A Little Knowledge Is A Dangerous Thing:

Model Specification, Data History, and CDO (Mis)Pricing*

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June 16, 2009

ABSTRACT

The revaluation of collateralized debt obligations (CDOs) plays a significant role in the ongoing 2007-2009 credit crisis. Starting in August 2007, a large amount of initially AAA rated CDO securities are substantially downgraded, some directly to junk grade. This paper explores two structural sources of CDO mispricing: modeling difficulty and data limitation. Simulating the frailty correlated default model of Duffie, Eckner, Horel, and Saita (2008), we show that CDO mis-pricing can be partly attributed to model misspecifications, as well as limited availability of historical data on CDO collateral assets. This simulation result is consistent with empirical evidence on historical performance of a sample of 279 CDOs. The frailty model estimated with adequate historical data would have reduced the amount of AAA rated CDO securities by 12% on average. The frailty model has predictive power for the subsequent downgrading of AAA rated CDO tranches. Our study addresses practical issues on financial innovations and provides guidance for corresponding risk management.

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*We thank for remarks from Andrew Carverhill, Wing Suen and seminar participants at the university of Hong Kong, 11th Bowles Symposium at Georgia State University, 19th Annual Derivative Securities and Risk Management Conference at Federal Deposit Insurance Corporation (FDIC). We are especially grateful to Andreas Eckner for providing the filtered frailty factor.

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ABSTRACT

The revaluation of collateralized debt obligations (CDOs) plays a significant role in the ongoing 2007-2009 credit crisis. Starting in August 2007, a large amount of initially AAA rated CDO securities are substantially downgraded, some directly to junk grade. This paper explores two structural sources of CDO mispricing: modeling difficulty and data limitation. Simulating the frailty correlated default model of Duffie, Eckner, Horel, and Saita (2008), we show that CDO mis-pricing can be partly attributed to model misspecifications, as well as limited availability of historical data on CDO collateral assets. This simulation result is consistent with empirical evidence on historical performance of a sample of 279 CDOs. The frailty model estimated with adequate historical data would have reduced the amount of AAA rated CDO securities by 12% on average. The frailty model has predictive power for the subsequent downgrading of AAA rated CDO tranches. Our study addresses practical issues on financial innovations and provides guidance for corresponding risk management.

JEL Classification: G12; G13; E43; E44

Keywords: CDO; Model Specification; Data History; Default Correlation; Frailty

I. Introduction

The ongoing 2007-2009 credit crisis has unprecedented impact on the financial industry.[1] At the center of this crisis is the previously little known financial innovation called collateralized debt obligations (CDOs). CDOs are debt claims with various seniorities against collateral asset pools. Senior claimholders will not suffer loss until the subordinated tranches are exhausted. Due to such prioritized structure and other credit enhancement such as insurance, most CDO senior tranches had AAA credit rating prior to the credit crisis. CDO issuance started in 1987 but remained dormant till 1997 (issuance $17 billion), since then the market grew rapidly to reach annual issuance of $520.6 billion in 2006, according to Securities Industry and Financial Markets Association (SIFMA). CDO issuance peaked in 2007Q2 ($178.6 billion) and afterwards declined exponentially (2009Q1 issuance $0.8 billion). Prior understanding about CDO valuation turned out to be spurious as evidenced by overwhelming write-downs.[2] This study examines potential structural causes of CDO mispricing.

The innovative nature of CDOs makes it difficult to pin down the exact reasons for this valuation failure. On one hand, given the short history of the product and modeling difficulties, Duffie (2007) doubts the capability for anyone to evaluate CDOs with comfortable accuracy. On the other hand, regulators and media have rushed to cry fouls to CDO underwriters and credit rating agencies, who brought CDOs to the marketplace. While some market participants likely deserve more blames than others, "careful research is needed to distinguish the relative importance of the bad incentives view and the mispricing view" as these two views have distinctly different implications for regulation and risk management going forward (Allen (2008)).

Given the limitations in modeling techniques and historical data, large losses do not automatically imply risk management failures (Stulz (2008)). This argument is particularly relevant for the current setting of CDOs which are collateralized on pool of

[1] Among the top five Wall Street investment banks, Lehman Brothers declared bankruptcy on September 15, 2008, Bear Stearns was acquired by J. P. Morgan on March 16, 2008, Merrill Lynch was acquired by Bank of America on September 14, 2008, Goldman Sachs and Morgan Stanley converted into bank holding companies on September 21, 2008.

[2] For instance, on July 28, 2008, Merrill Lynch sold $30.6 billion in notional value U.S. super senior ABS CDOs to an affiliate of Dallas, Texas based private equity firm Lone Star Funds for $6.7 billion, or 22 cents on a dollar. (Merrill Lynch also financed 75% of the sale through a loan with recourse only on those CDOs.)

default-risky assets. Accurate valuation of CDOs requires modeling the joint distribution of those assets, especially the default correlation. Defaults are rare events. Hence, default correlation is hard to measure. Furthermore, even single-obligor credit risk analysis proves to be difficult. There is little consensus on the best practices on portfolio credit risk modeling. In this paper, we examine the impact of data limitation and model specification on portfolio credit risk evaluation and CDO mispricing.[3]

Traditional portfolio credit risk models such as Vasicek (1987) assume that default correlation is driven only by observable common factors. However, recent studies show that such approach significantly under-estimate the actual default correlation (Das, Duffie, Kapadia, and Saita (2007), Christoffersen, Ericsson, Jacobs, and Jin (2009)). Based on this observation, Duffie, Eckner, Horel and Saita (DEHS, 2008) propose a frailty correlated default model, in which the *latent* "frailty" factor is unobservable and time-varying. The frailty approach is popular for modeling mortality rates in acturial science (see, e.g., Wang and Brown (1998)). Duffie, Eckner, Horel and Saita (2008) show that their model performs well in matching historical default patterns.

We first show, via simulations, that the DEHS frailty model provides a good estimation approach. We obtain convergence of the maximum likelihood estimates of the parameters determining the default intensity process. Furthermore, the frailty factor time series can be successfully filtered out through Bayesian analysis. The filtered conditional posterior joint distribution of the dynamic frailty factor has a mean path highly correlated with the hypothetic "true" path. After validating the DEHS model, we conduct more simulations to examine the effects of model specification and data history on portfolio credit risk analysis. We focus on the tail risk that is most relevant to CDO senior tranches often rated AAA.

Our simulation results on model specification substantiate the importance of frailty factor to portfolio credit risk valuation. At AAA level, expected portfolio default loss rate is 5.4% higher with frailty consideration than without frailty. Hence, ignoring frailty factor would result in 5.4% extra AAA tranche size. Moral hazard caused by implicit government support has been proposed as a possible cause of excessive risk-taking by financial institutions. For example, when the market drops, the Federal Reserve may cut interest rate to alleviate financing costs and boost market sentiment. This phenomenon

[3]The issues on conflicts of interest and CDO security design are discussed by Griffin and Tang (2008), Nicolo and Pelizzon (2008).

is most prominent during Allan Greenspan's tenure and coined as "Greenspan Put". However, our simulation result shows that such consideration of "Greenspan Put" has little effect on portfolio credit risk valuation when frailty factor is present. Lastly, rapid growth of credit derivatives market may be accompanied by potential structural breaks in default modeling. We find modest effect of such consideration jointly with frailty factor. When structural break exists in the data generating process, portfolio default loss rate is underestimated by 2.4% when we assume that there is no break in the historical data for the model estimation.

CDO market has short history and data could be a concern for back testing. Additionally, data quality can be problematic due to misrepresentation of collateral information (e.g., Keys et al (2008)). It is difficult to evaluate the impact of data limitation using real data for obvious reasons. Simulation analysis is our preferred approach. We find that more data may not always be desirable as the estimation results are state dependent. While the non-monotonicity of data length effect is to be further researched, it is comforting to see that the estimation approach is robust to data noise.

Having examined potential impacts of model and data on CDO valuation, we apply the DEHS frailty model to historical CDO data. Our sample contains 279 CDOs issued between October 1997 and December 2004. There are 65 collateralized bond obligations (CBOs), 94 collateralized loan obligations (CLOs), 103 CDOs collateralized with asset-backed securities (ABS CDOs) including most mortgage-back securities, 17 CDOs collateralized with other CDO tranche securities (CDO^2s).

Our empirical findings are consistent with the simulation results. The no-frailty model generates lower portfolio default rates and hence higher AAA tranche sizes than a credit rating agency result. The no frailty model underestimates default rates by 7% on average. However, the frailty factor increases the portfolio default rate at AAA level by 19%. Therefore, accounting for frailty factor the AAA rated CDO tranches could shrink by 12%. If the frailty model is indeed useful, we would observe subsequent downgrades of CDO AAA tranches when the frailty model indicates higher risk than rating agency model. As of the rating data available date December 2008, most CDO rating changes at AAA level occurred to ABS CDOs. We find in the ABS CDO group, that the frailty model has 21% extra predictive power for future downgrades. About 80% of ABS CDO AAA rated tranches with high risk according to frailty model are subsequently downgraded, in contrast to 0% for low risk group.

We make three contributions to the literature. First, we shed light on the potential structural causes of CDO mispricing. We show that model specification and data limitation can have substantial effects on CDO valuation. Our empirical evidence supports the simulation results. Second, although model uncertainty is well studied in equity markets and portfolio allocation (e.g., Garlappi, Uppal, and Wang (2007)), we take it to the credit derivatives market and present its strong impact. Model inaccuracy is probably one of the biggest factors for the CDO valuation failure. Third, our study provides a good framework for analyzing financial innovations, which will likely continue and the same model and data issues would appear repeatedly. Therefore, our research provides preliminary direction for future risk management practice.

Our study is built upon Duffie, Eckner, Horel and Saita (2008). We add to existing studies in the following ways. While Longstaff and Rajan (2008) argue that historical CDO prices are well explained, Brennan, Hein, and Poon (2008) and Coval, Jurek, and Stafford (2008) show that substantial mispricing can arise in the CDO structuring process. Our finding of systematic mispricing due to limitation in historical data and model misspecification provides a justification for the above seemingly conflicting findings. Fender, Tarashev, and Zhu (2008) also show that CDO can be overvalued relative to equivalent corporate bonds. Eckner (2008), Feldhtter (2008) and Heitfield (2008) use MCMC for CDO pricing. Our study differs by the economic motivation. Finally, our paper makes similar points to the discussion by Coval, Jurek, and Stafford (2009).

The rest of this paper is organized as follows. Section II reviews the setting of our study and relevant literature. Section III describes the frailty correlated default model and our simulation method. Our simulation results on the effects of model specification, data history and their interaction are discussed in Sections IV. Empirical analysis using historical CDO data is provided in Section V. Section VI concludes the paper.

II. CDO Primer and Literature Review

The prototype of a CDO originates in 1987 at junk bond powerhouse Drexel Burnham Lambert (bankrupt in February 1990). The resurgence in current format is mostly attributed to Credit Suisse First Boston in 1997 (notably Christopher Ricciardi).[4] The CDO market experienced explosive growth in recent years before its collapse in 2008. CDOs are investment conduits holding credit securities as collateral assets and issuing secured notes as liabilities with prioritized payment structure. They belong to the category of pay-through asset-backed securities (ABS).[5] Major collateral asset types include corporate loans and bonds but other types include credit card debt and credit derivative contracts. Most CDOs have multiple tranches where parts of the tranches are sold to different investors. However, single-tranche CDOs ("bespoke" CDOs) are often structured specifically for a particular investor need. Except static deals, CDO assets are administered by collateral managers. CDO operations are overseen by trustees.

A. CDO Structuring, Rating, and Pricing

CDOs serve the economic purpose of balancing the supply and demand of credit market. Investment banks underwrite CDOs, like IPOs and SEOs, mostly in the format of full commitment. The underwriters often provide bridge loans to purchase and warehouses to store collateral assets before deal closing. Underwriters also often provide liquidity facilities (such as revolving loans, swaps, and put options) for the CDO. CDOs can be initiated by collateral asset originator, manager, or underwriter. The liability structure of the CDO is mostly determined by the underwriter (with agreement of the manager) according to investor demand and rating requirement.

CDO market has been a rated market from the beginning. With the estimated distribution of expected portfolio default loss,[6] the tranche must withstand the scenario default rate (SDR) of the desired rating. SDR is the portfolio default rate (with some adjustment based on default experience) for which the default probability exceeding this portfolio default rate is no greater than that of historical corporate bond default rate with the same rating. In practice, many investors rely on the ratings for CDO pricing.

[4]The development of the credit derivatives market in general is largely attributed to J. P. Morgan (notably Blythe Masters), which invented credit default swaps (CDS) that fueled the synthetic CDO market.

[5]CDOs are distinguishable from traditional ABS in two aspects. First, CDO structure and collateral assets are much more diverse than traditional ABS. Second, CDO liability structure is more complex with trigger events to retire the senior tranches and other credit enhancements.

[6]CDOs are constructed from underlying portfolio characterized by collateral credit quality, maturity and correlation. The cash flows are tranched into different classes. The credit quality of each class depends on the recovery rate as well as credit enhancements. The valuation of CDO, therefore, starts with and depends heavily on the accurate assessment of the credit risk of the collateral portfolio.

All three major rating agencies (S&P, Moody's, and Fitch) employ simulation methods when rating CDOs. Two different approaches are often used to derive credit portfolio value from individual collateral assets. The structural approach (e.g., "copula" approach, used especially by S&P) assumes asset value processes are correlated, and a firm defaults when its asset value falls below some default threshold. Asset value is simulated with imposed correlations. Credit portfolio value is determined after all assets are simulated. Repeating the simulation multiple times results in a distribution of the portfolio value. Reduced form approach assumes default occurs suddenly and unpredictable. Default intensity can be linked to firm-specific and market-wide variables. The number of defaults in the collateral portfolio follows a given distribution (e.g., Binomial as used by Moody's diversity score system). Portfolio loss rate is drawn repeatedly from this default distribution. After obtaining the distribution of portfolio loss rate, different scenarios are defined referring to different ratings through "idealized default probability" concept. Those scenario default rates are key to obtaining desired ratings for the CDO tranches.

The purchase price for CDO notes are mostly at par. The coupon rate on each tranche is the most visible pricing indicator. However, coupon rate, rating and tranche size are jointly determined. The credit spread of a given rating is easily agreeable. Hence, the most critical pricing component is tranche size (equivalently the risk level of the tranche). We focus on tranche size throughout the paper and use rating, pricing, and valuation interchangeably.

B. Portfolio Credit Risk and CDO Valuation

Credit risk portfolio valuation is difficult due to non-normal distributions. The simulation approach used in practice is often criticized for oversimplification and lack of economic intuition. Closed-form solutions can only be obtained under strong assumptions such as the Vasicek (1987) model, which is based on Merton (1974) distance-to-default (DD) model applied to correlated collateral assets. Merton DD model regards equity value as a call option on the firm's underlying assets with a strike price set at the face value of debt. The firm's asset value and asset volatility are inferred from the equity value following an iterative procedure. Then Default Probability is calculated as the normal cumulative distribution function of a Z-score depending on the above variables. However, the tractability and accuracy of the Merton DD model are tightly restricted by the underlying strong assumptions.

One recent successful extension of the Merton model to CDO valuation is Coval, Jurek, and Stafford (2008). Conditioning on the realization of market return, a factor structure is added to Merton (1974) model. Firm asset values are exposed to a common market factor, which introduces the default correlation. Different from other one factor models (Vasicek(1987) etc.), no restriction is imposed to the common factor distribution. Eom, Helwege and Huang (2004) compare the structural models for corporate bonds spread from the empirical perspective. While the implied bond spreads from the Merton model tend to underestimate the spreads realized in the market, other structural models, however, seem to on average suffer from the overestimation problem. Moreover, although most structural models generally overestimate especially for the high leveraged firms, they are even more likely to underestimate the relatively safety bonds at the same time.

Andreou and Ghysels (2008) emphasize the effect of structural shifts to the credit risk structural model. Instead of fixing the structural model parameter at some points, they point out that it is necessary to take into account the structural parameter variation. Failing to incorporate this effect may result in biased inferences. An optimal sequential quality control procedural with minimum detecting time for monitoring the structural breaks is suggested, which is extremely useful for monitoring the corporation stability during financial distress. Moreover, with a good finite sample behavior as indicated in their simulation, the suggested procedure could be used for the quality control of the credit models.

