Proceedings of the Institution of Civil Engineers
Structures & Buildings 152
November 2002 Issue 4
Pages 361–369

Paper 12560

Received 30/01/2001
Accepted 10/06/2002

Keywords:
beams & girders/concrete structures/design methods & aids

A. K. H. Kwan
Department of Civil Engineering, The University of Hong Kong

J. C. M. Ho
Department of Civil Engineering, The University of Hong Kong

H. J. Pam
Department of Civil Engineering, The University of Hong Kong

Flexural strength and ductility of reinforced concrete beams

A. K. H. Kwan, J. C. M. Ho and H. J. Pam

In the design of reinforced concrete beams, especially those made of high-strength concrete and those in earthquake-resistant structures, both the flexural strength and ductility need to be considered. From the numerical results obtained in a previous study on the post-peak behaviour and flexural ductility of reinforced concrete beams, the interrelation between the flexural strength and the flexural ductility that could be simultaneously achieved was evaluated and plotted in the form of charts. Using these charts, a new method of beam design called 'concurrent flexural strength and ductility design' that would allow engineers to consider both the strength and ductility requirements at the same time before deciding on whether to use high-strength concrete or add compression reinforcement has been developed. For application to cases in which the concrete grade is prescribed, a simpler method of first determining the limits of steel ratios that would satisfy the ductility requirement and then designing the reinforcement details according to the strength requirement has also been proposed. Examples are presented to illustrate the application of these methods.

NOTATION

A_{sc}, A_{st}	areas of compression and tension reinforcement
b, d	breadth and effective depth of beam section
d_1	depth of compression reinforcement
d_n	neutral axis depth
E_c, E_s	Young's moduli of concrete and steel reinforcement
f_c	cylinder compressive strength of concrete
f_{co}	*in situ* uniaxial compressive strength of concrete
f_y	yield strength of steel reinforcement
h	total depth of beam section
M	resisting moment of beam section
M_p	peak resisting moment of beam section
P	applied axial load to beam section
x	distance from neutral axis
ε_{co}	strain in concrete at peak stress
ε_p	residual plastic strain in steel reinforcement
ε_s	strain in steel reinforcement
ε_y	yield strain of steel reinforcement
μ	ductility factor
μ_{min}	specified minimum ductility factor
ρ_b	balanced steel ratio of beam section
ρ_{bo}	balanced steel ratio of beam section with no compression steel
ρ_c, ρ_t	compression steel ratio ($\rho_c = A_{sc}/bd$) and tension steel ratio ($\rho_t = A_{st}/bd$)
σ_c, σ_s	stresses in concrete and steel reinforcement
σ_{sc}, σ_{st}	stresses in compression and tension reinforcement
φ	curvature of beam section
φ_u, φ_y	ultimate and yield curvatures of beam section
$()_{max} =$	maximum value of ()

1. INTRODUCTION

In the design of a reinforced concrete beam, both the flexural strength and ductility need to be considered. Although usually more attention is paid to the flexural strength and only a simple check is carried out to ensure that a certain minimum level of flexural ductility is provided by keeping the beam under-reinforced, this does not mean that the flexural ductility is unimportant. From the structural safety point of view, ductility is at least as important as strength. A good ductility would provide the beam with a much better chance of survival when it is overloaded, subjected to accidental impact or attacked by a severe earthquake.

In recent years, because of the relatively high strength/weight ratio and other obvious advantages, high-strength concrete is becoming more and more popular.[1,2] However, there is one major problem with high-strength concrete; it is generally more brittle than normal-strength concrete. It has been found during an experimental study by Pam *et al.*[3] that a reinforced high-strength concrete beam, if not properly designed, could fail in a rather brittle manner. Thus, particular attention is required, when designing a high-strength concrete beam, to ensure that a minimum level of flexural ductility is provided. However, the study by Pam *et al.* has also indicated that just keeping the beam under-reinforced might not be sufficient to ensure that the high-strength concrete beam would be provided with the same minimum level of flexural ductility that is normally provided in a normal-strength concrete beam.

Ductility is particularly important in earthquake-resistant structures. Some engineers hesitate to use high-strength concrete in any structure located in a seismic region because of its higher brittleness. In actual fact, the ductility performance of a reinforced concrete member does not increase or decrease in direct proportion to the ductility of the concrete used and is dependent also on other parameters such as the reinforcement details. With proper detailing, it should be possible to design a high-strength concrete member to have at least the same ductility as that of a similar normal-strength concrete member. For instance, in the case of a column, the ductility could be restored to a higher level by adding more confining reinforcement[4] and, in the case of a beam, the ductility could be increased by adding compression reinforcement, as will be seen in this paper. However, theoretically, an earthquake-resistant structure should be provided with a better than normal ductility. This would require careful ductility design of each individual member, which is not going to be easy, especially when high-strength concrete is used.

The ductility demand of a structural member is dependent on the level of impact or earthquake load that it would be subjected to, how early it would yield before maximum impact/earthquake response and the structural form, etc. Hence, the ductility demand may vary from one member to another member. The provision of a fixed minimum level of ductility to all members is overly simplistic. But, to provide different levels of ductility to suit different situations, quantitative analysis of the ductility of each individual member is required. However, even in the simple case of a reinforced concrete beam, there is no simple method for direct evaluation of flexural ductility. In order to determine the flexural ductility of a beam section, it is first necessary to analyse the complete moment–curvature relation of the section and then calculate the amount of inelastic curvature that the section can sustain before the onset of flexural failure. Whatever the method of analysis employed, a non-linear structural analysis using the actual stress–strain curves of the constitutive materials is required. Up to now, only limited analysis of the complete moment–curvature relation of reinforced concrete sections has been carried out[5–8] and as a result there have been few data on the flexural ductility of reinforced concrete beams.

