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HEDGING AND NONLINEAR RISK EXPOSURE

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Hedging and Nonlinear Risk Exposure *

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This paper documents some empirical evidence of nonlinear spot-futures exchange rates relationships and develops an expected utility model of an exporting firm to examines the associated economic implications. The model shows that the firm should export more (less) and adopt an over (under) hedge in an unbiased currency futures market if the spot-futures exchange rates relationship is convex (concave) rather than linear. When fairly priced currency options on futures are available, the firm should use them in conjunction with the currency futures so as to achieve better hedging against its nonlinear exchange rate risk exposure. This provides a rationale for the hedging role of options when the underlying uncertainty is nonlinear in nature.

JEL classification: D21; D81; F31

Keywords: Nonlinear exchange rate risk; Production; Futures; Options

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This paper documents some empirical evidence of nonlinear spot-futures exchange rates relationships and develops an expected utility model of an exporting firm to examine the associated economic implications. The model shows that the firm should export more (less) and adopt an over (under) hedge in an unbiased currency futures market if the spot-futures exchange rates relationship is convex (concave) rather than linear. When fairly priced currency options on futures are available, the firm should use them in conjunction with the currency futures so as to achieve better hedging against its nonlinear exchange rate risk exposure. This provides a rationale for the hedging role of options when the underlying uncertainty is nonlinear in nature.

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1. Introduction

Exchange rate risk management has received increasing attention nowadays when more and more companies of all sizes and of all industries source and sell abroad. Corporations take it very seriously in response to the continuing incidence of exchange rate fluctuations of major currencies (see Meese, 1990) and to its high visibility driven by accounting rules and regulations.[1] Indeed, as documented in a survey by Rawls and Smithson (1990), it is ranked by financial managers as one of their primary objectives.

Given the real-world prominence of exchange rate risk management, there have been a great many papers concerning the production and hedging decisions of a risk-

[1]The Financial Accounting Standards Board requires transaction and translation gains and losses to be reported in income statements. The Securities and Exchange Commission, on the other hand, requires statements explaining the impact of exchange rate movements on reported profits to be included in annual reports.

averse exporting firm facing exchange rate uncertainty (see, e.g., Katz and Paroush, 1979; Benninga *et al.*, 1985; Kawai and Zilcha, 1986; Broll and Zilcha, 1992; Adam-Müller, 1997; and Viaene and Zilcha, 1998). In models of this process, a linear regression of the spot exchange rate on the futures exchange rate is usually employed to capture the presence of basis risk.[2] One rather robust result emanated from the extant literature is that the exporting firm should adopt a full hedge in an unbiased currency futures market so as to eliminate completely the hedgeable component of its exchange rate risk exposure. A corollary of this full-hedging result is that other hedging instruments such as currency options would be redundant. Indeed, as shown by Lapan *et al.* (1991) in the context of the competitive firm under output price uncertainty *à la* Sandmo (1971), options are part of the optimal hedging strategy only when futures prices and/or options premiums are perceived as biased. In this regard, options are appealing more as a speculative device to exploit private information on output price distributions, and less so as a hedging instrument.

The purpose of this paper is to re-examine the optimality of a full hedge in an unbiased currency futures market and the hedging role of currency options by allowing a nonlinear spot-futures exchange rates relationship. We first document empirically that nonlinear spot-futures exchange rates relationships prevail in many currencies. We then consider theoretically how this nonlinearity affects the exporting firm's optimal production and hedging decisions when the firm has access to an unbiased currency futures market. Specifically, we show that the firm should export more (less) and adopt an over (under) hedge if the spot-futures exchange rates relationship is convex (concave) rather than linear. When fairly priced currency options on fu-

[2]As pointed out by Briys *et al.* (1993), the direction of regressability of spot and futures prices turns out to be quite important in analyzing the effects of basis risk. For the alternative specification in which the futures price is regressable on the spot price, see also Paroush and Wolf (1986, 1989) and Broll *et al.* (1995).

tures are available, we show that they would be used in conjunction with the currency futures by the firm. The asymmetric payoff profiles of options, as opposed to linear futures contracts, are more suitable for hedging against nonlinear risk.

Despite the widespread use and importance of options in risk management, as well as the vast academic and practitioner literature on option pricing, the optimality of options being a hedging instrument remains largely unexplored. Moschini and Lapan (1992) show that production flexibility of the competitive firm under output price uncertainty *à la* Sandmo (1971) leads to an *ex post* profit function that is convex in output prices, thereby inducing the firm to use options for hedging purposes. Sakong *et al.* (1993) and Moschini and Lapan (1995) show that production uncertainty provides another rationale for the use of options as a hedging instrument. The usefulness of options under production uncertainty is related to the multiplicative interaction between price and yield uncertainty, which affects the curvature of the firm's profit function. Lence *et al.* (1994) show that forward-looking firms would use options as a hedging instrument because they care about the effects of future output prices on profits from future production cycles. This paper offers yet another rationale for the hedging role of options when the underlying uncertainty is nonlinear in nature.

The organization of the paper is as follows. The next section presents some empirical evidence of nonlinear spot-futures exchange rates relationships. Section 3 delineates the basic model of a risk-averse exporting firm facing nonlinear exchange rate uncertainty. Section 4 examines the optimal production and hedging decisions of the firm in the presence of an unbiased currency futures market. Section 5 derives the optimal hedging strategy of the firm when the firm can also trade fairly priced call options on futures. The final section concludes.

2. Evidence of nonlinearities

In this section, we shall present some empirical evidence of nonlinear spot-futures exchange rates relationships. The data on daily spot and three-month futures exchange rates are supplied by Datastream. Daily spot and three-month futures exchange rates for the Australian dollar (AUD), British pound (GBP), Canadian dollar (CAD), Deutsche mark (DEM), French franc (FRF), and Japanese yen (JPY), all against the U.S. dollar, are taken over the period beginning on December 31, 1993 and ending on May 31, 1999. Spot exchange rates are closing prices in New York while futures exchange rates are closing prices in the International Money Market (Chicago).

To detect nonlinearities between spot and futures exchange rates, we propose the following regression model:

$$e_t = \alpha + \beta f_t + \gamma f_t^2 + \epsilon_t \tag{1}$$

where e_t and f_t are the spot and futures exchange rates at time t, α, β, and γ are scalars, and ϵ_t is a noise term. To avoid running spurious regressions, we need to check two things. First, do the time series of e_t, f_t, and f_t^2 contain unit roots, i.e., are they I(1)? Second, if e_t, f_t, and f_t^2 are indeed I(1), are they cointegrated?

