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Flexural strength enhancement of confined reinforced concrete columns

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As part of a continuing research study, this paper proposes a new design aid to calculate the actual moment capacity of confined reinforced concrete columns. Up to now the moment capacity of a reinforced concrete column is calculated based on the code's guidelines for an unconfined section. As most reinforced concrete columns contain transverse or confining reinforcement, which will enhance the column moment capacity, the actual moment capacity will be much higher than the unconfined moment capacity. This additional flexural strength will increase the shear force demand in the column, and if it is not accounted for in the design will jeopardise the column to fail in shear. In this study the actual moment capacity of a confined reinforced concrete column is obtained by multiplying the moment capacity calculated using the BS 8110 method with the proposed flexural strength enhancement factor. By using regression analysis, an equation for the flexural strength enhancement factor is derived as the function of all the parameters that have effects on the moment capacity. An example is presented to show the accuracy of the proposed method.

NOTATIONS

A_c area of concrete core measured to outside of perimeter hoops
A_g gross area of cross-section
A_s area of longitudinal steel
b width of cross section
c_x concrete core dimension parallel to x-axis (measured to centre line of confining steel)
c_y concrete core dimension parallel to y-axis (measured to centre line of confining steel)
d_c diameter of circular confining steel (centre-to-centre)
E_c initial tangent modulus of concrete
E_{sec} secant modulus of concrete at peak stress
f stress in concrete corresponding to strain ε
f_{cc} confined concrete compressive strength
f_{co} peak stress of unconfined concrete stress-strain curve $= 0{\cdot}75 f_{cu}$
f_{cu} unconfined concrete compressive cube strength
f_l effective lateral confining pressure
f_y yield strength of longitudinal steel
f_{ys} yield strength of confining steel
h height of cross section
k_e confinement effectiveness coefficient
M moment
M_{BS} design moment capacity evaluated using BS 8110 equivalent rectangular concrete stress block taking into account partial safety factors for strength of materials
M_c maximum moment capacity evaluated from the moment-curvature curve based on the modified Mander *et al.* concrete stress-strain model
M_{ref} ultimate moment capacity evaluated using BS 8110 equivalent rectangular concrete stress block without partial safety factors for the strength of materials
P compressive axial load on columns
R ratio of strain increase to stress increase due to confinement
R^2 coefficient of correlation
r $E_c/(E_c - E_{sec})$
s vertical spacing of confining steel (centre-to-centre)
s' vertical clear spacing between confining steel
w_i clear distance between each longitudinal steel bar supported by the corner of a hoop or cross tie
x $\varepsilon/\varepsilon_{cc}$
α, β coefficients of regression line
ε strain in concrete corresponding to stress f
ε_{cc} strain corresponding to peak stress in confined concrete
ε_{co} strain corresponding to peak stress in unconfined concrete
ρ ratio of longitudinal steel area to gross-section area
ρ_c ratio of longitudinal steel area to nominal concrete core area (measured to centre line of confining steel)
ρ_s volumetric ratio of confining steel (measured to outside of confining steel)
ω analytical flexural strength enhancement factor
ω_d design flexural strength enhancement factor

1. INTRODUCTION AND RESEARCH SIGNIFICANCE

Columns play a very important role in building structures in resisting the gravity loads as well as lateral loads caused by wind or earthquake. In resisting the gravity and lateral loads, columns will be subjected to high axial compressive and shear forces. Therefore, a significant amount of transverse reinforcement, which has a double function in resisting shear forces as well as providing confinement to the concrete core, must be provided in reinforced concrete columns. The presence of transverse reinforcement and axial compressive load to a certain extent will cause enhancement in the flexural strength capacity of the reinforced concrete column. The effect of axial load towards strength enhancement has been implicitly taken into account in the calculation of the column section unconfined moment capacity. However, the effects of transverse or confining reinforcement towards the enhancement of the section moment capacity have never been considered in the design of reinforced concrete columns.

