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Research on Classroom Practice:

A monograph for Topic Study Group 24, ICME 11

The Introductory Chapter

(Prepared by the Editorial Board: Ida Ah Chee Mok (Chief), Guy Brousseau (Advisor), Ma. Guadalupe Cabañas, Ricardo Cantoral-Uriza, Hélia Oliveira, João Pedro da Ponte, Filippo Spagnolo)

For the first time of its history, ICME, the International Congress on Mathematical Education, in its eleventh edition opened a study group on the topic "Research on Classroom Practices" held in Monterrey, Mexico, July 6 - 13, 2008. The topic was a very comprehensive topic. The team of organization chose to start with an open attitude to welcome the texts that participants of the Congress thought to be able to belong to the topic. Consequently, the majority of the presentations in the Congress represented very diversified cultures, models and perspectives. While devoting much time on the presentations of the diverse practices, only a little was discussed concerning the issues pertaining to the topic. To advance a little, the participants of the group were further invited to exchange ideas by preparing this special issue.

While reporting of the classroom practices delivered in different projects, systems and cultures, the documents provide opportunities for us to have a closer understanding of the variety of reasons for these studies. The documents also allow us to reflect on the bases and the methods that legitimate the contributions of observations of classroom practice in research on mathematics teaching and to reflect on the uses of these observations and of their results in education systems.

Studying Teachers and Teaching

The majority of the papers discussed interactions between the teacher and the students in classroom episodes situated in different contexts. Some consisted of data from projects involving intervention based on the collaboration between researchers and the teachers (e.g., Lezama, Martinho and Ponte, Oliveira). Some consisted of data from project involving non-participant observation (e.g., Mok, Ho, Hesiquio and Bustamante). Some common notions emerge from these reports. There is an agreement that students should be encouraged to express their thoughts and ideas in lessons. This is in line with the trend of recent research where multiple frameworks for analysis and promoting the class interactions are used although there are always more possibilities than we can include with our limited effort (e.g., Hufferd-Ackles, Fuson, and Sherin, M.G., 2004; Kieran, 2001; Ryve, 2006; Sfard, 2001; Clarke, Keitel and Shimizu, 2006). Besides, there is a consensus that the teacher plays an important role in shaping the classroom discourse. Classroom discourse in lessons can appear in different forms. The style and contents may vary according to the specific purposes of the mathematical tasks, the teachers' actions and beliefs, and of course the students' participation. Very often, some tasks may have aimed to promote students to voice their thoughts in exploration. In these cases the teachers put focus on encouraging students' participation and talking about their ideas, overcoming obstacles they came across in class discussion. Therefore, the teachers do in fact play an important role in a variety of situations and their ways of leading or facilitating the class discourse matter.

Hesiquio and *Bustamante* reported the analysis of the explanations given by a teacher, for the notion of similarity, in a high school geometry class. Episodes were selected to show how the teacher approaches the concept of similarity via different problems geometrical relationships. In some instances of class dia-

logues, the teacher sought to minimize potential conflicts during the negotiation through routines and obligations.

Ho compared the classroom practices of two Grade 5 teachers to address the extent they reflect the intents of a mathematical problem solving curriculum. Episodes of the observed lessons were coded into categories of heuristic-instruction, teaching of concepts and skills, going over assigned work and student activities. Classroom talk, particularly in the heuristic-instruction and student activities, revealed two contrasting approaches best described as traditional and more reform-minded. The salient features identified in the two approaches are important to the understandings of what it means to enact a mathematical problem solving curriculum.

Lezama analysed the interaction between teachers and students in a lesson and presents how stages of didactic engineering could be built for the notion of exponential function. In the experience, the teachers were a determining factor because how they received and intervened the design of the activity, and interacted with the students in the situation were important. The author provides evidence of how students, when faced with the difficult task of interacting with the mathematical content of the situation, direct their interactions towards the teacher confronting complex communication situations such as those the author has named "ambiguous interactions". The author found that the teacher would always generate an interaction policy but subject to two basic criteria. A *centering*, aimed at returning the group to the subject of the situation if it digressed, and an *unblocking* if for any reason the group were stuck for some time on a problem or showed signs that they would not be able to get around it.

