Iterative Correction of Frequency Response Mismatches in Time-interleaved ADCs: A Novel Framework and Case Study in OFDM Systems

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***Abstract*—In this paper, we study a versatile iterative framework for the correction of frequency response mismatch in time-interleaved ADCs. Based on a general time varying linear system model, we establish a flexible iterative framework, which enables the development of various efficient iterative correction algorithms. In particular, we study the Gauss-Seidel iteration in detail to illustrate how the correction problem can be solved iteratively, and show that the iterative structure can be efficiently implemented using Farrow-based variable digital filters with few general-purpose multipliers. Simulation results show that the proposed iterative structure performs better than conventional compensation structures. Moreover, a preliminary study on the BER performance of OFDM systems due to TI ADC mismatch is conducted.**

***Index Terms*—Farrow structures, Frequency response mismatch, Iterative methods, OFDM, Time-interleaved analog-to-digital converters, Variable digital filters.**

I. INTRODUCTION

In modern wireless communication systems such as software defined radios and wideband OFDM-related systems, the performance of analog-to-digital converters (ADCs) are of paramount important owing to the requirements of high sampling rate and low power consumption usually encountered in practical systems [1]. The overall performance of a communication system often hinges on these critical components. In general, the ADC performance is determined by various limitations in a given process technology, e.g. IC fabrication [2]. In order to stay with the current technology while meeting the increasing requirements of modern communication systems, new structures for improving the performance of current signal converters is an important problem in both research and industrial communities. One promising ADC scheme that is capable of offering high sampling rate is time-interleaved (TI) ADCs [3], in which an array of ADCs works in parallel at a low or median sampling rate. If the outputs of the ADC array are combined appropriately, much higher sampling rate can be achieved. However, any small channel mismatches between sub-ADCs cause a significant degradation in performance [4].

A particular type of mismatch in TI ADCs is the time-skew errors between different channel ADCs. Previously, there were numerous successful attempts in correcting the timing mismatch errors in TI ADCs [5] – [10]. More recently, some research works have focused on a more general problem of frequency response mismatch, for which each channel is assumed to have its own magnitude and phase characteristics [11] – [15]. Among these works, the compensation structure studied in [11] and a similar one in [12] are especially attractive for real-time applications because of their relatively lower reconfigurable complexity. In particular, the authors in [11] developed a system model describing the general relationship between the input and output signals, analyzed the error signal due to frequency response mismatches in detail, and demonstrated how the cascade of compensation filters improve the accuracy of the compensated output stage-by-stage. Two major advantages of this approach are that its implementation complexity is independent of the number of channels, and its scalability enables one to obtain compensated outputs with different resolutions.

In the paper, we investigate the problem in another direction by considering it as an inversion problem of a time varying linear system, and propose a versatile framework for the development of iterative methods and structures to correct the frequency response mismatches in TI ADCs. In order to facilitate the real-time implementation, we focus on the general iterative framework allowing the problem to be solved in a sample-by-sample manner. Under the proposed framework, it can be shown that the compensation structure in [11] indeed corresponds to a simplification of a classical iterative method called Richardson iteration (RI). Therefore, it is expected that the proposed framework should enjoy all the advantages of the compensation structure in [11]. On the other hand, to reduce the implementation complexity, it is possible to employ other iterative methods with faster convergence rate, such as Gauss-Seidel iteration (GSI) and successive over relaxation (SOR). As an illustration, the usefulness of the proposed framework is demonstrated by studying the GSI in detail. Simulation results show that the GSI is able to converge to the desired solution at a convergence rate two times faster than the RI. Moreover, thanks to rich theoretical analysis of iterative methods in mathematical communities [16], [17], lots of useful results, such as sufficient conditions for convergence, can be applied and further elaborated to analyze the performance of the proposed framework.

Because of the time varying nature of the system model, we further realize the proposed iterative framework using variable digital filters (VDFs). Like the timing mismatch compensation in [8] – [10], the VDF can be designed to accommodate any possible frequency response mismatches which are described by a polynomial approximation. Therefore, the resulting structure usually consists of a number of fixed subfilters and a few tuning parameters. Major advantages of the proposed structure are that the VDF coefficients involved can be varied online to cope with possibly changing system, and more importantly it can be implemented as the well-known Farrow

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structure with a limited number of variable multipliers required to implement the tuning parameters [18].