The authors have recently developed a new method of analysing the complete moment–curvature behaviour of reinforced concrete beams that not only uses the actual stress–strain curves but also takes into account the stress-path dependence of the constitutive properties of the materials.[9,10] Analysis of reinforced concrete beams using this method revealed that at the post-peak stage the neutral axis depth keeps on increasing and, beyond a certain point, the strain in the tension reinforcement starts to decrease. To cater for such strain reversal, the stress-path dependence of the stress–strain relation of the steel reinforcement must be taken into account. In fact, the numerical results had indicated that the negligence of the stress-path dependence of the material properties in the previous analysis methods developed by others[5–8] could lead to significant errors in the moment–curvature relation and flexural ductility. Using this newly developed analysis method, a parametric study has been carried out to evaluate the effects of various structural parameters on the flexural ductility of reinforced normal- and high-strength concrete beams.

In this study, a new design method for reinforced concrete beams that would allow concurrent consideration of the flexural strength and ductility requirements has been developed. It is based on rigorous non-linear flexural analysis of singly and doubly reinforced concrete beam sections using the newly developed method. Using the results of the analysis, design charts correlating the flexural strength and ductility of the beam sections to various structural parameters have been produced. Using these charts, the concrete grade, tension steel ratio and compression steel ratio that would simultaneously satisfy the given flexural strength and ductility requirements can be determined directly. For application to cases in which the concrete grade is prescribed, a simpler design method of first determining the limits of tension and compression steel ratios that would satisfy the flexural ductility requirement and then designing the reinforcement details according to the flexural strength requirement has also been developed.

2. METHOD OF ANALYSIS

Since details of the method of analysis used have been given previously,[9,10] only an outline of the method is presented here.

Three basic assumptions are made in the analysis

- plane sections before bending remain plane after bending
- the tensile strength of the concrete may be neglected
- there is no bond-slip between the reinforcement bars and the concrete.

They are all commonly accepted and are nearly exact except in deep beams or in localised areas near cracks. For convenience, the sign conventions adopted are such that all strain and stress quantities are positive, as listed in the following

- compressive strain and stress in concrete are positive
- compressive strain and stress in compression reinforcement are positive
- tensile strain and stress in tension reinforcement are positive.

Referring to Fig. 1 and denoting the curvature of the beam by φ, the strain developed in the beam section is given by

$$\varepsilon = \varphi x \qquad (1)$$

Having obtained the strain values, the stresses developed in the concrete and the steel reinforcement may be evaluated from their respective stress–strain curves.

For the concrete, the complete stress–strain curve model developed by Attard and Setunge,[11] which has been shown to

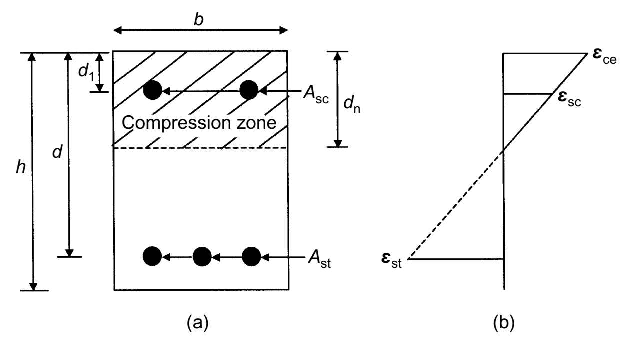

Fig. 1. A beam section subjected to bending moment: (a) beam section; (b) strain distribution

be applicable to a broad range of concrete strengths from 20 to 130 MPa, is used. The stress–strain curve is given by

$$\boldsymbol{\sigma}_c/\boldsymbol{f}_{co} = \frac{A(\boldsymbol{\varepsilon}_c/\boldsymbol{\varepsilon}_{co}) + B(\boldsymbol{\varepsilon}_c/\boldsymbol{\varepsilon}_{co})^2}{1 + (A-2)(\boldsymbol{\varepsilon}_c/\boldsymbol{\varepsilon}_{co}) + (B+1)(\boldsymbol{\varepsilon}_c/\boldsymbol{\varepsilon}_{co})^2} \quad (2)$$

in which $\boldsymbol{\sigma}_c$ and $\boldsymbol{\varepsilon}_c$ are the stress and strain in the concrete, $\boldsymbol{f}_{co}$ is the peak stress and $\boldsymbol{\varepsilon}_{co}$ is the strain at peak stress. It should be noted that the peak stress $\boldsymbol{f}_{co}$ is actually the *in situ* uniaxial compressive strength of the concrete, which may be determined from the standard cube or cylinder strengths using appropriate correction factors. The parameters A and B defining the shape of the curve are as given by Attard and Stewart.[12] Some typical stress–strain curves so derived are shown in Fig. 2.

For the steel reinforcement, a bilinear stress–strain curve is employed. To cater for strain reversal, the stress-path dependence of the stress–strain relationship is taken into account by assuming that the unloading path follows the initial elastic slope, as in Fig. 3. When the strain is increasing, the stress in the steel is given by

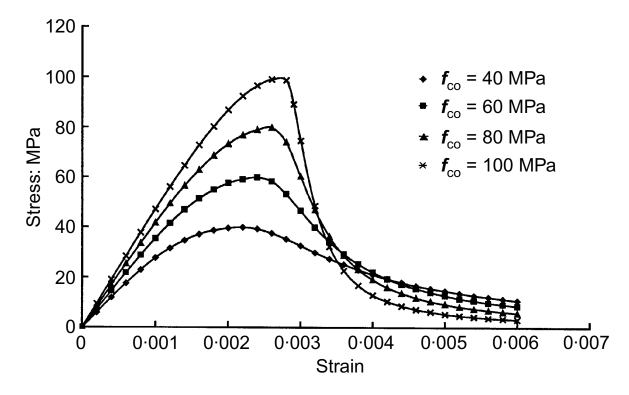

Fig. 2. Stress–strain curves of concrete derived from Attard and Setunge's model[11]