Alternative to structural models is the so-called reduced form models, which are widely used in assessing portfolio risk as well, e.g., Duffie, Saita and Wang (2007). The main difference between these two approach are the nature of the event that triggers default and the model fitting. While the structural models identify default when distance to default falls below certain barrier, Duffie, Saita and Wang (2007)'s model assumes that defaults occur randomly with a probability determined by firm specific distance to default, trailing stock return and macroeconomic variables including interest rates and market-wide stock returns. As many as the choices are, model uncertainty problem is widely accepted. This uncertainty problem undoubtedly will affect the CDO valuation accuracy.

Coval, Jurek and Stafford (2008) show that CDO senior tranche is inaccurately priced, and senior tranche investors should have required higher risk premium than that indicated by the "unreliable" ratings. The mis-prcing comes from the economic catastrophe feature of CDO and many other structured products which default only under extreme bad economic states. This default clustering feature in bad economic states acts as an extra source of risk for senior CDO tranches. Rating agencies, however, ignore this economic catastrophe feature in practice. Investors therefore should not rely on credit ratings for CDO pricing or risk assessing, for the information contained is insufficient. To correct the CDO pricing failure, Coval, Jurek and Stafford (2008) develop a state contingent framework based on a modified Merton's (1974) structural model.

Coval, Jurek, and Stafford (2009) provide a detailed discussion of the structured products market and the valuation/rating failure. Additional to the economic catastrophe feature as discussed in Coval, Jurek and Stafford (2008), they claim that small model error can be significantly magnified by the pooling and tranching structure of structured products. The model error could be either inaccurate assumption for the default correlations or default probability of collateral assets. The largest impact can be found in the more complicated CDO^2.

CDO mispricing may also be related to *local thinking* as addressed by Gennaiolio and Shleifer (2008). Using a nearly Bayesian Model of decision making, they suggest that decision makers are likely to be misled and make mistake when evaluating the probability based on the representativeness. Generally, the representativeness consists of the more frequent and common evens, and moderate mistake is witnessed for the probability estimation. When there is a mismatch between the representative and frequency, however, the probabilities of the hypothesis with the rare event as representativeness tend to be severely underestimated by a local thinker with limited memory. Hence, CDO credit risk, with infrequent representativeness, might be severely underestimated by a local thinker.

C. Default Correlation and Frailty Factor

Although agency conflicts may arise during the security design (Mehran and Stulz (2007), SEC (2008)), structured finance instruments, CDOs in particular, can be useful investment tools as long as the default correlation is low, as shown by DeMarzo (2005) and Leland (2007). However, default correlation is hard to measure and this part contributed mostly to the failure of CDO valuation (Brunnermeier (2009), BIS (2008), Crouhy, Jarrow, and Turnbull (2008), Hull (2008), Plosser (2009), S&P (2007)). For such low occurrence events, Bayesian approach is particularly appealing (Kiefer (2009), McNeil and Wendin (2007), Glasserman and Li (2005), Loffler (2003)). Therefore, when assessing credit risk of structured finance instruments such as CDOs, it is necessary to consider both the firm specific default predictors, and more challengingly, the default correlation.

A conventional portfolio loss risk model assumes that default correlation comes only from the observable factors. Even with the benefits of various firm-specific and macroeconomic covariates, however, Das, Duffie, Kapadia, and Saita (2007) finds empirical evidence that defaults are more clustered than suggested by conventional models based merely on observable factors. Bonfim (2009), Koopman, Kraussl, Lucas, and Monteiro (2009), Jimenez and Mencia (2009), Berd, Engle, and Voronov (2007) all stress the importance of not only the observable factors, but also the unobservable factors which generate extra correlation. Model uncertainty is discussed by Cont (2006) and Rajna (2000). DEHS (2008) provides a new model for corporate default intensity with a time varying common latent factor, and in the presence of a firm specific unobservable covariate. They find that the prediction power of a general credit model will be increased dramatically by incorporating an common unobservable covariate. Compared with the traditional method, this model is especially good for the default clustering estimation. However, this refined pricing model still suffers from parameter uncertainty. Limited data history further deteriorates the estimation accuracy. While Bayesian approach is employed to solve the parameter uncertainty, no research has been done for the effect of limited data history.

In the spirit of DEHS (2008), we use dynamic frailty model as the benchmark model for portfolio loss estimation. We depart from DEHS (2008) by assuming different scenarios of data structure. This can be achieved by controlling the data generating process. If data history matters, we would expect to see the portfolio-loss estimation results affected dramatically by different data structures. This finding has an important implication for assessing the credit risk of senior CDO tranches, which confines their loss to tail default distribution of the underlying collaterals.

III. Frailty Correlated Default Model and Simulation Method

One firm default status may have impact on another firm's default probability. Acharya, Schaefer, and Zhang (2008) document the impact of GM and Ford downgrade on the entire market constituents, even though some of them are completely unrelated to GM and Ford. Jorion and Zhang (2007) conduct a larger scale analysis over bankruptcies and find similar results. The reason for these seemingly unrelated firms sharing a default factor can be learning, as argued by Collin-Dufresne, Goldstein, and Helwege (2003) and Giesecke (2004) or market structure as argued by Allen and Carletti (2006).

A. Dynamic Frailty Model

Motivated by the definitive finding of excessive default clustering in Das, Duffie, Kapadia, and Saita (2007), Duffie, Eckner, Horel and Saita (2008) propose a frailty correlated default model, in which default intensity of firm i at time t takes a proportional hazard specification as

$$\begin{aligned}\lambda_{it} &= \Lambda(S_i(X_t), \theta) \\ &= \exp(\alpha + \beta \cdot V_t + \gamma \cdot U_{it} + Y_t + Z_i). \end{aligned} \tag{1}$$

Default events are driven by three types of factors:

1. Observable macroeconomic factors (V_t), including market-wide stock returns and interest rates. Demchuk and Gibson (2006) argue that stock market performance is an important credit spread determinant.

2. Observable firm-specific factors (U_{it}) such as a firm's "distance-to-default" and trailing stock return. Duffie, Saita, and Wang (2007) examine the predictive power of these observable factors and achieve the highest out-of-sample prediction accuracy ratio among then available models.

3. Unobservable common frailty factor Y_t and firm heterogeneous frailty factor Z_i.

Conditional independence of default arrivals is regained under the assumption that additional default clustering could be captured by the frailty factors. They apply an iterative procedure combining Monte Carlo integration and maximum likelihood estimation to compute the intensity parameter θ. Gibbs sampler is employed to draw the posterior distribution of the latent frailty factors.[7] Details of the model are provided in Appendix A and the estimation algorithm are provided in Appendix B.

B. *Simulation Approach*

Simulation study helps understand the full picture of model performance. Misspecification of a model leads to biased estimation and might eventually produce deflected prediction. For example, when common-frailty-driven defaults are not accounted for, we underestimate the possible extreme losses of a credit portfolio.[8] Historical data plays a critical role in empirical model performance. With limited data, model estimation may be sensitive to data structures.[9] Long run means of the underlying factors are often challenging to tie down if monthly data is employed.

To assess the model specification and data history effects, we simulate a series of data structures, each of which accompanies with a specific economic scenario. The number of firms simulated is 2800 and the history lasts for 25 years. To keep akin to the factor dynamics implied in the real historic data, we employ the same Gaussian first-order vector autoregressive model for the observable factors in Duffie, Saita and Wang (2007) and the same Ornstein-Uhlenbeck process with long run mean 0 for the common frailty factor as specified in Duffie, Eckner, Horel and Saita (2008). The time step is taken to be one month. Here we give a brief review of this factor time series model. The maximum likelihood parameter estimation, which is provided in Appendix B of Duffie, Saita and Wang (2007), is listed in Appendix C.

A simple arbitrage-free two-factor affine term-structure model is specified for the

[7] Z_i is difficult to pin down given the size of the data used in their paper. Furthermore, its presence does not qualitatively change the significance of Y_t. Therefore, this unobservable firm heterogeneity is opted out of their final model for portfolio credit risk evaluation.

[8] As shown in the realized portfolio loss quantile test in Figure 9 of Duffie, Eckner, Horel and Saita (2008)

[9] Public firms rarely go to default. Among the total 2793 companies in Duffie, Eckner, Horel and Saita (2008), 496 defaults occurred during the 25 years from 1979 to 2003.

three month treasury rates (r_{1t}) and 10-year treasury rates (r_{2t}).

$$r_{t+1} = r_t + k_r(\theta_r - r_t) + C_r\varepsilon_{t+1}, \tag{2}$$

where θ_r is the long-run mean of interest rates, C_r is a 2×2 matrix, and $\varepsilon_1, \varepsilon_2 \ldots$ are independent standard normal vectors.

For firm specific factors of distance to default D_{it} and log-assets V_{it}, and trailing 1-year S&P 500 return,[10]

$$\begin{aligned}\begin{bmatrix} D_{i,t+1} \\ V_{i,t+1} \end{bmatrix} = \begin{bmatrix} D_{it} \\ V_{it} \end{bmatrix} + \begin{bmatrix} k_D & 0 \\ 0 & k_V \end{bmatrix} \left(\begin{bmatrix} \theta_{iD} \\ \theta_{iV} \end{bmatrix} - \begin{bmatrix} D_{it} \\ V_{it} \end{bmatrix} \right) \\ + \begin{bmatrix} b \cdot (\theta_r - r_t) \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma_D & 0 \\ 0 & \sigma_V \end{bmatrix} \eta_{i,t+1},\end{aligned} \tag{3}$$

$$S_{t+1} = S_t + k_s(\theta_s - S_t) + \xi_{t+1}, \tag{4}$$

where θ_{iD}, θ_{iV} are long-run means for firm i's distance to default and log assets, respectively. η_{it} is the two dimensional innovation vector.

Correlation among the observable factors is modeled as

$$\begin{aligned} \eta_{i,t} &= Az_{it} + Bw_t, \\ \xi_t &= \alpha_S u_t + \gamma_S w_t, \end{aligned} \tag{5}$$

where z_{it} and w_t are independent two-dimensional standard normal vectors and u_t are independent standard normals.

For tractability and parsimony, the mean-reverting speed k_D of distance to default is assume to be homogeneous across all firms. Distance to default is an asset volatility adjusted measure of leverage and its volatility σ_D does not vary by firm as implied by Merton's theory. Asset volatility σ_V and its mean-reverting speed k_V are also assumed to be homogeneous across firms. However, a common targeted leverage ratio leads to unrealistic estimated term structure of future default probabilities. Duffie, Saita and Wang (2007) instead estimate θ_{iD} firm by firm, with the cross-sectional distribution

[10] Firm asset value is solved using the Merton's model. For more details, refer to Merton (1974), Crosbie and Bohn (2002) and Vassaulou and Xing (2004).

displayed in figure 12 of their paper. In our simulation study, we load the long run means of distance to default and log assets for each firm in the following way.

As reported in Duffie, Saita and Wang (2007), the estimated θ_{iD} across the whole firm set has a median of 3.1, with interquartile range of 1.4-4.8. A careful inspection of figure 12 reveals that the interval 0.0-8.0 of θ_{iD} covers most of the firms except those at the extreme lower or upper tail of the distribution. And within this interval, θ_{iD} is approximately linear to the rank of firm i, which means that θ_{iD} might be uniformly distributed on this range. Accordingly, we parameterize θ_{iD} as

$$\theta_{iD} \sim \mathbf{U}(0.0, 8.0), \tag{6}$$

where $\mathbf{U}$ denotes the uniform distribution.

Long run means of log assets are not reported in Duffie, Saita and Wang (2007). Here we turn to Bharath and Shumway (2008) who apply a similar estimation procedure and provide the quartiles of estimates for an augmented firm set.[11] The reported asset value ranges from 1.52 to 22949.32 (log asset value from 0.4 to 10.0).[12] For simplicity, we assume in our simulation that θ_{iV} is uniformly distributed on this interval:

$$\theta_{iV} \sim \mathbf{U}(0.4, 10.0). \tag{7}$$

To generate time series for the observable factors, we need to make further assumptions of the starting value of the factor processes and the entry time of each firm. Following the common practice, all factors are assumed to start at their long run means. We are left with roughly 1400 active firms at the end of the data period after subtracting the number of defaults and merger-acquisition from the total number of firms in Duffie, Saita and Wang (2007).[13] Campbell, Hilscher and Szilagyi (2008) provide average number of active firms in each year from 1963 to 2003 in a lager data set. This number increases from 4342 in 1980 to 7833 in 2003. Proportional to this growth rate, in the simulation we assume that 800 firms exist at the beginning of the data period. The

[11] Duffie, Saita and Wang (2007) consider 2770 industrial firms from 1980 to 2004, with 497 defaults identified. Bharath and Shumway (2008) examine all firms in the intersection of the Compustat Industrial file—Quarterly data and CRSP daily stock return for NYSE, AMEX and NASDAQ between 1980 and 2003, excluding the financial firms. They obtain total 1449 defaults.

[12] Winsorized at the 1st and 99th percentiles by Bharath and Shumway.

[13] We do not deduct "other exits" since most of this type are various data gaps.

other 2000 firms enter evenly in the following 25 years.

Once time series for distance to default and log assets are available, we can get the face value of debt (L_t) and market value of equity (W_t) of each firm i by sequentially solving the following two equations. Let V_t denotes asset value at time t, then

$$D_t = \frac{\ln(V_t/L_t) + (\mu_A - \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}; \tag{8}$$

$$W_t = V_t\Phi(d_1) - L_t e^{-rT}\Phi(d_2), \tag{9}$$

where $d_1 = \frac{\ln(V_t/L_t)+(r+\frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}$, $d_2 = d_1 - \sigma_A\sqrt{T}$, $\Phi(\cdot)$ is the standard normal cumulative distribution function. r is the risk free interest rate measured as the 3-month treasury rates.

We take the forecast horizon to be 12 months. We avoid solving for the asset volatility in virtue of the assumption of their homogeneity across all firms as specified in equation 3. Its value is fixed at the maximum likelihood estimate of 0.1169. Some scholars provide various approaches to estimate the expected asset drift rate μ_A.[14] In this paper, we deviate from all these methods by making use of the mean-reversion property of the log assets process. We calculate μ_V as the expected mean-reversion during the next period.

$$\mu_V - \frac{1}{2}\sigma_V^2 = k_V(\theta_V - \ln(V_t)) \tag{10}$$

Taking μ_V and σ_V into equation 8, we can directly derive the debt value L_t. The time series for firm's market equity follows from the call-option pricing formula as stated in equation 9. It is unrealistic to assume a constant level of face value of debt in a time period as long as 25 years. Combining the assumptions of leverage targeting and mean-reverting asset process, we allow a firm to dynamically adjust their outstanding debt, as suggested by Collin-Dufrensne and Goldstein (2001).[15]

[14] Vassalou and Xing (2004) calculate firm specific average returns on each stock. Bharath and Shumway(2008) estimate previous year asset return. Hillegeist, Keating, Cram and Lundstedt (2004) use previous year asset return, but replace it with the risk free interest rate if this return is negative. Campbell, Hilscher and Szilagyi (2008) use 0.06, an empirical proxy for equity premium, plus risk free risk as an estimate.

[15] Collin-Dufrensne and Goldstein (2001) and Duffie, Saita and Wang (2007) show that dynamic debt adjustment and leverage targeting could generate more realistic term structure of default probabilities.

Now we come to the determination of the exit time for each firm. There are three major types of exits defined in Duffie, Saita and Wang (2007), defaults, merger-acquisition and "other exits". Each type of exit will not restrict the intensity parameter estimation of the other types.[16] Since "other exits" are mostly data gaps of various types, they are less relevant for our study and we exclude them for simplicity. As argued in Duffie, Saita and Wang (2007), merger-acquisition has relatively little effect on default hazard rate and merger-acquisition itself need not prevent future default if debts are not paid back immediately. Here we do not consider merger-acquisition exits, either.

