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Effects of long-term time-dependent behaviour on dynamic properties of cable-stayed bridges

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Abstract

A structural health monitoring system installed on a bridge provides the necessary data for engineers to evaluate its integrity, durability and reliability through the observation of changes in bridge properties caused by any damage or deterioration. However, the time-dependent behaviour of construction materials such as concrete and steel cables also causes changes in structural characteristics. If these are not taken into account properly, false alarms may result. This paper presents a systematic and efficient method to study the effects of long-term time-dependent behaviour due to concrete creep, concrete shrinkage and cable relaxation on the dynamic properties of cable-stayed bridges. The finite element model of the cable-stayed bridge is built up with beam elements and proper cable elements considering their geometric nonlinearity and time-dependent effects. The long-term time-dependent analysis is carried out using an efficient single-step finite element method using the age-adjusted elasticity modulus and shrinkage-adjusted elasticity modulus for concrete, and the relaxation-adjusted elasticity modulus for steel cables. Then the dynamic properties of the bridge can be obtained by the subspace iteration method. The effects of long-term time-dependent behaviour including concrete creep, concrete shrinkage and cable relaxation on the dynamic properties of typical cable-stayed bridges are examined in detail.

Keywords: cable-stayed bridges, FEM, free vibration analysis, single-step method, time-dependent

1 Introduction

Cable-stayed bridges have become very popular over the last four decades not only because of their structural efficiency but also their aesthetic appearance. Structural health monitoring (SHM) systems are often installed on such bridges to monitor their safety through the observation of changes in bridge properties caused by damage or deterioration. However the time-dependent behaviour can redistribute internal forces and change the geometry, resulting in changes in bridge properties. If these are not considered properly, false positive or false negative alarms of SHM systems may arise. So it is necessary to take into account such effects carefully in order to build a reliable monitoring system.

A reliable method for time-dependent analysis of concrete structures is to use time integration with the finite element method (Ghali et al., 2002). As it is time-consuming, various single-step methods have been proposed by incorporating *age-adjusted elasticity modulus* (AAEM) based on a stress relaxation function of concrete (Ghali et al., 2002), *shrinkage-adjusted elasticity modulus* (SAEM) by allowing for the interaction between concrete creep and shrinkage (Au et al., 2009), and *relaxation-adjusted elasticity modulus* (RAEM) on the basis of a stress relaxation of tendons (Si et al., 2009).

However, the effects of time-dependent behaviour on the dynamic performance of concrete structures have received little attention. Sapountzakis and Katsikadelis (2003) adopted an effective AAEM as a time-dependent tangential modulus to study the creep and shrinkage effects on dynamic analysis of reinforced concrete slab-and-beam structures, and showed that the natural frequencies decreased with time. As there has been little work on the time-dependent effects on dynamic properties of cable-stayed bridges, it is desirable to develop a systematic and efficient method to investigate the effect of time-dependent behaviour due to concrete ageing, creep and shrinkage, and cable relaxation on the dynamic properties of such structures.

2 Methodology for analysis of time-dependent behaviour

2.1 Equivalent creep coefficient for steel cables

The intrinsic stress relaxation $\Delta\sigma_{pr}$ in a steel cable is the loss of stress at constant strain. It depends on both the duration of sustained tension t (hour) and the ratio of the initial stress σ_{p0} to the "yield" strength of steel f_{py}. The equation proposed by Magura et al. (1964) for stress-relieved cables is

$$\frac{\Delta\sigma_{pr}}{\sigma_{p0}} = -\frac{\log(t)}{10}\left(\frac{\sigma_{p0}}{f_{py}} - 0.55\right) \tag{1}$$