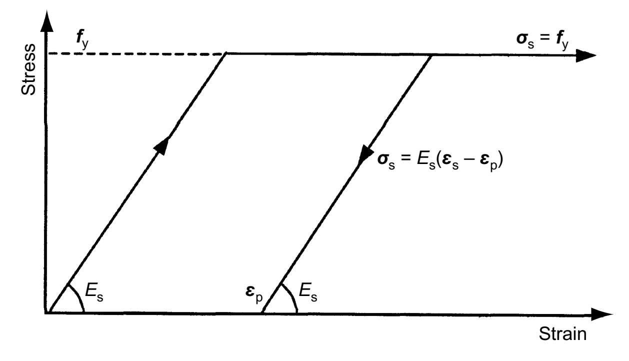

Fig. 3. Stress–strain curve of steel with allowance for stress-path dependence

- at elastic stage

$$\boldsymbol{\sigma}_s = E_s \times \boldsymbol{\varepsilon}_s \quad (3a)$$

- after yielding

$$\boldsymbol{\sigma}_s = \boldsymbol{f}_y \quad (3b)$$

in which $\boldsymbol{\sigma}_s$ and $\boldsymbol{\varepsilon}_s$ are the stress and strain in the steel, respectively, E_s is the Young's modulus and $\boldsymbol{f}_y$ is the yield stress. On the other hand, when the strain is decreasing, the stress in the steel becomes

$$\boldsymbol{\sigma}_s = E_s(\boldsymbol{\varepsilon}_s - \boldsymbol{\varepsilon}_p) \quad (4)$$

where $\boldsymbol{\varepsilon}_p$ is the residual strain at the end of the last strain increasing cycle.

The stresses developed in the beam section must satisfy the following axial and moment equilibrium conditions

$$\boldsymbol{P} = \int_0^{d_n} \sigma_c b \, dx + \sum A_{sc}\boldsymbol{\sigma}_{sc} - \sum A_{st}\boldsymbol{\sigma}_{st} \quad (5)$$

$$M = \int_0^{d_n} \boldsymbol{\sigma}_c bx \, dx + \sum A_{sc}\boldsymbol{\sigma}_{sc}(d_n - d_1) + \sum A_{st}\boldsymbol{\sigma}_{st}(d - d_n) \quad (6)$$

in which $\boldsymbol{P}$ is the axial load and M the resisting moment.

The moment–curvature relation of the beam section is analysed by applying prescribed curvature to the section incrementally in small steps starting from zero. For a given curvature, the strains developed in the section are first evaluated based on an assumed or the previous value of the neutral axis depth. From the strains so obtained, the stresses developed in the concrete and the steel are determined from their respective stress–strain curves. Axial equilibrium of the section is then checked. Normally, the axial equilibrium condition is not immediately satisfied and there is an unbalanced axial force. An iterative procedure of successively adjusting the neutral axis depth until the unbalanced axial force is negligibly small is used to satisfy the axial equilibrium condition. Having determined the neutral axis depth, the resisting moment of the section is evaluated from the moment equilibrium condition. This gives a pair of curvature and resisting moment values. The numerical process is repeated for each prescribed curvature value and continued until the curvature is large enough for the resisting moment to increase to the peak and decrease to less than 50% of the peak value. From the moment–curvature curve, the flexural strength M_p is determined as the resisting moment at the peak and the flexural ductility evaluated from the yield and ultimate curvatures as will be explained later. A computer program for the above analysis has been developed using MathCad 7 Professional Edition.

3. PARAMETRIC STUDY

3.1. Sections analysed

The beam sections analysed are the same as the one shown in Fig. 1. These beam sections are given constant dimensions of $b = 300$ mm, $h = 600$ mm, $d = 550$ mm, and $d_1 = 50$ mm. For parametric study, the *in situ* concrete compressive strength $\boldsymbol{f}_{co}$ is varied from 30 to 100 MPa to cover both normal- and high-strength concretes, the compression steel ratio ρ_c ($\rho_c = A_{sc}/bd$) is varied from 0 to 1·5% to cover singly and doubly reinforced sections, and the tension steel ratio ρ_t ($\rho_t = A_{st}/bd$) is varied from 0 to 1·5 times the balanced steel ratio to cover under-reinforced and over-reinforced sections. On the other hand, the steel reinforcement is assumed to have constant properties with $\boldsymbol{f}_y = 460$ MPa and $E_s = 200$ GPa.

3.2. Moment–curvature curves

Some selected moment–curvature curves of the sections analysed are shown in Fig. 4. It is seen that the moment–curvature curves of under- and over-reinforced sections have very different shapes. In the case of an under-reinforced section, the moment–curvature curve is almost linear before the peak moment is reached and there is a fairly long yield plateau at the post-peak stage before the resisting moment drops more rapidly until complete failure, indicating a ductile mode of failure. However, in the case of an over-reinforced section, the moment–curvature curve is more like a single smooth curve with a sharp peak indicating a brittle mode of failure.

3.3. Failure mode and balanced steel ratio

Three failure modes have been observed:

- tension failure under which the tension reinforcement yields before the concrete fails
- compression failure under which the tension reinforcement remains unyielded even when the concrete has failed completely
- balanced failure under which the tension reinforcement just yields when the concrete fails.

The tension steel ratio that leads to balanced failure is called balanced steel ratio and denoted hereafter by ρ_b.

The balanced steel ratio may be evaluated by a trial-and-error process of analysing beam sections with different tension steel ratios and checking whether the tension reinforcement has ever yielded. It has been found during such analysis that at a relatively low tension steel ratio, the tension reinforcement yields right at the point of peak moment. However, at a relatively high tension steel ratio close to the balanced steel ratio, the tension reinforcement does not yield at the point of peak moment, but rather yields within the yield plateau range after the point of peak moment. So long as the tension reinforcement yields before the beam section fails completely, regardless of when it yields, the beam section is regarded as an under-reinforced section. If the tension reinforcement just yields before strain reversal as the beam section is loaded until complete failure, the beam section is regarded as a balanced section and its tension steel ratio taken as the balanced steel ratio.

