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<th>Capturing propagation of infected program states</th>
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ABSTRACT
Coverage-based fault-localization techniques find the fault-related positions in programs by comparing the execution statistics of passed executions and failed executions. They assess the fault suspiciousness of individual program entities and rank the statements in descending order of their suspiciousness scores to help identify faults in programs. However, many such techniques focus on assessing the suspiciousness of individual program entities but ignore the propagation of infected program states among them. In this paper, we use edge profiles to represent passed executions and failed executions, contrast them to model how each basic block contributes to failures by abstractly propagating infected program states to its adjacent basic blocks through control flow edges. We assess the suspiciousness of the infected program states propagated through each edge, associate basic blocks with edges via such propagation of infected program states, calculate suspiciousness scores for each basic block, and finally synthesize a ranked list of statements to facilitate the identification of program faults. We conduct a controlled experiment to compare the effectiveness of existing representative techniques with ours using standard benchmarks. The results are promising.

Categories and Subject Descriptors
D.2.5 [Software Engineering]: Testing and Debugging — Debugging aids

General Terms: Experimentation, Verification

Keywords
Fault localization, edge profile, basic block, control flow edge

1. INTRODUCTION
Coverage-based fault-localization (CBFL) techniques \cite{2,8,15,21,23,24} have been proposed to support software debugging. They usually contrast the program spectra information \cite{14} (such as execution statistics) between passed executions and failed executions to compute the fault suspiciousness \cite{24} of individual program entities (such as statements \cite{15}, branches \cite{21}, and predicates \cite{18}), and construct a list of program entities in descending order of their fault suspiciousness. Developers may then follow the suggested list to locate faults. Empirical studies \cite{2,15,18,19} show that CBFL techniques can be effective in guiding programmers to examine code and locate faults.

During program execution, a fault in a program statement may infect a program state, and yet the execution may further propagate the infected program states \cite{9,22} a long way before it may finally manifest failures \cite{23}. Moreover, even if every failed execution may execute a particular statement, this statement is not necessarily the root cause of the failure (that is, the fault that directly leads to the failure) \cite{9}.

Suppose, for instance that a particular statement $S$ on a branch $B$ always sets up a null pointer variable. Suppose further that this pointer variable will not be used in any execution to invoke any function, until another faraway (in the sense of control dependence \cite{5} or data dependence) statement $S'$ on a branch $B'$ has been reached, which will crash the program. If $S$ is also exercised in many other executions that do not show any failure, $S$ or its directly connected branches cannot effectively be pinpointed as suspicious. In this scenario, existing CBFL techniques such as Tarantula \cite{15} or SBI \cite{24} will rank $S'$ as more suspicious than $S$. Indeed, in the above scenario, exercising $B'$ that determines the execution of $S'$ always leads to a failure \cite{24}. Thus, the branch technique proposed in \cite{24}, for example, will rank $B'$ as more suspicious than $B$, which in fact is directly connected to the first statement $S$. The use of data flow analysis may reveal the usage of...
propagation-based CBFL technique in this paper. Computationally expensive. We propose a steady and efficient propagation of fault suspiciousness of one branch or statement to other branches of these concrete program states by a "transfer function" of the model. By solving the set of equations, our model always constructs homogeneous equations and ensures that the condition of being solvable by standard mathematics techniques such as Gaussian elimination [29]. By solving the set of equations, our technique obtains the suspiciousness score for each basic block. We finally rank the basic blocks in descending order of suspiciousness scores, and assign a rank for each statement.

We conduct controlled experiments to compare the effectiveness of existing representative techniques with ours on four real-life medium-sized programs. The empirical results show that our technique can be more effective than peer techniques. For each edge, we contrast such a state abstraction in the mean pass profile with that in the mean failed profile to assess the fault suspiciousness of the edge. In our model, to backtrack how much every basic block [3] contributes to the observed program failures, we set up a system of linear algebraic equations to express the propagation of the suspiciousness scores of a basic block to its predecessor block(s) via directly connected control flow edges — for each edge, we split a fraction of the suspiciousness score to be propagated to a predecessor basic block.

Our model always constructs homogeneous equations and ensures that the number of equations is the same as the number of variables. Such a constructed equation set satisfies a necessary condition of being solvable by standard mathematics techniques such as Gaussian elimination [29]. By solving the set of equations, our model obtains the suspiciousness score for each basic block. We finally rank the basic blocks in descending order of their suspiciousness scores, and assign a rank for each statement. We conduct controlled experiments to compare the effectiveness of existing representative techniques with ours on four real-life medium-sized programs. The empirical results show that our technique can be more effective than peer techniques.

The main contribution of this work is twofold: (i) To the best of our knowledge, the work is the first that integrates the propagation of program states to CBFL techniques. (ii) We use four real-life medium-sized programs flex, grep, gzip and sed to conduct experiments on our technique and compare them with five other

### Table 1: Faulty version v2 of program schedule and fault localization comparison.

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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>susp. rank</td>
<td>susp. rank</td>
<td>susp. rank</td>
<td>susp. rank</td>
<td>susp. rank</td>
</tr>
<tr>
<td>if (block_queue) {</td>
<td>e1</td>
<td>b1</td>
<td>0.50</td>
<td>9</td>
<td>0.29</td>
<td>9</td>
<td>0.71</td>
</tr>
<tr>
<td>count = block_queue-&gt;mem_count + 1;</td>
<td>e2</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = (int) (count*ratio);</td>
<td>e3</td>
<td>b2</td>
<td>0.71</td>
<td>7</td>
<td>0.50</td>
<td>7</td>
<td>0.71</td>
</tr>
<tr>
<td>proc = find_nth(block_queue, n);</td>
<td>e4</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>if (proc) {</td>
<td>e5</td>
<td>b3</td>
<td>0.71</td>
<td>7</td>
<td>0.50</td>
<td>7</td>
<td>0.71</td>
</tr>
<tr>
<td>block_queue= del_ele(block_queue, proc);</td>
<td>e6</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prio = proc-&gt;priority;</td>
<td>e7</td>
<td>(fault)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prio_queue[prio] =</td>
<td>e8</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>append_ele[prio_queue[prio], proc];</td>
<td>e9</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>} // next basic block</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tbody>
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<tr>
<th>Edge</th>
<th>e1</th>
<th>e2</th>
<th>e3</th>
<th>e4</th>
<th>e5</th>
<th>e6</th>
</tr>
</thead>
<tbody>
<tr>
<td>susp.</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>rank</td>
<td></td>
<td>0.53</td>
<td>0.71</td>
<td>0.03</td>
<td>0.71</td>
<td>0.41</td>
</tr>
<tr>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>N/A</td>
</tr>
<tr>
<td>% of code examined according to the ranking of statements</td>
<td>78%</td>
<td>78%</td>
<td>78%</td>
<td>100%</td>
<td>44%</td>
<td></td>
</tr>
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Figure 1: Faulty version v2 of program schedule and fault localization comparison.
techniques, namely Tarantula, SBI, Jaccard, CBI, and SOBER. The empirical results show that our technique is promising.

