdots, h_{N_t}^k]^H$ is the channel vector between user k and the BS, modeled according to complex Gaussian distribution with zero mean and covariance $\sigma^2 \boldsymbol{I}$, where $\boldsymbol{I}$ refers to unit matrix, and z_k denotes additive white Gaussian noise with zero mean and unit variance.

In the case when the number of beams is equal to the number of transmit antennas, i.e., $N = N_t$, the above model corresponds to the conventional multi-beam orthogonal transmission [5]. In this paper, we concentrate on the cases with $N > N_t$ where the beamforming vectors $\boldsymbol{b}_n$, $n = 1, \cdots, N$, are no longer orthogonal to each other.

B. Maximum SINR Scheduling Strategy

As to the scheduling strategy at the BS, each user calculates the received signal-to-interference plus noise ratios (SINRs) on N different beams, and feeds back the maximum SINR and its corresponding beam index to the BS. More specifically, according to (1), the received SINR of user k on beam n is given by

$$\gamma_{k,n} = \frac{|\boldsymbol{h}_k^H \boldsymbol{b}_n|^2}{\frac{N}{\rho} + \sum\limits_{l=1,\, l\neq n}^{N} |\boldsymbol{h}_k^H \boldsymbol{b}_l|^2}, \tag{2}$$

where $|x|$ stands for the amplitude of x. For user k, the maximum SINR among N beams is determined as $\hat{\gamma}_k = \max_{n=1,\cdots,N} \gamma_{k,n}$, and the corresponding beam-index is $\hat{n}_k = \arg\max_{n=1,\cdots,N} \gamma_{k,n}$. Therefore, the feedback pertaining to user k is the pair $(\hat{\gamma}_k, \hat{n}_k)$.

After receiving all the feedbacks at the BS, the user with the maximum SINR, among all users whose $\hat{n}_k = n$, is chosen to be served on beam n. More precisely, the maximum SINR achieved through beam n is determined as

$$\hat{\gamma}_{\max,n} = \max_{\hat{\gamma}_k:\, \hat{n}_k = n} \hat{\gamma}_k \tag{3}$$

and the index of the user to be served on beam n in the next time slot is given by

$$\hat{k}_n = \arg \max_{\hat{\gamma}_k:\, \hat{n}_k = n} \hat{\gamma}_k. \tag{4}$$

Remark 1: This study focuses on the case with $N_r = 1$ receive antenna for each user but it can be generalized to the cases with $N_r > 1$ in a straightforward manner. For example, a combining strategy such as maximum ratio combining can be exploited to treat each terminal with $N_r > 1$ as a single-dimensional receiver [11].

III. Distribution Function of Received SINR

In general, increasing the number of transmit beams enhances spatial multiplexing gain but also increases the total inter-beam interference, thus degrading achievable sum-rate. Aiming at assessing this tradeoff, we analytically investigate the distribution function of the received SINR in this section, and the sum-rate analysis is provided in the next section.

With the principle of orthogonal projection, the beamforming vector $\boldsymbol{b}_l$ can be expressed in terms of $\boldsymbol{b}_n$, where $l \neq n$, via their cross-correlation coefficient $\delta_{l,n} \triangleq \left|\boldsymbol{b}_l^H \boldsymbol{b}_n\right|$, that is,

$$\boldsymbol{b}_l = \delta_{l,n} \boldsymbol{b}_n + \sqrt{1 - \delta_{l,n}^2}\, \boldsymbol{b}_{l,n}^{\perp}, \quad 1 \leq l, n \leq N, \tag{5}$$

where $\boldsymbol{b}_{l,n}^{\perp}$ is the orthonormal vector of $\boldsymbol{b}_l$ to $\boldsymbol{b}_n$. Substituting (5) into (2), the received SINR of user k on beam n can be rewritten as