Transverse reinforcement in reinforced concrete columns has several functions, they are (*a*) to resist shear forces, (*b*) to restrain longitudinal reinforcement from buckling and (*c*) to confine concrete core. All of these functions will contribute in enhancing the flexural ductility and strength of the reinforced concrete column. Therefore, the ductility and moment capacity of a reinforced concrete column will increase with increasing amount of the confining steel. The increase in ductility is always beneficial to a column regardless of any implications as the result of it. However, the increase in moment capacity from the original design value, will underestimate the shear demand in the column. Depending on the amount of confining steel and the magnitude of axial compressive load, the increase in moment capacity can be as high as 40% of the unconfined moment capacity. The increase in moment capacity will automatically increase the shear demand. It has been widely known that shear failure is much more brittle than bending failure, and thus should be prevented. It has to be noted that as the result of underestimation in the shear capacity, shear failure may happen prior to bending failure. To prevent this from happening, the actual confined moment capacity that takes into account the confining steel must be used instead of the unconfined moment capacity in determining the shear reinforcement in the column.

Most of the structural design codes[1,2] require that reinforced concrete columns be provided with a certain amount of transverse or confining steel. However, up until now, they do not provide guidelines as to how to determine the actual moment capacity of confined reinforced concrete columns.

This paper proposes a method to estimate the actual moment capacity of confined reinforced concrete columns. The major parameter investigated is the flexural strength enhancement factor, which is defined as the ratio of confined moment capacity to unconfined moment capacity. The flexural strength enhancement factor is a function of a series of variables, i.e. amount and yield strength of confining steel, amount of longitudinal steel, concrete compressive strength and magnitude of axial compressive load. The amount of confining steel is expressed in terms of volumetric ratio with respect to the volume of the concrete core, the amount of longitudinal steel is expressed in terms of area ratio with respect to the gross cross-section area, and the axial compressive load is expressed in terms of axial load level with respect to the full axial load capacity.

The unconfined moment capacity, which is served as the reference, is calculated based on the equivalent rectangular concrete stress block recommended by BS 8110,[2] whereas the confined moment capacity is calculated based on the confined concrete stress-strain relationship by Mander *et al.*,[3] which is modified to facilitate the comparison with the reference moment. Moment-curvature analysis is used to obtain the maximum value for the confined moment capacity. The cases considered include all column sections that have various combinations of the concerned variables, they are

- concrete cube strength, ranging from 25 to 60 MPa
- longitudinal steel ratio, ranging from 1 to 6%
- volumetric ratio of transverse or confining steel ranging from 1 to 3·5%
- yield strength of transverse or confining steel, either 250 or 460 MPa
- axial compressive load level, ranging from 0·1 to 0·6.

The proposed method in this study to calculate the flexural strength enhancement factor is expected to become a powerful design aid for structural design engineers in calculating more exactly the actual moment capacity of confined reinforced concrete columns.

2. ANALYTICAL STRESS-STRAIN CURVES OF CONFINED REINFORCED CONCRETE SECTIONS

One of the available stress-strain models for confined reinforced concrete sections is chosen and modified in this study to be used in generating the moment-curvature curves for reinforced concrete columns.

The chosen model was proposed by Mander *et al.*[3] The major parameters in the concrete stress-strain curve are effective lateral confining pressure, f_l, and confined concrete strength, f_{cc}, which are expressed in equations (1) and (2), respectively.

$$f_l = 0{\cdot}5 k_e \rho_s f_{ys} \tag{1}$$

$$f_{cc} = f_{co}\left(-1{\cdot}254 + 2{\cdot}254\sqrt{1 + \frac{7{\cdot}94 f_l}{f_{co}}} - 2\frac{f_l}{f_{co}}\right) \tag{2}$$

where k_e = confinement effectiveness coefficient, which will be explained further
ρ_s = volumetric ratio of confining steel (measured to outside of confining steel)
f_{ys} = yield strength of confining steel
f_{co} = peak stress of unconfined concrete stress-strain curve.

In the original model of Mander *et al.*[3] stress-strain curve, the value of f_{co} is taken to be the concrete compressive cylinder strength. In order to obtain a fair comparison between the unconfined concrete stress-strain curves of Mander *et al.* and BS 8110, the value of f_{co} is modified and replaced by $0{\cdot}75 f_{cu}$, where f_{cu} is concrete compressive cube strength. This modified value is obtained by approximating (within 5% error) the first moment of area about its origin of both the stress-strain models for strain up to 0·0035 and for concrete cube strength ranging from 25 to 60 MPa. The first moment of area is used instead of the area, as the former is closely proportional to the section moment capacity.