It is assumed that the creep coefficient is independent of age and the modulus of elasticity E_s remains constant. Given the type of tendon and based on the intrinsic stress relaxation, the creep coefficient of a tendon stressed at time t_0 can be worked out at regular time intervals Δt as (Au and Si, 2009):

$$\overline{\varphi}_s(\Delta t) = \frac{-\Delta\sigma_s(t_0)}{\sigma_s(t_0) + \Delta\sigma_s(t_0)/2} \tag{2}$$

$$\overline{\varphi}_s\left[(k+1)\Delta t\right] = \frac{\sigma_s(t_0)\overline{\varphi}_s(k\Delta t)}{\sigma_s(t_0)+\Delta\sigma_s(t_0)/2} - \frac{\Delta\sigma_s(t_0+k\Delta t)\left[1+\overline{\varphi}_s(\Delta t)/2\right] - \Delta\sigma_s(t_0)\overline{\varphi}_s\left[(k-1)\Delta t\right]/2}{\sigma_s(t_0)+\Delta\sigma_s(t_0)/2}$$
$$-\frac{\sum_{i=2}^{n}\frac{\Delta\sigma_s\left[t_0+(i-1)\Delta t\right]}{2}\left\{\overline{\varphi}_s\left[(k-i+2)\Delta t\right] - \overline{\varphi}_s\left[(n-i)\Delta t\right]\right\}}{\sigma_s(t_0)+\Delta\sigma_s(t_0)/2} \quad (k = 1,2\ldots n) \tag{3}$$

Defining an ageing coefficient $\chi_s(t,t_0) = \overline{\chi}_s(t-t_0)$ to take into account time-dependent effects due to creep which is also assumed to be independent of age, then the RAEM $\overline{E}_s(t-t_0)$ (Si et al., 2009), which is essential to the single-step method of analysis involving steel cables, can be defined as

$$\overline{E}_s(t-t_0) = \frac{E_s}{1+\overline{\chi}_s(t-t_0)\overline{\varphi}_s(t-t_0)} \tag{4}$$

2.2 Single-step method for time-dependent behaviour of cable-stayed bridges

To carry out the single-step finite element analysis of a cable-stayed concrete bridge for time-dependent behaviour, one may use the AAEM (Ghali et al., 2002) to account for concrete creep, the SAEM (Au et al., 2009) to account for interaction between shrinkage and creep of concrete, and the RAEM (Si et al., 2009) to account for relaxation of steel cables.

The AAEM $\overline{E}_{cc}(t,t_0)$ to account for concrete creep (Ghali et al., 2002) can be expressed as

$$\overline{E}_{cc}(t,t_0) = \frac{E_c(t_0)}{1+\chi_{cc}(t,t_0)\varphi_c(t,t_0)} \tag{5}$$

where $E_c(t_0)$ is the elastic modulus at time t_0, $\chi_{cc}(t,t_0)$ is the ageing coefficient and $\varphi_c(t,t_0)$ is the creep coefficient of concrete at time t after loading at time t_0. The incremental load vector $\{\Delta q^e\}_{cc}$ of a concrete member from time t_0 to t can be derived as

$$\{\Delta q^e\}_{cc} = [\bar{k}(t,t_0)]_{cc}\{\Delta\delta\}_{cc} + \{\Delta f(t,t_0)\}_{\varphi c} \tag{6}$$

in terms of the incremental creep displacement vector $\{\Delta\delta\}_{cc}$, the stiffness matrix $[\bar{k}(t,t_0)]_{cc}$ and the incremental load vector due to creep effect $\{\Delta f\}_{\varphi c}$ calculated by using the AAEM.

The SAEM $\bar{E}_{cs}(t,t_0)$ to account primarily for shrinkage and its interaction with creep (Au et al., 2007) is similarly given in terms of an ageing coefficient $\chi_{cs}(t,t_0)$ by

$$\bar{E}_{cs}(t,t_0) = \frac{E_c(t_0)}{1+\chi_{cs}(t,t_0)\varphi_c(t,t_0)} \tag{7}$$

Then the incremental load vector $\{\Delta q^e\}_{cs}$ from time t_0 to t can be expressed as

$$\{\Delta q^e\}_{cs} = [\bar{k}(t,t_0)]_{cs}\{\Delta\delta\}_{cs} + \{\Delta f(t,t_0)\}_{cs} \tag{8}$$

In terms of the incremental shrinkage displacement vector $\{\Delta\delta\}_{cs}$, the stiffness matrix $[\bar{k}(t,t_0)]_{cs}$ and the incremental load vector due to shrinkage $\{\Delta f(t,t_0)\}_{cs}$ formed by SAEM.