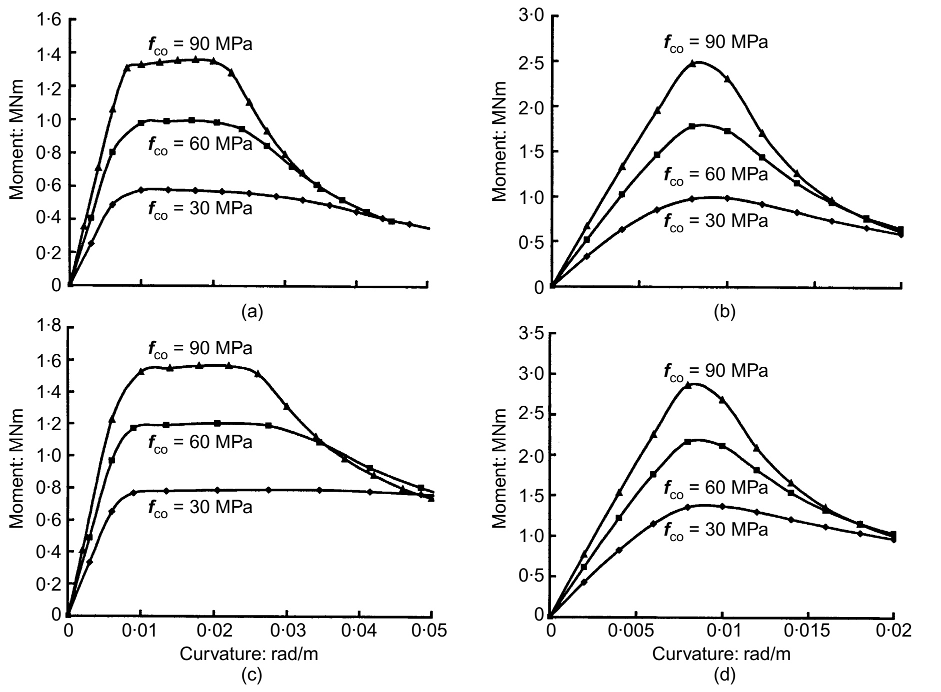

Fig. 4. Complete moment–curvature curves of some beam sections analysed: (a) under-reinforced and with no compression reinforcement ($\rho_t/\rho_b = 0{\cdot}5$, $\rho_c = 0\%$); (b) over-reinforced and with no compression reinforcement ($\rho_t/\rho_b = 1{\cdot}5$, $\rho_c = 0\%$); (c) under-reinforced and with compression reinforcement ($\rho_t/\rho_b = 0{\cdot}5$, $\rho_c = 1\%$); (d) over-reinforced and with compression reinforcement ($\rho_t/\rho_b = 1{\cdot}5$, $\rho_c = 1\%$)

The balanced steel ratio, ρ_b, so obtained for a given concrete strength is found to increase linearly with the compression steel ratio, ρ_c, and is given by

$$\rho_b = \rho_{bo} + \rho_c \tag{7}$$

where ρ_{bo} is the balanced steel ratio of the beam section when no compression reinforcement is provided. The values of ρ_{bo} for different concrete grades are listed in the second column of Table 1. It is seen that ρ_{bo} increases with the concrete grade but not in direct proportion.

3.4. Flexural ductility

The flexural ductility is measured in terms of a ductility factor, μ, given by

$$\mu = \varphi_u/\varphi_y \tag{8}$$

where ϕ_u and φ_y are the

f_{co}: MPa	ρ_{bo}: %	Maximum value of $(\rho_t - \rho_c)/\rho_{bo}$	Maximum value of $(\rho_t - \rho_c)$: %
30	3·19	0·75	2·39
40	3·95	0·68	2·67
50	4·69	0·62	2·93
60	5·39	0·58	3·15
70	6·06	0·55	3·35
80	6·70	0·53	3·53
90	7·30	0·51	3·69
100	7·87	0·49	3·83

Table 1. Balanced steel ratios and maximum values of $(\rho_t - \rho_c)$ for $\mu_{min} = 3{\cdot}32$

ultimate curvature and yield curvature, respectively. The ultimate curvature, φ_u, is taken as the curvature at which the resisting moment has, after reaching the peak, dropped to 0·80 of the peak moment. On the other hand, the yield curvature, φ_y, is defined as the curvature at the hypothetical yield point of an equivalent elasto-plastic system whose equivalent elastic stiffness is taken as the secant stiffness at 0·75 of the peak moment before the peak moment is reached and yield strength is taken as the peak moment; the yield curvature so defined is actually equal to the curvature at 0·75 of the peak moment divided by 0·75.

The values of μ so evaluated are plotted against the corresponding values of tension steel ratio, ρ_t, in Fig. 5. It is seen that at a given concrete grade, the ductility factor decreases with the tension steel ratio but increases with the compression steel ratio. However, the effect of the concrete grade is more complicated. At given compression and tension steel ratios, the ductility factor seems to increase slightly with the concrete grade albeit a higher grade concrete should be less ductile. This is because the major factor affecting the flexural ductility is actually the degree of the beam section being under-reinforced or over-reinforced. As the concrete grade increases, the balanced steel ratio, ρ_b, also increases and consequently the tension steel to balanced steel ratio ρ_t/ρ_b is reduced, leading to an increase in the degree of being under-reinforced or a decrease in the degree of being over-reinforced. The increase in flexural ductility due to the reduction in the ρ_t/ρ_b ratio has outweighed the decrease in flexural ductility due to the reduction in ductility of the concrete.

In Reference 10, the ductility factor, μ, has been correlated to the concrete grade, tension steel ratio and compression steel ratio by regression analysis and the following formula for direct evaluation of flexural ductility derived

$$\mu = 10{\cdot}7(\boldsymbol{f}_{co})^{-0{\cdot}45}[(\rho_t - \rho_c)/\rho_{bo}]^{-1{\cdot}25} [1 + 95{\cdot}2(\boldsymbol{f}_{co})^{-1{\cdot}1}(\rho_c/\rho_t)^3] \quad (9)$$

in which ρ_t should be taken as equal to ρ_b when ρ_t is greater than ρ_b. Within the ranges of structural parameters studied, the values of μ obtained by this formula are accurate to within 10% error. It can be seen from this formula that at a given ρ_t/ρ_b or $(\rho_t - \rho_c)/(\rho_{bo})$ ratio, that is, at a given degree of the section being under- or over-reinforced, the flexural ductility decreases as the concrete grade increases.