Besides, we also carry out a preliminary study related to the effect of the limited TI ADC resolution due to the abovementioned mismatch errors on the performance of an OFDM receiver. Simulation results indicate that the ADC resolution improved by the proposed iterative corrector can be carefully selected to obtain a near optimal performance with minimum complexity.

The paper is organized as follows: Section II describes the problem of frequency response mismatches occurred in TI ADCs. The equivalent time varying linear model, and two particular examples of the iterative framework, namely the GSI and RI, for signal reconstruction are then presented in Section III. Section IV is devoted to the realization of the linear model using VDFs. The efficient implementation of the GSI using Farrow structure is also discussed. In Section V, the convergence condition of the GSI is studied. After that, two detailed examples, including a preliminary study of the OFDM system performance related to the ADC resolution, are given in Section VI to illustrate the usefulness of the proposed approach. Finally, conclusion is drawn in Section VII.

II. Background

In a M-channel TI ADC, M medium-speed (or low-speed) ADCs are operated in parallel, but the sampling instants between two adjacent ADCs differ by one system clock period. Ideally, if M ADCs are functionally identical and the channel outputs are combined appropriately, we obtain an equivalent ADC, which should have the same precision as the channel ADCs, but offering a speed M times faster. However, any small mismatches between M ADCs lead to degraded performance.

Fig. 1 shows the M-channel TI ADC with frequency response mismatches, where $s_c(t)$ is the input continuous-time (CT) signal, $F_n(j\Omega)$, for $n = 0,1\cdots,M-1$, are frequency responses of the channel filters, and $y[n]$ is the output sequence. Note that typical example of channel frequency response is a linear phase shift in timing mismatch problem [5] – [10]. Also, $F_n(j\Omega)$ can be treated as an M-periodic time-varying filter, i.e. $F_n(j\Omega) = F_{n+M}(j\Omega)$ for all n.

The mismatches occur in the TI ADC when at least one channel frequency response is different form others. Usually, it is required that all channel frequency responses should be matched to a desired time-invariant frequency response $\overline{F}(j\Omega)$ such that $F_n(j\Omega) = \overline{F}(j\Omega)$, for $n = 0,1\cdots,M-1$ [11]. This results in an equivalent single channel ADC shown in Fig. 1(b), where $s_c(t)$ is filtered by $\overline{F}(j\Omega)$ before sampling to obtain $x[n]$. As suggested in [11], such frequency distortion can be compensated, say via equalization in communication systems, which is commonly encountered in the single channel ADC. Therefore, we will focus in this paper on how to find $x[n]$ given $y[n]$.

In what follows, we will establish a discrete-time (DT)

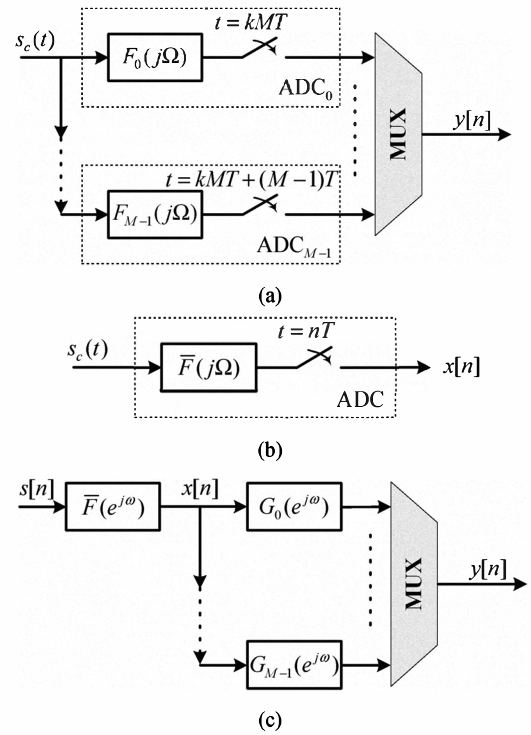

Fig. 1: (a) M-channel time-interleaved ADC with channel frequency response mismatches, (b) its equivalent circuit when all the channel responses are matched to $\overline{F}(j\Omega)$, and (c) its equivalent discrete-time model.