$$\gamma_{k,n} = \frac{X}{\frac{N}{\rho} + \sum\limits_{\substack{l=1 \\ l\neq n}}^{N} \left[\delta_{l,n}^2 X + \left(1 - \delta_{l,n}^2\right) Y_l + \delta_{l,n} \sqrt{1 - \delta_{l,n}^2}\, Z_l\right]}, \tag{6}$$

where

$$X \triangleq \left|\boldsymbol{h}_k^H \boldsymbol{b}_n\right|^2, \tag{7}$$

$$Y_l \triangleq \left|\boldsymbol{h}_k^H \boldsymbol{b}_{l,n}^{\perp}\right|^2 \tag{8}$$

and

$$Z_l \triangleq \boldsymbol{h}_k^H \boldsymbol{b}_n \left(\boldsymbol{h}_k^H \boldsymbol{b}_{l,n}^{\perp}\right)^H + \boldsymbol{h}_k^H \boldsymbol{b}_{l,n}^{\perp} \left(\boldsymbol{h}_k^H \boldsymbol{b}_n\right)^H. \tag{9}$$

Based on the theory of optimal Grassmannian line packing [12], [13], in order to guarantee that the correlation between any two beamforming vectors is as small as possible, beamforming vectors must be symmetric, i.e., $\delta_{l,n} = \delta_0$, where $1 \leq l \leq N$ and $l \neq n$. Define

$$\alpha \triangleq (N-1)\delta_0^2 \tag{10}$$

denoting the total inter-beam interference from the other $N-1$ beams on beam n. Let $\beta \triangleq 1 - \delta_0^2$, $\eta \triangleq \delta_0 \sqrt{1 - \delta_0^2}$,

$$Y \triangleq \sum_{l=1,\, l\neq n}^{N} Y_l \tag{11}$$

and

$$Z \triangleq \sum_{l=1,\, l\neq n}^{N} Z_l. \tag{12}$$

Then, when the inter-beam interference are symmetric, (6) can be rewritten as

$$\gamma_{k,n} = \frac{X}{\frac{N}{\rho} + \alpha X + \beta Y + \eta Z}. \tag{13}$$

Since $\boldsymbol{h}_k$ is a complex Gaussian random vector and $\boldsymbol{b}_n$ and $\boldsymbol{b}_{l,n}^{\perp}$ are normalized constant vectors, it is clear that X in (7) and Y_l in (8) are of exponential distribution with unit mean and unit variance. Consequently, Y in (11) is of chi-square distribution with $2(N-1)$ degrees of freedom. Accordingly, the probability density functions (PDFs) of X and Y are given by

$$f(x) = \exp(-x), \quad x \geq 0 \tag{14}$$

and

$$f(y) = \frac{1}{\Gamma(N-1)} y^{N-2} \exp(-y), \quad y \geq 0, \tag{15}$$

respectively, where $\Gamma(x)$ denotes the Gamma function. Furthermore, the PDF of Z is summarized in the following lemma.

Lemma 1: The PDF of Z in (12) is given by

$$f(z) = \frac{1}{2^N \Gamma(N)} z^{N-1} W_{0,\, N-\frac{1}{2}}(2z), \quad -\infty < z < +\infty, \tag{16}$$

with zero mean and variance $2(N-1)$, where $W_{\kappa,\,\mu}(z)$ denotes the Whittaker function [14, Eq.13.14.3].

Proof: See [15]. ■

Due to the high complexity of the PDF of Z in (16), the exact distribution function of $\gamma_{k,\,n}$ in (13) is hard to obtain. However, it is noted that ηZ has zero mean and small variance $2(N-1)\eta^2 = 2(N-1)\delta_0^2(1-\delta_0^2)$ with moderate N, since δ_0 is usually very small ($\delta_0^2 \ll 1$) in order to avoid inter-beam interference. So, the effect of ηZ on received SINR $\gamma_{k,\,n}$ is negligible, and (13) can be approximately given by

$$\gamma_{k,\,n} \approx \frac{X}{\frac{N}{\rho} + \alpha X + \beta Y}. \tag{17}$$

Notice that, when $N = N_t$, $\alpha = 0$ and $\beta = 1$, (17) reduces to the exact expression of received SINR under orthogonal transmission [16]. Consequently, our analysis is general and applicable to either orthogonal or non-orthogonal OBF.