We calculate default intensity as

$$\lambda_{it} = e^{\alpha+\beta_1 D_{it}+\beta_2 R_{it}+\beta_3 r_t+\beta_4 S_t+y_t}, \tag{11}$$

where R_{it} is the trailing 1-year stock return. (α, β) = (-1.029, -1.201, -0.646, -0.255, 1.556), the real data estimates reported in Table II of Duffie, Eckner, Horel and Saita (2008). A hypothetic frailty path, which remains latent in reality, is generate with mean-reverting speed 0.03 and volatility 0.15, which come from the marginal frailty parameter posterior distribution in Figure 6 in Duffie, Eckner, Horel and Saita(2008).

For firm i, the conditional probability of survival from entry time t_i to some future time s_i before the data cutoff date T_i is given by

$$p_i(t_i, s_i) = e^{-\sum_{t=t_i}^{s_i} \lambda_{it}\Delta t} \tag{12}$$

Δt equals to one month.

Default time is simulated using the Inverse-CDF method offered in Duffie and Singleton (1999). For each firm i, we draw a uniform random number U. Default time τ is determined as

$$\tau = \inf\{s_i : p_i(t_i, s_i) \le U, t_i \le s_i \le T_i\} \tag{13}$$

If $p_i(t_i, T_i) > U$, the firm never defaults in our data period.

Now we can put our factor time series and default timing data into the frailty model to estimate the default intensity parameters. By extending the factor time series with

[16] See Proposition 2 of Duffie, Saita and Wang (2007).

the pre-specified model, we can evaluate the credit risk of any portfolio constructed on the underlying firms in our data set.

C. Validation Tests

As shown in Figure 5 of Duffie, Eckner, Horel and Saita (2008), the latent factor plays a crucial role in the tail of the probability density of predicted number of defaults in future 5 years. Common source of current level of and future shocks to this latent factor enlarges the risk of heavily clustered defaults remarkably. Thus the filtered-out latent factor path and the mean-reverting speed, κ, and volatility, η, which govern its time series dynamics, are of gravity to assess the modeled correlation risk. Maximum likelihood estimates of the default intensity parameters converges to the true data generating process when the number of firms and number of time periods become large. It is helpful to do some convergence test first when working with limited real data.

According to the doubly stochastic assumption, estimation of the factor time series model could be separated from estimation of the default intensity parameters. We focus on the the default intensity estimation and also check the posterior distribution of the filtered frailty factor through the Bayesian analysis. Using the simulation approach described in the previous section, we simulate one set of the observable macro economic factors and firm specific factors, and one hypothetic frailty path. Then for 100 times, we draw a new U, the default trigger, for each firm and let default time be determined accordingly. It corresponds to 100 different realizations of the firm-default history. The maximum number of defaults recorded is 648 and minimum 573. We estimate the frailty model for each realization. The mean frailty path is shown in Figure 1 and the parameter estimation is provided in Table I.

We can see that the mean filtered frailty paths tightly follow the “true” frailty path. Correlation between filtered frailty and “true” frailty ranges from 0.87 to 0.96. Estimated intensity parameter is close to the true data generating process. Root mean square error of estimated intensity parameter is moderate and of similar magnitude to the standard error of parameter estimation provided in Table II of Duffie, Eckner, Horel and Saita (2008). It is relatively safe to conclude that the model appropriately pins down the intangible risk embedded in the latent frailty factor and the intensity parameter estimation is not likely to be heavily skewed given available 25 years’ firm-default history.

IV. Simulation Results

Using the dynamic-frailty model as a benchmark, we demonstrate in this section the effects of various model misspecifications and data limitations to the portfolio credit risk assessment.

A. *Model Specification*

A.1. No-frailty versus Dynamic-frailty Model

Even for single-obligor credit risk modeling, there is no consensus on the best performing model. Model failure has been recorded in nearly all areas. The seminal work of Vasicek (1987) on portfolio credit risk is shown to be inaccurate for heterogenous asset pools (Hanson, Pesaran, and Schuermann (2008)). More seriously, default correlation is assumed to be driven only by observable factors. This counterfactual assumption is widely adopted until recently. However, we believe existing CDOs are almost all evaluated based on this low correlation assumption.

In order to understand to what extent CDO mispricing might be due to an omitted latent factor, we formally conduct analysis of portfolio default rate prediction with no-frailty and dynamic-frailty model. The simulated 25 years data of observable factors is summarized in Panel A of Table II and the estimated no-frailty and dynamic-frailty model is listed in Panel B of Table II. The total across firm default intensities from the no-frailty and dynamic-frailty models are plotted in Figure 2 together with the number of defaults in each year. We form a portfolio with all active firms at the end of year 25. Figure 3 shows the portfolio's future five years default rate distribution generated by no-frailty and dynamic-frailty model. Panel C of Table II shows the quantiles of the default rate distribution. The 0.95, 0.99 and 0.999 quantiles for no-frailty model prediction is 14.29%, 17.33% and 21.01%, respectively, and 11.66%, 13.41%, and 15.55% for dynamic-frailty model prediction, respectively. If we take the 0.999 quantile as the SDR for AAA rating, frailty factor will bring down the AAA tranche size by 5.4%.

A.2. "Greenspan Put" Effect

In the US market, many investors believe that they are protected by a Greenspan put.[17] Whenever the market undergoes a crisis, the Fed will step in and cut interest rates to inject liquidity to the market. Generally, this kind of monetary policy creates a correlation between the short term interest rate and stock market performance.

In Duffie, Eckner, Horel and Saita (2008), interest rates are assumed to be independent of stock market index trailing return. To incorporate this Greenspan effect, we impose a correlation (lag one) between the interest rates and the market return by introducing correlation ρ between the innovation terms of 3 month interest rate $\varepsilon_{1,t+1}$ and S&P 500 trailing return ξ_t. Historically, the lag one correlation between the innovations of 3 month interest rate and S&P 500 return is about 0.18 during the 10 years from 1997 to 2006. This period is the last ten years when Greenspan was in office of Federal reserve, and also a time when the CDO market experienced exponential growth. For illustration purpose, we opt for an correlation of 0.3.

Based on the simulated historical time series (with correlation), we compare the model prediction with and without Greenspan effect (ρ equals to 0.3 and 0 respectively). Summary of factor time series are provided in Panel A of Table III, estimated frailty model in Panel B, and quantiles of predicted portfolio default rate in Panel C. Figure 4 shows the portfolio's future five years default rate distribution generated by dynamic-frailty model with correlated and uncorrelated macro factors. We can see that the portfolio default rate is slightly higher when correlation is introduced. The difference is 0.09%, 0.14% and 0.14% for 0.95, 0.99, and 0.999 quantiles, respectively. In spite of the intention to stabilize the economy, the insignificant change in risk profiles suggested that there is little real effect in a sense of protecting firms from defaults. This insignificant tail effect of model with Greenspan effect might be interpreted by the "Greenspan put". Based on the perception of Greenspan put protection in (extreme) bad economic state, firms tend to overtake risk. This moral hazard effect to some extend offsets the original purpose of Greenspan monetary policy. Our results also implies that the assumption of zero correlation between interest rates and stock market return in the dynamic-frailty model does not have significant effect to the default estimation results.

[17] *Greenspan put may be encouraging complacency*, Wall Street Journal, December 08, 2000.

A.3. Structural Break

Andreou and Ghysels (2008) emphasize the effect of structural shifts to the credit risk structural model. Instead of fixing the structural model parameter at some points, they point out that it is necessary taking into account of the structural parameter variation. Failed to considering the effects, may results in biased inferences based on the credit model. In this section, we examine the effect of parameter structural breaks for default prediction.

To demonstrate potential structural break, the 25 years of historical data is split into the former 20 years and the latter 5 years. For the former 20 years, default time are simulated use intensity parameters (α, β) = (-1.029, -1.000, -0.646, -0.255, 1.556). For the latter 5 years, we change to (α, β) = (-1.029, -1.400, -0.646, -0.255, 1.556). Thus, a structure break with respect to the intensity parameter of distance to default is embedded in the historical data. The factor time series are summarized in Panel A of Table IV. We estimate the frailty model as though there is no break and results are provided in Panel B of Table IV. We investigate the structural break effect by comparing the default prediction from estimated model assuming no break and the true model governing firm defaults in the last 5 years. Predicted default rate distribution is shown in Figure 5 and quantiles of default distribution are provided in Panel C of Table IV.

We find modest effect of structural break with the presence of frailty factor. The 0.95, 0.99 and 0.999 quantiles from the true model is 14.27%, 17.48% and 21.48%, respectively, compared to 13.05%, 15.78% and 19.08% from the model assuming no break. The difference is 2.4% at the 0.999 quantile.

B. Data History Effects

B.1. Length of Available Data

Data availability is an important concern when we are doing maximum likelihood estimation. Data structure might shift from decade to decade, even year by year. Limited historic data in deficiency of typical economic states leads to biased estimation results and sometimes is detrimental for prediction. The current credit crunch is frequently imputed to the financial innovations, like CDO and CDS, which are organized from models based on very short time-horizon data history. We lay out a data history test, exploiting the dynamic frailty model, to see how the availability of historical data will affect the tails of the predicted default rate distribution.

We truncate our dataset generated with frailty in the former section A.1. and save the last 10 and 5 years. 5-10 years is close to the actual data history of large volume issuance of CDO or CDS. We estimate the dynamic-frailty model with the limited data history, with results presented in Panel A of Table V. We can see that as we have shorter available historical data, parameter estimates become less significant. The frailty factor dies away in the 5 years case since it has a low volatility of 0.07 and a very high mean-reverting speed close to 1.

The predicted default rate distribution is shown in Figure 6. Quantiles in Panel B of Table V show that the model estimated with 10 years' data underestimates the right rail most, 5.4% for the 0.999 quantile, while the model with 5 years' data underestimates the left tail most, 1.1% for the 0.05 quantile. The non-monotonic data length effect is of the same magnitude as the difference between the no-frailty and dynamic-frailty models.

B.2. Realization of Economic States

Business cycle tends to affect all firms in the economy. Co-movements of firms are very important to assess the risk profile of super-senior CDO tranches which only suffer when extreme loss occurs. In common CDO rating practice, the dynamics of macroeconomic factors are seldom taken into consideration. Here we demonstrate the effect different economic conditions might impose on portfolio default loss. We assign one set of targeted distance to default and long run asset level to each firm and maintain the same level for each firm in the following simulation. Starting from the same macroeconomic condition, we repeat the data simulation process for several times. Each simulation represents a random realization of firm default history. The realization with smallest default number is picked out as good economic state, and largest as the bad economic state. The frailty model is estimated for both states. The results are presented in table VI and figure 8 shows the predicted default rate distribution.

Firm quality is controlled to be the same in a sense that each firm has the same targeted distance to default and long run asset level in each simulation. However, we experience a larger number of defaults in the bad economic state. The default rate quantiles in Table VI shows that instead of reversing to lower default rate in the future, the bad economy predicts a 4.3% higher default rate at the 0.999 quantile.

B.3. Data Quality

Duffie and Lando (2001) suggests that noise in issuer's asset value due to the incomplete accounting information reshapes the term structure of credit spreads of corporate bonds. Firms might jump to default even in a very short maturity. The same is true for distance to default. The estimation of distance to default might deviate from the true value since we could only get periodic and noisy accounting data. Distance to default estimation error is also a source of the frailty factor. Since systematic biases in distance to default estimation has already been captured by the common frailty variable, here instead we focus on whether the frailty model can work properly if the estimation contains some white noise. The usual way to perform noise test is to engage the signal to noise ratio, which is the proportion of the standard deviation of the signal to that of the noise. We add a 10% and 20% firm specific noise to the true distance to default simulated in former section A.1, corresponding to signal to noise ratio 10 and 5, respectively.[18]

The estimated model and predicted default rate quantiles are provided in Panel A and Panel B of Table VII, respectively. Figure 9 shows the predicted default rate distribution. We can see that the intensity parameter for distance to default all remains highly significant, although slowly decreasing in absolute value as the noise becomes louder. Comparing the quantiles predicted without noise to the those predicted with signal to noise ratio 10 and 5, the differences are all less than 2%. The frailty model displays robustness to unsystematic noise in distance to default estimation.

To sum up, common frailty factor, historical data length and macroeconomic state have large effects on the predicted portfolio default rate. However, in the presence of frailty factor, model prediction is relatively robust to structural breaks in default intensity process and noise in distance to default estimation. Correlation between macroeconomic factors has actually little impact on the default rate distribution.

[18]The noise is assume to be mean-reverting following Duffie and Lando (2001).

V. Empirical Evidence

Our simulation results suggest the potential effects of model specification and data history on CDO valuation. We conduct corresponding empirical analysis in this section. We first describe our sample CDO data. The empirical method is demonstrated in a case study. Specifically, we carry out credit risk evaluation on CDO AAA tranches using both the no-frailty model and dynamic-frailty model. We scrutinize the ability of the benchmark dynamic-frailty model to predict subsequent downgrading of the senior AAA rated CDO tranches over our sample CDOs.

A. Data Description

Our sample contains 279 CDOs issued between October 1997 and December 2004.[19] The distribution according to collateral asset type is as follows: 65 CBOs, 94 CLOs, 103 ABS CDOs and 17 CDO^2s. We obtain the first report after the ramp-up of the asset portfolio with the following collateral asset characteristics:

- Closing date (CDate): The date the CDO is purchased by investors.
- Number of obligors (N): Number of distinct obligors for the collateral asset portfolio.
- Weighted average rating (WAR): Average credit rating of the collateral asset portfolio, weighted by the par amount of each asset.
- Weighted average maturity (WAM): Average maturity of the collateral asset portfolio, weighted by the par amount.
- Default measure (DM): The average expected default rate of collateral assets, weighted by the par amount and annualized with average asset maturity.
- Variability measure (VM): The annualized standard deviation of collateral asset default rates, which measures the dispersion of underlying assets without consideration of correlation.

Table VIII describes CDO name, closing date, number of obligors, weighted average rating, weighted average maturity for each CDO in our sample. We also list the number

[19]We stop the data in 2004 due to the availability of frailty factors. Additionally, the CDO market has explosive growth with some irregular activities during the 2005-2007 period. Consequently, nonstructural factors could drive CDO pricing after 2004.

of notches downgraded as of December 2, 2008 for the initially AAA rated tranches and the SDRs at report date for the initial rating. For the downgraded notches, number 0 denotes never downgraded and number 1-19 correspond to downgrading from AAA to AA+ all the way down to CC. We have 10 CDOs downgraded in our sample CBOs, 0 in CLOs, 54 in ABS CDOs and 2 in CDO^{2}s. SDR is the required subordination, or the percentage of portfolio loss rate a CDO tranche at a given rating level must sustain without causing a cash flow event of default. The probability of default in the assets portfolio exceeding this percentage is no greater than historical default probability of corporate bonds with the same rating. For example, if the portfolio default distribution is as the one with frailty in figure 3 and the average realized default probability for AAA rated corporate bond is 0.1%, then the SDR for AAA tranche is 21.01%, the 0.999 quantile. Once SDR for a desired tranche rating is available, the tranche size can be determined as no greater than 1-SDR.[20]

B. A Case Study and Methodology

We first illustrate our CDO valuation method via an example case analysis. All 4 types of CDOs are valuated in a similar way. The chosen CDO is called *Independence I*. This CDO is collateralized with various ABS securities including CMBS, RMBS, ABS, and CDO. We demonstrate how we evaluate this ABS CDO and how the frailty model generates results to predict ultimate downgrade.

Independence I is issued by Independence I CDO, Ltd. (a special purpose vehicle registered in Cayman Islands) and co-issued by Independence I CDO Inc. (a special purpose vehicle registered in Delaware). (The Independence series continue to Independence VII issued on March 28, 2006.) The closing date is December 7, 2000 according to Moody's and December 12, 2000 according to S&P. Credit Suisse is the lead underwriter and counterparty for interest rate swap agreements. The collateral manager is Independence Fixed Income Associates Inc. (renamed Declaration Research and Management LLC. in 2003). From Moody's New Issue report dated April 13, 2001, the collateral pool is fully ramped by March 12, 2001 (about 65% complete at closing date).