The rest of this paper is organized as follows: Section 2 shows a motivating example. Section 3 presents our technique. Section 4 presents further discussion on our technique, followed by an experimental evaluation in Section 5 and a literature review in Section 6. Section 7 concludes the paper.

2. MOTIVATING EXAMPLE

This section shows an example on how the modeling of propagation of infected program states via control flow edges helps locate faults effectively.

Figure 1 shows a code excerpt from the faulty version v2 of the program schedule (from SIR [10]). The code excerpt manages a process queue. It first calculates the index of a target process, and then moves the target process among priority queues. We seed an extra “+1” operation fault into statement s2 in Figure 1. It may cause the program to select an incorrect operation for subsequent processing, which will lead to a failure.

This code excerpt contains two “if” statements (s1 and s5), which divide the code excerpt into three basic blocks [5] (namely, b1, b2, and b3). The basic block b1 contains only one statement s1. The result of evaluating “block queue” in s1 determines whether the statements s2 to s8 are skipped. The basic blocks b2 contains statements s2, s3, s4, and s5. The basic block b3 contains statements s6, s7, and s8. We also depict the code excerpt as a control flow graph (CFG) in Figure 1. In this CFG, each rectangular box represents a basic block and each arrow represents a control flow edge that connects two basic blocks. For example, e2 indicates that the decision in s1 has been evaluated to be true in an execution, so it transmits from b1 to b2. Since the fault lies in b2, we use a weighted border to highlight the rectangular box b2. Note that we add a dummy block b4 (as a dashed rectangular box) and an edge e6 (as a dashed arrow) to make this CFG more comprehensible.

We examine the program logic, and observe that many failures are caused by the execution of b2 followed by b3, rather than merely executing b2 without executing b3. Even if b2 is executed and results in some infected program state, skipping b3 will not alter the priority queue, and thus the effect of the fault at s2 is less likely to be observed through subsequent program execution. On the other hand, executing b2 followed by b3 means that an infected program state (such as incorrect values for the variables count, n, or proc) at b2 is successfully propagated to b3 through the edge e4. Since previous studies suggest the comparison of execution statistics to assess the suspiciousness scores of statements, they will be more likely to result in a wrong decision — b3 will appear to be more suspicious than b2. The following serves as an illustration.

In this example, we use seven test cases (dubbed t1 to t7). Their statement and edge execution details are shown in Figure 1. A cell with the “•” notation indicates that the corresponding statement is exercised (or an edge is traversed) in the corresponding test execution. For instance, let us take the first test case t1 (8 successful one, referred to as “passed”). During its execution, the basic blocks b1, b2, and b4 are exercised; moreover, the control flow edges e1, e2, and e5 are traversed. Other test cases can be interpreted similarly. The passed/fail status of each test case is shown in the “Pass/Fail status” row. We apply Tarantula [24], Jaccard [1], and SBI [24] to compute the suspiciousness score of every statement, and rank statements in descending order of their scores. Presuming that the programmer may check each statement according to their ranks until reaching the fault [15][24], we thus compute the effort of code examination to locate this fault [15][24]. We show their effectiveness in the “sus.” and “rank” columns, and the row “% of code examined according to the ranking of statements” of Figure 1. We observe that b3, rather than b2, is deemed to be the most suspicious basic block if we apply Tarantula or SBI. When applying Jaccard, b2 and b3 are equally deemed to be the most suspicious basic blocks. As a result, the fault cannot be effectively located by any of these techniques. To locate the fault, each examined technique needs to examine 78% of the code.

Intuitively, the execution of b3 may lead to a failure, and yet it is not the fault. On the other hand, b2 contains the fault, but its execution does not lead to a failure as often as b3. Since existing techniques find the suspicious program entities that correlate to failures, they give higher ranks to those statements (such as b3) whose executions frequently lead to failures, but give lower ranks to those statements (such as b2) whose executions less often lead to failures. If we always separately assess the fault suspiciousness of individual statements (such as b2 and b3) and ignore their relations, this problem may hardly be solved.

Since executing an edge can be regarded as executing both the two basic blocks connected by the edge, do edge-oriented techniques somehow capture the relationships among statements and perform effectively on this example? We follow [21] to adopt hr, an edge-oriented technique (which we will refer to as Branch in this paper) to work on this example. Branch assesses the suspiciousness scores of control flow edges (say, e1 and e2), and then associate their suspiciousness scores with statements that are directly connected (in sense of incoming edges or outgoing edges). Branch first ranks e2 and e4 as the most suspicious edges (both having a suspiciousness score of 0.71). In a program execution, traversing e2 means to enter the true branch of e1 followed by executing b2, and traversing e4 means having executed b2 and will continue to execute b3. We observe that executing b2 generates infected program states (on variables count, n, and proc), which propagate to b3 through e4. We further observe that these two highly ranked edges precisely pinpoint the fault location. When associating edges to statements, the rules in Branch only propagate the edge suspiciousness to those statements within the same block as the conditional statement for the edge. However, a fault may be several blocks away from the edge and the loop construct may even feedback a faulty program state. For this example, Branch assigns identical suspiciousness scores to all statements and they cannot be distinguished from one another. As a result, 100% code examination effort is needed to locate the fault. Since the core of Branch is built on top of Ochiai [1], we have iteratively replaced this core part of Branch by Tarantula, Jaccard, and SBI. However, the fault-localization effectiveness results are still unsatisfactory (100% code to be examined). In practice, the propagation of infected program states may take a long way, such as a sequence of edges, before it may finally result in failures. We need a means to transfer over the edges information about infected program states and failures.