Martinho and *Ponte* discuss the conceptions, practices and reflections about practices of a mathematics teacher with respect to classroom communication and their change during the activity of a collaborative project between the researcher and two teachers. To promote the communication dynamics in the classroom, the teacher was expected to stimulate students' interest and to have the ability to enrich the interaction between the teacher and students. By studying the reflections of a very experienced teacher, the project through providing the possibility to plan, review and assess new teaching experiences is reported to have strengthened the teacher's understanding of communication issues in her classroom, putting her practices under scrutiny, and developing richer communication processes between her and her students.

Mok presents a description of an event of a Shanghai mathematics lesson to show how the teacher may play a role as a mathematical "enculturator", an important influential person in the shaping a culture about the nature of mathematics and how mathematics should be learned in the classrooms. The theoretical point of departure is that classroom practice is a process of "mathematical enculturation", developed by Bishop (1991). The process is a dynamic, intentional, shaping process, which refers to what happens in mathematics classroom. Many of these features found in the class interaction directed by the teacher such as the Socratic style the teacher's questioning and the demand in precision of expressing the content and use of language are essential in shaping the students' understanding and appreciation of the mathematical objects as well as the culture of mathematics learning.

Oliveira analysed the teacher's role in supporting students' work in mathematical investigations, in particular, with one task which involves generalisation in sequences in the 8th grade, and the constraints the teacher faces when performing this role. Regarding her support to students' generalisation, some helpful steps were observed: display more steps in the sequence; draw a different representation, usually a table; establish a relation between the term and its position; look for familiar sequences in the numbers; and, use a proper variable. From the results it appears that to understand the teacher's role in this situation it is necessary to take into account: i) the nature of the other tasks that have been solved in previous lessons and its relation to this one; ii) the teacher's perspectives about what counts as generalisation and the

different possible representations involved in the task; and, iii) finally, the compatibility of the goals established by the teacher for the task and for supporting students' work.

One of the obstacles in the evolution of this concept has been the relationship between figurative and numerical aspects.

Soto-Johnson, Cribari and *Wheeler* analyzed preservice elementary teachers' written reflections and investigated how written reflections influence their learning of geometry. Their findings suggest that the participants performed better on tasks when they participated in written reflections, and participants, who wrote reflections at the beginning of the semester, produced stronger reflections. By a crossover quasi-experimental research design, the results of the study indicated incorporating reflections into the mathematics classroom increased participants' achievement on related tasks, allowed preservice teachers an opportunity to reflect on the learning and teaching of mathematics, and served as a further assessment of student understanding.

Studying Students' Understanding and Behavior

Some authors chose to focus on other aspects of what happened inside the lessons. They looked for didactical experiences that enhance students' performances or understanding of students' conceptions. Many of these studies are about approaches to teach a certain topic while enhancing the exploratory and investigative nature of the mathematical processes in the classroom practice.

Arias and *Araya* describes two didactical contracts that characterize the identified typical interactions in two math classes of tenth grade, during the study of the polynomial factorization. They introduce a new kind of contract, different from those already considered: the "potential adidactical contract", intending to promote better mathematical performances in the students. The newly named situation is characterized by: the teacher proposes a problematic-situation which allows the students to construct the new knowledge as the optimal solution; the responsibility "to get in" the knowledge to the class is not exclusive of the teacher; and students formulate and validate strategies to solve a problem.

Cabanas and *Cantoral-Uriza* examined university students' perceptions of how the area on plane regions is conserved, compared and measured and also the role of the teacher in this process. Classroom interactions during the solution of a learning situation which involves geometrical transformations (convex polygons) and includes a semi-structured interview were analysed. Teacher's interventions are studied from his: open questions; questions about the conclusion at which students arrived; questions about the conclusion or method or procedure; intervention to confront an argument or procedure, or perhaps, as persuasion. Students´ interventions are studied from their: arguments to convince about their conclusion; intervention to reject or confront a procedure or argument, or perhaps, to assign meaning to a mathematical object. The findings show how the students perceive area conservation, area comparison and area measurement. The teacher's role becomes complex when he confronts students' arguments. The results of this study indicate that almost all students´ arguments are referred to concepts, properties, relations and processes associated with the tasks.