As pointed out by Granger and Newbold (1974) and Phillips (1986), in regressions involving data characterized by unit roots, the conventional significant tests are misleading in that they tend to reject the hypothesis of no relationship when, in fact, there might be none. To detect for the presence of unit roots, we employ the augmented Dickey-Fuller (DF) test with and without a time trend (Dickey and Fuller, 1979, 1981). The augmented DF test with a time trend performs the following regression on each time series, y_t:

$$\Delta y_t = a + bt + \phi y_{t-1} + \sum_{i=1}^{n} \phi_i \Delta y_{t-i} + v_t$$

where Δ is the first difference operator, and n is the number of lags chosen to ensure the stationarity of the residual series, v_t. The null hypothesis that $\phi = 0$ (the presence of a unit root) is tested against the alternative hypothesis that $\phi < 0$. Table 1 reports the augmented DF test statistics for various time series of spot, futures, and squared futures exchange rates (variable names ended with S, F, and F2, respectively). At 10% level of significance, we reject the null hypothesis of the presence of a unit root for all variables in Table 1.[3]

(Insert Table 1 here)

To test for cointegration among the time series of e_t, f_t, and f_t^2. we follow Engle and Granger (1987) to first run the following cointegrating regression:

$$\hat{e}_t = \hat{\alpha} + \hat{\beta} f_t + \hat{\gamma} f_t^2$$

where $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ are the OLS estimates of the true α, β, and γ, respectively. We then estimate ϵ_t in equation (1) by the residuals from the above cointegrating regression:

$$u_t = e_t - \hat{\alpha} - \hat{\beta} f_t - \hat{\gamma} f_t^2$$

If the time series of u_t contains no unit roots, we can conclude that the time series of e_t, f_t, and f_t^2 are cointegrated. Table 2 reports the augmented DF test statistics for various time series of residuals from the cointegrating regressions. At 10% level of significance, we reject the null hypothesis of no cointegration for the Australian dollar,

[3] The squared futures exchange rate for the British pound is marginally rejected to be I(1) at 10% level of significance but not at 5% level of significance. Thus, we do not exclude the British pound from the cointegration tests in Table 2.

British pound, Deutsche mark, and Japanese yen. When a time trend is added, all six currencies are cointegrated at 10% level of significance.

(Insert Table 2 here)

Stock (1987) demonstrates that if the time series of the underlying variables are cointegrated, the OLS estimators would be superconsistent in that their sampling distributions collapse onto the true values at an even faster rate than in the case where all the classical assumptions hold.[4] Table 3 reports the OLS estimations of various spot-futures exchange rates relationships. For the Canadian dollar and French franc, we follow the cointegration test results in Table 2 to include a time trend variable to the OLS estimations. Except the Australian dollar, the British pound, Deutsche mark, French franc, and Japanese yen all reveal a nonlinear spot-futures exchange rates relationship at 1% level of significance and the Canadian dollar at 5% level of significance.

(Insert Table 3 here)

A few remarks are in order. First, nonlinearities between spot and futures exchange rates seem to be a prevalent phenomenon in our sample. Second, these nonlinear relationships are equally likely to be convex (i.e., a positive γ) or concave (i.e., a negative γ). Third, the order of magnitude of the nonlinear components is by and large relatively small.

[4] It is worth mentioning that the superconsistency result of Stock (1987) is a large-sample result. In small samples, the OLS estimators are biased and this bias is related to $1 - R^2$, where R^2 is the coefficient of determination for the cointegrating regression (see Banerjee *et al.*, 1986).

3. The model

Motivated by the empirical observations in the previous section, we shall develop a theoretical model which examines the economic implications of nonlinear spot-futures exchange rates relationships. To this end, we consider a competitive exporting firm which has a planning horizon over a single period, starting at time 0 and ending at time T. To begin, the firm has to choose an output level, Q, to be produced in its home country. The entire output will then be sold to a foreign country at time T at a price denominated in the foreign currency. The selling price per unit is exogenously given and is normalized to unity for simplicity. The firm's production process gives rise to a cost function, C, compounded to time T, with $C(0) \geq 0$, $C' > 0$, and $C'' > 0$.

The spot exchange rate at time T, denoted by $\tilde{e}$, specifies the amount of the domestic currency that can be exchanged per unit of the foreign currency at that time.[5] Since the firm does not know *ex ante* the *ex post* realization of $\tilde{e}$, it faces exchange rate risk exposure of Q. The firm, however, can trade infinitely divisible futures contracts and options on futures (including calls and puts) which call for delivery of the domestic currency per unit of the foreign currency at time $\hat{T}$.[6] Because payoffs of any combinations of futures, calls, and puts can be replicated by any two of these three financial instruments (see, e.g., Cox and Rubinstein, 1985; and Sercu and Uppal, 1995), one of them is redundant. It is, therefore, no loss of generality to restrict the firm to use futures contracts and call options on futures only.[7]

Let f_0 be the known futures exchange rate at time 0. On the expiration date,

[5] Throughout the paper, a tilde (~) always signifies a random variable.

[6] A call (put) option on futures gives the holder the right, but not the obligation, to buy (sell) the underlying futures contract at a given strike price on a pre-specified date (see, e.g., Cox and Rubinstein, 1985; and Sercu and Uppal, 1995).

[7] The case in which the firm uses futures and puts and that in which the firm uses calls and puts can be analyzed analogously.

time $\hat{T}$, the futures exchange rate is $\tilde{f}$ which is uncertain. If the expiration dates of the futures contracts and the call options on futures do not match the maturity of the foreign currency cash flow to which the firm is exposed (i.e., $\hat{T} \neq T$), basis risk exists because $\tilde{e}$ need not equal $\tilde{f}$. Given this maturity mismatch, hedging with the futures contracts and the call options on futures is *de facto* imperfect and is referred to as a delta hedge.[8]

Consistent with the empirical study in Section 2, we model the basis risk by assuming that $\tilde{e}$ is regressable on $\tilde{f}$ in the following quadratic manner:

$$\tilde{e} = \alpha + \beta\tilde{f} + \gamma\tilde{f}^2 + \tilde{\epsilon} \tag{2}$$

where α, β, and γ are scalars, and $\tilde{\epsilon}$ is a zero-mean, finite-variance, random variable independent of $\tilde{f}$. If $\gamma = 0$, we are back to the extant literature that $\tilde{e}$ and $\tilde{f}$ are linearly related. If $\gamma \neq 0$, the nonlinear component, $\gamma\tilde{f}^2$, is convex or concave depending on whether γ is positive or negative, respectively. We refer to the case when $\gamma > 0$ as the case wherein the firm faces convex exchange rate risk exposure. Likewise, the firm is said to face concave exchange rate risk exposure when $\gamma < 0$. To simplify notation, we set $\alpha = 0$ and $\beta = 1$ as no additional insights are gained with arbitrarily chosen α and β.