In equations (1) and (2), it should be noted that: (*a*) for an unconfined section, the value of f_l becomes zero and f_{cc} becomes equal to f_{co} or $0{\cdot}75f_{cu}$; and (*b*) it is presented here only for circular or square confinement, in which the confining pressure is similar in all directions.

The parameter k_e in equation (1) is the ratio of the smallest effectively confined concrete core area, which is located midway between two layers of confining steel, to the nominal concrete core area, which is measured to the centre line of the confining steel. The effectively confined concrete core area is obtained by assuming that the core concrete spalls in the form of a series of second-degree parabolas between the clear distance of the longitudinal steel bars supported by the corner of a hoop or cross tie in the horizontal plane, and between the clear spacing of the confining steel in the vertical plane. Mander *et al.* assumed that the angles between the slope of the tangents at the ends of the parabola and its base line to be 45°. Based on all these assumptions, k_e was derived. Equations (3a) and (3b) express k_e respectively for circular and rectangular confinement.

$$k_e = \frac{1 - 0{\cdot}5\dfrac{s'}{d_c}}{1 - \rho_c} \tag{3a}$$

$$k_e = \frac{\left(1 - \dfrac{\sum w_i^2}{6c_xc_y}\right)\left(1 - \dfrac{s'}{2c_x}\right)\left(1 - \dfrac{s'}{2c_y}\right)}{1 - \rho_c} \tag{3b}$$

where s' = clear spacing between confining steel
d_c = diameter of circular confining steel (measured to the centre line of the steel)
ρ_c = ratio of longitudinal steel area to nominal concrete core area (measured to centre line of confining steel)
w_i = clear distance between each longitudinal steel bar supported by the corner of a hoop or cross tie
c_x = concrete core dimension parallel to x-axis (measured to centre line of confining steel)
c_y = concrete core dimension parallel to y-axis (measured to centre line of confining steel).

Note that for a square confinement, the value of c_x is equal to c_y.

The strain corresponding to the peak stress is expressed as

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + R\left(\frac{f_{cc}}{f_{co}} - 1\right)\right] \tag{4}$$

where ε_{co} which is equal to 0·002, is the strain corresponding to the peak stress for unconfined concrete and R, which is an empirical value, is the ratio of the strain increase to stress increase due to confinement. According to Mander *et al.*,[3] the value of R can vary from 3 to 6, and a value of 5 is recommended for analytical modelling purposes.

The entire confined concrete stress-strain relationship is expressed following the Popovics' model[4] in the form of:

$$f = \frac{f_{cc}xr}{r - 1 + x^r} \tag{5}$$

where $x = \varepsilon/\varepsilon_{cc}$
ε = strain in concrete corresponding to f
$r = E_c/(E_c - E_{sec})$
E_c = initial tangent modulus of concrete
E_{sec} = secant modulus of concrete at peak stress, equal to f_{cc}/ε_{cc} for confined concrete and f_{co}/ε_{co} for unconfined concrete.

In equation (5) the confined concrete strength, f_{cc}, and thus the effective lateral confining pressure, f_l, control the shape of the descending branch of the stress-strain curve, or in other words the ductility of the concrete.

The reasons for adopting the modified Mander *et al.* concrete stress-strain model are: (*a*) only one single equation defines both the ascending and descending branches of the stress-strain curve; (*b*) the model can also be used for unconfined concrete sections; (*c*) the model can be applied to any shape of concrete member section confined by any kind of transverse reinforcement (spirals, cross ties, circular or rectangular hoops); and (*d*) there is no residue stress at the tail end of the stress-strain curve. The last phenomenon resembles more the behaviour of confined concrete under compression at large deformation.

The concrete stress-strain curves for various volumetric ratios of confining steel together with the concrete stress-strain curve recommended in BS 8110[2] are shown in Fig. 1. All the stress-strain curves were obtained based on the assumed reinforced concrete section described in Fig. 2.

Comparison of the stress-strain curves in Fig. 1 reveals that:

- The ductility increases with the volumetric ratio of confining steel.
- The maximum stress increases with the volumetric ratio of confining steel.
- The strain corresponding to the peak stress increases with the volumetric ratio of confining steel.
- Compared to the unconfined modified stress-strain curve of Mander *et al.*, the stress-strain curve of BS 8110 under-estimates the ductility and value of maximum stress. It should be noted that the first moment of area up to the strain of 0·0035 of these two curves about the origin are approximately the same. The error is only about 0·4% for f_{cu} = 50 MPa.