4. CONCURRENT FLEXURAL STRENGTH AND DUCTILITY DESIGN METHOD

4.1. Interrelation between flexural strength and ductility

From the above parametric study, it is evident that the major factors affecting the flexural strength and ductility of a reinforced concrete beam section are the concrete grade, tension steel ratio and compression steel ratio. In the case of a singly reinforced section, at a fixed concrete grade, the use of a higher tension steel ratio leads to a higher flexural strength but a lower flexural ductility. Hence, the increase in flexural strength is achieved at the expense of a lower flexural ductility. On the other hand, the use of a lower tension steel ratio leads to a higher flexural ductility but a lower flexural strength, and therefore the increase in flexural ductility is achieved at the expense of a lower flexural strength. The simultaneous achievement of both high flexural strength and high flexural ductility is

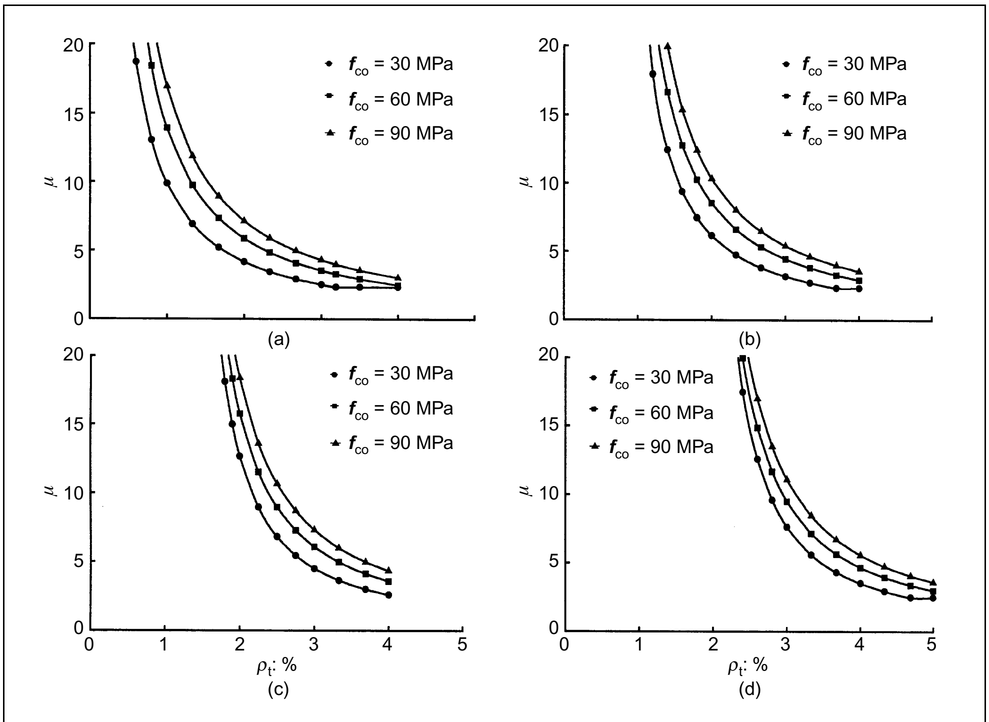

Fig. 5. Ductility factor μ plotted against tension steel ratio ρ_t: (a) $\rho_c = 0\%$; (b) $\rho_c = 0{\cdot}5\%$; (c) $\rho_c = 1\%$; (d) $\rho_c = 1{\cdot}5\%$

not easy. For a given concrete grade, there is a limit to the flexural strength and ductility that could be achieved at the same time.

4.2. Effect of using high-strength concrete

The interrelation between the flexural strength and ductility that could be simultaneously achieved for a given concrete grade can be revealed by plotting the flexural ductility against the flexural strength for tension steel ratios varying from 0 to 6% as shown in Fig. 6. Different concrete grades yield different flexural ductility–flexural strength curves. Each of these curves actually demarcates the limit of flexural strength and ductility that could be achieved at the same time. One important observation is that the curve for a higher grade concrete is generally on the upper right side of that of a lower grade concrete. This implies that the use of a higher grade concrete could extend the limit of flexural strength and ductility that could be simultaneously achieved. In other words, the use of a higher grade concrete could increase the flexural ductility at the same flexural strength, increase the flexural strength at the same flexural ductility, or increase at the same time both the flexural strength and the flexural ductility.

4.3. Effect of adding compression reinforcement

At a fixed concrete grade, the addition of compression reinforcement without increasing the tension reinforcement would produce a significant increase in flexural ductility but little increase in flexural strength. However, if accompanied by an increase in tension reinforcement, the addition of compression reinforcement could also produce a significant increase in flexural strength although the net increase in flexural ductility would be reduced. Its overall effect is best revealed by plotting the flexural ductility against the flexural strength for different compression steel ratios as in Fig. 7. From these curves, it is evident that, like the use of high-strength concrete, the addition of compression reinforcement could substantially extend the limit of flexural strength and ductility that could be simultaneously achieved. However, the addition of compression reinforcement would also lead to significant increase in the cost of construction, which may or may not be justified depending on the situation.

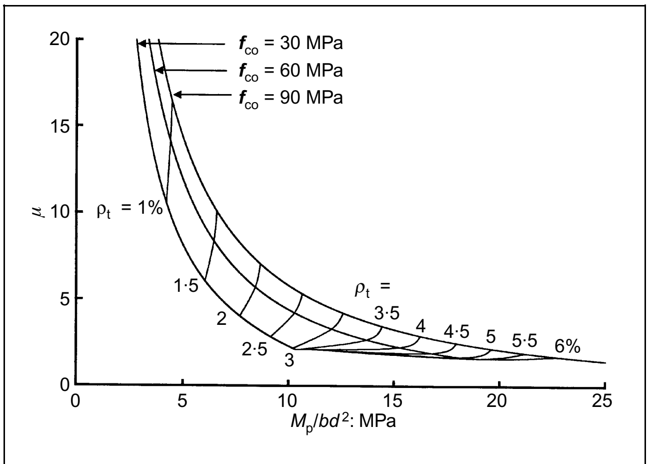

Fig. 6. Flexural ductility plotted against flexural strength for each concrete grade