Di Paola and *Spagnolo* via experimental teaching studied the cognitive behaviour of pupils (3-10 years old) to the resolution of algebraic and pre-algebraic problems that were designed with a cultural reference to the Chinese language. They carried out a parallel analysis and comparison between Italian and Chinese pupils in different mathematical contexts. In the paper they reported how the different types of logical argumentative structure of the problems could engrave on the resolution behaviour of the students such as the types of solutions and the passages of the decisive algorithm and errors; how these registers can influence their process of learning of the specific algebraic concepts.

Lo Cicero and *Spagnolo* reported an experimental study that aimed for improving students' reading and the understanding of function graphs, through didactic activities in the laboratory, using a sensor motion. Particularly, there were improvements in the reading of maxima of functions and of interval sizes on 10 year-old students. The experimentation consisted of a laboratorial lesson1 of two hours, where the students

compared reading and prediction of the graphs realised with the motion sensor and the software Logger Lite2.

Matos and *Ponte* identified the contributions of a teaching unit based on exploratory and investigation tasks about functional relationships in fostering grade 8 students' algebraic thinking. This teaching unit of this study addressed several mathematics curriculum topics – numerical sequences, functions, and 1st degree equations; aiming to promote the development of algebraic thinking, based on exploring functional relationships. Results indicate that working in the proposed tasks and combining students' group work with classroom discussions promoted the development of meaning for the algebraic language. The work in this unit also encouraged the students to construct new concepts and enhanced their algebraic knowledge and thinking.

Saraiva and *Teixeira* reported a three month-long study developed in one grade 11 class and presented secondary school students' understanding of function, in an exploratory and investigative approach; and to identify the students' difficulties concerning that concept. The students were able to manipulate the symbols, and operate with them, but this was not sufficient for the structural understanding of function. The authors further argue that despite the difficulties found in the study, the students' mathematical activity using exploratory and investigative mathematical tasks with good interaction favored meaningful learning on functions.

Methodological Issues and Other Themes

Both the reports of Brousseau and Chopin present the work of COREM (*Centre d'Observation et de Recherches sur l'Enseignement des Mathématiques* - Center for Observation and Research on the Teaching of Mathematics), created by the IREM (at the Université de Bordeaux).

Brousseau presents some results obtained from the observation of classroom practices carried out in COREM. Making certain effects of the didactical contract evident made it possible to predict and follow for 25 years the development of a phenomenon unleashed by massive direct evaluation and class observation. Inspired by the example of the COREM, this text prepares the introduction of a genuine deontology of classroom observation.

Chopin discusses the status and the methods of the observation of classroom practice. The verb "to observe" don't easily account for the complexity of the activity really embedded in. Many levels are included: this of the scientific question which always controls the kind of observation to make; this of the ethical question provided from the particular nature of the subject of the study. The author reports how to develop few aspects of these questions considering the way how they have structured the creation and the management of an experience of observation of mathematics teaching, in a school situated in the region of Bordeaux, in France: the COREM.

Gill and *O'Donoghue* carried out a qualitative study of Service mathematics in Irish universities in order to examine the context, the practice and experience of Service mathematics teaching in Irish universities, for developing a meaningful characterization of Service mathematics in Irish Universities today.

Also an effort was made to define more clearly the teaching/learning contract that exists between the actors in this sphere of activity. In addition the authors built on this preliminary profile to develop a theoretical model of Service mathematics teaching using Bernstein's curriculum theory (1996) and aspects of Realistic Mathematics Education (Treffers, 1993). In this paper the authors utilize this characterization to define more clearly the classroom practices of university service mathematics classes in Ireland. The authors conclude that Service mathematics may be characterised as a *pedagogic* discourse within the discipline of mathematics. Service mathematics is essentially a pedagogical task demanded by client departments such as Engineering and Business.

Conclusion

The term "classroom practices" is used in United States for a long time ago, to appoint all that occurs in the classrooms, and which can be determined by agreements - standards - between professors and the society[1]. Thus all the teachers, of all disciplines, in all classes and for all pupils of the world implement classroom practices in each one of their courses. In addition, all those that accompany them, form them or direct them evoke, observe, describe, criticize or recommend i.e. study these practices, identify the good features in the practices and seek for factors sustaining success and failure. Although studies of this nature are not for distinguish the better from the worse, comparison sometimes will bring about some judgment.