model between $x[n]$ and $y[n]$. Suppose that $s_c(t)$ is a bandlimted CT signal with maximum frequency $f_{\max}$, and the sampling rate $f_s = 1/T$ is greater than the Nyquist rate $2f_{\max}$. Then, the equivalent DT relations of $s[n] = s_c(nT)$, $x[n]$ and $y[n]$ in Figs. 1(a) and 1(b) can be expressed as

$$x[n] = \sum_{k=-\infty}^{\infty} s[k] \cdot \bar{f}(n-k), \text{ and} \tag{1}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot g_n(n-k), \forall n, \tag{2}$$

where $\bar{f}(n_0)$ and $g_n(n_0)$ are respectively the DT impulse responses of the desired channel filter $\overline{F}(e^{j\omega})$ and the time varying filter

$$G_n(e^{j\omega}) = F_n(e^{j\omega}) / \overline{F}(e^{j\omega}), \tag{3}$$

with $F_n(e^{j\omega}) = F_n(j\Omega)$ and $\overline{F}(e^{j\omega}) = \overline{F}(j\Omega)$, $|\omega| = |\Omega T| \le \pi$. Fig. 1(c) shows an equivalent DT model of Fig. 1(a). It is noted that the above model and the iterative framework proposed in Section III are very general, and they are valid for arbitrary M.

To find $x[n]$ given $y[n]$ in Eqn. (2), we have to consider a practical realization of $g_n(n_0)$, which has an infinite impulse response. Moreover, it is usually accompanied by the assumption that $s_c(t)$ is slightly oversampled. More precisely, the discrete-time Fourier transform of $s[n]$ is zero for $\alpha\pi \le |\omega| \le \pi$, $0 < \alpha < 1$.

Let $h_n[n_0]$ be the corresponding approximation of the ideal

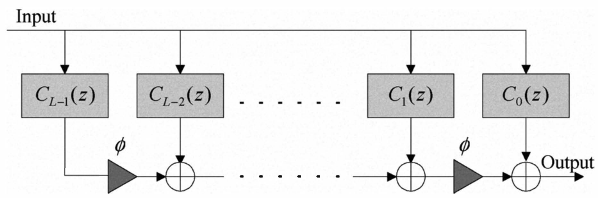

Fig. 2: Farrow structure for implementing a VDF.

impulse response $g_n(n_0)$. Assume that the frequency response of $h_n[n_0]$ is designed to approximate $G_n(e^{j\omega})$ in the frequency band of interest (i.e. $0 \le |\omega| \le \alpha\pi$), then Eqn. (2) can be approximated as

$$y[n] \approx \sum_{k=n-N_{h2}}^{n+N_{h1}} x[k] \cdot h_n[n-k], \tag{4}$$

where N_{h1} and N_{h2} are positive integers. When both N_{h1} and N_{h2} are finite, $h_n[n_0]$ can be realized as a FIR filter. On the other hand, if N_{h1} and/or N_{h2} are infinite, $h_n[n_0]$ may alternatively be realized as an IIR filter. For simplicity, in the rest of the paper, we will mainly focus on the FIR case.

III. Versatile Iterative Framework

Consider the matrix form of (4):

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}, \tag{5}$$

where $\boldsymbol{y} = [y[-\infty], \cdots, y[\infty]]^T$, $\boldsymbol{x} = [x[-\infty], \cdots, x[\infty]]^T$ and $[\boldsymbol{A}]_{n,k} = a_{n,k} = h_n[n-k]$, $n,k = \cdots,-1,0,1,\cdots$. The problem at hand is to recover the uniform sequence $\boldsymbol{x}$, given its mismatched output $\boldsymbol{y}$. In other words, we want to solve the system of linear equations in (5). For the sake of presentation, $\{y[n]\}$ and $\{x[n]\}$ are assumed to be discrete signals with finite and sufficiently large number of samples N for $n = 0,1,\cdots,N-1$. Thus, $\boldsymbol{y}$ and $\boldsymbol{x}$ now become $(N \times 1)$ vectors and $\boldsymbol{A}$ is a $(N \times N)$ matrix. Also, $h_n[n_0]$ is assumed to be noncausal. For practical implementation, it can be easily made causal by introducing appropriate delays.