1 In this paper, we use the term SBI to denote Yu et al.’s approach [24] of applying Liblit et al.’s work CBI [18] at statement level. At the same time, we still keep the term CBI when referring to the original technique in [18].
3. OUR MODEL

3.1 Problem Settings

Let \( P \) be a faulty program, \( T = \{ t_1, t_2, \ldots, t_k \} \) be a set of test cases associated with passed executions, and \( T' = \{ t'_1, t'_2, \ldots, t'_k \} \) be a set of test cases that are associated with failed executions. In the motivating example, for instance, \( P \) is version v2 of schedule, \( T = \{ t_1, t_2, t_4, t_5, t_7 \} \), and \( T' = \{ t_3, t_6 \} \). Similarly to existing work (such as [15]), the technique assesses the fault suspiciousness of each statement of \( P \) and can also produce a list of statements of \( P \) in descending order of the suspiciousness scores.

3.2 Preliminaries

We use \( G(P) = (B, E) \) to denote the control flow graph (CFG) [3] of the program \( P \), where \( B = \{ b_1, b_2, \ldots, b_n \} \) is the set of basic blocks (blocks for short) of \( P \), and \( E = \{ e_1, e_2, \ldots, e_m \} \) is the set of control flow edges (edges for short) of \( P \), in which each edge \( e_i \) goes from one block to another (possibly the same block) in \( B \). Thus, we sometimes write \( e_i \) as \( edg(b_{h1}, b_{h2}) \) to denote an edge going from \( b_{h1} \) to \( b_{h2} \); this edge \( e_i \) is called the incoming edge of \( b_{h2} \) and the outgoing edge of \( b_{h1} \). The block \( b_{h2} \) is a successor block of \( b_{h1} \), and the block \( b_{h1} \) is a predecessor block of \( b_{h2} \). We further use the notation \( edg(*) \) and \( edg(b_i, *) \) to represent the set of incoming edges and the set of outgoing edges of \( b_i \), respectively. In Figure 1, for instance, edges \( e_4 \) and \( e_5 \) are the outgoing edges of block \( b_2 \), and block \( b_3 \) is a successor block of \( b_2 \) with respect to edge \( e_4 \); \( edg(b_2, *) = \{ e_4, e_5 \} \) is the set of outgoing edges of block \( b_3 \), and \( edg(*, b_3) = \{ e_4 \} \) is the set of incoming edges of block \( b_3 \).

An edge is said to have been covered by a program execution if it is traversed at least once. For every program execution of \( P \), whether an edge is covered in \( E \) can be represented by an edge profile. Suppose \( t_k \) is a test case. We denote the edge profile for \( t_k \) by \( P(t_k) = (\theta(e_1, t_k), \theta(e_2, t_k), \ldots, \theta(e_m, t_k)) \), in which \( \theta(e_i, t_k) \) means whether the edge \( e_i \) is covered in the corresponding program execution of \( t_k \). In particular, \( \theta(e_i, t_k) = 1 \) if \( e_i \) is covered by the execution, whereas \( \theta(e_i, t_k) = 0 \) if \( e_i \) is not covered. Take the motivating example in Figure 1 for illustration. The edge profile for test case \( t_1 \) is \( P(t_1) = (\theta(e_1, t_1), \theta(e_2, t_1), \theta(e_3, t_1), \theta(e_4, t_1), \theta(e_5, t_1), \theta(e_6, t_1)) = (1, 1, 0, 0, 1, 0) \).

3.3 CP: Our Fault-Localization Model

We introduce our fault-localization model in this section.

A program execution passing through an edge indicates that the related program states have propagated via the edge. Therefore, we abstractly model a program state in a program execution as related program states have propagated via the edge. Consequently, the edge profiles in passed executions and failed executions are the corresponding edge profiles of all passed executions, and another mean edge profile for those of all failed executions. Such mean edge profiles represent the central tendencies of the program states in the passed executions and failed executions, respectively. \( CP \) contrasts the two mean edge profiles to assess the suspiciousness of every edge. The formula to compute the suspiciousness score is given in equation (2) and explained with the aid of equation (3).

During a program execution, a block may propagate program states to adjacent blocks via edges connecting to that block. In Section 3.3.2, we use equation (4) to calculate the ratio of the propagation via each edge, and use such a ratio to determine the fraction of the suspiciousness score of a block propagating to a successor block via that edge. We use backward propagation to align with the idea of backtracking from a failure to the root cause. For each block, \( CP \) uses a linear algebraic equation to express its suspiciousness score by summing up such fractions of suspiciousness scores of successor blocks of the given block. Such an equation is constructed using equation (5) or (6), depending on whether the block is a normal or exit block. By solving the set of equations (by Gaussian elimination), we obtain the suspiciousness score for each block involved.

As presented in Section 3.3.3, \( CP \) ranks all blocks in descending order of their suspiciousness scores, then assigns a rank to each statement, and produces a ranked list of statements by sorting them in descending order of their suspiciousness scores.

3.3.1 Calculating the Edge Suspiciousness Score

In Section 2, we have shown that edges can provide useful correlation information for failures. However, the size of \( T \) may be very different from that of \( T' \). To compare the sets of edge profiles for \( T \) with those for \( T' \), we propose to compare their arithmetic means (that is, central tendencies).

If an edge is never traversed in any execution, it is irrelevant to the observed failures. There is no need to compute the propagation of suspicious program states through that edge. We thus exclude them from our calculation model in Sections 3.3.1 and 3.3.2.

In our model, we use the notation \( P_1 = (\theta_1(e_1), \theta_1(e_2), \ldots, \theta_1(e_m)) \) to denote the mean edge profile for \( T \), and \( P_2 = (\theta_2(e_1), \theta_2(e_2), \ldots, \theta_2(e_m)) \) for \( T' \), where \( \theta_i(e) \) and \( \theta_i(e) \) for \( i = 1 \) to \( m \) are calculated by:

\[
\theta_i(e) = \frac{1}{1 \leq t_k \leq m} \sum_{t_k \in T} [\theta(t_k, e)] ; \quad \theta_i(e) = \frac{1}{1 \leq t_k \leq m} \sum_{t_k \in T'} [\theta(t_k, e)] .
\]

Note that the variables \( u \) and \( v \) in equation (1) represent the total numbers of passed and failed test cases, respectively. Intuitively, \( \theta_1(e) \) and \( \theta_2(e) \) stand for the probabilities of an edge being exercised in a passed execution and failed execution, respectively, over the given test set. For example, \( \theta_1(e_4) = (\theta(e_4, t_1) + 3 \theta(e_4, t_3) + \theta(e_4, t_4)) / 2 = (1 + 0) / 2 = 0.5 \) and, similarly, \( \theta_2(e_4) = 0 \).