These defined classroom practices are the subject since centuries of abundant literature. Their perception by the population is the base of the agreement by which this one agrees to delegate the education of his/her children to the teachers - those who insert the children in the culture of their society. They thus vary from one culture with another. The decision of ICME11 to devote a study group to such general holistic and protean topic, was consequently at the same time natural and daring. The team which agreed the challenge to organize this group initially considered the academic way. It would have been a question of inviting the participants to determine, by preliminary studies, which one could understand by "classroom practices specific to mathematics", to present the various methods of research on this subject (theoretical observations, experiments, research, objects of the studies, methodology of research, etc.), to propose ways of approaching, in a scientific way, problems arising from the search for at the same time effective and negotiable school practices with the society, and so on.

The stake was to obtain more homogeneous exchanges in order to determine common knowledge and the most promising ways. Team TSG24 gave up rather quickly this impracticable prospect for a "study group". It could only produce theoretical discussions which would move away the experts themselves, whereas their presence is essential in the debates that ICME could organize thereafter. It appears more appropriate to collect initially all that the participants would find relevant to present on this topic. The very long presentation of the topic on the site had more for object to widening the range of the proposals than to limit it. This empirical method might have faltered into the danger of the opening of a new "Babel's tower". Nonetheless, it appeared on the contrary much more convivial and its success obliged the organizers to divide the contributors in two parallel rooms.

[1] Standards in Classroom Practice, research synthesis (2002), Chapter 2, Mathematics standards in classroom practice, Judy E. Florian with the assistance of Ceri B. Dean
www.mcrel.org/PDF/Synthesis/5012RR_RSStandardsClassroomPractice.pdf
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This special issue is the result of almost four years of joint efforts dated from the initial invitations for the study group. It shows what we could collect and produce better on the subject. We hope that the variety, the quality and the interest of the papers presented here will inspire, in new researchers, the desire also for working on this topic.

The collection of the articles is not a sufficient testimony of the work of TSG24 because it does not reflect the examined questions, individually or collectively nor the reflections which they caused. With the great regret of the leading team they cannot be presented here because they did not lead to common texts. (Let us give some examples of them in appendix).

The harvest is very modest, but it is only because it was impossible to hold a reasonable time for their public discussion and for their formulation. To report with meticulousness a simple episode of class requires much work and a considerable time. Moreover, to analyze it can refer to knowledge resulting from many disciplines, some not very familiar with all the participants. We thus preferred to devote the serviceable time to listen to us mutually to provide the human foundations of a future work. A final word, it should well be admitted that the empirical way does not promise fast progress in the constitution of an organized field of research. It may even be the worst for the progress towards a constitution. However, the generous sharing of the participants of their devoted effort in their own culture and system produces a crop of another nature.

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Appendix

Distribution of the articles according to some criteria illustrate the concerns

Types of study	
Criteria allotted at least once to one of the articles presented	number
1. Classroom practices within well-defined didactical systems	12
2. Classroom practices promoted by research projects	11
3. Analytical accounts of empirical lessons based on observations of classroom practices	11
4. Comparison of classroom practices between different systems	2
5. Perspectives (theoretical, socio-cultural, political) informing different classroom practices and analysis of these	9
6. Classroom practices for the teaching of specific topics	12
7. Classroom practices for specific mathematical processes such as problem solving, investigation, projects, basic skills…	8
8. To gather, document, bring closer and compare studies on classroom practice independently of the reasons	0
9. To present and compare classroom practices following different conditions or mathematical topics	3
10 To present projects of multimedia-library to preserve the observations of classes (video recordings, transcriptions of lessons,…	0
11. To reflect on the bases and the methods which legitimate contributions observations of classroom practice in research…	5
12. To reflect on the uses of these observations and of their results in education systems	1
13 Condition for observation of classroom practices	1

A few written questions exchanged between the members of TSG24

- What are the major attributes qualifying good mathematics classroom practice in different cultural systems?
- Comment dire si une pratique de classe est meilleure qu'une autre
- To what extent, may some apparently important characteristics in some cases become less important in some other cases?
- Are there evidences of common teacher practices in the observed mathematics classroom?
What is different in his/her practice?
- How does the teacher organize the interplay between students and teaching situations?
- Can we identify, among the texts of TSG24, identical phenomena about mathematics teaching put in light from different theoretical tools?

Others appeared only orally.

- How to determine classroom practices which can answer a requirement of individual results?
- Can the professors use classroom practices which would not be understood by the population?
- Does there exist an ethics of the classroom practices and an ethics of research on the classroom practices?