For high-speed applications, directly inverting $\boldsymbol{A}$ to find $\boldsymbol{x}$ is undesirable due to high arithmetic complexity. In this paper, we propose to solve the problem using iterative methods. For efficient implementation, we are interested in those which can be realized in a sample-by-sample manner. Most of them take the form of

$$\boldsymbol{x}^{(m+1)} = \boldsymbol{G}\boldsymbol{x}^{(m)} + \boldsymbol{f}, \tag{6}$$

where $\boldsymbol{G}$ and $\boldsymbol{f}$ are derived from $\boldsymbol{A}$ and $\boldsymbol{y}$, and $\boldsymbol{x}^{(m)}$ denotes the solution in the m-th iteration. The next step is to determine the partitioning of $\boldsymbol{A}$ to form $\boldsymbol{G}$.

As an illustration, we particularly consider the Gauss-Seidel iteration (GSI) as follows

$$\boldsymbol{x}^{(m+1)} = (\boldsymbol{D}-\boldsymbol{L})^{-1}\boldsymbol{U}\boldsymbol{x}^{(m)} + (\boldsymbol{D}-\boldsymbol{L})^{-1}\boldsymbol{y}, \tag{7}$$

where $\boldsymbol{D}$, $-\boldsymbol{L}$ and $-\boldsymbol{U}$ are respectively the diagonal, negative strictly lower triangular and negative strictly upper triangular parts of the matrix $\boldsymbol{A}$, and therefore $\boldsymbol{A} = \boldsymbol{D} - \boldsymbol{L} - \boldsymbol{U}$. The

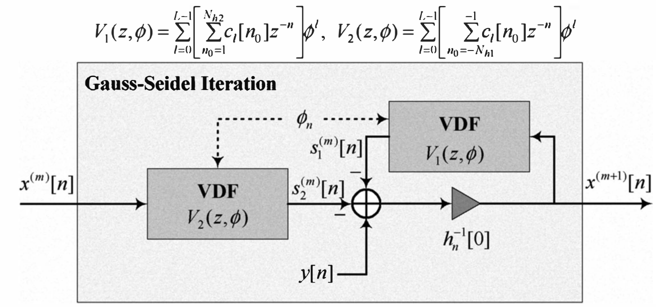

Fig. 3: VDF-based correctors in the form of the m-th Gauss-Seidel iteration.

equivalent time domain representation of (7) can be written as

$$x^{(m+1)}[n] = h_n^{-1}[0]\left(y[n] - \sum_{k=n-N_{h2}}^{n-1} x^{(m+1)}[k] \cdot h_n[n-k] \right.$$
$$\left. - \sum_{k=n+1}^{n+N_{h1}} x^{(m)}[k] \cdot h_n[n-k] \right), \quad n = 0,\cdots,N-1. \tag{8}$$

It should be noted that Eqn. (6) provides a general framework for solving the reconstruction problem using iterative methods, which greatly extends the previous works in [11] and [12]. For example, consider an alternative decomposition of $\boldsymbol{A}$ such that $\boldsymbol{G} = \boldsymbol{I} - \mu\boldsymbol{A}$ and $\boldsymbol{f} = \mu\boldsymbol{y}$ for some scalar μ . One then gets

$$\boldsymbol{x}^{(m+1)} = (\boldsymbol{I} - \mu\boldsymbol{A})\boldsymbol{x}^{(m)} + \mu\boldsymbol{y}, \tag{9}$$

which is known as Richardson iteration (RI). After a careful examination, it is noticed that the RI reduces to the compensation structure in [11] when $\mu = 1$. Of course, other similar iterative methods such as Jacobi iteration (JI), successive over relaxation (SOR) and so on can also be used for the tradeoff between the implementation complexity and convergence rate. For simplicity, we only focus on the GSI, which converges significantly faster than the RI as illustrated in Section VI later.