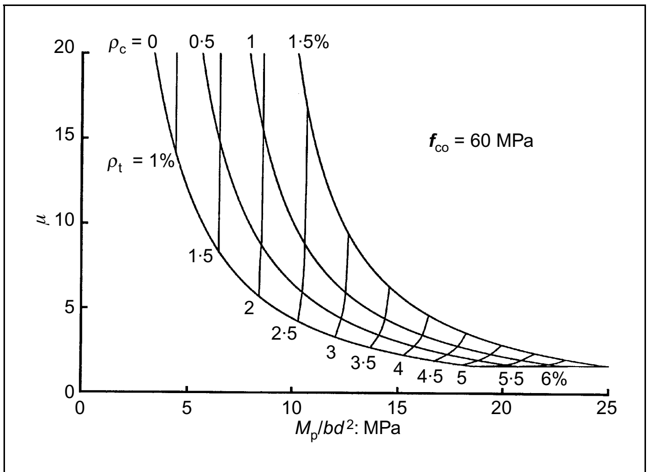

Fig. 7. Flexural ductility plotted against flexural strength for each compression steel ratio

4.4. Concurrent flexural strength and ductility design

Figures 6 and 7 can be used as design charts for a new method of designing reinforced concrete beams, which the authors have named 'concurrent flexural strength and ductility design'. This new design method allows concurrent consideration of the flexural strength and ductility requirements during the design of reinforced concrete beams before deciding on whether to use high-strength concrete and/or add compression reinforcement. For ease of application, Figs 6 and 7 are combined together and further refined to form four design charts, as shown in Fig. 8. For given flexural strength and ductility requirements in terms of $M_p/(bd^2)$ and μ, the concrete strength and steel ratios that would meet these requirements can be obtained directly from the charts by plotting the point with x-coordinate equal to the required flexural strength and y-coordinate equal to the required flexural ductility on the charts.

However, for a given set of flexural strength and ductility requirements, there are several design options, as can be seen from Fig. 8. The use of chart 1 would produce a beam design with no compression reinforcement while the use of the other charts would produce beam designs with compression reinforcement provided. There could be many different combinations of concrete strength and compression steel ratio ranging from a high concrete strength plus a low compression steel ratio to a low concrete strength plus a high compression steel ratio that would meet the given set of strength and ductility requirements. Since the addition of compression reinforcement is generally quite costly, it is recommended that chart 1 (for $\rho_c = 0\%$) should first be used. If the flexural strength and ductility requirements could not be simultaneously satisfied despite the use of a high-strength concrete, then the size of the beam section should be enlarged or some compression reinforcement should be added. Engineering judgement taking into consideration both the implications of changing the member size and the increase in cost due to addition of compression reinforcement is needed. If it is decided that the size of the beam section is to remain unchanged and compression reinforcement is to be added, the required compression steel ratio can be determined by using successively chart 2 (for $\rho_c = 0{\cdot}5\%$), chart 3 (for $\rho_c = 1{\cdot}0\%$) and chart 4 (for $\rho_c = 1{\cdot}5\%$). If the flexural strength and ductility requirements could not be met even when a

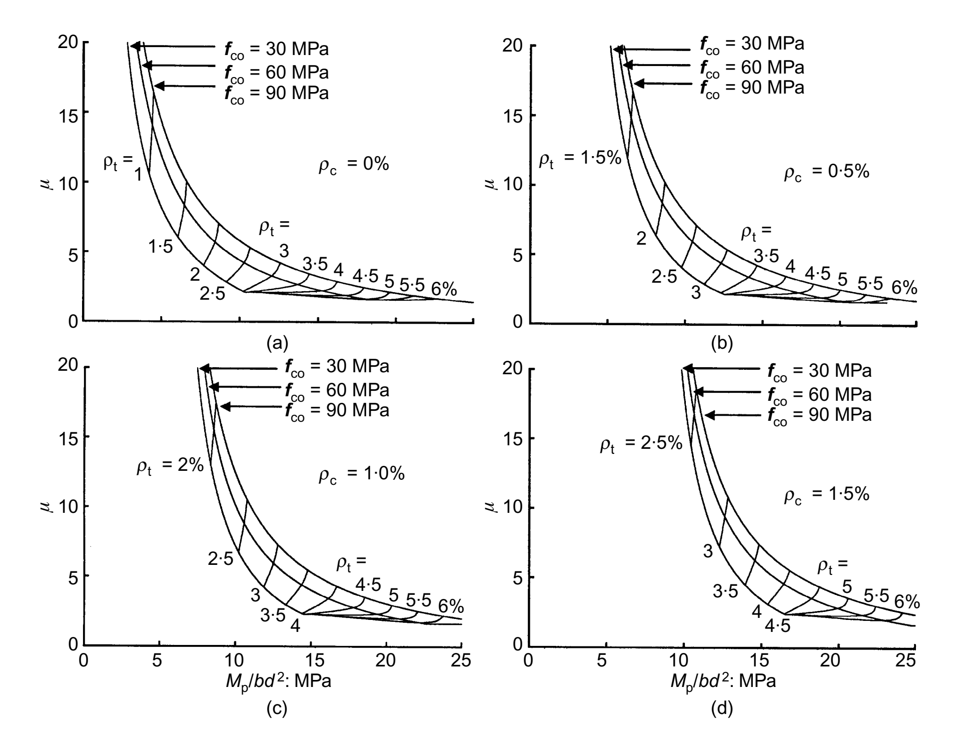

Fig. 8. Design charts for concurrent flexural strength and ductility design: (a) chart 1 $\rho_c = 0\%$; (b) chart 2 $\rho_c = 0{\cdot}5\%$; (c) chart 3 $\rho_c = 1\%$; (d) chart 4 $\rho_c = 1{\cdot}5\%$

compression steel ratio of 1·5% is used, then there is no other option apart from increasing the size of the beam section; a compression steel ratio of greater than 1·5% is not recommended not only because of the cost implication, but also because such a high compression steel ratio would lead to very congested steel reinforcement in the beam.