Independence I has an initial principle amount of US$300 million with the following capital structure: Class A first priority senior secured notes $223.5 million (74.5%),

[20]In some cases, a larger fraction is achieved through other credit enhancements such as insurance.

Class B second priority senior secured notes \$50 million, Class C Mezzanine secured notes \$15 million, and preference share 11.5 million.[21] Moody's initially assigns AAA rating to Class A tranche, followed by the Class B with Aa3, and Class C with Baa2. Preference shares are not rated. S&P assigns AAA rating to Class A. However, S&P does not rate Class B, Class C and preference shares. Fitch also provides Class A with AAA rating, Class B with AA-, Class C with BBB and preference shares not rated. Class A of this CDO is subsequently downgraded to AA- rating on August 30, 2004 and further downgraded to A- on November 16, 2005 by S&P. Fitch downgrades Class A to A on March 7, 2006, further to BB on March 9, 2009. Moody's downgrades Class A to AA2 on February 18, 2005, to Baa2 on February 2, 2007, and further to B1 and placed under review for possible downgrade on April 22, 2009.

The collateral asset characteristics reported on December 26, 2003 before any downgrade are as follows: the collateral asset portfolio contains 95 assets from 83 obligors, with weighted average rating of BBB-, weighted average maturity of 8.45 years, average expected asset default rate 0.0112, variability of the default rate of 0.0162. For the AAA rating of this collateral portfolio, a rating agency derives scenario default rate of 29.2% using a default rate threshold of 0.0073.

To apply the no-frailty model and the dynamic-frailty model to this ABS CDO, we adopt the maximum likelihood estimation of the factor times series dynamics and default intensity parameters provided in Duffie, Saita and Wang (2007) and Duffie, Eckner, Horel, and Saita(2008). The frailty factor estimation available is up to the end of year 2003. For a CDO with closing date in the year 2004 (may be initiated in 2003), We extend this factor to date through the OU process dynamics starting from the end month of 2003. The 3-month treasury bill rate and S&P 500 index are obtained from the Board of Governors of the Federal Reserve system and CRSP database, respectively. We choose the WAM of the CDO as the prediction horizon and we assume each obligor has an equal amount of principal in the asset pool.

Collateral pools of CDOs consist of various types of assets such as corporate bonds, leveraged loans, sovereign debts, ABS tranches and CDO tranches. Our exemplificative ABS CDO is comprised of 41.8% commercial mortgage-backed securities, 23% residential mortgage-backed security assets, 21.4% asset-backed securities, and 13.8% CDO assets.

[21]Those numbers are provided by Moody's new issue report. S&P record has preference share size \$12 million.

It is prohibitive to estimate the distance to default and asset process for these complex securitized products. Instead, some rating agencies use average default probability of the same rating cohort to proxy for the default probability of the same type of assets and assume pairwise correlation among the obligors based on industry sector and geographic region. For example, S&P's CDO Evaluator and Fitch's VECTOR give default probability based on asset type, rating and maturity.[22] Furthermore, for CBOs and CLOs, the obligor might be a private firm and even such rating is not available. Therefore, in our study, we do not go into obligor-by-obligor estimation for distance to default and asset processes. For simplicity and tractability, we make use of the portfolio average expected default rate (DM) and variability of default rate (VM). We assume that default probability of each obligor in the collateral portfolio is log-normally distributed with mean DM and standard deviation VM times square root of N, the number of obligors. We choose log-normal distribution in light of non-negative default probabilities and right-skewed default rate distribution. Then we equally draw N quantiles of the log-normal distribution on the interval $(0, 1)$ and assign these quantiles as the default rates of the obligors.

The sampled default probability of obligor i, DP_i, is transformed into targeted distance to default, θ_{iD}, through the inverse cumulative normal distribution function Φ^{-1}:

$$\theta_{iD} = -\Phi^{-1}(DP_i), \tag{14}$$

For simplicity, we assume that long run means of the assets of each obligor is uniformly distributed on some quartile range of the asset values as estimated in Bharath and Shumway (2008). Specifically, we choose for CBOs the uppermost quartile, 6.3-10.0, in view of the fact that CBOs mostly consist of bonds issued by relative large-cap companies. We choose for CLOs the 0.25-0.5 quartile, 3.3-4.7, since most underlying assets of CLOs are leveraged loans from small and median size firms. For ABS CDOs and CDO^{2}s, we do not have a strong prior reason to choose a particular asset span and simply use the interquartile range, 3.3-6.3.[23] Empirical evidence in Titman and Wessels (1988), Rajan and Zingales (1995), and Fama and French (2002) shows that larger firms

[22]For each asset type, default probabilities across all ratings and for typical maturities are estimated from historical default data of that specific type. Sometimes adjusted default probabilities from other asset types are used when the historical data is scarce for a recent innovation.

[23]For ABS CDOs and CDO^{2}s, our SDR prediction is not sensitive to the asset span when it is shifted down to the lower interquartile 0.4-4.7 or up to the upper interquartile 4.7-10.0.

tend to have higher leverage. Thus we assign in reverse order the long run means of assets to targeted distance to default for each obligor. Then a larger obligor in our sample has a lower targeted distance to default.

Given the firm specific factors starting from its long run means, we apply the no-frailty model and dynamic-frailty model to generate the portfolio default rate distribution for each CDO and decide the SDRs accordingly. The results are provided in Table VIII. For *Independence I*, the SDR is 29.2% by rating agency, 25.3% predicted by the no-frailty model and 51.8% predicted by the dynamic-frailty model. Our benchmark dynamic-frailty model allows a AAA tranche of 48.2%. However, the SDR by rating agency allows for 70.8%. Here we construct a variable R_i for CDO i as:

$$R_i = SDR_{DF,i} - SDR_i \tag{15}$$

where $SDR_{DF,i}$ is the SDR predicted by the dynamic-frailty model and SDR_i is the SDR by rating agency. R_i denotes the deviation of AAA tranche size with the dynamic-frailty model prediction as the benchmark. A positive R_i implies aggressive rating and negative R_i implies conservative rating. Therefore, higher R_i corresponds to riskier AAA rated tranche. For our case *Independence I*, R_i equals to 22.6%, which reveals its high risk-bearing. Note that the Class A tranche with initial AAA rating from all three rating agencies is eventually downgraded to BB credit rating.

C. Empirical Results

The estimated SDRs from the no-frailty model and dynamic-frailty model are presented in Table VIII and plotted by closing date in Figure 10 together with the SDRs by rating agency. The no-frailty model estimation is highly correlated to the SDRs by rating agency, with correlation 0.90 for CBO, 0.85 for CLO, 0.89 for ABS CDO and 0.74 for CDO^2, respectively. Compared with no-frailty model estimation, on average the SDRs by rating agency overestimate by 10% for CBO, 7% for CLOs, 5%for ABS CDO and 14% for CDO^2, respectively. However, when frailty factor is considered, on average the SDRs by rating agency underestimate by 11% for CBO, 14% for CLOs, 12%for ABS CDO and 4% for CDO^2, respectively. Over all 279 CDOs in the sample, dynamic-frailty model predicts SDRs on average 12% higher than those by rating agency, while the no-frailty model predicts 7% lower.

According to our empirical results, if we only consider observable factors for our portfolio credit risk evaluation, the SDRs by rating agency, which are the primary determinants of the assigned rating, overestimated the portfolio risk on average for all four types of CDOs. For CDO^2s, overestimation is most prominent. Once the extra source of risk, the common frailty which systematically affects the whole economy, is taken into account, risks of all 4 types of CDOs are underestimated by the SDRs from rating agency. ABS CDOs are most directly related to the subprime mortgage crisis and have experienced widespread downgradings even for AAA rated tranches. Of the 103 ABS CDO in our sample, 54 are downgraded by one or more rating agencies. 10 downgrades are recorded in our sample of 65 CBOs and 94 CLOs. The dynamic-frailty model predicts the large risk underestimation, over 10%, for CBOs and CLOs. We expect to see more downgrades of these two types in the future. The frailty factor is very important to understand the risks embedded in the AAA tranches. It brings down the AAA tranche size by about 19% when added to the model.

Since most downgrades recorded in our sample happen to ABS CDO, we conduct a downgrading prediction study with respect to ABS CDOs in our sample. R_i of ABS CDOs ranges from -0.29 to 0.28. We separate the ABS CDOs into 4 categories according their risk level as measured by R_i. The 1st low risk category is with R_i from -0.3 to 0. The 2nd risk category is with R_i from 0 to 0.1. The 3rd risk category is with R_i from 0.1 to 0.2. The 4th high risk category is with R_i from 0.2 to 0.3. As shown in Figure 11, 5 CDOs falls in the 1st low risk category, 17 in the 2nd, 71 in the 3rd and 10 in the last high risk category. The average rate of downgrades for CDOs in the low risk category is 0, in contrast with average downgrading rate of 24%, 46% and 80% for the following three riskier categories, respectively. Our benchmark dynamic-frailty model exhibits high power to separate out the safest and riskiest CDOs.

To better understand the model performance, we draw the power curve of our prediction in Figure 12. As in the standard accuracy ratio test, the horizontal axis of the power curve denotes the riskiest x fraction of CDOs, ranked by R_i. The vertical axis gives the fraction of downgrades included in the riskiest x fraction. Accuracy ratio is measured as twice the area between the power curve and the 45 degree line. In our case, the highest value in theory is 0.56. Our dynamic-frailty model prediction gives an accuracy ratio of 0.21. It does have power to predict subsequent downgrading of AAA tranches. For a CDO with an median size of \$600 million and maturity 7 years, the average 12% AAA tranche inflation will lead to an underpayment of about $1.5 million to the investors if this 12% should actually be rated as AA and yield spread is 30 basis points between AAA and AA rated CDO tranches. The accuracy ratio of 21% will on average gain the investors $0.32 million, comparable to CDO rating fees (usually around $0.5 million).

The power curve also tells us that our dynamic-frailty model does not do very well in the median risk range. This might reflect the fact that in reality the ultimate AAA tranche size comprises some final adjustments to the amount approved by the SDR from rating agency model results. In our case study of *Independence I*, the AAA tranche size is 74.5% while the SDR by rating agency allows for 70.8%. Another possible reason is that rating agencies make tradeoffs between rating stability and timeliness. Altman and Rijken (2005) confirms the exclusive focus of agency ratings on the permanent component of credit quality and disregard of the temporary component. Our empirical results implies that we should see more downgrades of CDO AAA tranches in the future. Further research with more available data samples and longer rating history will surely give us a more comprehensive understanding.

Since higher R_i denotes more risk, the AAA tranch is expected to be downgraded to a lower credit rating when the deviation is higher. Here we do a regression of the AAA tranche downgraded notches on the risk proxy, R_i. The downgraded notches (DG) are shown in Table VIII.

$$DG_i = a + b \cdot R_i + \varepsilon_i \tag{16}$$

We expect b to be positive and significant. For the 103 ABS CDOs, b equal to 18.6 with t-statistic 2.3. b is significant at the 5% confidence level. R square for this regression is 5%.

VI. Conclusion

One of the most remarkable episodes of the ongoing 2007-2009 credit crisis is the widespread downgrading of top rated (often cases AAA) CDO securities and overwhelming write-downs resulting from CDO revaluation. In this paper we analyze the structural causes of CDO mispricing. Our simulation results suggest that model mis-specification and data quality can have substantial effects on CDO valuation. The frailty default factor identified by Duffie, Eckner, Horel, and Saita (2008) is especially important. Ignoring the frailty factor might inflate the AAA tranche of a CDO by about 5.4%. Consequently, the AAA tranche would have been rated much lower had we considered the frailty factor and other data issues.

We conduct empirical analysis on 279 CDOs issued between October 1997 and December 2004. Our no-frailty model obtains CDO portfolio default rate at AAA level close to rating agency estimates, except for CDO^2s. However, considering frailty factor will raise the AAA portfolio default rate by 21% for CBOs, 21% for CLOs, 17% for ABS CDOs, 18% for CDO^2s relative to no-frailty model. Furthermore, the relative risk increase for ABS CDOs which experienced the most AAA downgrades in our sample can predict future downgrade as of December 2, 2008 (with an accuracy ratio of 0.21). Hence, the information content in the frailty factor is economically significant.

Understanding the pricing of CDO is useful for future regulatory policies and risk management strategies, as future financial innovation will likely be accompanied by similar issues on model specification and data quality. The frailty model and Bayesian estimation approach discussed in this paper will be useful for portfolio credit risk analysis as default data is scarce. Prior belief can shape the result in significant ways. Exploring the economic sources of the frailty factor and formation of prior belief about default correlation is left for future research.

Appendices

A Dynamic Frailty Model

Following Duffie, Eckner, Horel, and Saita(2008), we define a Markov state Vector X_t which include both firm-specific and macroeconomic covariates U_{it}, V_t and Y_t. If X_t is fully observable, the default intensity λ_{it} for a given firm i at time t will be

$$\begin{aligned}\lambda_{it} &= \Lambda(S_i(X_t);\theta) \\ &= e^{\alpha+\beta\cdot V_t+\vartheta\cdot U_{it}+Y_t}\end{aligned} \tag{17}$$

where $S_i(X_t)$ represents the component of X_t that is relevant to the default intensity of firm i, θ represents the parameter vector for default intensity to be estimated.

The likelihood of the data at the parameters (γ,θ) is given by

$$\begin{aligned}&\mathcal{L}(\gamma,\theta|W,Y,D) \\ &= \mathcal{L}(\gamma|W)\mathcal{L}(\theta|W,Y,D) \\ &= \mathcal{L}(\gamma|W)\prod_{i=1}^{m}(e^{-\sum_{t=t_i}^{T_i}\lambda_{it}\Delta t}\prod_{t=t_i}^{T_i}[D_{it}\lambda_{it}\Delta t+(1-D_{it})]).\end{aligned} \tag{18}$$

where η is the parameter governing the dynamics of V_t, U_{it}.

However, given that Y_t is not observable to the econometrician, the likelihood is then

$$\begin{aligned}&\mathcal{L}(\gamma,\theta|W,D) \\ &= \int \mathcal{L}(\gamma,\theta|W,y,D)p_Y(y)dy \\ &= \mathcal{L}(\gamma|W)\int \mathcal{L}(\theta|W,y,D)p_Y(y)dy \\ &= \mathcal{L}(\gamma|W)E[\prod_{i=1}^{m}(e^{-\sum_{t=t_i}^{T_i}\lambda_{it}\Delta t}\prod_{t=t_i}^{T_i}[D_{it}\lambda_{it}\Delta t+(1-D_{it})])|W,D].\end{aligned} \tag{19}$$

where D_i is the vector of default indicators. That is, for company i, $D_i = 0$ before default, and 1 when default. $p_Y(y)$ represents the unconditional probability density of the unobservable common factor Y. Here, we assume that Y is independent of W.

The model parameters are estimated through a combination of EM algorithm and the Gibbs sampler.

B Parameter Estimation

For estimating the default intensity parameter $\theta = (\beta, \eta, \kappa)$, a combination of Markov chain Monte Carlo (MCMC) and the expectation-maximization (EM) algorithm is employed. This combination has advantage for the Maximum likelihood parameter estimation in the model with incomplete information. The detailed steps include

Step 1. Get the maximum likelihood estimator of the intensity model with only observable covariates $\hat{\beta}$. That is the MLE from equation 18 without considering the effect of unobservable covariate Y.

Step 2. Assign an initial estimate value for θ, as suggested by Duffie, Eckner, Horel and Saita (2008), at $\theta^{(0)} = (\hat{\beta}, 0.05, 0)$.

Step 3. Draw n independent sample path for the frailty factor $Y^{(1)}, \ldots, Y^{(n)}$ from $p_Y(\cdot|W, D, \theta^l)$, that is the conditional density of Y's OU process. This can be down with MCMC, specifically Gibbs sampler, while taking the l^{th} estimate value for θ^l as well as the observable covariates W and D as given.