At time T, the profit of the firm, $\tilde{\Pi}$, is given by

$$\tilde{\Pi} = (\tilde{f} + \gamma\tilde{f}^2 + \tilde{\epsilon})Q - C(Q) + (f_0 - \tilde{f})H + [p - \max\,(\tilde{f} - k,\ 0)]Z \tag{3}$$

where we have used equation (2) with $\alpha = 0$ and $\beta = 1$, H is the number of the futures contracts sold (purchased if negative), and Z is the number of the call options

[8]The qualitative results of this paper remain unchanged if we consider a cross-hedge vis-à-vis a delta hedge when futures contracts and options on futures are available for a 'related' currency, rather than for the foreign currency. For cross-hedging, see, e.g., Anderson and Danthine (1981), Broll *et al.* (1998), and Broll and Wong (1999).

on futures written (purchased if negative) with the strike price, k, at the option premium, p. The firm possesses a von Neumann-Morgenstern utility function, U, defined over its time T profit with $U' > 0$ and $U'' < 0$, indicating the presence of risk aversion. The firm is an expected utility maximizer and has to solve the following *ex ante* decision problem:

$$\max_{Q,\ H,\ Z} \ \mathrm{E}[U(\tilde{\Pi})] \tag{4}$$

where E is the expectation operator and $\tilde{\Pi}$ is defined in equation (3). Throughout the paper, we assume that both the futures contracts and the call options on futures are fairly priced so that $f_0 = \mathrm{E}(\tilde{f})$ and $p = \mathrm{E}[\max\ (\tilde{f} - k,\ 0)]$. Furthermore, we set the strike price, k, equal to the time 0 futures exchange rate, f_0.

4. Export and hedging decisions with currency futures only

In this section, we consider the case where the currency futures are the only hedging instruments available to the firm, i.e., $Z \equiv 0$. Using this restriction, the first-order conditions for program (4) are given by

$$\mathrm{E}\{U'(\tilde{\Pi}^*)[\tilde{f} + \gamma\tilde{f}^2 + \tilde{\epsilon} - C'(Q^*)]\} = 0 \tag{5}$$

$$\mathrm{E}[U'(\tilde{\Pi}^*)(f_0 - \tilde{f})] = 0 \tag{6}$$

where $\tilde{\Pi}^* = (\tilde{f} + \gamma\tilde{f}^2 + \tilde{\epsilon})Q^* - C(Q^*) + (f_0 - \tilde{f})H^*$ and $(Q^*,\ H^*)$ is the optimal solution to equations (5) and (6).

In the case of linear risk exposure (i.e., $\gamma = 0$), it is easily verified that $H^* = Q^*$ solves equation (6). To see this, substituting $H^* = Q^*$ into the left-hand side of equation (6) and using the independence of $\tilde{f}$ and $\tilde{\epsilon}$ yields

$$\mathrm{E}\{U'[(f_0 + \tilde{\epsilon})Q^* - C(Q^*)]\}\mathrm{E}(f_0 - \tilde{f}) = 0$$

where the equality follows from the unbiasedness of the currency futures market. This is simply the well-known full-hedging theorem derived in the hedging literature (see, e.g., Danthine, 1978; Holthausen, 1979; Katz and Paroush, 1979; and Benninga *et al.*, 1983). Indeed, as shown by Lence (1995), this utility-free hedge position is optimal as long as $\tilde{f}$ is conditionally independent of $\tilde{\epsilon}$, a much weaker requirement than independence (see Ingersoll, 1987).

We are interested in studying how the nonlinear component, $\gamma\tilde{f}^2$, albeit arbitrarily small, would affect the optimal export and hedging decisions of the firm relative to the case of linear risk exposure. In other words, we are interested in performing the comparative static exercise with respect to γ, starting at $\gamma = 0$. To this end, we totally differentiate equations (5) and (6) with respect to γ and evaluate the resulting equations at $\gamma = 0$ to yield (see Appendix 1 for the derivation):

$$\frac{\mathrm{d}Q^*}{\mathrm{d}\gamma} = \frac{\mathrm{E}[U'(\tilde{\Pi}^*)] + \mathrm{E}\{U''(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]\}Q^*}{\mathrm{E}[U'(\tilde{\Pi}^*)]C''(Q^*) - \mathrm{E}\{U''(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]^2\}}\mathrm{E}(\tilde{f}^2) \tag{7}$$

$$\frac{\mathrm{d}H^*}{\mathrm{d}\gamma} = \frac{\mathrm{d}Q^*}{\mathrm{d}\gamma} + \frac{\mathrm{Cov}(\tilde{f},\ \tilde{f}^2)}{\mathrm{Var}(\tilde{f})}Q^* \tag{8}$$

where $\tilde{\Pi}^* = (f_0 + \tilde{\epsilon})Q^* - C(Q^*)$. Inspection of equation (8) leads to our first proposition.

Proposition 1 If the currency futures market is unbiased, the presence of convex (concave) exchange rate risk exposure induces the firm to opt for an over (under) hedge in the currency futures market.

Proof. Since $\mathrm{Cov}(\tilde{f},\ \tilde{f}^2) > 0$, it follows immediately from equation (8) that $\mathrm{d}H^*/\mathrm{d}\gamma > \mathrm{d}Q^*/\mathrm{d}\gamma$. □

Multiplying $-U'[\Pi^*(\epsilon)][f_0 + \epsilon - C'(Q^*)]$ to both sides of the above inequality and taking expectations with respect to the probability distribution function of $\tilde{\epsilon}$ yields

$$\mathrm{E}\{U''(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]\} > \frac{U''[\Pi^*(\hat{\epsilon})]}{U'[\Pi^*(\hat{\epsilon})]}\mathrm{E}\{U'(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]\} = 0$$

where the equality follows from equations (5) and (6). Using this and the fact that $C'' > 0$ and $U'' < 0$, equation (7) implies that $\mathrm{d}Q^*/\mathrm{d}\gamma > 0$. □

The intuition of Proposition 2 is as follows. Taking the variance of the firm's time T profit in equation (9) yields

$$\mathrm{Var}(\tilde{\Pi}) = (H-Q)^2\mathrm{Var}(\tilde{f}) + \gamma^2 Q^2 \mathrm{Var}(\tilde{f}^2) - 2\gamma Q(H-Q)\mathrm{Cov}(\tilde{f},\ \tilde{f}^2) + Q^2\mathrm{Var}(\tilde{\epsilon}). \quad (10)$$

Suppose that the firm does not change its production and hedging decisions when convex exchange rate risk exposure is introduced, i.e., the solution remains at $(Q^*,\ H^*)$ as if $\gamma = 0$. Given that the currency futures market is unbiased, we have $H^* = Q^*$. Inspection of equations (9) and (10) reveals that an infinitesimal increase in γ from zero increases the firm's time T profit by $\tilde{f}^2 Q^*$, while leaving the variance unchanged. Given DARA, the firm is willing to take on additional risk. This leads to increased production in response to convex exchange rate risk exposure. To reduce the associated higher risk level due to this aggressive production decision, the firm adjusts its short futures position upward by selling more futures. Similar intuition applies to the case under concave exchange rate risk exposure (i.e. an infinitesimal decrease in γ from zero).