IV. Implementation using Variable Digital Filter

The time varying nature of $h_n[n_0]$ naturally prompts us to consider the use of variable digital filters (VDFs), which are able to vary their characteristics online by adjusting a tuning parameter ϕ . The basic idea to realize $h_n[n_0]$ using the VDF is to represent its impulse response as a polynomial in ϕ :

$$h[n_0,\phi] = \sum_{l=0}^{L-1} c_l[n_0] \cdot \phi^l, \; n_0 = -N_{h1},\cdots,0,\cdots,N_{h2}, \tag{10}$$

where $h[n_0,\phi]$ is a general representation of $h_n[n_0]$, in which ϕ can be adjusted to be ϕ_n to accommodate the time varying nature of $h_n[n_0]$, L is the number of subfilter and $c_l[n_0]$ is the impulse response of the l-th subfilter. Furthermore, the z-transform of the VDF can be expressed as:

$$H(z,\phi) = \sum_{l=0}^{L-1} C_l(z)\phi^l = \sum_{l=0}^{L-1}\left[\sum_{n_0=-N_{h1}}^{N_{h2}} c_l[n_0]z^{-n}\right]\phi^l \tag{11}$$

where $C_l(z)$ is the z-transform of the l-th subfilters. This gives rise to the Farrow structure as shown in Fig. 2. It can be seen that the Farrow structure consists of digital subfilters with fixed coefficients and a limited number of multipliers to implement the tuning parameter ϕ.

We now consider the efficient implementation of the GSI using the Farrow structure mentioned above. First of all, with (10), we rewrite (8) as

$$x^{(m+1)}[n] = h_n^{-1}[0]\left(y[n] - s_1^{(m)}[n] - s_2^{(m)}[n]\right), \quad (12)$$

where $s_1^{(m)}[n] = \sum_{k=n-N_{h2}}^{n-1} x^{(m+1)}[k] \cdot h[n-k,\phi_n]$ and

$$s_2^{(m)}[n] = \sum_{k=n+1}^{n+N_{h1}} x^{(m)}[k] \cdot h[n-k,\phi_n].$$

It is seen that $s_1^{(m)}[n]$ can be obtained by feeding $x^{(m+1)}[n]$ into a VDF $V_1(z,\phi) = \sum_{l=0}^{L-1}\left[\sum_{n_0=1}^{N_{h2}} c_l[n_0] z^{-n}\right]\phi^l$ with appropriate values of ϕ_n. As for $s_2^{(m)}[n]$, we similarly define another VDF $V_2(z,\phi) = \sum_{l=0}^{L-1}\left[\sum_{n_0=-N_{h1}}^{-1} c_l[n_0] z^{-n}\right]\phi^l$. Fig. 3 shows the resulting VDF-based structure for implementing the m-th iteration of the GSI reconstruction algorithm.

V. Convergence Analysis

An important aspect of iterative methods is the conditions for convergence. It is well known that the iteration in (6) converges for any $\boldsymbol{f}$ and $\boldsymbol{x}^{(0)}$ iff the spectral radius of $\boldsymbol{G}$, $\rho(\boldsymbol{G})$, is less than one. However, due to large N and time-varying parameter ϕ_n (and hence $\boldsymbol{A}$) in general, it is difficult to derive a necessary and sufficient condition based on the spectral radius of $\boldsymbol{G}$. Therefore, sufficient conditions that guarantee convergence will be considered below.

For the GSI, it is convenient to use a simpler sufficient condition which states that the iterations converge for any $\boldsymbol{f}$ and $\boldsymbol{x}^{(0)}$ iff $\boldsymbol{A}$ is a diagonally dominant matrix [16]. That is $\left|a_{n,n}\right| > \sum_{n\neq k}\left|a_{n,k}\right|$, for all n, which is equivalent to

$$\left|h[0,\phi_n]\right| > \sum_{n_0\neq 0}\left|h[n_0,\phi_n]\right|, \text{ for all } \phi_n. \quad (13)$$

Therefore, the condition can be readily checked, since the subfilter coefficients of a given VDF are pre-determined.