Step 4. Maximization step. Define the intermediate quality

$$\begin{aligned} Q(\theta, \theta^{(l)}) &= E_{\theta^{(l)}}(\log \mathcal{L}(\theta|W, Y, D)) \\ &= \int \log \mathcal{L}(\theta|W, y, D) p_Y(y|W, D, \theta^{(l)})\, dy \end{aligned} \tag{20}$$

Based on the sample path for Y drawn in step 3, $Q(\theta, \theta^{(l)})$ can be approximated by

$$\hat{Q}(\theta, \theta^{(l)}) = \frac{1}{n} \sum_{j=1}^{n} \log \mathcal{L}(\theta|W, Y^{(j)}, D) \tag{21}$$

Then the new parameter estimate $\theta^{(l+1)}$ can be get by

$$Max \ \ \hat{Q}(\theta, \theta^{(l)}) = Max \ \ \frac{1}{n}\sum_{j=1}^{n} \log \mathcal{L}(\theta|W, Y^{(j)}, D) \tag{22}$$

Step 5. Back to step 3, and replace $\theta^{(l)}$ with the new estimator $\theta^{(l+1)}$. Proceed to step 4 to get $\theta^{(l+2)}$. Repeating step 3, 4, until the estimation of θ reasonable convergence.

The asymptotic standard errors for the parameter estimators can be calculate from the Hessian matrix of the expected complete-data likelihood.

C Correlation Structure of Observable Factors

This appendix lists the factor time series models estimated by Duffie, Saita and Wang (2007).

$$k_r = \begin{pmatrix} 0.03 & -0.021 \\ -0.027 & 0.034 \end{pmatrix}, \quad \theta_r = \begin{pmatrix} 3.59 \\ 5.47 \end{pmatrix},$$

$$C_r = \begin{pmatrix} 0.5639 & 0 \\ 0.2247 & 0.2821 \end{pmatrix},$$

$$b = (0.0090 \quad -0.0121)', \quad k_D = 0.0355, \sigma_D = 0.346$$

$$k_V = 0.015, \sigma_V = 0.1169$$

$$AA' + BB' = \begin{pmatrix} 1 & 0.448 \\ 0.448 & 1 \end{pmatrix}, \quad BB' = \begin{pmatrix} 0.448 & 0.0338 \\ 0.0338 & 0.0417 \end{pmatrix},$$

$$k_S = 0.1137, \alpha_S = 0.047, \theta_S = 0.1076,$$

$$\gamma_S = (0.0366 \ \ 0.0134)'.$$

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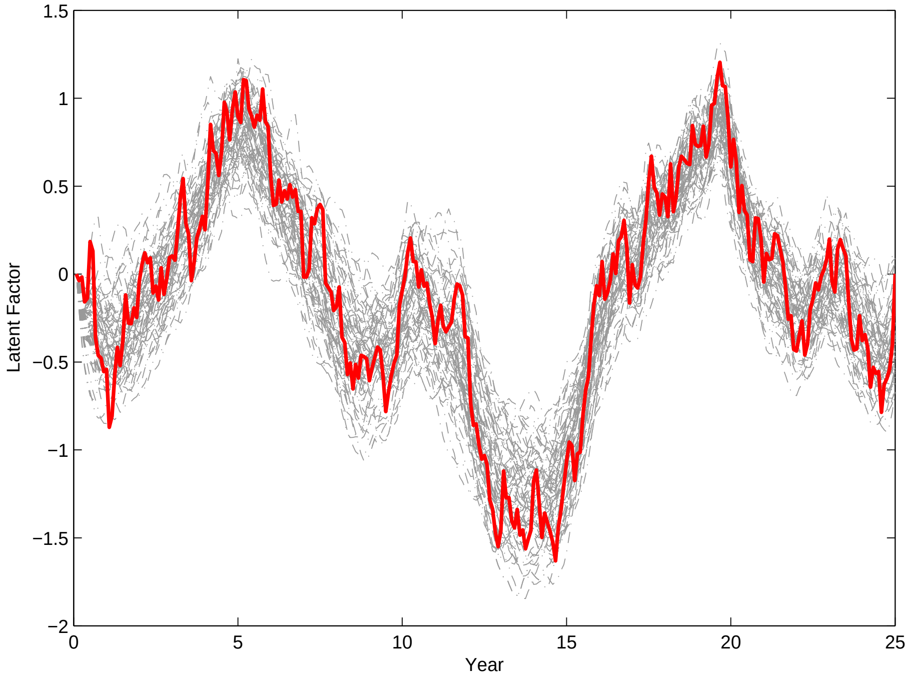

Figure 1: Hypothetic and filtered frailty. The solid line is the simulated hypothetic frailty variable. The dash-dotted lines are the conditional posterior mean of the filtered latent frailty variables given different realization of default time for each firm.

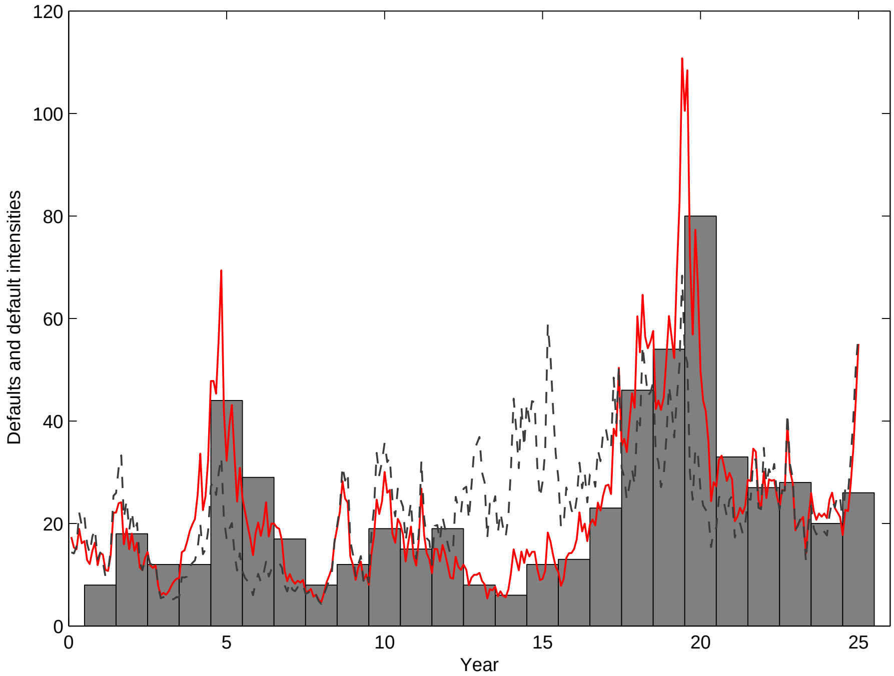

Figure 2: Defaults and default intensities. The bars represents number of defaults in each year. The solid and dashed lines represent the estimated default intensities aggregated across firms each month with (a) dynamic-frailty model (solid line), (b) no-frailty model (dashed line), respectively.

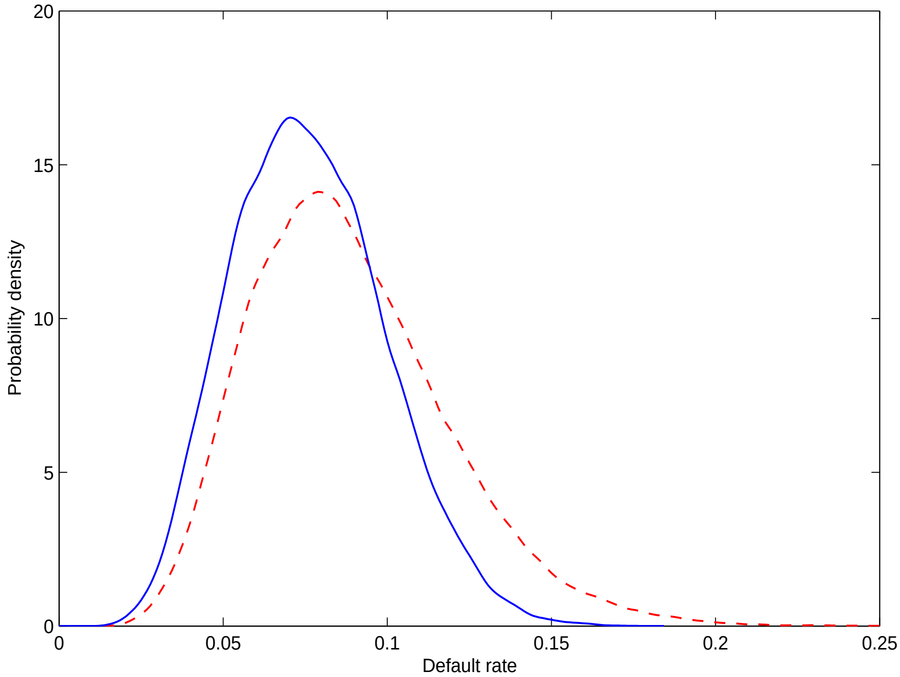

Figure 3: Portfolio default rate distribution with and without frailty factor. The conditional probability density of default rate within 5 years, for the portfolio formed by all active firms at the 25th-year end, from (a) no-frailty model (solid line), (b) dynamic-frailty model (dashed line). We apply Gaussian kernel smoothing (with bandwidth 5) to the Monte-Carlo generated empirical distribution.

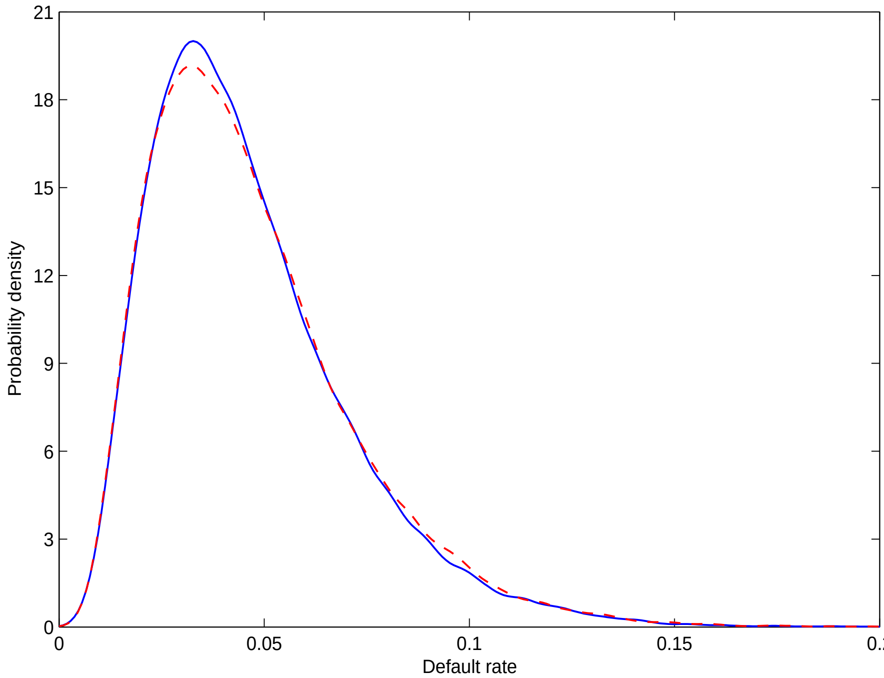

Figure 4: Portfolio default rate distribution with and without Greenspan effect. The conditional probability density of default rate within 5 years, for the portfolio formed by all active firms at the 25th-year end, in (a) a model with positively correlated short term interest rate and stock market performance (solid line), (b) a model without such correlation (dashed line). We apply Gaussian kernel smoothing (with bandwidth 5) to the Monte-Carlo generated empirical distribution.

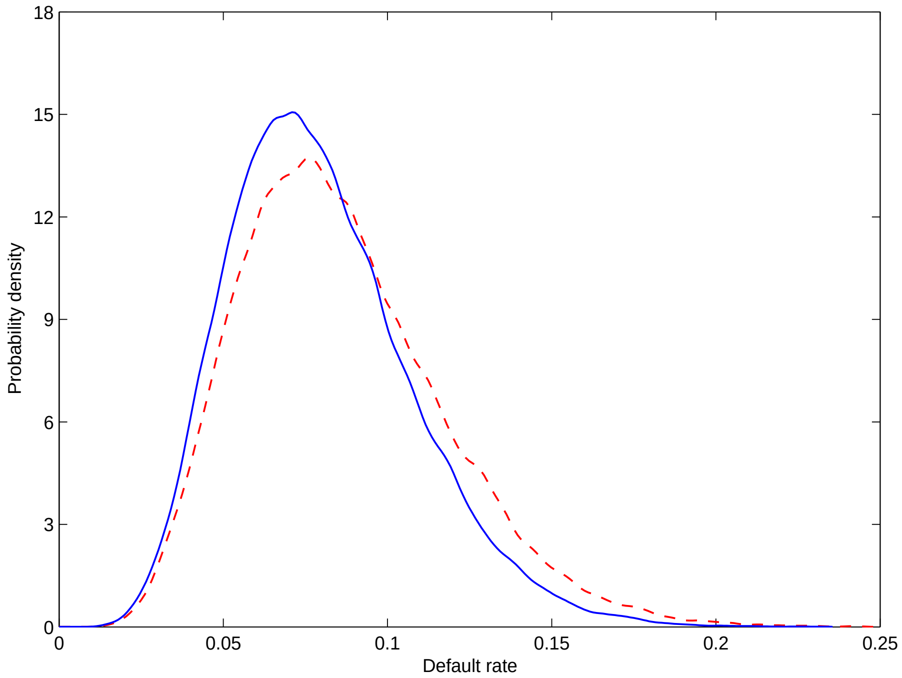

Figure 5: Portfolio default rate distribution with and without structural break in the intensity parameter for distance to default. The conditional probability density of default rate within 5 years, for the portfolio formed by all active firms at the 25th-year end, in (a) a model with structural break in data (solid line), (b) a model without such break (dashed line). We apply Gaussian kernel smoothing (with bandwidth 5) to the Monte-Carlo generated empirical distribution.

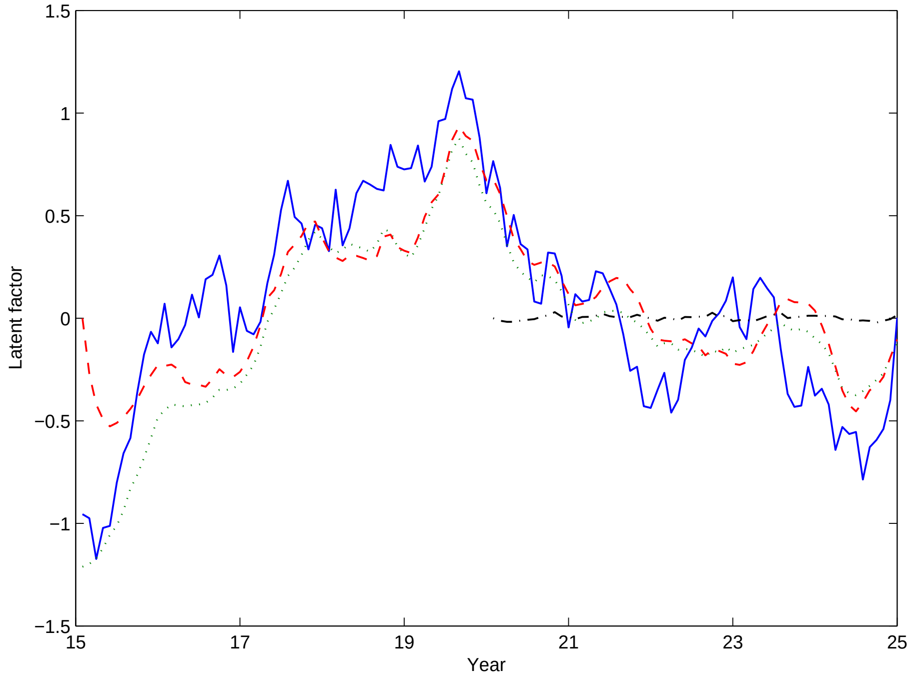

Figure 6: Filtered frailty with different lengths of available historic data. The posterior mean of the filtered latent frailty variables conditioned on available historic data of (a) 25 years (dotted line), (b) 10 years (dashed line), (c) 5 years (dash-dotted line). The solid line is the simulated hypothetic frailty variable.