With the help of (14) and (15), after some further mathematical derivations, the CDF and PDF of $\gamma_{k,\,n}$ in (17) can be shown as

$$F_{\Gamma_{k,\,n}}(\gamma) = 1 - \exp\left(-\frac{N\gamma}{\rho(1-\alpha\gamma)}\right)\left(1 + \frac{\beta\gamma}{1-\alpha\gamma}\right)^{-(N-1)} \tag{18}$$

and

$$\begin{aligned} f_{\Gamma_{k,\,n}}(\gamma) = & \\ & \exp\left(-\frac{N\gamma}{\rho(1-\alpha\gamma)}\right)\left(1+\frac{\beta\gamma}{1-\alpha\gamma}\right)^{-N} \\ & \times (1-\alpha\gamma)^{-2}\left[\frac{N}{\rho}\left(1+\frac{\beta\gamma}{1-\alpha\gamma}\right) + (N-1)\beta\right], \end{aligned} \tag{19}$$

respectively. As a special case, the orthogonal OBF with $N = N_t$ implies $\delta_0 = 0$ and thus $\alpha = 0$ and $\beta = 1$. Putting $\alpha = 0$ and $\beta = 1$ into (18) and (19), they reduce to the exact distribution functions of received SINR under orthogonal transmission [16].

Since the maximum SINR in (3) is given by $\hat{\gamma}_{\max,\,n} = \max_{k=1,\cdots,K} \gamma_{k,\,n}$, by using the results from order statistics, the CDF and PDF of $\hat{\gamma}_{\max,\,n}$ in (3) are given by

$$F_{\Gamma_{\max,\,n}}(\gamma) = F^K_{\Gamma_{k,\,n}}(\gamma) \tag{20}$$

and

$$f_{\Gamma_{\max,\,n}}(\gamma) = K\, f_{\Gamma_{k,\,n}}(\gamma) F^{K-1}_{\Gamma_{k,\,n}}(\gamma), \tag{21}$$

respectively.

IV. Asymptotic Analysis on Achievable Sum-rate

Based on the obtained distribution function of received SINR, the achievable sum-rate and the sum-rate scaling law are investigated in this section.

For beam n with received SINR $\hat{\gamma}_{\max,\,n}$, the instantaneous data rate can be calculated by the Shannon formula $\ln(1+\hat{\gamma}_{\max,\,n})$ in the unit of nat/s/Hz. Moreover, since there are N beams in total, the achievable sum-rate is given by

$$\begin{aligned} R &= \sum_{n=1}^{N} \mathcal{E}\left\{\ln(1+\hat{\gamma}_{\max,\,n})\right\} && (22)\\ &= \sum_{n=1}^{N} \int_0^{+\infty} \ln(1+\gamma) K f_{\Gamma_{k,\,n}}(\gamma) F^{K-1}_{\Gamma_{k,\,n}}(\gamma)\, \mathrm{d}\gamma && (23)\\ &= KN \int_0^{+\infty} \ln(1+\gamma) f_{\Gamma_{k,\,n}}(\gamma) F^{K-1}_{\Gamma_{k,\,n}}(\gamma)\, \mathrm{d}\gamma, && (24) \end{aligned}$$

where (21) was exploited to reach (23). Due to the complicated expressions of $f_{\Gamma_{k,\,n}}$ and $F_{\Gamma_{k,\,n}}$ in (18) and (19) respectively, the integration in (24) has no closed-form expression and has to be evaluated numerically. In order to gain insights into the sum-rate, we instead derive the limiting distribution of $\hat{\gamma}_{\max,\,n}$ as $K \to \infty$. However, notice that the limiting distribution cannot be obtained by directly applying $K \to \infty$ in (20), since for any $F_{\Gamma_{k,\,n}}(\gamma) < 1$, (20) reduces to 0 as $K \to \infty$ and hence (20) is a degenerate distribution. Below, the asymptotic theory of extreme order statistics is exploited to attain a non-degenerate limiting distribution for $\hat{\gamma}_{\max,\,n}$ such that the sum-rate scaling law is obtained, which explicitly reveals the effect of the number of transmit beams and the inter-beam interference on the achievable sum-rate.