Based on the RAEM defined in Eq. 4, the incremental load vector $\{\Delta q^e\}_s = [\Delta f_1 \;\; \Delta f_2]_s^{\mathrm{T}}$ can be expressed in terms of the stiffness matrix $[\bar{k}(t,t_0)]_s$, the incremental nodal displacement vector $\{\Delta u\}_s$ and the incremental load vector due to cable relaxation $\{\Delta f(t,t_0)\}_{\varphi s}$ as (Si et al., 2009)

$$\{\Delta q^e\}_s = [\bar{k}(t,t_0)]_s\{\Delta u\}_s + \{\Delta f(t,t_0)\}_{\varphi s} \tag{9}$$

where the stiffness matrix $[\bar{k}(t,t_0)]_s$ is given in terms of the cross sectional area A_s and length l_s by

$$[\bar{k}(t,t_0)]_s = \frac{\bar{E}_s(t-t_0)A_s}{l_s}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{10}$$

The incremental load vector $\{\Delta f(t,t_0)\}_{\varphi s}$ is given in terms of the cable force $\bar{N}_s(t_0)$ at time t_0 by

$$\{\Delta f(t,t_0)\}_{\varphi s} = \frac{\bar{E}_s(t-t_0)}{E_s}\bar{\varphi}_s(t-t_0)\begin{Bmatrix} \bar{N}_s(t_0) \\ -\bar{N}_s(t_0) \end{Bmatrix} \tag{11}$$

2.3 Modelling of cable-stayed bridges for free vibration analysis

A long-span cable-stayed concrete bridge consists of different kinds of structural components with time-independent nonlinear effects including interaction between axial force and bending, sag effects caused by weight of stay cable and large displacements (Curley and Shepherd, 1996). The bridge deck and pylons are modelled as Bernoulli-Euler beam elements (Au et al., 2001b). Each stay cable is modelled as a single truss element with an equivalent modulus E_{eq} (Au et al., 2001b) given by

$$E_{eq} = \frac{E_{ca}}{1+(wH_{ca})^2 A_{ca}E_{ca}/12T^3} \tag{12}$$

where H_{ca} is the horizontal projected length, A_{ca} is the cross-sectional area, E_{ca} is the effective modulus of elasticity, w is the weight per unit length and T is the updated cable tension of the cable.

An efficient approach to consider the "P-delta effect" of the bridge girders and towers is to adopt the geometric stiffness matrix $[k]_G$ of each beam element (McGuire et al., 2002) to modify its elastic stiffness matrix $[k]_E$. The total stiffness matrix $[k]_T$ (Zienkiewicz and Taylor, 1989) becomes

$$[k]_T = [k]_E + [k]_G \tag{13}$$

After building up the global matrices, free vibration analysis of the cable-stayed bridge can be carried out (Cook et al., 2001) using the instantaneous moduli of elasticity.

3 Case studies

3.1 Dynamic properties of a cable-stayed concrete cantilever

The dynamic properties of a hypothetical cable-stayed concrete cantilever as shown in Figure 1 under long-term effects are studied using the proposed method. The cantilever is 10m in length with a square cross section of 1m×1m. Its weight density is 25kN/m^3. The characteristic compressive strength of concrete is f_{ck} =36MPa. Wet curing is conducted until T_s=3 days after which shrinkage begins. The relative humidity is taken as 80% throughout. The stay cable is a stress relieved tendon with a cross sectional area A_s =250mm^2, Young's Modulus E_s=200GPa, weight density ρ_s =78kN/m^3 and initial tension P_0=210kN applied at time t_0 =28 days referring to the age of cantilever. The initial prestressing ratio is 0.8. The parameters of CEB-FIP Model Code 1990 (CEB, 1993) are adopted and the sag effect of cable and geometric nonlinearities of the cantilever are ignored. The cantilever is modelled by 4 identical beam elements and the cable is modelled by a truss element. First, the initial natural frequencies of this structure at Day 28 are calculated without any time-dependent behaviour. Then the dynamic properties after 300 days are calculated for various cases, namely (a) Case A: concrete ageing only; (b) Case B: concrete ageing and cable relaxation only; (c) Case C: concrete ageing, creep, shrinkage and cable relaxation; and (d) Case D: cable relaxation only. The first three frequencies and their percentage differences from the initial frequencies are shown in Table 1.