It is remarked that the choice of $\overline{F}(e^{j\omega})$ plays an important role on the performance of iterative methods. A simple way is to choose $\overline{F}(e^{j\omega})$ such that $G_n(e^{j\omega})$ is close to one. This serves two main purposes. First, $\boldsymbol{A}$ becomes more diagonally dominance and hence the abovementioned convergence condition can be easily guaranteed. Second, it is known that the diagonal dominant matrix $\boldsymbol{A}$ enhance the convergence rate of the iterative framework in (5). Therefore, the implementation complexity can be reduced with less number of iterations. This also agrees with the suggestion in [11], wherein $\overline{F}(e^{j\omega})$ is chosen as the average response of $F_n(e^{j\omega})$ through the analysis in frequency domain. However, it should be noted that the resulting spectrum after iterative correction would be close to $\overline{F}(e^{j\omega})X(e^{j\omega})$ instead of $X(e^{j\omega})$. Consequently, an additional compensation filter may be needed to compensate for $\overline{F}(e^{j\omega})$, if this distortion cannot be tolerated. In this regard, one may choose $\overline{F}(e^{j\omega})=1$ to achieve perfect reconstruction in exchange for increased number of iterations and hence implementation complexity. Fortunately, under the proposed iterative framework, we are able to examine the problem more flexibly and efficiently from the view point of iterative methods for solving linear system. For instance, we may use instead other efficient iterative methods with faster convergence rate (e.g. successive over relation) to solve the problem. Due to page limitation, we will report the above issue in a future work.

VI. Design Examples

A. Bandwidth Mismatches of TI ADC

In this subsection, we will investigate the performance of the proposed iterative structure by means of computer simulations. For comparison purpose, we will also consider the promising compensation structure proposed in [11]. As we mentioned in Section III, this structure can be regarded as the RI with $\mu = 1$.

As an illustration, we will consider bandwidth mismatches in an M-channel TI ADC. The corresponding channel frequency response is given by

$$F_n(j\Omega) = \frac{1}{1 + j\frac{\Omega}{\Omega_n^c}} \frac{1 - e^{-(\Omega_n^c T + j\Omega T)}}{1 - e^{-(\Omega_n^c T + jM\Omega T)}}, \quad (14)$$

where Ω_n^c is a time varying cutoff frequency. Interested readers are referred to [11] and [14] for more details. In the simulation below, we will choose $M = 4$ and $\Omega_n^c = [1, 0.95, 0.93, 0.9]\pi / T$. Fig. 4(a) shows the uncorrected output spectrum for a multi-cosine input signal. It can be seen that the largest aliasing component is about -39.37 dB.

To determine $h[n_0,\phi_n]$ in (3), the desired frequency response is chosen as

$$\overline{F}(j\Omega) = \frac{1}{1 + j\frac{\Omega}{0.95\pi/T}}, \quad (15)$$

which approximates the average response of $F_n(e^{j\omega})$ as discussed in Section V. In discrete time domain, the desired response of the VDF can be written as

$$H_d(\omega,\phi) = \frac{F(e^{j\omega},\phi)}{\overline{F}(e^{j\omega})} = \frac{1 + j\frac{\omega}{0.95\pi}}{1 + j\frac{\omega}{\phi\pi}} \frac{1 - e^{-(\phi\pi + j\omega)}}{1 - e^{-(\phi\pi + jM\omega)}}, \quad (16)$$

where ϕ is the tuning parameter defined as $\phi = \Omega^c T / \pi$. The VDF design method in [19] is employed to solve the following problem:

$$\min \max_{(\omega,\alpha)\in\Psi}\left|H(e^{j\omega},\phi) - H_d(\omega,\phi)\right|, \quad (17)$$

where Ψ collectively denotes the frequency and tuning range of interest. According to the maximum input frequency and Ω_n^c defined earlier, Ψ is chosen as $\omega \in [-0.9\pi, 0.9\pi]$ and $\phi \in [0.9, 1]$. A VDF has been designed with the following specifications: $N_{h1} = N_{h2} = N_h = 23$ and number of subfilters $L = 5$. Fig. 4(b) and 4(c) show respectively the compensated