In order to obtain the sum-rate scaling law, which refers to the achievable sum-rate with large K [17], we first derive the limiting distribution for $\hat{\gamma}_{\max,\,n}$ as $K \to \infty$. From the asymptotic theory of extreme order statistics, it is known that, if $\hat{\gamma}_{\max,\,n}$ is suitably normalized, its limiting distribution must be one of the three types of extreme-value distributions, namely, Fréchet, Weibull and Gumbel distributions [18]. The von Mises's criteria are sufficient conditions to determine which limiting distribution $\hat{\gamma}_{\max,\,n}$ belongs to, and our result is summarized in the following lemma.

Lemma 2: For the received SINR $\gamma_{k,\,n}$ of user k with respect to beam n, with CDF $F_{\Gamma_{k,\,n}}(\gamma)$ and PDF $f_{\Gamma_{k,\,n}}(\gamma)$ given by (18) and (19) respectively, as the number of users $K \to \infty$, the limiting distribution of the maximum SINR in (3) is of the Gumbel distribution. That is,

$$\lim_{K\to\infty} F_{\Gamma_{\max,\,n}}(\gamma) = H_{3,\,0}\left(\frac{\gamma-a}{b}\right), \tag{25}$$

where

$$H_{3,\,0}(\gamma) = \exp\left(-e^{-\gamma}\right), \tag{26}$$

the normalizing parameters a and b are given by

$$a = \frac{\rho \ln K}{c + \rho\alpha \ln K} + \mathcal{O}(\ln\ln K) \tag{27}$$

and

$$b = \frac{c^2 \left(c + \rho\beta \ln K\right)}{(c + \rho\alpha \ln K)^2 \left[\frac{N}{\rho}\left(c + \rho\beta \ln K\right) + c(N-1)\beta\right]}, \quad (28)$$

respectively. In (27) and (28), the notation $f(x) = \mathcal{O}\left(g(x)\right)$ is defined as $\lim_{x\to\infty} |f(x)/g(x)| < \infty$, and $c \triangleq N+\rho(N-1)\beta$.

Proof: See [15]. ■

Applying *Lemma 2* in (22) yields:

$$\begin{aligned} R &= \sum_{n=1}^{N} \mathcal{E}\left\{\ln\left(1+\hat{\gamma}_{\max,\,n}\right)\right\} \\ &\leq \sum_{n=1}^{N} \ln\left(1+\mathcal{E}\{\hat{\gamma}_{\max,\,n}\}\right) \quad (29) \\ &= N\ln\left(1+a+b\Upsilon\right) \quad (30) \end{aligned}$$

where the Jensen's inequality was exploited to derive (29) owing to the fact that the logarithmic function is strictly concave; the limiting distribution (25) was exploited to reach (30) and $\Upsilon = 0.5772\cdots$ is the Euler's constant. Moreover, substituting (27) and (28) into (30) and as $K \to \infty$, (30) reduces to a simple sum-rate scaling law and it is given in the following theorem.

Theorem 1: When the total inter-beam interference α for each beam is identical, and as $K \to \infty$, the sum-rate of non-orthogonal OBF scales as

$$R \sim N \ln\left(1 + \frac{1}{\alpha}\right), \quad (31)$$

where the notation $f(K) \sim g(K)$ is defined as $\lim_{K\to\infty} f(K)/g(K) = 1$.