Table 1. The long-term dynamic properties of cable-stayed cantilever after 300 days

Mode	Initial *f*	Case A		Case B		Case C		Case D	
	(Hz)	*f* (Hz)	Dif.(%)	*f* (Hz)	Dif.(%)	*f* (Hz)	Dif.(%)	*f* (Hz)	Dif.(%)
1	6.910	7.194	4.11	7.296	5.59	7.297	5.60	6.885	-0.35
2	41.779	43.629	4.43	44.296	6.02	44.299	6.03	41.775	-0.01
3	117.333	122.536	4.43	124.412	6.03	124.422	6.04	117.331	-0.00

Table 1 shows that the natural frequencies increase under concrete ageing effect and its combination with other time-dependent deformations, while they decrease slightly under the effect of cable relaxation only. It indicates that concrete ageing has the most important influence on the long-term dynamic properties among various time-varying factors by comparing Cases A, B and C. The results from Cases B and C demonstrate that cable relaxation has more effect on the dynamic characteristics than concrete creep and shrinkage. Comparison of the results of various cases shows that the interaction among various time-varying factors is greater than their individual effects. Therefore it is desirable to take into account the interaction among various time-varying factors when long-term analysis of dynamic behaviour is performed.

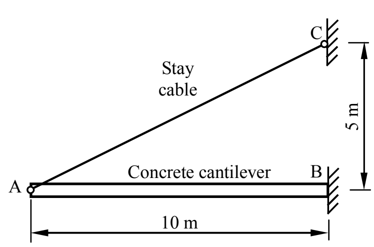

Figure 1. A cable-stayed cantilever

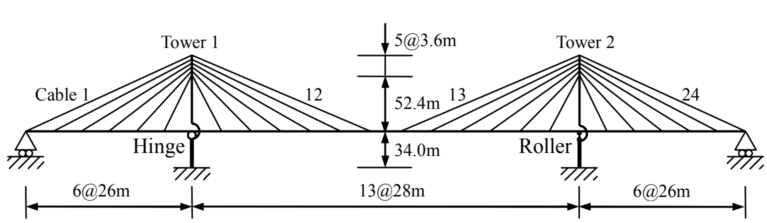

Figure 2. A cable-stayed concrete bridge

3.2 Dynamic properties of a cable-stayed bridge considering time-dependent behaviour

A cable-stayed concrete bridge from Au et al. (2001a) with a main span of 364m, as shown in Figure 2, is modelled as a two-dimensional structural system for dynamic analysis. The bridge is hinge supported on Tower 1 but roller-supported on Tower 2. The corresponding properties of the bridge deck and towers are given in Table 2. The properties of stay cables and the finite element discretisation are the same as those of Au et al. (2001a). The characteristic compressive strength of the concrete used in the deck and towers is f_{ck} =40MPa. Wet curing is carried out until T_s =3 days after which shrinkage begins. The parameters of CEB-FIP Model Code 1990 (CEB, 1993) are adopted.

Table 2. Properties of deck and towers of a cable-stayed bridge

Part of structures	A (m^2)	I (m^4)	E_{c28} (MPa)	ρ (kg/m^3)
Bridge deck	6.00	4.1875	36 268	2550
Bridge tower – above deck	14.2	30.0	36 268	2550
Bridge tower – below deck	35.8	40.0	36 268	2550

Firstly, the initial natural frequencies of the bridge at Day 28 are calculated without considering geometric nonlinearities and any time-dependent behaviour. Secondly, the natural frequencies at Day 28 are obtained considering geometric nonlinearities (Case A). Then the dynamic properties after 300 days are obtained for various cases considering geometric nonlinearities, namely (a) Case B: concrete ageing only; (b) Case C: concrete ageing and creep only; and (c) Case D: concrete ageing, concrete creep and shrinkage. The first ten natural frequencies together with the percentage changes compared with the initial values are shown in Table 3.