5. Optimal hedging with currency futures and options

In this section, we resume the firm's ability to use the call options on futures to hedge against its exchange rate risk exposure. The first-order conditions for program (4) are given by

$$\mathrm{E}\{U'(\tilde{\Pi}^*)[\tilde{f} + \gamma \tilde{f}^2 + \tilde{\epsilon} - C'(Q^*)]\} = 0 \tag{11}$$

$$\mathrm{E}[U'(\tilde{\Pi}^*)(f_0 - \tilde{f})] = 0 \tag{12}$$

$$\mathrm{E}\{U'(\tilde{\Pi}^*)[p - \max\,(\tilde{f} - f_0,\ 0)]\} = 0 \tag{13}$$

where $\tilde{\Pi}^* = (\tilde{f} + \gamma \tilde{f}^2 + \tilde{\epsilon})Q^* - C(Q^*) + (f_0 - \tilde{f})H^* + [p - \max\,(\tilde{f} - f_0,\ 0)]Z^*$ and $(Q^*,\ H^*,\ Z^*)$ is the optimal solution to equations (11), (12), and (13).

In the case of linear risk exposure (i.e., $\gamma = 0$), we can show that $H^* = Q^*$ and $Z^* = 0$ solve equations (12) and (13). To see this, substituting $H^* = Q^*$ and $Z^* = 0$ into the left-hand side of equations (12) and (13), and using the independence of $\tilde{f}$ and $\tilde{\epsilon}$ yields

$$\mathrm{E}\{U'[(f_0 + \tilde{\epsilon})Q^* - C(Q^*)]\}\mathrm{E}(f_0 - \tilde{f}) = 0$$

$$\mathrm{E}\{U'[(f_0 + \tilde{\epsilon})Q^* - C(Q^*)]\}\mathrm{E}[p - \max\,(\tilde{f} - f_0,\ 0)] = 0$$

where the equalities follow from the joint unbiasedness of the currency futures and options markets. Thus, $H^* = Q^*$ and $Z^* = 0$ are indeed optimal, implying that the call options on futures play no hedging role in the case of linear risk exposure. Lapan *et al.* (1991) reach the same conclusion in the context of the competitive firm under output price uncertainty *à la* Sandmo (1971).

To study the effects of the nonlinear component, $\gamma \tilde{f}^2$, on the hedging decision of the firm, we use the same trick by performing the comparative static exercise with respect to γ, starting at $\gamma = 0$, conditional on the output level, Q^*. Totally differentiating equations (12) and (13) with respect to γ and evaluating the resulting equations at $\gamma = 0$ yields (see Appendix 2 for the derivation):

$$\frac{\mathrm{d}H^*}{\mathrm{d}\gamma} = \frac{\mathrm{Cov}(\tilde{f},\ \tilde{f}^2)}{\mathrm{Var}(\tilde{f})}Q^* - \frac{\mathrm{Cov}[\tilde{f},\ \max\,(\tilde{f} - f_0,\ 0)]}{\mathrm{Var}(\tilde{f})}\frac{\mathrm{d}Z^*}{\mathrm{d}\gamma} \tag{14}$$

$$\frac{\mathrm{d}Z^*}{\mathrm{d}\gamma} = \frac{\mathrm{Cov}[\tilde{f}^2,\ \max\ (\tilde{f} - f_0,\ 0)]\mathrm{Var}(\tilde{f}) - \mathrm{Cov}(\tilde{f},\ \tilde{f}^2)\mathrm{Cov}[\tilde{f},\ \max\ (\tilde{f} - f_0,\ 0)]}{\mathrm{Var}(\tilde{f})\mathrm{Var}[\max\ (\tilde{f} - f_0,\ 0)] - \mathrm{Cov}[\tilde{f},\ \max\ (\tilde{f} - f_0,\ 0)]^2} Q^* \tag{15}$$

where $\tilde{\Pi}^* = (f_0 + \tilde{\epsilon})Q^* - C(Q^*)$. Inspection of equations (14) and (15) reveals that the signs of $\mathrm{d}H^*/\mathrm{d}\gamma$ and $\mathrm{d}Z^*/\mathrm{d}\gamma$ depend only on the probability distribution function of $\tilde{f}$ and not on the firm's utility function. When $Z \equiv 0$, equation (14) reduces to equation (8), conditional on the output level being fixed at Q^*.

Equation (14) has the following intuitive interpretation. Conditional on the output level, Q^*, the firm's time T profit in equation (3) can be written as

$$\begin{aligned}\tilde{\Pi} &= [\tilde{f}Q^* + (f_0 - \tilde{f})H_1] + [\gamma\tilde{f}^2Q^* + (f_0 - \tilde{f})H_2] \\ &\quad + \{[p - \max\ (\tilde{f} - f_0,\ 0)]Z + (f_0 - \tilde{f})H_3\} + \tilde{\epsilon}Q^* - C(Q^*)\end{aligned}$$

where $H = H_1 + H_2 + H_3$. As in the previous section, the optimal hedge ratios for the first two risk components are $H_1/Q^* = 1$ and $H_2/\gamma Q^* = \mathrm{Cov}(\tilde{f},\ \tilde{f}^2)/\mathrm{Var}(\tilde{f})$. If the firm uses the call options on futures in response to nonlinear exchange rate risk exposure, the optimal hedge ratio for the third additional risk component equals the slope of the regression of $\max\ (\tilde{f} - f_0,\ 0)$ on $\tilde{f}$ and thus we have $H_3/Z = -\mathrm{Cov}[\tilde{f},\ \max\ (\tilde{f} - f_0,\ 0)]/\mathrm{Var}(\tilde{f})$.

In general, the sign of $\mathrm{d}Z^*/\mathrm{d}\gamma$ is determinate only when we know the underlying probability distribution function of $\tilde{f}$. We can, however, derive some unambiguous results by following Lapan *et al.* (1991) and Moschini and Lapan (1992, 1995) to focus on symmetric distributions. We state and prove the following proposition.