**Edge suspiciousness calculation.** We calculate the suspiciousness score of any given edge \( e_i \) using the equation

\[
\theta_i(e_i) = \frac{\theta_i(e_i) - \theta_1(e_i)}{\theta_2(e_i) + \theta_1(e_i)}
\]

which contrasts the difference between the two mean edge profiles. Intuitively, \( \theta_i(e_i) \) models the (normalized) difference between the probability of \( e_i \) being traversed in an average passed execution and the probability of an average failed execution. When \( \theta_i(e_i) \) is positive, it reflects that the probability of edge \( e_i \) being covered in \( P' \) is larger than that in \( P \). Since such an edge is more frequently exercised in failed executions than in passed executions, it may be more likely to be related to a fault. When \( \theta_i(e_i) = 0 \), the edge \( e_i \) has identical probabilities of being covered in \( P' \) and \( P \). Such an edge is deemed to be less likely to be related to a fault than an edge having a positive suspiciousness score. When \( \theta_i(e_i) \) is negative, it means that \( e_i \) is less frequently executed in \( P' \) than in \( P \).

In short, for an edge \( e_i \), the higher the values of \( \theta_i(e_i) \), the more suspicious the edge \( e_i \) is deemed to be, and the more suspicious the propagation of program states via \( e_i \) is deemed to be.
To understand why equation (2) is useful for ranking edges according to their suspiciousness, let \( \text{Prob}(e) \) denote the (unknown) probability that the propagation of infected program states via \( e \) will cause a failure. The proof in Appendix A shows that

\[
\text{Prob}(e) = \frac{v \cdot \theta^e(e)}{v \cdot \theta^e(e) + u \cdot \theta^0(e)}
\]

is the best estimate for this probability. It is also proved in Appendix B that, no matter whether equation (2) or (3) is chosen to estimate the fault-relevance of edges, sorting edges in descending order of the results always generates the same edge sequence (except tie cases). In other words, we can also determine the order of the suspiciousness of the edges through equation (3), or determine the probability that an edge causes a failure by using equation (2).

We will adopt equation (2) rather than equation (3) because the value range of equation (2), which is from \(-1\) to \(1\) and symmetric with respect to 0, favors follow-up computation. For example, equation (3) always generates positive values. However, summing up positive operands always generates an operation result that is greater than each operand, and hence there may not be a solution for the constructed equation set (see Section 3.3.2).

Take the motivating example as an illustration. \( \text{Prob}(e_1) = (2 \times 1.00) / (2 \times 1.00 + 5 \times 1.00) = 0.29 \). Similarly, \( \text{Prob}(e_2) = 0.50, \text{Prob}(e_3) = 0.00, \text{Prob}(e_4) = 1.00, \text{Prob}(e_5) = 0.33, \text{and} \text{Prob}(e_6) = 1.00 \). Furthermore, \( \theta^0(e_1) = (1.00 - 1.00) / (1.00 + 1.00) = 0.00 \). Similarly, \( \theta^0(e_2) = 0.43, \theta^0(e_3) = -1.00, \theta^0(e_4) = 1.00, \theta^0(e_5) = 0.11, \text{and} \theta^0(e_6) = 1.00 \). By sorting \( e_1, e_6 \) in descending order of their values using equation (2), we obtain \( \{e_4, e_6, e_2, e_5, e_1, e_3\} \), where \( e_4 \) and \( e_6 \) form a tie case. Based on the two most suspicious edges \( e_2 \) and \( e_6 \), we can focus our attention to trace from \( b_2 \) to \( b_3 \) via \( e_4 \), and then to \( b_4 \) via \( e_6 \). However, one still does not know how to rank the fault suspiciousness of the blocks (other than examining all three blocks at the same time). In the next section, we will show how we use the concept of propagation to determine the suspiciousness score of each block.

### 3.3.2 Solving Block Suspiciousness Score

By contrasting the mean edge profiles of the passed executions and the failed executions, we have computed the suspiciousness of edges in the last section. In this section, we further associate edges with blocks, and assess fault suspiciousness score of every block.

To ease our reference, we use the notation \( BR(b_i) \) to represent the suspiciousness score of a block \( b_i \).

Let us first discuss how a program execution transits from one block to another. After executing a block \( b_i \), the execution may transfer control to one of its successor blocks. Suppose \( b_i \) is a successor block of \( b_j \). The program states of \( b_j \) may be propagated to \( b_i \) via the edge \( \text{edg}(b_j, b_i) \). Rather than expensively tracking the dynamic program state prorogation from \( b_j \) to \( b_i \), we approximate the expected number of infected program states of \( b_j \) observed at \( b_i \) as the fraction of the suspiciousness score of \( b_j \) from that of \( b_i \). This strategy aligns with our understanding that we can only observe failures from outputs rather than from inputs.

**Constructing an equation set.** To determine the fraction mentioned above, we compute the sum of suspiciousness scores of the incoming edges of \( b_i \), and compute the ratio of propagation via the edge \( \text{edg}(b_j, b_i) \) as the ratio of the suspiciousness score of this particular \( \text{edg}(b_j, b_i) \) over the total suspiciousness score for all edges. The formula to determine this ratio is:

\[
W(b_j, b_i) = \frac{\theta^0(\text{edg}(b_j, b_i))}{\sum_{\forall \text{edg}(b_j, b_k)\neq \theta^0(\text{edg}(b_j, b_k))}}
\]

\( W(b_j, b_i) \) models the portion of the contribution of \( \text{edg}(b_j, b_i) \) with respect to the total contribution by all incoming edges of \( b_i \). Intuitively, it represents the ratio of the observed suspicious program states at \( b_j \) that may be prorogated from \( b_i \).

The fraction of the suspiciousness score that \( b_i \) contributes to \( b_j \) is, therefore, the product of this ratio and the suspiciousness score of \( b_j \) (that is, \( BR(b_j) \times W(b_i, b_j) \)).