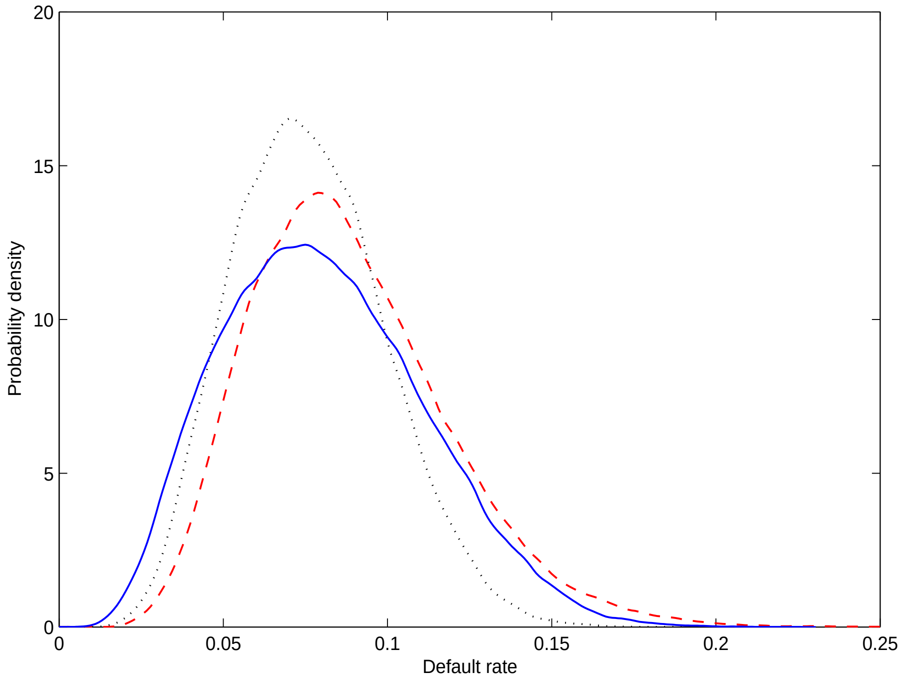

Figure 7: Portfolio default rate distribution with different lengths of available historic data. The probability density of default rate within 5 years, for the portfolio formed by all active firms at the 25th-year end, in a frailty model conditioned on available historic data of (a) 25 years (dashed line), (b) 10 years (dotted line), (c) 5 years (solid line). We apply Gaussian kernel smoothing (with bandwidth 5) to the Monte-Carlo generated empirical distribution.

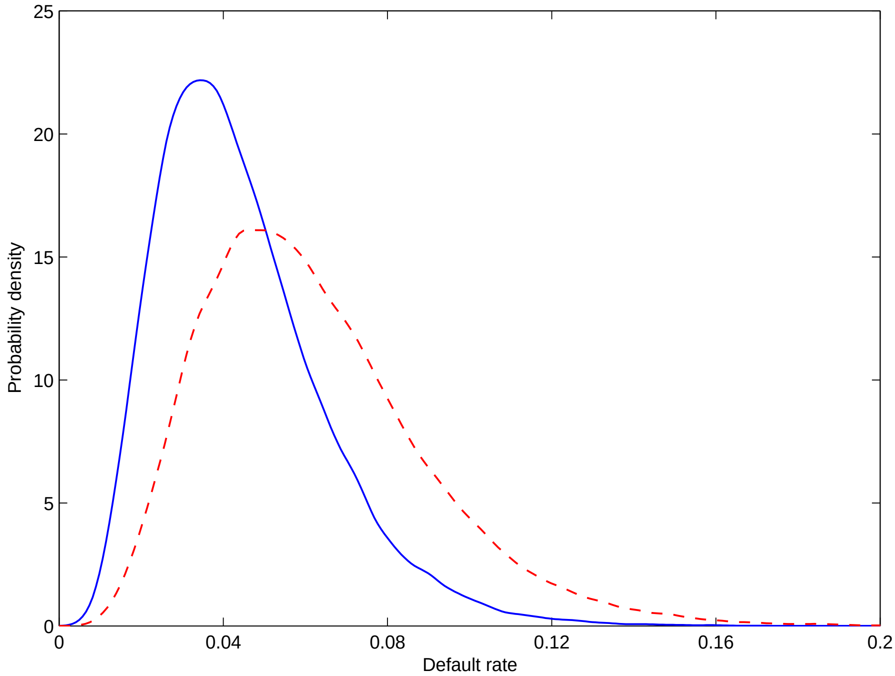

Figure 8: Portfolio default rate distribution with different economic states. The probability density of default rate within 5 years, from the portfolio formed by all active firms at the 25th-year end, in a frailty model conditioned on (a) a relatively good economic state (solid line), (b) a relatively bad economic state (dashed line). We apply Gaussian kernel smoothing (with bandwidth 5) to the Monte-Carlo generated empirical distribution.

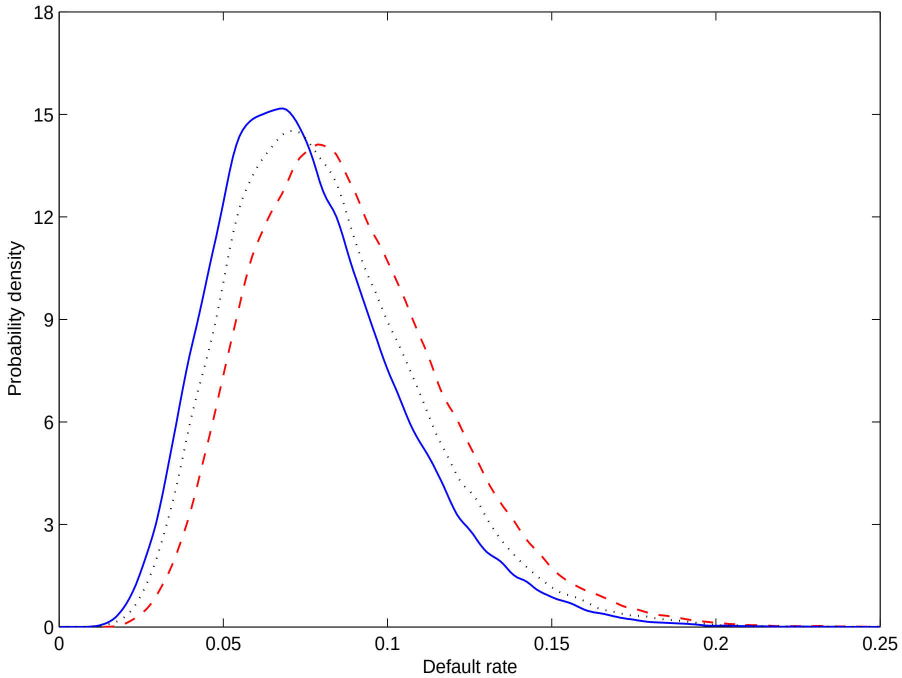

Figure 9: Portfolio default rate distribution with data estimation errors. The probability density of default rate within 5 years, from the portfolio formed by all active firms at the 25th-year end, in a frailty model conditioned on distance to default estimation errors with (a) signal to noise ratio 5 (dotted line), (b) signal to noise ratio 10 (solid line), (c) no noise (dashed line). We apply Gaussian kernel smoothing (with bandwidth 5) to the Monte-Carlo generated empirical distribution.

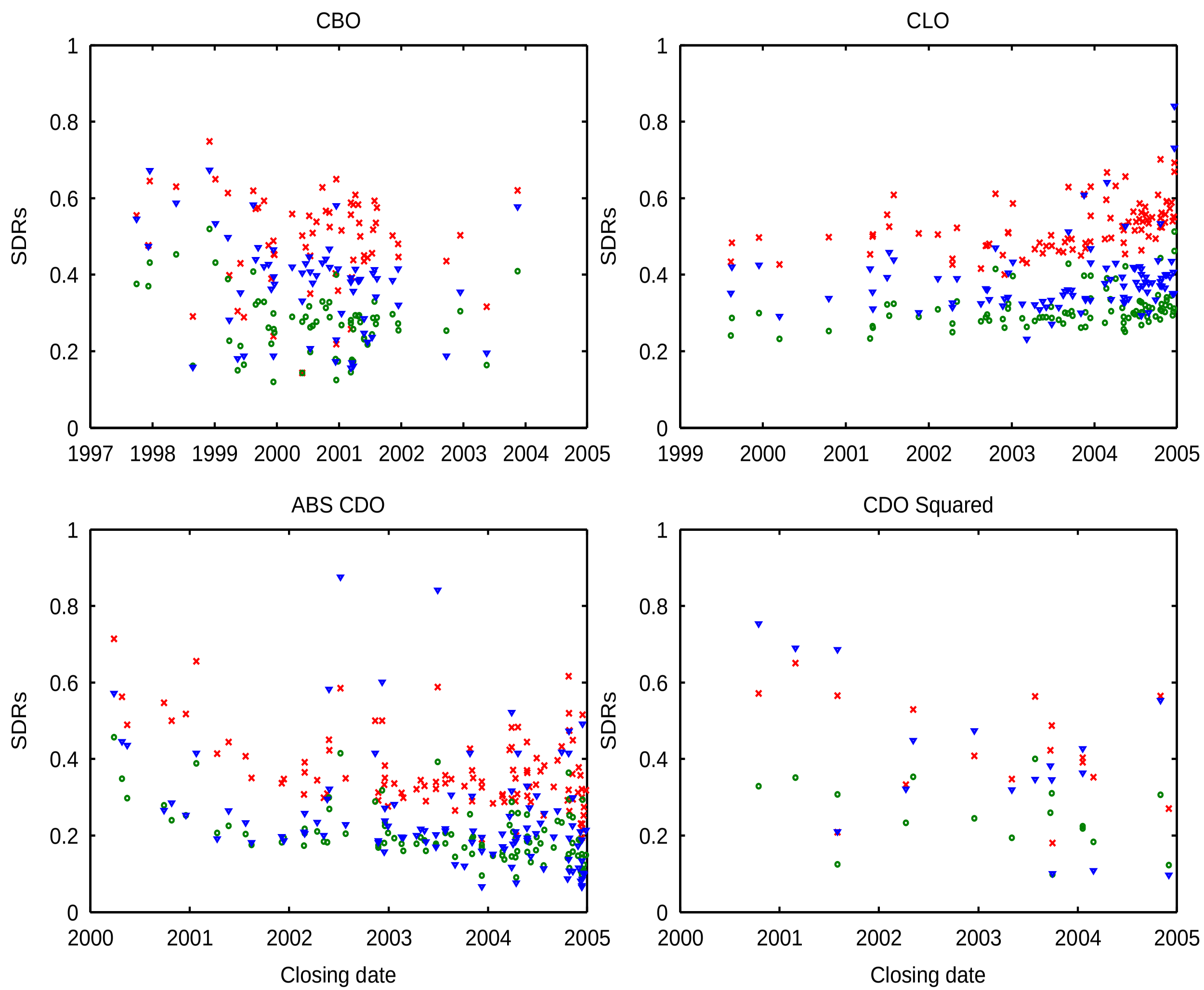

Figure 10: Scenario Default Rates for each CDO type. This figure compares the scenario default rates from (a) rating agency (triangle), (b) no-frailty model (circles), (c) dynamic-frailty model (crosses). Horizontal axis denotes the closing date of each CDO.

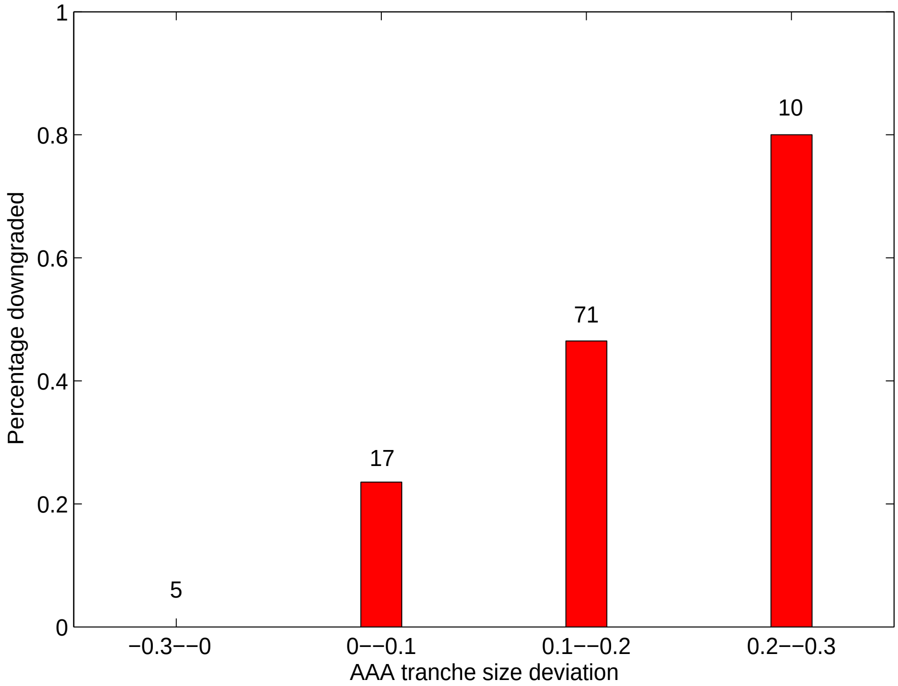

Figure 11: Prediction of CDO downgrading for different risk groups. This figure presents the CDO downgrading prediction with resect to ABS CDOs in our sample for three risk groups. The integers above the bar are the number of CDOs belonging to the relevant risk groups.

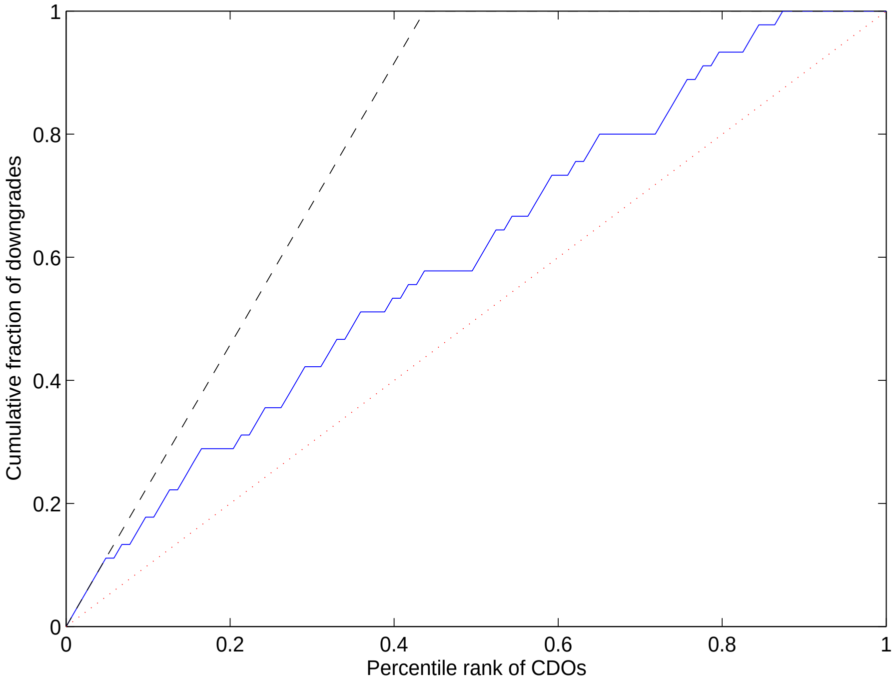

Figure 12: Average out-of-sample power curves for default prediction. The horizontal axis denotes the riskiest x fraction of CDOs, ranked with R_i. The power curve value for x gives the fraction of downgrades included in the riskiest x fraction. The solid line is the curve for our benchmark frailty model and the dashed line is the best prediction in theory.

Table I
Validation Test

This table reports the validation test of the intensity parameter estimation. The maximum and minimum numbers of defaults recorded are 648 and 573 respectively. RMSE denotes root mean squared error, and E-parameter denotes mean of the estimated parameters.