Proposition 3 If the currency futures and options markets are jointly unbiased and if the random futures exchange rate is symmetrically distributed, the presence of convex (concave) exchange rate risk exposure induces the firm to opt for a short (long) calls position in the currency options market.

Proof. Assume that $\tilde{f}$ is symmetrically distributed over support $[f_0 - \delta,\ f_0 + \delta]$ according to the cumulative distribution function, F, with mean f_0 and variance σ^2. Given this symmetry assumption and the joint unbiasedness of the currency futures and options markets, we have

$$\mathrm{Var}[\max\,(\tilde{f} - f_0,\ 0)] = \frac{\sigma^2}{2} - p^2 \tag{16}$$

$$\mathrm{Cov}[\tilde{f},\ \max\,(\tilde{f} - f_0,\ 0)] = \frac{\sigma^2}{2} \tag{17}$$

$$\mathrm{Cov}[\tilde{f}^2,\ \max\,(\tilde{f} - f_0,\ 0)] = f_0\sigma^2 + \int_{f_0}^{f_0+\delta} (f - f_0)^3\,\mathrm{d}F(f) - \sigma^2 p \tag{18}$$

$$\mathrm{Cov}(\tilde{f},\ \tilde{f}^2) = 2f_0\sigma^2 \tag{19}$$

Substituting equations (16) to (19) into equation (15) yields

$$\frac{\mathrm{d}Z^*}{\mathrm{d}\gamma} = \frac{4Q^*}{\sigma^2 - 4p^2}\int_{f_0}^{f_0+\delta} (f - f_0)[(f - f_0)^2 - \sigma^2]\,\mathrm{d}F(f) \tag{20}$$

Since $\int_{f_0}^{f_0+\delta}[(f - f_0)^2 - \sigma^2]\,\mathrm{d}F(f) = 0$, the integral in the right-hand side of equation (20) can be written as

$$\int_{f_0}^{f_0+\delta} (f - f_0 - \sigma)[(f - f_0)^2 - \sigma^2]\,\mathrm{d}F(f) = \int_{f_0}^{f_0+\delta} (f - f_0 - \sigma)^2(f - f_0 + \sigma)\,\mathrm{d}F(f)$$

which is strictly positive. Using the same equality, we have

$$\int_{f_0}^{f_0+\delta} [(f - f_0 - \sigma)^2 + 2\sigma(f - f_0 - \sigma)]\,\mathrm{d}F(f) = 0$$

which implies that

$$\frac{\sigma}{2} - p = \int_{f_0}^{f_0+\delta} [\sigma - (f - f_0)]\,\mathrm{d}F(f) = \frac{1}{2\sigma}\int_{f_0}^{f_0+\delta} (f - f_0 - \sigma)^2\,\mathrm{d}F(f) > 0$$

Thus, $\sigma^2 - 4p^2 = (\sigma - 2p)(\sigma + 2p) > 0$. It then follows from equation (20) that $\mathrm{d}Z^*/\mathrm{d}\gamma > 0$. □

The intuition of Proposition 3 is as follows. The firm's convex (concave) exchange rate risk exposure calls for a concave (convex) hedge position so as to offset the risk in a better way. A concave (convex) hedge position is created by shorting the futures contracts and concomitantly writing (buying) the call options on futures. Figure 1 shows how the combination of short futures and calls positions stabilizes the firm's time T profit when the firm faces convex exchange rate risk exposure. The combined payoff of the firm's hedge position approximates closer to the firm's nonlinear exchange rate risk exposure in a piecewise linear way.

(Insert Figure 1 here)

Moschini and Lapan (1992) show that production flexibility leads to *ex post* convex profit functions, thereby making options a useful instrument for hedging purposes. Sakong *et al.* (1993) and Moschini and Lapan (1995) show that production uncertainty, which affects the curvature of profit functions, provides another rationale for the hedging role of options. Lence *et al.* (1994) show that in a multiperiod setting options are used because future output prices affect profits from future production cycles. Proposition 3 adds to this scant literature on the hedging role of options in that when the underlying uncertainty is nonlinear in nature, the asymmetric payoff profiles of options (as opposed to linear futures contracts) are more suitable for hedging purposes.

6. Conclusions

In recent years, we have witnessed a growing literature on the optimal production and hedging decisions of a risk-averse exporting firm facing exchange rate uncertainty.

Unlike the extant literature which usually assumes a linear regression of the spot exchange rate on the futures exchange rate, this paper introduces a nonlinear spot-futures exchange rates relationship. This nonlinear specification is not as ad hoc as it appears because empirically some currencies do have spot-futures exchange rates relationships which reveal either convexity or concavity.

To examine the economic implications of a nonlinear spot-futures exchange rate relationship, this paper has developed an expected utility model of an exporting firm facing nonlinear exchange rate uncertainty. When the firm has access to an unbiased currency futures market, we have shown that it is optimal for the firm to export more (less) and adopt an over (under) hedge should the spot-futures exchange rates relationship be convex (concave) rather than linear. We have further shown that the firm would optimally use fairly priced currency options in conjunction with the futures contracts. The combined payoff of the firm's hedge position approximates closer to the firm's nonlinear exchange rate risk exposure in a piecewise linear way. This provides a rationale for the hedging role of options when the underlying uncertainty is nonlinear in nature.

Appendices

1. Derivation of equations (7) and (8)

Totally differentiating equations (5) and (6) with respect to γ, evaluating the resulting equations at $\gamma = 0$, and applying Cramer's rule yields

$$\frac{\mathrm{d}Q^*}{\mathrm{d}\gamma} = \frac{EU^*_{H\gamma}EU^*_{QH} - EU^*_{Q\gamma}EU^*_{HH}}{EU^*_{QQ}EU^*_{HH} - EU^*_{QH}EU^*_{HQ}} \tag{A.1}$$

$$\frac{\mathrm{d}H^*}{\mathrm{d}\gamma} = \frac{EU^*_{Q\gamma}EU^*_{QH} - EU^*_{H\gamma}EU^*_{QQ}}{EU^*_{QQ}EU^*_{HH} - EU^*_{QH}EU^*_{HQ}} \tag{A.2}$$

where $\tilde{\Pi}^* = (f_0 + \tilde{\epsilon})Q^* - C(Q^*)$, and

$$EU^*_{QQ} = \mathrm{E}\{U''(\tilde{\Pi}^*)[\tilde{f} + \tilde{\epsilon} - C'(Q^*)]^2\} - \mathrm{E}[U'(\tilde{\Pi}^*)]C''(Q^*) \tag{A.3}$$

$$EU^*_{HH} = \mathrm{E}[U''(\tilde{\Pi}^*)(f_0 - \tilde{f})^2] \tag{A.4}$$

$$EU^*_{QH} = EU^*_{HQ} = \mathrm{E}\{U''(\tilde{\Pi}^*)[\tilde{f} + \tilde{\epsilon} - C'(Q^*)](f_0 - \tilde{f})\} \tag{A.5}$$

$$EU^*_{Q\gamma} = \mathrm{E}[U'(\tilde{\Pi}^*)\tilde{f}^2] + \mathrm{E}\{U''(\tilde{\Pi}^*)[\tilde{f} + \tilde{\epsilon} - C'(Q^*)]\tilde{f}^2\}Q^* \tag{A.6}$$

$$EU^*_{H\gamma} = \mathrm{E}[U''(\tilde{\Pi}^*)(f_0 - \tilde{f})\tilde{f}^2]Q^* \tag{A.7}$$