Special Case: The preceding analysis can be applied in orthogonal OBF without inter-beam interference, i.e., $\delta_0 = 0$. In particular, putting $\alpha = 0$, $\beta = 1$ and $N = N_t$ into (27), (28), (30) and as $K \to \infty$ yields

$$R' \sim N_t \ln\ln K, \quad (32)$$

which is exactly the sum-rate scaling law under orthogonal transmission [5]. Furthermore, comparing (31) with (32), it is observed that, as $K \to \infty$, for non-orthogonal OBF, multi-user diversity gain vanishes since R is independent of K, and the sum-rate is dominated by the number of beams N and the total inter-beam interference α. However, for orthogonal OBF, multi-user diversity gain always benefits the sum-rate since there is no inter-beam interference.

The theorem above reveals that the sum-rate of non-orthogonal transmission increases proportionally to the number of transmit beams N, but it is offset by the total inter-beam interference α. In other words, for fixed N_t and N, non-orthogonal OBF is an interference-limited system. Clearly, in order to achieve maximum R with fixed N and N_t, α must be kept as small as possible.

In general, the $N_t \times N$ non-orthogonal beamforming matrix with $\delta_{l,\,n} = \delta_0$, where $1 \leq l \leq N$ and $l \neq n$, is equivalent to the optimal Grassmannian frame in the complex space $\mathbb{C}^{N_t}$ [12]. The optimal Grassmannian frame is equiangular tight frame and its frame correlation achieves the lower bound provided by the Rankin inequality [12]:

$$\delta_0 = \sqrt{\frac{N - N_t}{N_t(N-1)}}. \quad (33)$$

Accordingly, for fixed N, the minimum total inter-beam interference is given by

$$\alpha_{\min} = (N-1) \times \left(\sqrt{\frac{N-N_t}{N_t(N-1)}}\right)^2 = \frac{N}{N_t} - 1. \quad (34)$$

Practical non-orthogonal Grassmannian beamforming matrices achieving the smallest inter-beam interference $\alpha_{\min}$ are detailed in [12], [13], [19], [20]. Applying (34) in *Theorem 1* yields the following corollary.

Corollary 1: For fixed N_t and N, when the inter-beam interference reaches the minimum $\frac{N}{N_t} - 1$, the sum-rate of non-orthogonal OBF scales as

$$R \sim N \ln\left(\frac{N}{N - N_t}\right). \quad (35)$$

Based on *Corollary 1*, it can be shown that the sum-rate monotonically decreases with increasing N for fixed N_t, but increases with N_t for fixed N. More precisely, when N_t is fixed, the first-order derivative of R in (35) with respect to N is given by

$$\frac{\mathrm{d}R}{\mathrm{d}N} = \ln\left(\frac{N}{N-N_t}\right) - \frac{N_t}{N-N_t} < 0, \quad (36)$$

where we exploited the inequality that $\ln x < x - 1$ whenever $x > 1$ [14, Eq.(4.5.4)]. On the other hand, when N is fixed and $N > N_t$, the first-order derivative of R with respect to N_t is shown as

$$\frac{\mathrm{d}R}{\mathrm{d}N_t} = \frac{N}{N - N_t} > 0. \quad (37)$$

An intuitive explanation of the monotonicity of R with respect to N and N_t is as follows. In the complex space $\mathbb{C}^{N_t}$, for fixed N_t, increasing N yields larger interference among N vectors. Nevertheless, if N is fixed, increasing N_t reduces the interference among N vectors.

V. Simulation Results and Discussions

In this section, numerical results based on the above analyses and Monte-Carlo simulation results are presented. All transmissions are over Rayleigh fading channels with zero mean and unit variance.