In general, a block \( b_i \) may have any number of successor blocks. Hence, we sum up such a fraction from every successor block of \( b_i \) to give \( BR(b_i) \), the suspiciousness score of \( b_i \), thus:

\[
BR(b_i) = \sum_{\forall \text{edg}(b_j, b_i)} [BR(b_j) \cdot W(b_j, b_i)]
\]

Let us take the motivating example for illustration: \( b_2 \) has two outgoing edges connecting to two successor blocks \( b_3 \) and \( b_4 \), respectively. Its suspiciousness score \( BR(b_2) \) is, therefore, calculated as \( BR(b_3) \cdot W(b_2, b_3) + BR(b_4) \cdot W(b_2, b_4) \). The propagation rate \( W(b_2, b_4) \) is calculated as \( \frac{\theta^0(e_4) / \theta^0(e_4) = 0.11 / (-1.00 + 0.11 + 1.00) = 1} \). The propagation rate \( W(b_2, b_3) \) is calculated as \( \frac{\theta^0(e_4) / \theta^0(e_4) = 1.00 / 1.00 = 1} \). 

**Handling exception cases.** We normally calculate the suspiciousness score of a block via its successor blocks. Let us consider an exception case where a block may have no successor. For a block containing, say, a return, break, or exit() function call, or for a block that crashes, the program execution may leave the block, or cease any further branch transitions after executing the block. In our model, if a block is found not to transit the control to other successor blocks in the same CFG (as in the case of a return statement or call function), we call it an *exit* block. Since exit blocks have no successor block, we do not apply equation (5) to calculate its suspiciousness score. Instead, we use the equation

\[
BR(b_i) = \sum_{\forall \text{edg}(b_j, b_i)} [BR(b_j) \cdot W(b_j, b_i)]
\]

which sums the suspiciousness scores of all the incoming edges to calculate the suspiciousness score of the exit block. Consider the motivating example again. There is no successor for \( b_3 \) in the CFG in Figure 1. Hence, \( BR(b_3) = \theta^0(e_3) + \theta^0(e_5) + \theta^0(e_6) = -1.00 + 0.11 + 1.00 = 0.11 \). (We should add that there are alternative ways to model the suspiciousness score for an exit block, such as using the formulas for block-level CBFL techniques.)

We have constructed an equation of \( BR(b_i) \) for every block \( b_i \) (including exit blocks). In other words, we have set up an equation set containing \( n \) homogenous equations (one for each block) and \( n \) variables as the left-hand side of each equation (one for each suspiciousness score of that block). In such a case, the equation set satisfies a necessary condition of being solvable by existing efficient algorithms for solving equation sets (such as Gaussian elimination [29], which we also adopt in the experiment in Section 5).

In the motivating example, we can set up an equation set \( BR(b_4) = 0.11, BR(b_3) = 9 \times BR(b_4), BR(b_2) = BR(b_4) + BR(b_3), BR(b_1) = -9 \times BR(b_4) + BR(b_2) \). We can solve it to give \( BR(b_4) = 0.11, BR(b_3) = 1.00, BR(b_2) = 1.11, \) and \( BR(b_1) = 0.11 \).
3.3.3 Synthesizing a Ranked List of Statements
After obtaining the suspiciousness score for every block, we further group those statements not in any block (such as global assignment statements) into a new block, and give it a lower suspiciousness score than that of any other ranked block. We also merge those blocks having identical suspiciousness scores, and produce a ranked list of blocks in descending order of their suspiciousness scores. All the non-executable statements and statements that are not exercised by any given executions are consolidated into one block, which is appended to the end of the ranked list and given the lowest suspiciousness score. Finally, one may assign ranks for statements. Following previous work [15], the rank of a statement is assigned as the sum of total number of statements in its belonging block and the total number of statements in the blocks prior to its belonging block. The CP column of Figure 1 shows the ranks of statements by our method, which only needs 44% code examination effort to locate a fault.

4. DISCUSSIONS
In previous work, a tie-break strategy (see [21][24]) is further employed to optimize the baseline fault-localization techniques. They give a better ranking list when one follows the ranking list to locate faults. However, they do not modify the computed suspicious scores of the program entities by those techniques. Our technique can also be optimized in exactly the same way.

In our technique, we capture the propagation of infected program states via CFG. In fact, other flow-graph representations of program executions, such as program dependency graphs [3] or data flow graphs, may be employed to replace CFG. We do not iteratively show how to adapt each of them in our technique.

The space and time complexity of our technique is analyzed as follows: With the same problem settings (u passed executions, v failed executions, n blocks, and m edges), the space complexity is mainly determined by the space needed to maintain the mean edge profiles for the passed executions and the failed executions, and the suspiciousness scores for edges, which are O(um), O(vm), and O(m), respectively. Therefore the space complexity of our technique is O(um + vm). The time complexity is determined by the time used to solve the set of equations. If Gaussian elimination is adopted, the time complexity of our technique will be O(n^3).

5. EXPERIMENTAL EVALUATION
In this section, we conduct a controlled experiment to evaluate the effectiveness of our technique.

5.1 Setup of Experiment

5.1.1 Subject Programs
We use four UNIX programs, namely, flex, grep, gzip, and sed, as subject programs. They are real-life medium-sized programs, and have been adopted to evaluate other CBFL techniques (as in [14][23][26]). We downloaded the programs (including all versions and associated test suites) from SIR [10] on January 10, 2008. Each subject program has multiple (sequentially labeled) versions. Table 1 shows the real-life program versions, numbers of lines of executable statements (LOC), numbers of applicable faulty versions, and the sizes of the test pools. Take the program flex as an example. The real-life versions include flex-2.4.7 to flex-2.5.4, and have 8571 to 10124 lines of executable statements. 21 single-fault versions are used in the experiment. All these faulty versions share a test suite that consists of 567 test cases. Following [12], we apply the whole test suite as inputs to individual subject programs.

5.1.2 Peer Techniques
In our experiment, we select five representative peer techniques to compare with our technique. Tarantula [24] and Jaccard [1] are two statement-level techniques. They are often chosen as alternatives for comparison in other evaluations of fault-localization techniques. CBI [18] and SOBER [19] are predicate-level techniques. Since they make use of predicates (such as branch decisions) to locate suspicious program positions, which are related to the edge concept in our technique, we decide to compare these techniques with ours. Note that CBI originally proposed to use random sampling to collect predicate statistics to reduce overhead. In our evaluation on CBI, we sample all the predicates (as in [19]) via gcov. In Yu et al.’s work [24], CBI is modified to become a statement-level technique (SBI [24]), and we also include SBI for comparison with our technique. Note that a tie-breaking strategy is included in Tarantula as stated in [24]. CP uses no tie-breaking strategy in our experiment.