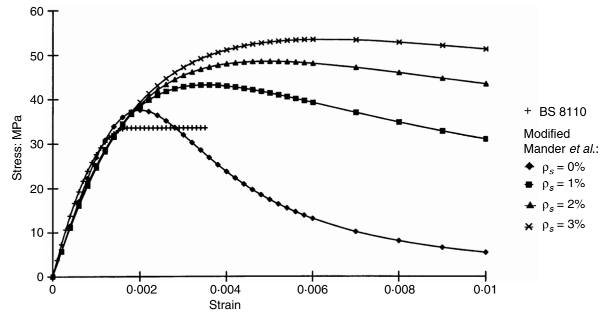

Fig. 1. Concrete stress-strain curves of BS 8110[2] and modified Mander *et al.*[3] for f_{cu} = 50 MPa

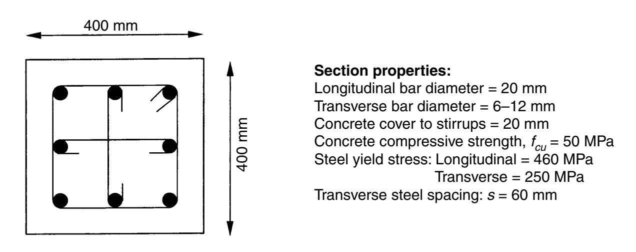

Fig. 2. Properties of the assumed reinforced concrete section for stress-strain curves

3. ANALYTICAL MOMENT-CURVATURE AND AXIAL LOAD–MOMENT INTERACTION CURVES FOR CONFINED REINFORCED CONCRETE COLUMNS

Using the previously discussed stress-strain curves for concrete and the elasto-plastic stress-strain model for longitudinal reinforcement as shown in Fig. 3, for a specific ratio of confining steel, a series of analytical moment-curvature curves can be generated for various axial compressive load levels. The assumptions and procedure used in generating the moment-curvature curves are outlined in the following.

Assumptions.

(*a*) The strain is linear across the height of the section, that is, plane bending.
(*b*) The tensile strength of the concrete is ignored.
(*c*) The concrete spalls at a strain of 0·004.
(*d*) The initial tangent modulus of the concrete, E_c, is adopted from ACI 318-99,[1] which is equivalent to $4230\sqrt{f_{cu}}$.
(*e*) For the elasto-plastic stress-strain curve of the longitudinal reinforcement, the yield stress and Young's modulus are 460 and 200 000 MPa, respectively.
(*f*) In determining the location of the neutral axis, convergence is assumed to have been reached when the difference between the calculated axial load and the applied axial load is within 1% of the latter value.

Procedure.

(*a*) Assign a value to the extreme concrete compressive fibre strain (normally starting with a very small value).

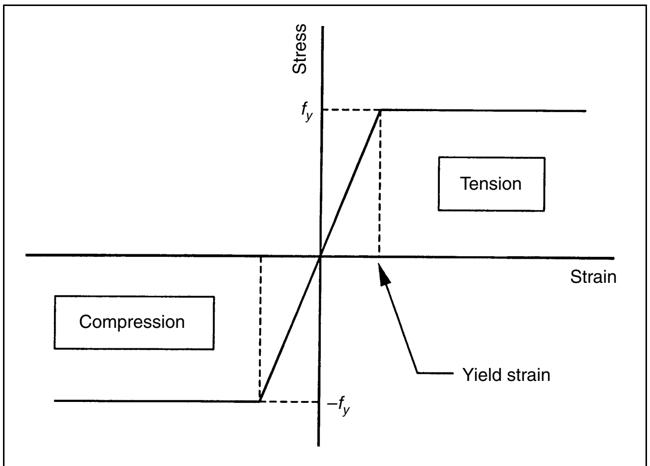

Fig. 3. Stress-strain relationship (symmetrical in tension and compression) for longitudinal reinforcement