	True parameter	E-parameter	RMSE
constant	-1.029	-0.990	0.201
distance to default	-1.201	-1.171	0.047
trailing stock return	-0.646	-0.583	0.098
3-month T-bill rate	-0.255	-0.265	0.045
trailing S&P 500 return	1.556	1.540	0.255
latent-factor volatility η	0.150	0.161	0.016
latent-factor mean reversion κ	0.030	0.031	0.005

Table II
No-frailty Model versus Dynamic-frailty Model

This table reports the frailty factor to default rate prediction. Panel A reports summary statistics for time series of observable factors. Panel B reports the Maximum likelihood estimates of default intensity parameters with and without frailty respectively. Panel C presents the percentiles of predicted default distribution. Total number of firms alive at the beginning of the prediction is 2170.

Panel A: Summary statistics

			Quantiles				
Variable	Mean	Std.	Min	0.25	Median	0.75	Max
distance to default	4.70	2.46	-3.40	2.81	4.76	6.59	12.86
trailing stock return(%)	13.98	72.01	-81.70	-29.11	-1.07	37.15	317.76
3-month T-bill rate	5.11	1.56	1.62	4.01	5.10	6.03	10.52
trailing S&P 500 return(%)	10.37	13.90	-24.59	0.35	9.12	20.54	47.44

Panel B: Maximum likelihood estimates of intensity parameters

	With frailty		Without frailty	
	Coefficient	t-statistic	Coefficient	t-statistic
constant	-1.046	-5.4	-0.828	-5.1
distance to default	-1.115	-31.8	-1.070	-30.3
trailing stock return	-0.732	-7.2	-0.812	-7.5
3-month T-bill rate	-0.253	-7.3	-0.325	-10.7
trailing S&P 500 return	1.756	5.9	1.538	5.1
latent-factor volatility η	0.147	10.3		
latent-factor mean reversion κ	0.029	5.1		

Panel C: Percentiles of predicted default rate distribution

	0.05	0.15	0.50	0.90	0.95	0.99	0.999
with frailty(%)	4.61	5.81	8.48	12.86	14.29	17.33	21.01
without frailty(%)	4.06	5.16	7.47	10.69	11.66	13.41	15.55

Table III
"Greenspan put" Effect

This table reports the Greenspan effect to default rate prediction. Panel A reports summary statistics for time series of observable factors used for parameter estimation. Panel B reports the Maximum likelihood estimates of default intensity parameters. Panel C presents the percentiles of predicted default rate distribution. Total number of firms alive at the beginning of the prediction is 2150.

Panel A: Summary statistics

	Quantiles						
Variable	Mean	Std.	Min	0.25	Median	0.75	Max
distance to default	4.72	2.41	-3.27	2.92	4.78	6.54	13.68
trailing stock return(%)	15.28	71.82	-88.16	-27.78	0.12	38.47	475.37
3-month T-bill rate	5.16	1.77	0.99	3.90	5.13	6.43	9.46
trailing S&P 500 return(%)	11.21	13.37	-19.05	0.81	11.54	21.21	53.12

Panel B: Maximum likelihood estimates of intensity parameters

	Coefficient	Std. Error	t-statistic
constant	-0.998	0.176	-5.7
distance to default	-1.176	0.035	-33.3
trailing stock return	-0.618	0.092	-6.7
3-month T-bill rate	-0.256	0.032	-8.0
trailing S&P 500 return	1.451	0.343	4.2
latent-factor volatility η	0.161	0.019	8.3
latent-factor mean reversion κ	0.027	0.005	5.5

Panel C: Percentiles of predicted default rate distribution

	0.05	0.15	0.50	0.90	0.95	0.99	0.999
with Greenspan effect(%)	1.63	2.33	4.19	7.91	9.35	12.47	16.42
without Greenspan effect(%)	1.63	2.33	4.19	7.91	9.26	12.33	16.28

Table IV
Structural Break in Intensity Parameter for Distance to Default

This table reports the effect of distance to default break to default rate prediction. Panel A reports summary statistics for time series of observable factors. Panel B reports the Maximum likelihood estimates of default intensity parameters. Panel C presents the percentiles of predicted default distribution. Total number of firms alive at the beginning of the prediction is 2123.

Panel A: Summary statistics

	Quantiles						
Variable	Mean	Std.	Min	0.25	Median	0.75	Max
distance to default	4.67	2.55	-2.83	2.79	4.80	6.61	12.86
trailing stock return(%)	11.87	65.09	-78.06	-27.22	0.00	33.64	261.72
3-month T-bill rate	5.11	1.56	1.62	4.01	5.10	6.03	10.52
trailing S&P 500 return(%)	10.37	13.90	-24.59	0.35	9.12	20.54	47.44

Panel B: Maximum likelihood estimates of intensity parameters

	Coefficient	Std. Error	t-statistic
constant	-1.054	0.165	-6.4
distance to default	-1.174	0.040	-29.3
trailing stock return	-0.546	0.092	-5.9
3-month T-bill rate	-0.257	0.032	-8.0
trailing S&P 500 return	1.665	0.281	5.9
latent-factor volatility η	0.172	0.017	10.2
latent-factor mean reversion κ	0.037	0.006	6.2

Panel C: Percentiles of predicted default rate distribution

	0.05	0.15	0.50	0.90	0.95	0.99	0.999
with break(%)	4.00	5.13	7.63	11.73	13.05	15.78	19.08
without break(%)	4.19	5.46	8.2	12.81	14.27	17.48	21.48

Table V
Length of Available Data

This table reports limitation in data availability to default rate prediction. Panel A reports the maximum likelihood estimates of intensity-model parameters with 25 years, 10 years, and 5 years of historical data. Panel B reports the percentiles of predicted default distribution at year 25. Total number of firms alive at the beginning of the prediction is 2170.

Panel A: Maximum likelihood estimates of intensity parameters

	25 years		10 years		5 years	
	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
constant	-1.046	-5.4	-1.6	-6.5	0.065	0.1
distance to default	-1.115	-31.8	-1.126	-24.1	-1.164	-14.9
trailing stock return	-0.732	-7.2	-0.674	-5.2	-0.405	-2.3
3-month T-bill rate	-0.253	-7.3	-0.149	-2.8	-0.457	-4.0
trailing S&P 500 return	1.756	5.9	1.136	2.6	0.525	0.7
latent-factor volatility η	0.147	10.3	0.204	7.0	0.074	0.8
latent-factor mean reversion κ	0.029	5.1	0.039	3.1	0.979	1.0

Panel B: Percentiles of predicted default rate distribution

	0.05	0.15	0.50	0.90	0.95	0.99	0.999
25 years (%)	4.61	5.81	8.48	12.86	14.29	17.33	21.01
10 years (%)	4.06	5.16	7.47	10.69	11.66	13.41	15.58
5 years (%)	3.50	4.79	7.83	12.21	13.46	15.67	18.36

Table VI
Realization of Economic States

This table reports the effect of random realization of economic states to default rate prediction. Panel A reports the Maximum likelihood estimates of intensity-model parameters for good and bad economic states. Panel B presents the percentiles of predicted default rate distribution. Total number of firms alive at the beginning of the prediction is 2065 for bad economic state and 2212 for good economic state.

Panel A: Maximum likelihood estimates of intensity parameters

	Good state		Bad state	
	Coefficient	t-statistic	Coefficient	t-statistic
constant	-1.217	-8.3	-0.749	-5.1
distance to default	-1.182	-32.1	-1.118	-33.3
trailing stock return	-0.612	-6.6	-0.558	-6.8
3-month T-bill rate	-0.227	-9.8	-0.315	-11.4
trailing S&P 500 return	1.580	4.9	1.117	3.6
latent-factor volatility η	0.165	12.0	0.178	12.3
latent-factor mean reversion κ	0.030	7.2	0.032	7.0

Panel B: Percentiles of predicted default rate distribution

	0.05	0.15	0.50	0.90	0.95	0.99	0.999
Good economic state(%)	1.76	2.49	4.07	7.14	8.27	10.71	14.20
Bad economic state(%)	2.57	3.54	5.76	9.83	11.23	14.43	18.50

Table VII
Data Estimation Error

This table reports the data estimation error effect with (a) signal to noise ratio (S-to-N ratio) 10, (b) signal to noise ratio 5. Panel A reports the Maximum likelihood estimates of intensity-model parameters. Panel B presents the percentiles of predicted default rate distribution.

Panel A: Maximum likelihood estimates of intensity parameters

	S-to-N ratio 5		S-to-N ratio 10	
	Coefficient	t-statistic	Coefficient	t-statistic
constant	-1.168	-5.9	-1.214	-6.4
distance to default	-1.046	-30.9	-1.105	-31.6
trailing stock return	-0.838	-7.8	-0.748	-7.3
3-month T-bill rate	-0.241	-6.8	-0.247	-7.1
trailing S&P 500 return	1.701	5.8	1.603	5.4
latent-factor volatility η	0.149	9.6	0.149	9.8
latent-factor mean reversion κ	0.032	5.8	0.028	5.2

Panel B: Percentiles of predicted default rate distribution

	0.05	0.15	0.50	0.90	0.95	0.99	0.999
S-to-N ratio 5(%)	4.01	5.21	7.74	12.03	13.46	16.50	19.98
S-to-N ratio 10(%)	3.69	4.79	7.19	11.34	12.76	15.71	19.35
No noise (%)	4.61	5.81	8.48	12.86	14.29	17.33	21.01

Table VIII: Empirical Results for Scenario Default Rate Prediction

This table reports CDOs' weighted average rating (WAR); closing date(CDate); weighted average maturity (WAM); number of obligers (N); scenario default rate(%) from (a) rating agency (SDR), (b) no-frailty model (SDR NF), (c) dynamic-frailty model (SDR DF); Notches for AAA tranche downgrading (DG). Averages of the SDRs are provide at the bottom of the tables for each CDO type.

Panel A: CBO

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DG
Nomura	97/10	CCC	4.5	149	54.5	37.6	55.5	0
Captiva	97/12	B	3.8	46	47.4	37.0	47.6	2
CHYPS	97/12	CCC	3.8	51	67.2	43.1	64.5	19
BEA	98/05	CCC	4.1	96	58.7	45.3	63.0	19
Conseco	98/08	BBB-	4.6	124	15.8	16.1	29.0	0
BEA	98/12	CCC	4.0	50	67.3	52.0	74.8	0
CHYPS	99/01	CCC+	3.8	60	53.3	43.2	65.0	0
Juniper	99/03	CCC+	4.9	101	49.7	38.8	61.4	0
Federated	99/03	B+	3.7	119	28.1	22.7	39.8	0
Emerald	99/05	BB	3.4	120	18.0	15.0	30.4	0
Cedar	99/06	B+	3.9	136	35.2	21.3	42.9	0
KNIGHT	99/06	BB+	3.6	135	18.8	16.5	28.9	0
Admiral	99/08	CCC	4.9	76	58.3	40.8	62.0	0
UBS B.	99/09	B-	4.7	90	43.9	32.2	57.2	0
INA	99/09	B-	4.8	88	47.1	33.0	57.5	15
Talcott N.	99/10	B-	5.2	146	42.1	32.9	59.2	0
FCIII	99/11	B	3.5	42	42.6	26.2	47.6	0
Centennial	99/12	B+	4.4	141	36.2	21.9	38.9	0
Triton	99/12	A-	6.8	117	18.8	12.0	23.9	0
NW Inv.	99/12	B	4.9	84	46.5	29.9	48.8	0
Bingham	99/12	B+	5.9	97	39.5	25.8	45.4	2
Inner H.	99/12	B+	5.4	157	37.5	24.8	45.2	0
Juniper	00/04	B	5.5	107	42.0	29.0	55.9	1
Arlington	00/06	B+	5.5	107	40.4	27.8	50.2	0
CAESAR	00/06	BB+	1.2	14	33.1	14.3	14.3	0
Equus	00/06	B-	4.6	107	42.9	29.0	47.2	0
Wilbraham	00/07	B	5.5	101	44.7	31.7	55.3	0
JWS	00/07	B+	5.7	118	40.7	26.3	44.9	0
Coliseum	00/07	BB+	4.9	91	20.7	19.8	35.1	0
Madison Ave.	00/08	B+	5.7	120	37.8	26.7	50.8	0
Nicholas-A.	00/08	B+	5.5	65	39.8	27.7	53.8	0
Chartwell	00/09	B	5.8	88	43.1	33.0	62.8	0
Muzinich	00/10	B	5.3	99	44.1	31.3	56.7	0
Capstan	00/11	B	5.6	64	46.6	32.8	56.3	0
Magma	00/11	B	5.2	102	41.9	28.9	52.5	0
Lone Star	00/12	BBB-	5.5	106	17.2	17.9	40.3	0
PPM America	00/12	BB+	1.7	48	22.9	12.5	21.9	0
Blue Eagle	00/12	B	3.9	20	58.0	40.0	65.0	0
Signature 5	00/12	BB-	3.4	104	41.5	17.3	35.9	0
VALEO	01/01	BB	7.3	97	29.8	26.8	51.5	0
Berkeley St.	01/03	B+	5.8	128	38.1	28.1	58.8	0
Liberty Squ.	01/03	B+	6.1	106	39.1	27.4	55.7	0
Phoenix	01/03	BBB-	3.3	62	15.6	14.5	25.8	2
Madison Ave.	01/03	BBB-	5.1	107	16.9	17.8	39.3	2
Hampden	01/03	BBB-	4.9	139	16.1	17.3	43.9	0
Centurion	01/03	B+	5.4	202	35.6	25.7	58.3	0
Canyon	01/04	B	5.9	143	41.4	29.4	60.8	0
Nicholas	01/04	B+	5.9	72	38.4	29.2	58.3	0
Mammoth	01/05	B+	5.9	129	38.5	29.4	53.5	0
Liberty Squ.	01/05	B+	6.2	106	38.8	27.6	50.0	0
Nova	01/05	BB+	5.1	62	24.7	23.4	43.5	0
Valeo	01/05	BB+	6.0	91	28.5	23.1	45.1	0
Balboa	01/06	BB+	6.6	120	22.3	21.7	44.4	0
Clearwater	01/07	BB+	7.1	171	23.6	24.4	45.6	2
Melchior	01/07	B+	5.4	87	40.3	28.7	51.7	0
Concerto	01/07	B+	6.1	97	41.3	33.0	59.3	0
Robeco	01/08	BB-	6.3	129	34.1	27.1	53.5	0
TCW	01/08	B+	6.1	139	38.9	28.8	57.6	0
Cashel	01/11	B+	5.5	101	38.5	29.7	50.2	0

(Continued)

Panel A-Continued

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DG
Signature 6	01/12	B+	5.5	115	41.5	27.2	48.1	0
Callidus	01/12	BB-	5.3	126	32.0	25.5	44.6	2
Cardinal	02/09	BBB-	5.9	71	18.7	25.4	43.5	0
Canyon	02/12	B+	6.3	131	35.4	30.5	50.3	0
Rendite	03/05	BB+	3.7	98	19.5	16.3	31.6	0
Prado	03/11	B-	4.7	44	57.7	40.9	62.0	0
Average					37.4	27.6	48.8	