5.1.3 Effectiveness Metrics
Each of Tarantula, SBI, Jaccard, and CP produces a ranked list of all statements. For every technique, we check all the statements in ascending order of their ranks in the ordered list, until a faulty statement is found. The percentage of statements examined (with respect to all statements) is returned as the effectiveness of that technique. This metric is also used in previous studies [14][24]. We note also that statements having the same rank are examined as a group.

CBI and SOBER generate ranked lists of predicates. To the best of our knowledge, the metric T-score [20] is used to evaluate their effectiveness in previous studies [19]. T-score uses a program dependence graph to calculate the distance among statements. Starting from some top elements in a ranked list of predicates, T-score conducts breadth-first search among the statements to locate a fault. The search terminates when it encounters any faulty statement, and the percentage of statements examined (with respect to all statements) is returned as the effectiveness of that technique [20]. Since it is reported in [19] that the “top-5 T-score” strategy gives the highest performance for CBI and SOBER, we follow suit

<table>
<thead>
<tr>
<th>Program Description</th>
<th>LOC</th>
<th>No. of single-fault versions</th>
<th>No. of test cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>flex</td>
<td>4081-5159</td>
<td>17</td>
<td>809</td>
</tr>
<tr>
<td>grep</td>
<td>8053-9089</td>
<td>17</td>
<td>809</td>
</tr>
<tr>
<td>gzip</td>
<td>4081-5159</td>
<td>17</td>
<td>809</td>
</tr>
<tr>
<td>sed</td>
<td>4756-9289</td>
<td>17</td>
<td>370</td>
</tr>
</tbody>
</table>

Following the documentation of SIR [10] and previous experiments [12][15][18][19][19], we exclude any single-fault version whose faults cannot be revealed by any test case. This is because both our technique and the peer techniques used in the experiment [1][18][19][24] require the existence of failed executions. Moreover, we also exclude any single-fault version that fails for more than 20% of the test cases [10][12]. Besides, as Jones et al. have done in [15], we exclude those faulty versions that are not supported by our experimental environment and instrumentation tool (we use the Sun Studio C++ compiler and gcov to collect edge profile information on program versions). All the remaining 110 single-fault versions are used in the controlled experiment (see Table 1).
to choose the top-5 predicates and report the top-5 T-score results as their effectiveness in our experiment.

If a fault is in a non-executable statement (such as the case of a code omission fault), dynamic execution information cannot help locate the fault directly. To reflect the effectiveness of a technique, we follow previous studies (such as [14]) to mark the directly infected statement or an adjacent executable statement as the fault position, and apply the above metrics.

5.1.4 Experiment Environment and Issues
The experiment is carried out in a Dell PowerEdge 1950 server (4-core Xeon 5355 2.66GHz processors, 8GB physical memory, and 400GB hard disk) serving a Solaris UNIX with the kernel version Generic_120012-14. Our framework is compiled using Sun C++ 5.8.

When applying our technique, an exceptional case is that the denominator in equation (4) may be zero. For every occurrence of a zero denominator in the experiment, the tool automatically replaces it by a small constant. $10^{-10}$ is chosen as the constant, which is less than any intermediate computing result by many degrees of magnitude. We have varied this constant from $10^{-11}$ to $10^{-9}$ and compared the effectiveness results of CP, and confirmed that the results are the same.

In the experiment, the time needed to generate a ranked list for one faulty version is always less than 1 second. The mean time spent for one faulty version is about 0.455 seconds.

5.2 Data Analysis
In this section, we compare our technique with Tarantula, SBI, Jaccard, CBI, and SOBER, and report their effectiveness on the 110 single-fault program versions. In the following subsections, the data related to “Tarantula”, “SBI”, “Jaccard”, “CBI”, and “SOBER” are worked out using the techniques described in the papers [24], [24], [1], [18], and [19], respectively. The data related to “CP” are worked out using our technique. For every plot in

Figure 2 and Figure 3, we use the same set of x-axis labels and legends.

5.2.1 Overall Results
To evaluate the overall effectiveness of a technique, we first take the average of the effectiveness results on the four subject programs. The results are shown in Figure 2. In the plot in Figure 2(a), the x-axis means the percentage of code that needs to be examined in order to locate the fault (according to the effectiveness metrics). We also refer to it as the code examination effort in this paper. The y-axis means the mean percentage of faults located. Take the curve for CP in Figure 2(a) for illustration. On average, CP can locate 48.24% of all faults by examining up to 5% of the code in each faulty version. The curves of Tarantula, Jaccard, CBI, SBI, and SOBER can be interpreted similarly. Note that the effectiveness of Tarantula, SBI, and Jaccard are very close, and hence their curves in Figure 2 and Figure 3 almost completely overlap.

Figure 2(a) gives the overall effectiveness of CP, Tarantula, SBI, Jaccard, CBI, and SOBER. Each of the six curves starts at the point (0%, 0%) and finally reaches the point (100%, 100%). Obviously, it reflects the fact that no fault can be located when examining 0% of the code, while all the faults can be located when examining 100%. We observe from the figure that CP can locate more faults than CBI and SOBER in the range from 1% to 99% of the code affordable to be examined. Moreover, the figure also shows that CP can locate more faults than Tarantula, SBI, and Jaccard almost in the entire range of the first one third (from 2% to 33%) of the code examination effort.

When comparing the mean effectiveness, although one cannot meaningfully conclude the results from outlier segments (such as those data points beyond three standard deviations), previous studies such as [19] once reported the results on the first 20% code examination range. Therefore, we further zoom in (to the range of [0%, 20%]) as shown in Figure 2(b).
The figure shows that, if only 1% of the code is affordable to be examined, Tarantula, SBI, and Jaccard can locate 31.99% of all faults, CP can locate 24.50%, CBI can locate 8.54%, while SOBER cannot locate any fault. If 2% of the code is affordable to be examined, encouragingly, CP not only catches up with Tarantula, SBI, and Jaccard, but also exceeds them a lot. For example, CP, Tarantula, SBI, Jaccard, CBI, and SOBER locate 41.55%, 33.18%, 32.45%, 32.90%, 11.48%, and 0.00% of the faults in all faulty versions, respectively. In the remaining range (from 2% to 20%) in Figure 2(b), CP always locates more faults than the peer techniques. For example, when examining 8%, 9%, and 10% of the code, CP locates 55.31%, 57.86%, and 57.86% of the faults, respectively; Tarantula locates 38.75%, 40.67%, and 42.03% of the faults; SBI locates 38.75%, 40.67%, and 42.49%; and Jaccard locates 38.46%, 40.39%, and 41.75%. In summary, by examining up to 20% of the code, CP can be more effective than the peer techniques.