It follows from the independence of $\tilde{f}$ and $\tilde{\epsilon}$ and from the unbiasedness of the currency futures market that equations (A.3) to (A.7) can be written as

$$EU^*_{QQ} = \mathrm{E}\{U''(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]^2\} - \mathrm{E}[U'(\tilde{\Pi}^*)]C''(Q^*) + EU^*_{HH} \tag{A.8}$$

$$EU^*_{HH} = \mathrm{E}[U''(\tilde{\Pi}^*)]\mathrm{Var}(\tilde{f}) \tag{A.9}$$

$$EU^*_{QH} = -EU^*_{HH}, \tag{A.10}$$

$$EU^*_{Q\gamma} = \Big\{\mathrm{E}[U'(\tilde{\Pi}^*)] + \mathrm{E}\{U''(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]\}Q^*\Big\}\mathrm{E}(\tilde{f}^2) - EU^*_{H\gamma} \tag{A.11}$$

$$EU^*_{H\gamma} = -\mathrm{E}[U''(\tilde{\Pi}^*)]\mathrm{Cov}(\tilde{f},\ \tilde{f}^2)Q^* \tag{A.12}$$

Using equations (A.8), (A.9), and (A.10), the denominators in the right-hand side of equations (A.1) and (A.2) are given by

$$-\Big\{\mathrm{E}[U'(\tilde{\Pi}^*)]C''(Q^*) - \mathrm{E}\{U''(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]^2\}\Big\}EU^*_{HH} \tag{A.13}$$

Using equations (A.9), (A.10), and (A.11), the numerator in the right-hand side of equation (A.1) becomes

$$-\Big\{\mathrm{E}[U'(\tilde{\Pi}^*)] + \mathrm{E}\{U''(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]\}Q^*\Big\}\mathrm{E}(\tilde{f}^2)EU^*_{HH} \tag{A.14}$$

Using equations (A.8), (A.10), and (A.11), the numerator in the right-hand side of equation (A.2) becomes

$$\begin{aligned}&\Big\{\mathrm{E}[U'(\tilde{\Pi}^*)]C''(Q^*) - \mathrm{E}\{U''(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]^2\}\Big\}EU^*_{H\gamma}\\ &\quad -\Big\{\mathrm{E}[U'(\tilde{\Pi}^*)] + \mathrm{E}\{U''(\tilde{\Pi}^*)[f_0 + \tilde{\epsilon} - C'(Q^*)]\}Q^*\Big\}\mathrm{E}(\tilde{f}^2)EU^*_{HH}\end{aligned} \tag{A.15}$$

Dividing expression (A.14) by expression (A.13) yields equation (7). Dividing expression (A.15) by expression (A.13) and using equations (7), (A.9), and (A.12) yields equation (8). □

2. Derivation of equations (14) and (15)

Totally differentiating equations (12) and (13) with respect to γ, evaluating the resulting equations at $\gamma = 0$, and applying Cramer's rule yields

$$\frac{\mathrm{d}H^*}{\mathrm{d}\gamma} = \frac{EU^*_{Z\gamma}EU^*_{HZ} - EU^*_{H\gamma}EU^*_{ZZ}}{EU^*_{HH}EU^*_{ZZ} - EU^*_{HZ}EU^*_{ZH}} \tag{A.16}$$

$$\frac{\mathrm{d}Z^*}{\mathrm{d}\gamma} = \frac{EU^*_{H\gamma}EU^*_{HZ} - EU^*_{Z\gamma}EU^*_{HH}}{EU^*_{HH}EU^*_{ZZ} - EU^*_{HZ}EU^*_{ZH}} \tag{A.17}$$

where $\tilde{\Pi}^* = (f_0 + \tilde{\epsilon})Q^* - C(Q^*)$,

$$EU^*_{ZZ} = \mathrm{E}\{U''(\tilde{\Pi}^*)[p - \max\,(\tilde{f} - f_0,\ 0)]^2\} \tag{A.18}$$

$$EU^*_{HZ} = EU^*_{ZH} = \mathrm{E}\{U''(\tilde{\Pi}^*)(f_0 - \tilde{f})[p - \max\,(\tilde{f} - f_0,\ 0)]\} \tag{A.19}$$

$$EU^*_{Z\gamma} = \mathrm{E}[U''(\tilde{\Pi}^*)[p - \max\,(\tilde{f} - f_0,\ 0)]\tilde{f}^2\}Q^* \tag{A.20}$$

and EU^*_{HH} and $EU^*_{H\gamma}$ are defined in equations (A.9) and (A.12), respectively. It follows from the independence of $\tilde{f}$ and $\tilde{\epsilon}$ and from the joint unbiasedness of the currency futures and options market that equations (A.18), (A.19), and (A.20) can be written as

$$EU^*_{ZZ} = \mathrm{E}[U''(\tilde{\Pi}^*)]\mathrm{Var}[\max\,(\tilde{f} - f_0,\ 0)] \tag{A.21}$$

$$EU^*_{HZ} = \mathrm{E}[U''(\tilde{\Pi}^*)]\mathrm{Cov}[\tilde{f},\ \max\,(\tilde{f} - f_0,\ 0)]\} \tag{A.22}$$

$$EU^*_{Z\gamma} = -\mathrm{E}[U''(\tilde{\Pi}^*)]\mathrm{Cov}[\tilde{f}^2,\ \max\,(\tilde{f} - f_0,\ 0)] \tag{A.23}$$

Using equations (A.8), (A.9), and (A.10), the denominators in the right-hand side of equations (A.16) and (A.17) are given by

$$\mathrm{E}[U''(\tilde{\Pi}^*)]^2\{\mathrm{Var}(\tilde{f})\mathrm{Var}[\max\,(\tilde{f} - f_0,\ 0)] - \mathrm{Cov}[\tilde{f},\ \max\,(\tilde{f} - f_0,\ 0)]^2\} \tag{A.24}$$