Panel B: CLO

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DG
Aeries	99/08	B+	4.9	212	35.1	24.1	43.4	0
Highland	99/08	B-	4.6	223	42.0	28.7	48.3	0
First D.	99/12	B	4.8	187	42.4	29.9	49.7	0
Longhorn	00/03	BB-	5.0	129	29.1	23.3	42.6	0
Addison	00/10	B+	5.3	162	33.7	25.3	49.8	0
Sequils	01/04	B	3.4	137	41.5	23.4	45.3	0
New Alliance	01/04	B+	5.6	158	35.5	26.6	50.0	0
TCW	01/05	BB-	5.1	107	31.0	26.2	50.5	0
COPERNICUS	01/07	B+	5.2	62	39.3	32.3	55.6	0
LANDMARK	01/07	B	5.1	116	45.8	29.3	52.6	0
Highland	01/08	B-	4.9	179	43.9	32.4	60.9	0
Race Point	01/11	BB-	6.0	179	30.0	29.0	50.8	0
Endurance	02/02	B	4.9	154	39.0	31.0	50.5	0
Carlyle	02/04	BB-	5.5	244	31.4	25.0	42.6	0
Katonah	02/04	BB-	5.6	103	32.6	27.2	44.2	0
INTERCONTINENTAL	02/05	B+	6.2	109	38.9	33.0	52.3	0
Centurion	02/08	B+	5.0	277	32.4	27.8	41.6	0
Saratoga	02/09	B+	5.4	278	36.3	28.9	47.6	0
Landmark	02/09	B+	5.4	98	36.0	29.6	47.7	0
Castle Hill	02/09	B+	5.3	143	33.5	28.0	48.0	0
RMF	02/10	B	7.2	94	47.0	41.5	61.2	0
Venture	02/11	BB-	5.4	155	31.8	28.4	45.2	0
Castle Hill	02/12	B+	5.0	135	33.7	26.1	40.0	0
Gulf Stream	02/12	B+	5.7	90	34.1	31.1	51.1	0
1888	02/12	B+	5.4	142	40.4	32.4	50.9	0
LEOPARD	03/01	B+	6.8	58	43.3	39.7	58.6	0
Katonah	03/02	BB-	5.2	98	32.3	28.6	43.9	0
Longhorn	03/03	BB+	5.2	72	23.1	26.4	43.1	0
Race Point	03/04	BB-	5.5	208	32.1	27.9	46.6	0
ARES	03/05	BB-	5.2	118	30.9	28.8	48.3	0
Katonah	03/05	BB-	5.1	90	33.0	28.9	45.6	0
LCM I	03/06	BB-	5.2	97	31.6	28.9	47.4	0
Callidus	03/06	B+	5.4	155	33.2	31.6	50.3	0
Waveland	03/06	BB	5.0	101	27.0	28.7	47.5	0
NYLIM	03/07	BB-	5.3	117	31.4	28.2	46.2	0
Castle Hill	03/08	B+	5.2	140	34.6	27.2	45.9	0
Gulf Stream	03/08	B+	5.4	134	35.6	30.1	48.5	0
Clydesdale	03/09	B+	5.3	174	36.0	29.9	49.4	0
EUROCREDIT	03/09	B	7.4	70	51.1	42.9	62.9	0
Union Square	03/09	B+	5.3	128	36.0	30.5	49.3	0
Magnetite	03/09	B+	5.4	156	34.6	29.3	46.6	0
Ballyrock	03/11	BB	5.5	149	30.0	26.2	45.0	0
A4	03/11	CCC	3.4	68	60.8	39.7	61.0	0
Babson	03/11	B+	5.2	205	33.3	26.3	46.9	0
Venture	03/11	B+	5.8	186	33.7	30.1	48.2	0
Landmark	03/12	BB-	5.4	122	33.3	28.7	48.6	0
Aquilae	03/12	B+	7.2	73	46.8	39.7	63.0	0
Navigator	03/12	B	5.4	166	43.1	33.7	55.4	0
LightPoint	04/02	B+	4.1	142	37.7	27.5	49.3	0
Clarenville	04/02	B+	6.1	99	41.7	36.4	59.6	0
A3	04/02	CCC	3.8	87	64.1	39.1	66.7	0
Ares	04/03	B+	5.4	210	38.7	33.6	54.8	0
Celerity	04/03	BB-	5.5	131	33.5	30.5	49.6	0
Leopard	04/04	B+	6.8	68	42.9	39.0	63.2	0
Northwoods	04/05	B+	5.4	67	39.4	31.3	52.7	0
Boston Harbor	04/05	BB-	5.6	126	32.8	27.4	52.8	0
Champlain	04/05	B+	5.6	207	37.0	29.0	51.6	0
Long Grove	04/05	B+	4.8	198	34.1	25.8	48.3	0
CENTURION	04/05	B+	5.2	295	32.8	25.1	45.4	0
Jubilee	04/05	B	6.6	64	52.7	42.2	65.6	0
Babson	04/06	B+	5.5	195	33.7	28.7	53.8	0
Canyon	04/06	B+	5.5	117	41.8	29.9	56.4	0
Petrusse	04/06	B+	5.8	236	41.6	29.7	51.6	0
Carlyle	04/07	B+	5.7	210	38.0	30.5	53.8	0
AMMC	04/07	B+	5.3	137	36.4	29.2	54.5	0
Hudson	04/07	B	5.6	139	42.1	33.1	58.6	0
FIRST	04/07	BB-	5.0	123	29.3	26.8	46.3	0
WhiteHorse	04/07	B+	5.8	119	40.1	32.8	56.3	0
Signature	04/07	B+	5.2	83	41.5	30.1	51.8	0
Gulf Stream	04/08	B+	5.7	156	37.1	30.1	53.9	0

(Continued)

Panel B-Continued

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DG
Venture	04/08	B+	5.8	234	38.1	32.1	57.7	0
Veritas	04/08	B+	6.0	113	39.4	31.0	55.8	0
Clydesdale	04/08	B+	5.5	245	35.5	28.9	54.3	0
Velocity	04/08	BB-	5.4	130	30.1	27.7	50.0	0
Flagship	04/08	B+	5.3	178	37.7	31.5	53.4	0
Essex Park	04/09	B+	5.7	141	37.8	31.2	55.0	0
KC	04/09	B+	4.5	79	33.4	29.1	49.4	0
Navigator	04/10	B	5.5	171	43.6	34.7	60.8	0
BlackRock	04/10	B+	5.3	315	37.1	28.3	52.5	0
Landmark	04/10	B+	5.6	136	38.2	30.9	54.8	0
Adagio	04/10	B	7.7	70	53.3	44.3	70.1	0
NYLIM	04/10	B+	5.4	171	39.0	32.3	56.1	0
Babson	04/10	B+	5.3	337	37.0	30.6	52.3	0
LCM II	04/11	B+	5.3	162	36.5	30.2	53.7	0
Hewetts Island	04/11	B+	5.7	122	40.0	32.0	55.7	0
Wind River	04/11	B	5.3	174	40.0	33.3	59.2	0
Premium	04/11	B	5.5	132	40.0	34.1	59.0	0
Callidus	04/12	B+	5.8	183	39.5	31.7	57.4	0
Alzette	04/12	B	6.7	263	43.5	34.6	58.9	0
First	04/12	B+	5.2	126	35.0	29.4	54.0	0
Chatham	04/12	B+	5.7	227	40.6	30.4	55.1	0
Whitney	04/12	B+	6.1	151	35.0	31.1	55.1	0
Field PointI	04/12	CCC	3.5	39	73.1	46.2	69.2	0
Field PointII	04/12	CCC	3.6	39	84.0	51.3	66.9	0
Average					38.5	31.0	52.5	

Panel C: ABS

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DG
Bleecker	00/03	B-	6.83	35	57.2	45.7	71.4	15
Talon	00/04	B+	6.92	66	44.6	34.8	56.2	7
Phoenix	00/05	BB-	7.84	47	43.5	29.8	48.9	0
Varick	00/09	BB+	7.75	86	26.6	27.9	54.7	17
TIAA	00/12	BB+	6.68	104	28.5	24.0	50.0	2
Independence	00/12	BBB-	8.45	83	29.2	25.3	51.8	6
Saybrook	01/02	B-	6.54	90	41.5	38.9	65.6	3
NYLIM	01/04	BBB	7.56	87	19.1	20.7	41.4	0
Independence	01/07	BBB-	8.45	102	26.5	22.5	44.4	4
Arroyo	01/08	BBB	7.69	108	23.3	20.4	40.7	0
Putnam	01/11	BBB	7.23	134	18.1	17.5	35.1	0
MADISON	01/12	BBB	6.72	95	19.7	18.2	33.7	0
Helios	01/12	BBB-	6.31	72	18.6	19.4	34.7	1
Commodore	02/02	BBB	6.38	52	20.9	17.3	30.8	0
Capital G.	02/02	BBB	7.61	85	20.5	20.6	36.5	0
Trainer	02/02	BBB-	7.71	92	25.8	21.7	39.1	0
F.A.B.	02/04	BBB-	5.91	90	23.5	21.1	34.4	0
Independence	02/05	BBB+	7.64	87	20.0	18.4	29.9	0
TIAA	02/05	BBB	7.02	55	29.5	18.2	30.9	0
Anthracite	02/05	BB+	7.00	40	58.3	30.0	45.0	0
Aspen	02/05	BBB-	7.00	26	32.1	26.9	42.3	0
LNR	02/07	B+	7.00	41	87.5	41.5	58.5	0
ACA	02/07	BBB	7.33	83	22.8	20.5	34.9	0
Saybrook	02/11	BB	7.14	284	41.5	28.9	50.0	4
Charles	02/11	A-	6.87	89	18.7	16.9	29.2	10
ABS Capital	02/11	BBB+	6.75	109	18.2	17.4	31.2	4
Anthracite	02/12	BB	6.99	44	60.0	31.8	50.0	0
Mulberry	02/12	BBB+	7.53	111	15.7	18.0	33.3	16
Birch	02/12	BBB+	7.00	40	23.8	22.5	35.0	0
C-BASS	02/12	BBB	6.82	47	27.1	23.4	38.3	0
CMBS	02/12	AA	7.00	29	22.4	20.7	27.6	0
Longport	03/01	BBB	7.09	155	28.1	19.4	33.5	18
Trainer	03/02	A-	7.00	77	19.6	17.8	31.2	19
Northlake	03/02	BBB+	6.97	131	19.6	16.0	29.8	14
C-BASS	03/04	BBB+	7.00	56	20.0	17.9	32.1	0
TIAA	03/05	BBB+	7.04	87	21.6	19.5	34.5	4
Faxtor	03/05	BBB	6.90	91	21.2	18.7	33.0	0
ACA	03/05	A-	7.02	100	18.3	16.0	29.0	19
Independence	03/06	A-	7.04	115	20.2	17.4	32.2	19
Mulberry	03/06	BBB+	6.95	112	17.0	17.9	33.9	18
LNR	03/07	B+	7.00	51	84.1	39.2	58.8	0
C-BASS	03/07	BBB	6.94	87	21.8	20.7	35.7	0
FAB	03/07	BBB	5.99	89	21.2	18.0	33.7	0
N-Star	03/08	BBB	6.94	69	30.5	20.3	34.8	0
Coronado	03/09	A	7.01	132	12.3	14.4	26.5	0
Putnam	03/10	A	9.02	207	12.0	16.9	32.9	0
Saturn	03/10	BBB-	7.21	82	41.5	25.6	42.7	0
ACA	03/11	A-	6.97	138	18.3	15.2	29.0	19
TIAA	03/11	BBB	7.01	73	30.3	19.2	37.0	0
C-BASS	03/11	BBB+	6.98	66	21.2	19.7	35.0	0
Lakeside	03/12	AA	10.55	89	15.9	16.9	32.6	0
BLUE BELL	03/12	AAA	6.75	137	6.6	9.5	19.0	8
Commodore	03/12	A-	7.03	91	19.5	17.6	34.1	13
Trainer	04/01	AA-	6.90	95	15.1	14.7	28.4	0
Independence	04/02	A-	6.94	155	20.4	14.8	30.3	19
Alexander	04/02	A	7.00	127	17.1	15.7	30.7	9
Knollwood	04/03	A+	6.91	160	16.4	13.8	28.8	19
C-Bass	04/03	BBB	6.97	66	25.0	22.7	42.4	0
Newcastle	04/03	BBB-	6.99	58	31.6	25.9	43.1	0
Anthracite	04/03	BB+	7.00	58	52.2	28.8	48.3	0
Lakeside	04/03	AA+	9.09	145	11.7	14.5	29.7	0
FAB	04/04	BBB+	7.48	62	17.8	21.0	37.1	0
Vermeer	04/04	BBB+	7.02	83	20.9	19.3	34.9	0
Bluegrass	04/04	A	6.87	112	18.4	14.3	29.0	18
Klio	04/04	AAA	6.43	160	7.6	9.1	20.0	4
Saturn	04/04	A	7.25	107	19.5	15.9	30.8	0
Saturn	04/04	BBB-	7.20	89	41.5	25.8	48.3	3
FAXTOR	04/05	BBB	6.85	92	22.0	18.5	37.0	0
C-Bass	04/05	BBB-	6.87	98	32.9	25.5	44.4	0
ACA	04/05	A-	6.98	102	19.4	15.7	30.4	0

(Continued)

Panel C-Continued

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DG
Rhodium	04/05	BBB	6.78	66	18.8	19.7	36.4	0
Sandstone	04/06	A-	7.00	55	27.2	18.2	32.7	0
Whately	04/06	A	6.88	184	14.5	13.0	28.8	17
RFC	04/06	BBB+	6.97	93	20.4	16.1	33.3	0
N-Star	04/07	BBB	6.94	82	30.3	19.6	40.2	0
Acacia	04/07	BBB	6.93	95	23.2	17.9	36.8	0
Cascade	04/07	AA+	8.08	107	11.3	12.1	25.2	18
C-Bass	04/09	BBB	6.83	107	25.7	21.5	38.3	0
Bluegrass	04/09	BBB+	6.88	113	19.6	16.8	32.7	17
Newcastle	04/09	BBB-	6.75	63	26.5	23.8	39.7	0
Inman	04/10	BBB-	7.34	81	41.8	23.5	43.2	0
Klio	04/10	AA-	7.56	113	8.6	14.2	29.2	8
Pinnacle	04/10	BBB+	6.57	160	13.8	15.1	31.9	18
Sherwood	04/10	B+	7.31	198	41.5	36.4	61.7	19
Porter	04/10	BB	6.47	78	47.3	29.5	51.9	0
Laguna	04/10	AA+	7.96	218	10.7	11.5	26.4	0
Reservoir	04/10	BBB-	7.08	99	19.3	25.3	47.5	14
Acacia	04/11	BBB+	6.95	83	22.6	18.1	36.1	0
Whitehawk	04/11	A-	6.42	95	10.6	15.8	29.5	1
Hillcrest	04/11	BBB-	6.95	129	29.8	24.8	45.0	10
Trainer	04/11	A+	6.95	109	17.2	14.7	31.2	3
Jupiter	04/12	BBB+	6.98	106	11.5	18.9	37.7	2
C-Bass	04/12	A-	6.90	70	21.0	18.6	35.7	0
McKinley	04/12	AA+	7.16	104	8.1	10.6	23.1	19
Revelstoke	04/12	AAA	6.08	72	6.5	9.7	19.4	5
Cimarron	04/12	AAA	6.98	93	6.9	9.7	21.6	3
Belle	04/12	AA	9.18	190	13.4	14.2	30.0	17
Vermeer	04/12	A-	7.06	106	18.8	15.1	32.1	0
Witherspoon	04/12	AA+	6.85	154	8.6	11.0	23.1	0
Fairfield	04/12	BB	6.99	75	49.1	29.3	51.6	0
Margate	04/12	AA	6.82	229	10.1	10.9	25.3	4
Zenith	04/12	AA-	6.86	146	9.7	12.3	27.4	15
Ischus	04/12	A	6.96	107	21.3	15.0	31.8	0
Average					24.7	20.1	36.7	

Panel D: CDO^2

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DG
Coast	00/10	B+	6.4	70	75.3	32.9	57.1	0
Centurion	01/03	B-	6.7	151	69.0	35.1	65.1	0
Coast	01/08	BB-	6.7	66	68.6	30.8	56.5	0
Lafayette Sovereign	02/04	BB+	3.8	30	32.1	23.3	33.3	0
Lynx	02/05	BB	7.7	34	44.8	35.3	52.9	0
Zais	02/12	BBB-	7.0	49	47.4	24.5	40.8	0
Connecticut Valley	03/05	BBB	6.8	72	31.9	19.4	34.7	0
Porter Square	03/07	BB-	7.0	55	34.6	40.0	56.4	0
Vertical	03/10	A-	3.7	61	10.1	9.8	18.0	0
Tricadia	04/01	BBB-	6.6	69	36.3	21.9	39.1	0
Zais	04/01	BBB-	5.7	67	42.6	22.4	40.3	0
Vertical	04/03	AA	9.8	71	10.8	18.3	35.2	0
Tricadia	04/11	BB+	7.5	62	55.3	30.6	56.5	0
TABS	04/12	AA+	7.6	98	9.7	12.2	27.0	18
Lusitano	01/08	BBB-	3.2	72	21.0	12.5	20.8	0
Pro Rata	03/09	B+	3.9	104	38.2	26.0	42.3	0
Hamilton	03/09	B+	5.0	158	34.6	31.0	48.7	19
Average					38.9	25.1	42.6	