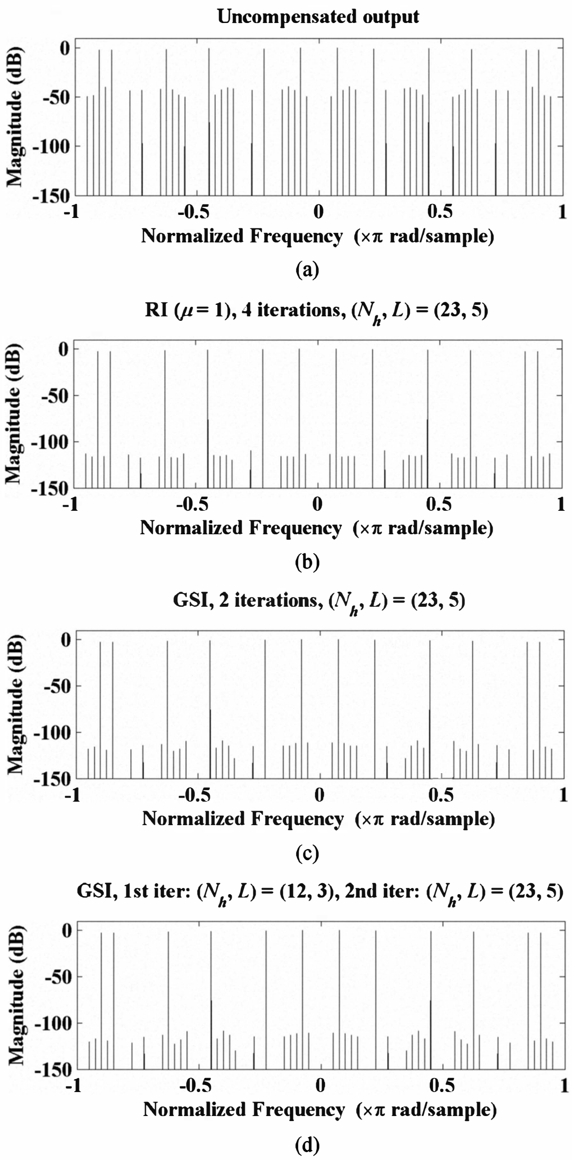

Fig. 4: (a) Uncorrected output spectrum. (b) Output spectrum obtained using the RI after 4 iterations. (c) Output spectrum obtained using the GSI after 2 iterations. (d) Output spectrum obtained using the GSI after 2 iterations, and a VDF of lower filter order and fewer subfilters is used in the first iteration.

spectra obtained using the RI with $\mu = 1$ (i.e. the structure in [11]) after 4 iterations and the GSI after 2 iterations. It can be seen that the largest spurs are reduced to -109.79 dB and -109.31 dB for the RI and GSI, respectively. This suggests that the convergence rate of GSI is about two times faster than that of RI, and hence illustrates the generality and usefulness of the proposed iterative framework.

Throughout the discussions earlier, the VDFs used in all iterations are assumed to be identical, which is merely a particular configuration of the iterative framework for the sake of presentation. In fact, it is possible to use VDFs with lower filter order and/or fewer subfilters in the first few iterations, so as to further reduce the overall implementation complexity. We notice that similar observation can be found in [10], but we are able to extend this idea to various iterative methods, including the RI, in a more general framework considered in this paper.

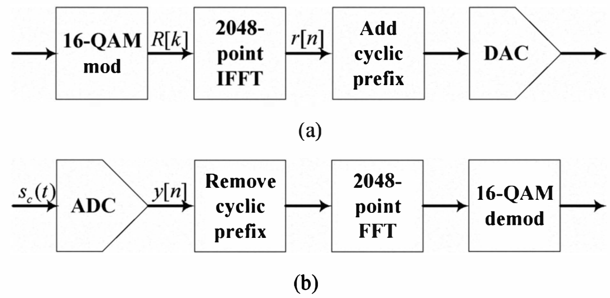

Fig. 5: (a) OFDM transmitter. (b) OFDM receiver.

Fig. 4(d) shows the compensated spectrum obtained using the GSI with two different VDFs, for which the VDF parameters used in the first iteration are $(N_h, L) = (12,3)$ and those in the second iteration are kept as $(N_h, L) = (23,5)$. It can be seen that the performance in such configuration is slightly degraded as compared with the result in Fig. 4(c). Furthermore, it can be shown that the ultimate performance of the iterative method is governed by the VDF used in the last iteration to a large extent. However, the underlying principles of such observation are omitted due to page limitation, and will be reported elsewhere.

B. Timing Mismatches of TI ADC in OFDM system

In this subsection, we will study the effect of timing mismatch errors in an OFDM system. As an illustration, we will employ the OFDM system model in [20], which is depicted in Fig. 5. At the transmitter side, the size of IFFT block is 2048. The IFFT input consists of 1800 16-QAM symbols $R[k]$ and the IFFT output is denoted as $r[n]$. After cyclic prefix addition and digital-to-analog conversion, the baseband signal is transmitted through an AWGN channel. At the receiver side, we assume that the received signal $s_c(t)$ is sampled using a non-ideal five-channel TI ADC to obtain $y[n]$ (c.f. Fig. 1(a)). With reverse operations as in transmitter, the final bit stream is obtained.