Using equations (A.12), (A.21), (A.22), and (A.23), the numerator in the right-hand side of equation (A.16) becomes

$$\begin{aligned}&\mathrm{E}[U''(\tilde{\Pi}^*)]^2Q^*\{\mathrm{Cov}(\tilde{f},\ \tilde{f}^2)\mathrm{Var}[\max\,(\tilde{f} - f_0,\ 0)]\\&\qquad -\mathrm{Cov}[\tilde{f}^2,\ \max\,(\tilde{f} - f_0,\ 0)]\mathrm{Cov}[\tilde{f},\ \max\,(\tilde{f} - f_0,\ 0)]\}\end{aligned} \tag{A.25}$$

Using equations (A.9), (A.12), (A.22), and (A.23), the numerator in the right-hand side of equation (A.17) becomes

$$\begin{aligned}&\mathrm{E}[U''(\tilde{\Pi}^*)]^2Q^*\{\mathrm{Cov}[\tilde{f}^2,\ \max\,(\tilde{f} - f_0,\ 0)]\mathrm{Var}(\tilde{f})\\&\qquad -\mathrm{Cov}(\tilde{f},\ \tilde{f}^2)\mathrm{Cov}[\tilde{f},\ \max\,(\tilde{f} - f_0,\ 0)]\}\end{aligned} \tag{A.26}$$

Dividing expression (A.26) expression (A.24) yields equation (15). Dividing expression (A.25) by expression (A.24) and using equation (15) yields equation (14). □

References

Adam-Müller, A.F.A. (1997). 'Export and Hedging Decisions under Revenue and Exchange Rate Risk: A Note', *European Economic Review*, **41**, 1412–26.

Anderson, R.W. and Danthine, J.-P. (1981). 'Cross Hedging', *Journal of Political Economy*, **89**, 1182–96.

Banerjee, A. Dolado, J.J., Hendry, D.F., and Smith, G.W. (1986). 'Exploring Equilibrium Relationship in Econometrics Through Static Models: Some Monte Carlo Evidence', *Oxford Bulletin of Economics and Statistics*, **48**, 253–77.

Benninga, S., Eldor, R., and Zilcha, I. (1983). 'Optimal Hedging in the Futures Market under Price Uncertainty', *Economics Letters*, **13**, 141–5.

Benninga, S., Eldor, R., and Zilcha, I. (1985). 'Optimal International Hedging and Output Policies in Commodity and Currency Forward Markets', *Journal of International Money and Finance*, 4, 537–52.

Briys, E., Crouhy, M., and Schlesinger, H. (1993). 'Optimal Hedging in a Futures Market with Background Noise and Basis Risk', *European Economic Review*, **37**, 949–60.

Broll, U., Wahl, J.E., and Zilcha, I. (1995). 'Indirect Hedging of Exchange Rate Risk', *Journal of International Money and Finance*, **14**, 667-78.

Broll, U. and Wong, K.P. (1999). 'Hedging with Mismatched Currencies', *Journal of Futures Markets*, **19**, Forthcoming.

Broll, U., Wong, K.P., and Zilcha, I. (1998). 'Multiple Currencies and Hedging', *Economica*, Forthcoming.

Broll, U. and Zilcha, I. (1992). 'Exchange Rate Uncertainty, Futures Markets and the Multinational Firm', *European Economic Review*, **36**, 815–26.

Cox, J. and Rubinstein, M. (1985). *Options Markets*, Prentice-Hall, Englewood Cliff, NJ.

Danthine, J.-P. (1978). 'Information, Futures Prices, and Stabilizing Speculation', *Journal of Economic Theory*, **19**, 79–98.

Dickey, D.A. and Fuller, W.A. (1979). 'Distribution of the Estimators for Autoregressive Time Series with a Unit Root', *Journal of American Statistical Association*, 74, 427–31.

Dickey, D.A. and Fuller, W.A. (1981). 'Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root', *Econometrica*, **49**, 1057-72.

Engle, R.F. and Granger, C.W.J. (1987). 'Co-integration and Error Correction: Representation, Estimation and Testing', *Econometrica*, **55**, 251-76.

Granger, C.W.J. and Newbold, P. (1974). 'Spurious Regressions in Econometrics', *Journal of Econometrics*, **2**, 111-20.

Holthausen, D.M. (1979). 'Hedging and the Competitive Firm under Price Uncertainty', *American Economic Review*, **69**, 989–95.

Ingersoll, J.E., Jr. (1987). *Theory of Financial Decision Making*, Rowman & Littlefield, Savage, MD.

Katz, E. and Paroush, J. (1979). 'The Effect of Forward Markets on Exporting Firms', *Economics Letters*, **4**, 272-4.

Lapan, H., Moschini, G., and Hanson, S.D. (1991). 'Production, Hedging, and Speculative Decisions with Options and Futures Markets', *American Journal of Agricultural Economics*, **73**, 66–74.

Lence, S.H. (1995). 'On the Optimal Hedge under Unbiased Futures Prices', *Economics Letters*, **47**, 385–8.

Lence, S.H., Sakong, Y., and Hayes, D.J. (1994). 'Multiperiod Production with Forward and Option Markets', *American Journal of Agricultural Economics*, **76**, 286–95.

Meese, R. (1990). 'Currency Fluctuations in the Post-Bretton Woods Era', *Journal of Economic Perspectives*, **4**, 117-34.

Moschini, G. and Lapan, H. (1992). 'Hedging Price Risk with Options and Futures for the Competitive Firm with Production Flexibility', *International Economic Review*, **33**, 607–18.

Moschini, G. and Lapan, H. (1995). 'The Hedging Role of Options and Futures under Joint Price, Basis, and Production risk', *International Economic Review*, **36**, 1025–49.

Paroush, J. and Wolf, A. (1986). 'Production and Hedging Decisions in Futures and Forward Markets', *Economics Letters*, **21**, 139–43.

Paroush, J. and Wolf, A. (1989). 'Production and Hedging Decisions in the Presence of Basis Risk', *Journal of Futures Markets*, **9**, 547–63.

Phillips, P.C.B. (1986). 'Understanding Spurious Regressions'. *Journal of Econometrics*, **33**, 311-40.

Rawls, S.W. and Smithson, C.W. (1990). 'Strategic Risk Management', *Journal of Applied Corporate Finance*, 1, 6–18.

Sakong, Y., Hayes, D.J., and Hallam, H. (1993). 'Hedging Production Risk with Options', *American Journal of Agricultural Economics*, **75**, 408–15.

Sandmo, A. (1971). 'On the Theory of the Competitive Firm under Price Uncertainty, *American Economic Review*, **61**, 65–73.