5. SIMPLIFIED DESIGN METHOD WHEN CONCRETE GRADE IS PRESCRIBED

Both the specified flexural strength and ductility requirements are minimum requirements. However, the flexural strength requirement and the flexural ductility requirement should not be treated in the same way. The provision of more than enough flexural strength would require the addition of more tension reinforcement and thus increase the cost of construction. Moreover, in the case of an earthquake-resistant structure, the provision of an excessive amount of flexural strength to the beam could violate the 'strong column–weak beam' design philosophy or increase the risk of having brittle shear failure. Hence, the provision of more than enough flexural strength should be avoided. On the other hand, regardless of whether the structure is an earthquake-resistant structure, the provision of a generous amount of flexural ductility would always improve the structural performance. The provision of more than enough flexural ductility may or may not increase the cost of construction depending on whether additional compression reinforcement is needed to provide the extra flexural ductility. If additional compression reinforcement is not needed, then the provision of more than enough flexural ductility would not increase the cost of construction. Therefore, the design strategy should be to provide just enough flexural strength and, as far as additional compression reinforcement is not required, generous flexural ductility.

When the concrete grade is already fixed, whether the flexural ductility requirement could be met can be checked simply by using equation (9) to evaluate the flexural ductility of the beam section. In most practical cases, due to the high cost of providing compression reinforcement, the compression steel ratio is generally smaller than one quarter of the tension steel ratio and under such a situation the last term in equation (9) is very close to 1·0. Replacing the last term in equation (9) by unity, a simplified equation for the ductility factor may be obtained as follows

$$\mu = 10{\cdot}7(\boldsymbol{f}_{co})^{-0{\cdot}45}[(\rho_t - \rho_c)/\rho_{bo}]^{-1{\cdot}25} \tag{10}$$

From this equation, the condition that must be met to satisfy the specified flexural ductility requirement may be derived as follows

$$\mu_{min} = 10{\cdot}7(\boldsymbol{f}_{co})^{-0{\cdot}45}[(\rho_t - \rho_c)/\rho_{bo})]^{-1{\cdot}25} \tag{11}$$

in which μ_{min} is the specified minimum ductility factor. Solving for the steel ratios, the maximum values of $(\rho_t - \rho_c)/\rho_{bo}$ and $(\rho_t - \rho_c)$ may be determined as

$$[(\rho_t - \rho_c)/\rho_{bo}]_{max} = 6{\cdot}66(\boldsymbol{f}_{co})^{-0{\cdot}36}(\mu_{min})^{-0{\cdot}8} \tag{12}$$

$$(\rho_t - \rho_c)_{max} = 6{\cdot}66(\boldsymbol{f}_{co})^{-0{\cdot}36}(\mu_{min})^{-0{\cdot}8}\rho_{bo} \tag{13}$$

Having determined the maximum difference between the tension and compression steel ratios that would satisfy the flexural ductility requirement, the reinforcement details of the beam section can then be designed according to the flexural strength requirement in the usual way.

In conventional design, it is generally considered good practice to limit the tension steel ratio in a singly reinforced beam section to not more than 75% of the balanced steel ratio. Since this practice was adopted a long time before the advent of high-strength concrete, presumably this applied mainly to beams made of normal-strength concrete. For beams made of normal-strength concrete with $\boldsymbol{f}_{co}$ equal to 30 MPa, this will give a minimum ductility factor of 3·32. Here, it is suggested that this same minimum flexural ductility should be regarded as an absolute minimum to be provided in all reinforced concrete beams even when the structure is not expected to resist earthquake loads. For beams in earthquake resistant structures, a higher flexural ductility requirement should be specified. In this regard, some general guidelines can be found in the American Code ACI 318M-95 and the New Zealand Standard NZS 3101: 1995.

The maximum values of $(\rho_t - \rho_c)/\rho_{bo}$ and $(\rho_t - \rho_c)$ that would meet the minimum flexural ductility requirement of $\mu_{min} = 3{\cdot}32$ for various grades of concrete are tabulated in Table 1. From these results, it can be seen that as the concrete grade increases, the maximum allowable value of $(\rho_t - \rho_c)/\rho_{bo}$ needs to be reduced to maintain minimum flexural ductility. However, since the value of ρ_{bo} increases with the concrete grade, the maximum allowable value of $(\rho_t - \rho_c)$ still increases with the concrete grade. The larger maximum allowable value of $(\rho_t - \rho_c)$ when high-strength concrete is used would allow the increase of flexural strength while maintaining a similar minimum level of flexural ductility, as illustrated in Table 2. It is only that the net percentage increase in flexural strength while maintaining similar flexural ductility is generally smaller than the corresponding percentage increase in concrete strength. For instance, in a singly reinforced section, when the concrete strength $\boldsymbol{f}_{co}$ is increased by 50% from 60 to 90 MPa, the flexural strength M_p/bd^2 is only increased by 21% from 12·55 to 15·14 MPa.

6. EXAMPLES

6.1. Example 1

A beam section with $b = 400$ mm, $h = 800$ mm, $d = 640$ mm, and $d_1 = 60$ mm is to be designed. The flexural strength and ductility requirements are given by: $M_p/bd^2 = 13{\cdot}5$ MPa and $\mu = 5{\cdot}0$. As a first attempt, chart 1 in Fig. 8 is used. Plotting the point (13·5, 5·0) on the graph, it is found that the required flexural strength and ductility cannot be simultaneously achieved even when a high-strength concrete with $\boldsymbol{f}_{co} = 90$ MPa is used. Thus, compression reinforcement has to be added. Plotting the point (13·5, 5·0) successively on chart 2, chart 3 and chart 4, two alternative solutions are found. Solution 1 is to use $\rho_c = 1\%$ and $\boldsymbol{f}_{co} = 60$ MPa. Solution 2 is to use $\rho_c = 1{\cdot}5\%$ and $\boldsymbol{f}_{co} = 30$ MPa. The choice between them is a matter of engineering judgement, taking into consideration the economy and simplicity of the overall design.