In order to focus only on the effect of the TI ADC mismatch error, we assume that all the necessary channel statistics are known. Also, we assume that the five sub-converters in the TI ADC exhibit time offsets $-\phi_n T$, $n = 0,1,2,3,4$, with respect to the ideal sampling time $t = nT$, wherein $\phi_n = [0, 0.15, -0.15, -0.15, 0.15]$. Similar to the discussions in [11], the corresponding time varying filter $F_n(j\Omega)$ can be expressed as

$$F_n(j\Omega) = e^{-j\Omega T\phi_n}. \tag{18}$$

Further, if we set $\overline{F}(e^{j\omega}) = 1$, then the desired response of the VDF in DT domain is given by

$$H_d(\omega, \phi) = e^{-j\omega\phi}. \tag{19}$$

This leads to a subclass of VDF, called variable fractional delay digital filter (VFDDF) which finds important application of sampling rate converters in software radio receivers [21]. To fulfill the specifications of the OFDM system mentioned above, the parameters of the VDF used for the proposed iterative corrector are as follows: $\omega \in [-0.9\pi, 0.9\pi]$, $\phi \in [-0.15, 0.15]$, $N_{h1} = N_{h2} = N_h = 10$, $L = 2$ in the first iteration and $L = 3$ in

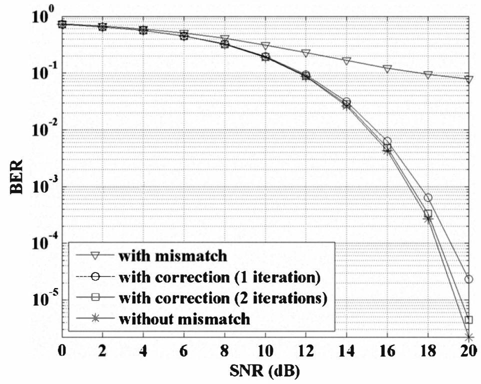

Fig. 6: BER performance before and after correction.

the second iteration.

As shown in Fig. 6, the simulation results evaluate the bit error rate (BER) against the signal-to-noise ratio (SNR) of the AWGN channel. We can see that the TI ADC mismatch errors lead to a significant noise floor, when comparing with the desired curve without mismatch, because the mismatch errors are dominated. On the other hand, after applying the proposed iterative corrector, the BER curve becomes closer to the desired one as the number of iteration increases. This suggests the usefulness of the proposed approach in correcting the mismatches of TI ADC iteratively. Moreover, the above experiment brings out an interesting issue on how the ADC resolution affects the BER performance of the 16-QAM OFDM system. A rough estimation of the ADC bit resolutions (approximated as 1 bit per 6 dB SNR for a sinusoidal input due to quantization noise [2]) gives respectively 2, 6 and 8 bits for the first three BER curves in Fig. 6. We can see that an ADC with 6 to 8 bits resolution is sufficient for a good BER performance. In practice, since there may exist other error sources, say coming from channel equalization, a higher bit resolution may be required. On the other hand, for larger SNR, additional iterations (and hence ADC resolutions) may be needed to achieve a better BER. Nevertheless, it is clear the quantization errors due to limited ADC resolutions are eventually masked by the noise floor of the AWGN channel.

VII. Conclusion

A versatile iterative framework for the correction of frequency response mismatches in TI ADCs has been proposed. While extension to other iterative methods of similar form such as successive over relaxation is possible, the Gauss-Seidel iteration has been studied in detail to illustrate how the problem can be efficiently solved iteratively based on a general time varying linear system model. Moreover, since the proposed iterative method can be efficiently realized using Farrow-based variable digital filters, the entire iterative procedure can be implemented without any multiplications, apart from the limited number of multipliers in the Farrow structure. Simulation result showed that the proposed method has better performance than conventional compensation structures. Also, the effect of ADC resolution on the BER performance of the 16-QAM OFDM system has been investigated.

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