(*b*) Assume a value of neutral axis depth measured from the extreme concrete compressive fibre (normally starting with one-half of the cross-section height).
(*c*) Calculate the strain and the corresponding stress at the centroid of each longitudinal reinforcement bar.
(*d*) Determine the stress distribution in the concrete compressive region based on the modified Mander *et al.*[3] stress-strain model for the given volumetric ratio of confining steel. The resultant concrete compressive force is then obtained by numerical integration of the stress over the entire compression region.
(*e*) Calculate the axial force from equilibrium and compare with the applied axial load. If the difference lies within the specified tolerance, the assumed neutral axis depth is adopted. The moment capacity and the corresponding curvature of the section are then calculated. Otherwise, a new neutral axis depth is determined from the iteration (using secant method) and steps (c) to (e) are repeated until it converges.
(*f*) Assign the next value, which is larger than the previous one, to the extreme concrete compressive fibre strain and repeat steps (b) to (e).
(*g*) Repeat the whole procedure until the complete moment-curvature curve is obtained.

Figure 4 shows a series of moment-curvature curves for different levels of axial compressive load that simulate the column axial loads from high to low storeys. Note that axial load level is defined as $P/f_{cu}A_g$, where P is compressive axial load acting on columns and A_g is gross area of cross section. The moment-curvature curves in Fig. 4 were derived using the reinforced concrete section described in Fig. 2 for a volumetric ratio of confining steel of 1%. It is observed in Fig. 4 that the ductility of the moment-curvature curves decreases when the axial load level increases, and the moment beyond the maximum point degrades more rapidly at higher axial load levels. For the low axial load levels of 0 and 0·1, the moment-curvature curves closely resemble an elasto-plastic shape. The maximum moment increases when the axial load level is increased to about 0·4, which is considered in this study as the 'balanced' level. The rate of moment increase is higher at low axial load levels, and the highest moment increase occurs when the axial load level is increased from 0·1 to 0·2. For axial load levels higher than the balanced axial load level, the maximum moment decreases as the axial load level is increased. It has to be noted in Fig. 4 that the small drop of moment immediately after the peak is due to the spalling of the cover concrete when the strain reaches larger than 0·004.

All of the above behaviours of the moment-curvature curves are consistent with those of the corresponding axial load–moment interaction curve, which is shown in Fig. 5. Shown also in Fig. 5 are the axial load–moment interaction curves for higher volumetric ratios of confining steel ($\rho_s = 2\%$ and 3%) and for unconfined columns based on BS 8110.[2]

Both the axial load and moment in Fig. 5 are non-dimensionalised. The former is equal to axial load level. Values of the maximum axial load capacity were obtained by assuming the concrete had reached the peak stress and the longitudinal steel had yielded, which can be expressed respectively for unconfined and confined column section as equations (6) and (7).

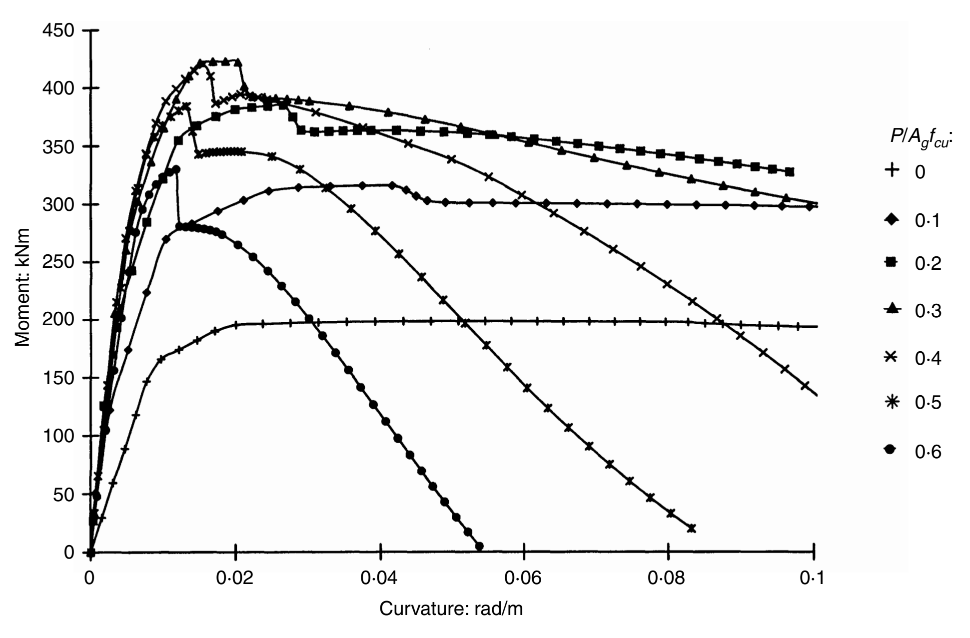

Fig. 4. Moment-curvature curves for $\rho_s = 1\%$ of different axial load levels (based on modified Mander *et al.* concrete stress-strain model[3])