In previous studies, a weighted average method has also been used [8]. For example, Chilimbi et al. [8] uses the total number of faults; for example, Chilimbi et al. [8] uses the total number of faults located in all programs as the y-axis (in the sense of Figure 2(b)), rather than the average percentage of faults located. To enable reader to compare previously published results with ours, we follow [8] to present such a plot as Figure 2(c). From this figure, we observe that if 2% to 16% of the code is examined, CP performs better than the other five techniques. However, Tarantula, SBI, and Jaccard catch up with CP gradually. The range (21% to 99%) is not shown in this paper owing to space limit, and yet we do observe that the effectiveness of CP, Tarantula, SBI, and Jaccard are very similar. More detailed statistical comparisons can be found in Section 5.2.3.

Overall, the experiment shows that CP can be effective. At the same time, it also shows that CP can be further improved.

### 5.2.2 Results on Individual Subject Programs

We further compare the effectiveness of CP against the peer techniques on each subject program. Figure 3 shows the corresponding results on the programs grep, gzip, sed, and grep, as depicted in Figure 2(a), the x-axis means the percentage of code examined, and the y-axis means the percentage of faults located by SBI within the given code examination effort (specified by the respective value on the x-axis). The curves for CP, Tarantula, Jaccard, CBI, and SOBER can be interpreted similarly.

The four plots in Figure 3 give the overall effectiveness of CP, Tarantula, SBI, Jaccard, CBI, and SOBER on each subject program. If 5% of the code has been examined, CP can locate faults in 47.61%, 52.94%, 21.81%, and 70.58% of the faulty versions of the programs grep, gzip, sed, and sed, respectively. On the other hand, Tarantula can locate 52.38%, 0.00%, 18.18%, and 70.58% of the faults, respectively; SBI can locate 52.38%, 0.00%, 20.00%, and 70.58%; and Jaccard can locate 52.38%, 0.00%, 21.81%, and 70.58%; CBI can locate 9.52%, 29.41%, 0.00%, and 35.29%; and SOBER can locate 0.00%, 5.88%, 0.00%, and 0.00%. The other points on the curves can be interpreted similarly.

Similarly to Section 5.2.1, let us discuss the first 20% of the code examination range. For grep and gzip, we observe that CP performs better than CBI or SOBER, and performs comparably with Tarantula, SBI, and Jaccard. For grep and sed, CP locates more faults than Tarantula, SBI, Jaccard, CBI, and SOBER within the first 20% code examination range. In summary, CP performs outstandingly in this range.

### 5.2.3 Statistics Analysis on Individual Faulty Versions

In this section, we further use popular statistics metrics to compare different techniques. Table 2 lists out the minimum (min), maximum (max), medium, mean, and standard derivation (stdev) of the effectiveness of these techniques, on the 110 single-fault versions. The effectiveness of each technique is evaluated using the same metric as in the previous section; therefore, the smaller the magnitude, the better is the effectiveness. We observe that in each row, CP gives the best (smallest) value among the six techniques, which further strengthens our belief that CP can be effective on locating faults.

To further find the relative merits on individual versions, we compute the difference in effectiveness between CP and each peer technique, and the results are shown in Table 3. Take the cell in column “CP–Tarantula” and row “<−5%” as an example. It shows that, for 42 (38.18%) of the 110 faulty versions, the code examination effort of using CP to locate a fault is less than that of Tarantula by more than 5%. Similarly, for the row “≥5%”, only

<table>
<thead>
<tr>
<th>Table 2: Statistics of effectiveness</th>
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<tr>
<td>min</td>
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<tr>
<td>max</td>
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<tr>
<td>medium</td>
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<tr>
<td>mean</td>
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<td>stdev</td>
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<table>
<thead>
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<th>Table 3: Statistics of differences in effectiveness</th>
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<td>---</td>
</tr>
<tr>
<td>Difference (percentage difference)</td>
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<td>&lt;−1%</td>
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<tr>
<td>−1% to 1%</td>
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<td>&gt;1%</td>
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<td>−5% to 5%</td>
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<td>&lt;−10%</td>
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<td>−10% to 10%</td>
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<td>&gt;10%</td>
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for locating them are 1.12%, 8.50%, 7.25%, 21.19%, 13.82%, and 88.2%, respectively. The last fault, which results in 6 failures among 567 test cases, is found in line 12193. It is an incorrect static variable definition. Since this fault is seeded in a global definition statement and the compiler tool gcov fails to log its execution, we mark its directly affected statement (say, line 12193) as the fault position. However, CP needs to examine 93.55% of the code to locate this fault. We scrutinize the case and find it to be a coincidence. For 7 out of 567 test cases that do not reveal failures, this branch statement is never covered. For the 6 test cases that reveal failures and the remaining 560 passed test cases, both the true branch and the false branch are covered. For more than 90% of the cases, the number of times that each branch is covered is very close to each other (with less than 5% difference). It is hard for CP to distinguish these two edges. We view that this practical scenario provides a hint for further improving CP, even though the current experiment shows that CP is promising.

5.3 Threats to Validity
We used gcov to implement our tool in which coverage profiling is completely conducted. The generation of the equation set by the tool is relatively straightforward. The equations are solved using a standard Gaussian elimination implementation. We have implemented the peer techniques ourselves and checked that the implemented algorithms adhere strictly to those published in the literature. We have also conducted trial runs on toy programs with limited test cases to assure the implementations of CP and other peer techniques.

Currently, we follow [19] to use T-score when evaluating CBI and SOBER. Some researchers have reported limitations in T-score (see [9], for example). A P-score has been proposed in [27] and may be considered for future studies. CP, Tarantula, SBI; and Jaccard produce ranked lists of statements, while CBI and SOBER generate ranked list of predicates. Consequently, the experiment has used two effectiveness metrics to report the results of different techniques. It is unsuitable to compare CP on a par with CBI and SOBER. In this connection, the comparison and the discussion of CP in relation to CBI and SOBER should be interpreted carefully.

We use flex, grep, gzip, and sed as well as their associated test suites to evaluate our technique. These programs are real-life programs with realistic sizes, and they have been used in previous studies [14][23][26]. It is certainly desirable to evaluate CP further on other real-life subject programs and scenarios.