Sercu, P. and Uppal, R. (1995). *International Financial Markets and the Firm*, Southern-Western College Publishing, Cincinnati, Ohio.

Stock, J.H. (1987). 'Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors', *Econometrica*, **55**, 1035-56.

Viaene, J.-M. and Zilcha, I. (1998). 'The Behavior of Competitive Exporting Firms under Multiple Uncertainty', *International Economic Review*, **39**, 591–609.

Table 1 Augmented Dickey-Fuller tests for unit roots

Variable	Without a time trend	With a time trend
AUDS	-0.899	-1.797
AUDF	-0.958	-1.807
AUDF2	-0.895	-1.776
GBPS	-2.485	-3.018
GBPF	-2.515	-2.827
GBPF2	-2.669*	-3.066
CADS	-1.549	-1.966
CADF	-1.322	-1.988
CADF2	-1.523	-2.080
DEMS	-0.979	-2.663
DEMF	-1.032	-2.686
DEMF2	-0.927	-2.471
FRFS	-1.131	-2.464
FRFF	-0.949	-2.397
FRFF2	-0.975	-2.390
JPYS	-1.361	-2.420
JPYF	-1.358	-2.447
JPYF2	-1.428	-2.481

Notes: * indicates that the test statistic rejects the null hypothesis of the presence of a unit root at 10% level of significance. The number of lags and the critical value of individual test statistic are produced by the unit root and cointegration procedure of SHAZAM 8.0.

Table 2 Cointegration tests for various currencies

Currency	Without a time trend	With a time trend
AUD	-3.596*	-4.278*
GBP	-4.606*	-4.614*
CAD	-2.184	-3.956*
DEM	-4.122*	-5.793*
FRF	-2.931	-4.162*
JPY	-6.174*	-6.817*

Notes: * indicates that the test statistic rejects the null hypothesis of the presence of a unit root at 10% level of significance. The number of lags and the critical value of individual test statistic are produced by the unit root and cointegration procedure of SHAZAM 8.0.

Table 3 Spot-futures exchange rates relationships for various currencies

Currency	$\hat{\alpha}$	Time trend	$\hat{\beta}$	$\hat{\gamma}$	Adj. R^2
AUD	0.002		0.986**	0.018	0.9990
GBP	0.426**		0.456**	0.174**	0.9969
CAD	-0.006	$-0.479 \times 10^{-5**}$	1.086**	-0.101*	0.9981
DEM	-0.031**		1.098**	-0.080**	0.9992
FRF	0.032	$-0.103 \times 10^{-5**}$	0.655**	0.929**	0.9990
JPY	-0.026**		1.046**	-0.027**	0.9989

Note: * and ** indicate that the coefficients are statistically significant at 5% and 1% levels of significance, respectively.

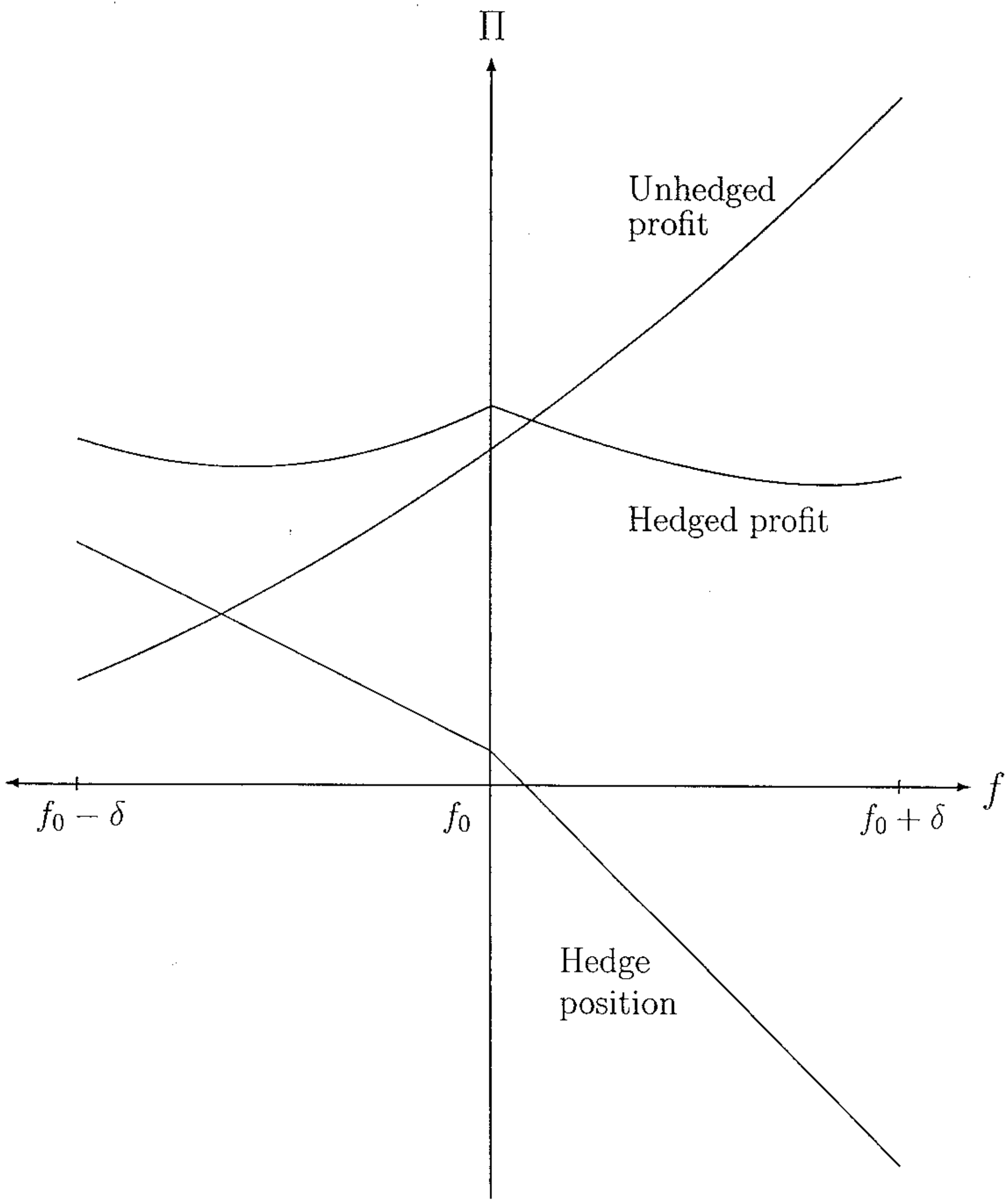

Fig. 1. Hedged profit under convex exchange rate risk exposure