6.2. Example 2

A beam section of $b = 300$ mm, $h = 600$ mm, $d = 550$ mm, and $d_1 = 50$ mm is to be designed. The concrete grade has been prescribed as $\boldsymbol{f}_{co} = 50$ MPa, and the strength and ductility requirements are: $M_p/bd^2 = 15{\cdot}0$ MPa and $\mu = 3{\cdot}32$. From Table 1, the maximum allowable value of $(\rho_t - \rho_c)$ is found to be 2·93%. Designing for the strength requirement in the usual way, the required values of ρ_t and ρ_c are determined as 3·76% and 0·83%, respectively.

7. CONCLUSIONS

The interrelation between the flexural strength and the flexural ductility that could be simultaneously achieved by a beam section has been evaluated and plotted for different concrete grades and compression steel ratios in the form of charts. From these charts, it can be seen that the use of a higher grade concrete could increase flexural ductility at the same flexural strength, increase flexural strength at the same flexural ductility, or increase both flexural strength and ductility. On the other hand, the addition of compression reinforcement without increasing the tension reinforcement could produce significant increase in flexural ductility but little increase in flexural strength, whereas the addition of compression reinforcement together with an increase in tension reinforcement could increase both the flexural strength and ductility.

Using these charts, a new beam design method called 'concurrent flexural strength and ductility design' has been developed. It allows engineers to consider both the strength and ductility requirements before deciding whether to use high-strength concrete and/or add compression reinforcement. For application to cases in which the concrete grade is prescribed, a simpler design method of first determining the maximum difference between the tension and compression steel ratios that would satisfy the flexural ductility requirement and then designing the reinforcement details according to the flexural strength requirement has also been developed. Examples have been given to illustrate the application of these newly developed methods.

For maintaining the same minimum level of flexural ductility that is normally provided in beams made of normal-strength concrete, it is proposed that the minimum required flexural ductility should be set at $\mu = 3{\cdot}32$. To meet this ductility requirement, the maximum allowable value of $(\rho_t - \rho_c)/\rho_{bo}$ needs to be reduced as the concrete grade increases but since the value of ρ_{bo} increases with the concrete grade, the maximum allowable value of $(\rho_t - \rho_c)$ still increases with the concrete grade. The larger maximum allowable value of $(\rho_t - \rho_c)$ when high-strength concrete is used would allow the flexural strength to be increased but the net percentage increase in flexural strength while maintaining minimum flexural ductility is generally

$\boldsymbol{f}_{co}$: MPa	Maximum value of ρ_t: %			Maximum value of M_p/bd^2: MPa		
	$\rho_c = 0\%$	$\rho_c = 0{\cdot}5\%$	$\rho_c = 1{\cdot}0\%$	$\rho_c = 0\%$	$\rho_c = 0{\cdot}5\%$	$\rho_c = 1{\cdot}0\%$
30	2·39	2·89	3·39	8·84	10·92	13·01
40	2·67	3·17	3·67	10·24	12·31	14·40
50	2·93	3·43	3·93	11·49	13·55	15·64
60	3·15	3·65	4·15	12·55	14·61	16·68
70	3·35	3·85	4·35	13·51	15·57	17·63
80	3·53	4·03	4·53	14·37	16·43	18·49
90	3·69	4·19	4·69	15·14	17·20	19·26
100	3·83	4·33	4·83	15·82	17·88	19·94

Table 2. Maximum tension steel ratios and flexural strength for $\mu_{min} = 3{\cdot}32$

smaller than the corresponding percentage increase in concrete strength.

8. ACKNOWLEDGEMENT

The work described was carried out with financial support provided by the Research Grants Council of Hong Kong (R.G.C. Project No. HKU 7012/98E).

REFERENCES

1. AMERICAN CONCRETE INSTITUTE. *State-of-the-Art Report on High Strength Concrete*. American Concrete Institute, Detroit, 1992, Committee 363 Report ACI 363-R92, p. 55.
2. CONCRETE SOCIETY. *Design Guidance for High Strength Concrete*. Concrete Society, 1998, Working Party Technical Report No. 49, p. 168.
3. PAM H. J., KWAN A. K. H. and ISLAM M. S. Flexural strength and ductility of reinforced normal- and high-strength concrete beams. *Proceedings of the Institution of Civil Engineers, Structures and Buildings*, 2001, **146**(4), 381–389.
4. YONG Y. K., NOUR M. and NAWY E. G. Behavior of laterally confined high-strength concrete under axial loads. *Journal of Structural Engineering, ASCE*, 1988, **114**, No. 2, 332–350.
5. FALCONER T. J. and PARK R. *Ductility of Prestressed Concrete Piles under Seismic Loading*. Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, 1982, Research Report No. 82-6, p. 121.
6. CARREIRA D. J. and CHU K. H. The moment–curvature relationship of reinforced concrete members. *ACI Journal*, 1986, **83**, No. 2, 191–198.
7. SAMRA R. M., DEEB N. A. A. and MADI U. R. Transverse steel content in spiral concrete columns subjected to eccentric loading. *ACI Structural Journal*, 1996, **93**, No. 4, 412–419.
8. SHEIKH S. A. and YEH C. C. Analytical moment–curvature relations for tied concrete columns. *Journal of Structural Engineering*, ASCE, 1997, **118**, No. 2, 529–544.
9. HO J. C. M., PAM H. J. and KWAN A. K. H. Theoretical analysis of complete moment–curvature behaviour of reinforced high-strength concrete beams. The Structural Design of Tall Buildings, 2003, (to be published).
10. PAM H. J., KWAN A. K. H. and HO J. C. M. Post-peak behaviour and flexural ductility of doubly reinforced high-strength concrete beams. *Structural Engineering and Mechanics*, 2001, **12**(5), 459–474.
11. ATTARD M. M. and SETUNGE S. The stress–strain relationship of confined and unconfined concrete. *ACI Materials Journal*, 1996, **93**, No. 5, 432–442.
12. ATTARD M. M. and STEWART M. G. A two parameter stress block for high-strength concrete. *ACI Structural Journal*, 1998, **95**, No. 3, 305–317.

Please email, fax or post your discussion contributions to the secretary by 1 May 2003: email: kathleen.hollow@ice.org.uk; fax: +44 (0)20 7799 1325; or post to Kathleen Hollow, Journals Department, Institution of Civil Engineers, 1–7 Great George Street, London SW1P 3AA.