As justified from (6) to (17), the received SINR in (6) is well-approximated by (17) and then its closed-form distribution functions follow in (18) and (19). The accuracy of this approximation is also demonstrated in Fig. 1, where the simulated PDF of received SINR in (6) is compared with the analytical PDF in (19), based on the Grassmannian beamforming with $N_t = 3$ and $\mathrm{SNR} = 5\,\mathrm{dB}$. From the upper panel, which is corresponding to the orthogonal OBF with $N = N_t = 3$, it is seen that the numerical results of (19) coincide perfectly with the simulation results, since the PDF in (19) with $\alpha = 0$ and $\beta = 1$ is the exact PDF of received

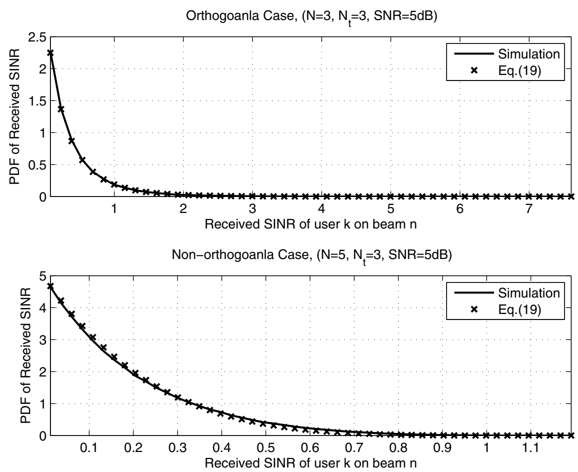

Fig. 1. PDF of the received SINR of user k on beam n with Grassmannian beamforming.

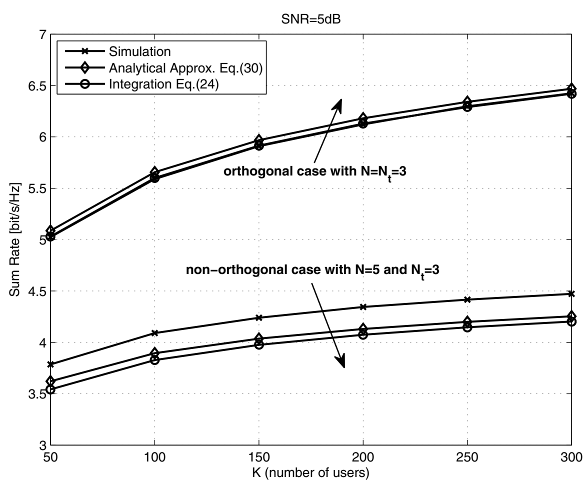

Fig. 2. The sum-rate of orthogonal and non-orthogonal OBFs with $N_t = 3$.

SINR under orthogonal transmission. Furthermore, the lower panel of Fig. 1 shows that, for the non-orthogonal OBF with $N_t = 3$ and $N = 5$, the numerical results of (19) are almost indistinguishable from the simulation results.

The sum-rate of orthogonal and non-orthogonal transmission is illustrated in Fig. 2, with the orthogonal OBF $N = N_t = 3$, and the non-orthogonal case $N = 5$, $N_t = 3$. For the orthogonal case, it is observed that the integration in (24) coincides perfectly with the simulation results. Moreover, it is seen that the analytical expression (30) is very tight with (24), in which the tiny difference is due to the Jensen's loss introduced in (29). On the other hand, for the non-orthogonal transmission, the numerical results of the integration in (24) is slightly smaller (about 6%) than the simulation results, due to the approximation introduced in (17). The analytical expression (30) is still very tight with (24). Furthermore, as expected, Fig. 2 shows that non-orthogonal OBF underperforms orthogonal case in terms of sum-rate, as inter-beam interference exists.

VI. Conclusion

The sum-rate of non-orthogonal Grassamnnian beamforming system with $N > N_t$ transmit beams was investigated in this paper. Our results show that non-orthogonal beamforming system is interference-limited. Also, when the inter-beam interference attains the minimum for fixed number of transmit antennas N_t and fixed number of transmit beams N, the sum-rate decreases monotonically with the number of transmit beams N.

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