6. RELATED WORK
Comparing program executions of a faulty program over different test cases and considering program states are frequently used fault-localization strategies. Delta debugging [9] isolates failure-causing inputs, produces cause effect chains and locates the root causes of failures. It considers a program execution (of a failure-causing inputs, produces cause effect chains and locates the root causes of failures. It switches a predicate’s decision at execution time to alter the original control flow of a failure-causing test case, aiming at locating a predicate such that a switch of the decision will produce correct output. Their latest version [14] works on the value set of all variables and the result looks promising.
Fault localization is a process to find the faults in failed programs. Existing coverage-based fault-localization approaches use the statistics of test case executions to serve this purpose. They focus on individual program entities, generate a ranked list of their suspiciousness, but ignore the structural relationships among statements.

In this paper, we assess the suspiciousness scores of edges, and set up a set of linear algebraic equations over the suspiciousness scores of basic blocks and statements, which abstractly models the propagation of suspicious program states through control flow edges in a back-tracking manner. Such equation sets can be efficiently solved by standard mathematical techniques such as Gaussian elimination. The empirical results on comparing existing techniques with ours show that our technique can be effective.

We have further conducted a case study on a multi-fault program to examine the effectiveness of our technique, and find cases that inspire future work. In order to further enhance the effectiveness of our approach, another prospective is to extend our edge profile technique to cover path profiles or data flow profiles as well.
APPENDIX A. Proof of equation (3)
Let \( \text{Prob}(e) \) be the probability that the propagation of infected program states via \( e \) causes a failure.

Let \( T = \{t_1, t_2, \ldots, t_n\} \) be a set of test cases associated with passed executions, and \( T' = \{t'_1, t'_2, \ldots, t'_m\} \) be a set of test cases associated with failed executions. Let \( \theta(e, t_i) \) denote whether the edge \( e \) is covered in the corresponding program execution of \( t_i \). We would like to estimate the value of \( \text{Prob}(e) \) from the subsets \( T_1 = \{t_i | \theta(e, t_i) = 1\} \subset T \) and \( T_2 = \{t'_i | \theta(e, t'_i) = 1\} \subset T' \) because the executions in these two subsets correlates with the traversal of \( e \). The expected number of failed executions in the sample set of \( T_1 \cup T_2 \) is \( \text{Prob}(e) \times |T_1 \cup T_2| \). This estimate is unbiased.

To maximize the value of \( \text{Prob}(e) \), we set the expected number of failed executions in the sample set to be equal to the actual number of failed executions. That is, we set \( \text{Prob}(e) \times |T_1 \cup T_2| = |T| \). We then solve for \( \text{Prob}(e) \) and obtain equation (3). The details of the proof are straightforward.

APPENDIX B. Proof that sorting of edges always produces the same sequence no matter whether equation (2) or (3) is used
We need an auxiliary function for the proof. We define a sign function such that \( \text{sgn}[x] = -1 \) if \( x < 0 \), \( \text{sgn}[x] = 0 \) if \( x = 0 \), and \( \text{sgn}[x] = 1 \) if \( x > 0 \).

Suppose \( e \) and \( e' \) are two edges in \( E \) satisfying the conditions (i) \( \theta(e) \neq 0 \lor \theta(e') \neq 0 \) and (ii) \( \theta(e) \neq 0 \lor \theta(e') \neq 0 \). We make use of the sign function to express their relative ranking order with respect to equation (3) as \( \text{sgn}[\text{Prob}(e) - \text{Prob}(e')] \). Similarly, we express their relative ranking order with respect to equation (2) as \( \text{sgn}[\theta^a(e) - \theta^a(e')] \).

Case 1 (\( \theta(e) = 0 \)). By equation (3), \( \text{Prob}(e) = 0 \). Also by equation (3),
\[
\text{Prob}(e) = \frac{\theta^a(e)}{\theta^a(e) + \theta^a(e')} \geq 0.
\]
Hence, \( \text{sgn}[\text{Prob}(e) - \text{Prob}(e')] = -\text{sgn}[\theta^a(e)] \). Similarly, by equation (2), \( \text{sgn}[\theta^a(e) - \theta^a(e')] = -\text{sgn}[\theta^a(e)] \). Thus, \( \text{sgn}[\text{Prob}(e) - \text{Prob}(e')] = \text{sgn}[\theta^a(e) - \theta^a(e')] \).

Case 2 (\( \theta(e') = 0 \)). Similarly to the proof of case 1, we have \( \text{sgn}[\text{Prob}(e) - \text{Prob}(e')] = \text{sgn}[\theta^a(e) - \theta^a(e')] \).

Case 3 (\( \theta(e) \neq 0 \land \theta(e') \neq 0 \)). Similarly to case (2), \( \text{sgn}[\theta^a(e) - \theta^a(e')] = \text{sgn}[\theta^a(e) - \theta^a(e')] \).

Case 4 (\( \theta(e) \neq 0 \land \theta(e') \neq 0 \land \theta^a(e) = 0 \)). We first discuss the value of \( \text{sgn}[\theta^a(e) - \theta^a(e')] \). Since none of \( \theta(e), \theta(e'), \theta^a(e), \) and \( \theta^a(e') \) is 0, each of them should be a positive number. Suppose \( a, b, \) and \( c \) are any positive numbers, and \( d \) is any number. We have
\[
\text{sgn}[\theta^a(e) - \theta^a(e')] = \text{sgn}[\theta^a(e) - \theta^a(e')] = \text{sgn}[\theta^a(e) - \theta^a(e')] = \text{sgn}[\theta^a(e) - \theta^a(e')]
\]
By setting \( a = u, b = v, c = w, \) and \( d = 0 \), we have \( \text{sgn}[\theta^a(e) - \theta^a(e')] = \text{sgn}[\text{Prob}(e) - \text{Prob}(e')] \). Similarly, by setting \( a = 1, b = 1, c = 2, \) and \( d = 1 \), we have \( \text{sgn}[\theta^a(e) - \theta^a(e')] = \text{sgn}[\theta^a(e) - \theta^a(e')] \). Hence, we obtain \( \text{sgn}[\text{Prob}(e) - \text{Prob}(e')] = \text{sgn}[\theta^a(e) - \theta^a(e')] \).

Hence, the relative ranking order of any two edges computed by equation (2) is the same as that computed by equation (3).