Table 3. Natural frequencies (Hz) of cable-stayed bridge with percentage changes shown in brackets

Mode	Initial	Case A	Case B	Case C	Case D
1	0.339	0.336 (-0.88%)	0.340 (0.29%)	0.342 (0.88%)	0.342 (0.88%)
2	0.469	0.462 (-1.49%)	0.470 (0.21%)	0.473 (0.85%)	0.473 (0.85%)
3	0.698	0.687 (-1.58%)	0.697 (-0.14%)	0.699 (0.14%)	0.699 (0.14%)
4	0.759	0.750 (-1.19%)	0.759 (0.00%)	0.759 (0.00%)	0.759 (0.00%)
5	0.855	0.843 (-1.40%)	0.856 (0.12%)	0.858 (0.35%)	0.857 (0.23%)
6	1.080	1.061 (-1.76%)	1.087 (0.65%)	1.089 (0.83%)	1.089 (0.83%)
7	1.270	1.252 (-1.42%)	1.290 (1.57%)	1.286 (1.26%)	1.286 (1.26%)
8	1.327	1.302 (-1.88%)	1.338 (0.83%)	1.336 (0.68%)	1.336 (0.68%)
9	1.388	1.364 (-1.73%)	1.403 (1.08%)	1.401 (0.94%)	1.401 (0.94%)
10	1.450	1.435 (-1.03%)	1.482 (2.21%)	1.488 (2.62%)	1.488 (2.62%)

Table 3 shows that geometric nonlinearities reduce the natural frequencies by a maximum of 1.9%. However, the natural frequencies increase under the combined effects of time-dependent behaviour and geometric nonlinearities by a maximum of 2.6%. It indicates that the time-dependent behaviour has more effect on dynamic properties of this bridge than geometric effects. Comparing Cases B, C and D shows that concrete ageing plays an important role in increasing the long-term natural frequencies. The effects of creep are higher than those of shrinkage. The interaction between concrete creep and ageing effects should be considered carefully for long-term performance analysis of concrete structures. It is also noted that the time-dependent behaviour and geometric nonlinearities often have opposite effects. Therefore they should be studied in detail for reliable damage identification in vibration-based structural health monitoring systems.

4 Conclusions

A systematic and efficient method is proposed to investigate the dynamic properties of cable-stayed bridges considering the effect of long-term time-dependent behaviour due to concrete ageing, creep and shrinkage together with cable relaxation. Apart from the use of finite element method that considers geometric nonlinearities, it also adopts an efficient single-step approach for analysis of time-dependent behaviour using the AAEM and SAEM for concrete and RAEM for cables. Free vibration analysis can be carried out using the updated internal forces and geometry of the bridge. Numerical examples are presented to illustrate the application of the proposed method as well as to investigate the behaviour of typical cable-stayed concrete bridges. Results from these investigations show that the natural frequencies, whether accounting for geometric effects or not, increase with time due to concrete ageing effect alone, its interaction with creep and shrinkage of concrete, and cable relaxation, or their combined effects. However the natural frequencies tend to decrease slightly with time when cable relaxation is considered alone. These results also indicate that concrete ageing has the most important influence on the dynamic properties among various time-varying factors. The interaction between concrete ageing effect and effect of concrete creep, cable relaxation or their combined effect are greater than their individual effects. The interaction between ageing effect and concrete shrinkage is negligible. Hence the interaction among various time-varying factors should be considered carefully during long-term dynamic analysis of concrete structures. Besides, the effect of time-dependent behaviour on dynamic properties varies from mode to mode. Therefore, the long-term dynamic characteristics due to time-dependent behaviour should be investigated in detail in order to ensure reliable damage identification in vibration-based structural health monitoring systems.

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