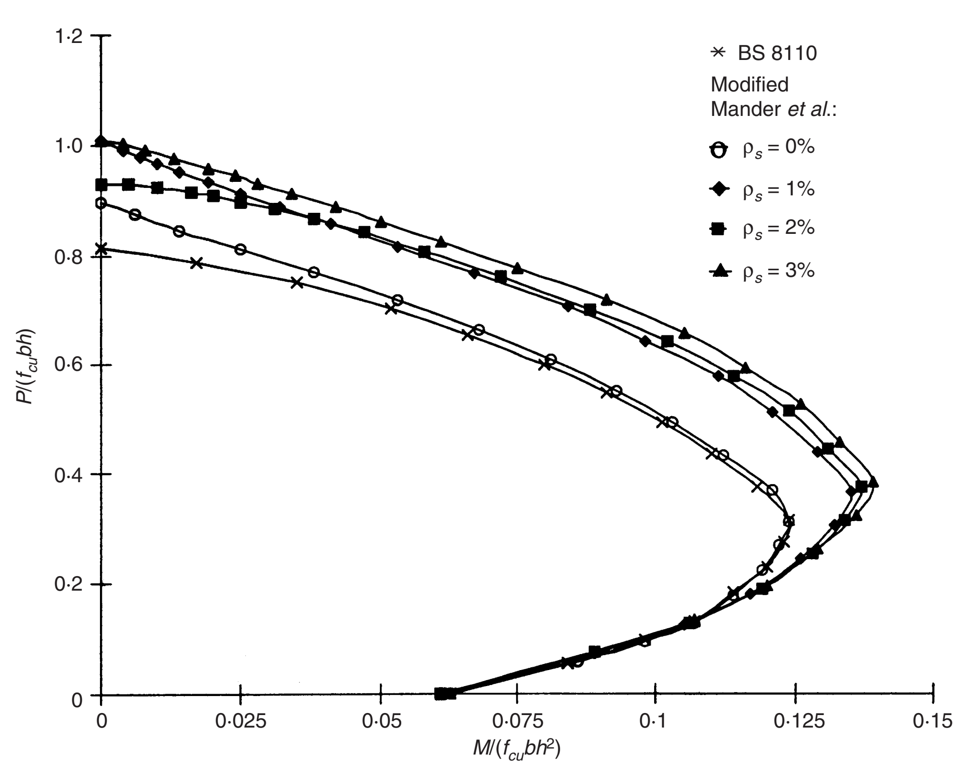

Fig. 5. Axial load–moment interaction curves

6	$P_{\text{unconfined}} = f_{co}A_g + f_yA_s$
7a	$P_{\text{confined}} = f_{cc}A_g + f_yA_s$
7b	$P_{\text{confined}} = f_{cc}A_c + f_yA_s$

where f_{co} = peak stress of unconfined concrete, which is equal to $0{\cdot}67f_{cu}$ and $0{\cdot}75f_{cu}$ for BS 8110 and modified Mander *et al.*[3] models, respectively

f_{cc} = peak stress of confined concrete

f_y = yield strength of longitudinal steel

A_g = gross area of cross section

A_c = area of concrete core measured to outside of perimeter hoops

A_s = area of longitudinal steel.

Equation (7a) is valid when the strain, which is uniform across the height of the cross section, is less than or equal to the spalling strain and equation (7b) is valid when the strain is larger than the spalling strain, beyond which the concrete cover is considered to have spalled.

The axial load–moment interaction curves were also calculated based on the column properties described in Fig. 2. It has to be noted that for the confined column sections, the moment and the extreme compressive fibre strain are taken at the peak point of the corresponding moment-curvature curve.

It is obvious from the axial load–moment interaction curves that the increase in moment happens from zero to the balanced axial load level, which ranges from about 0·32 for unconfined sections to about 0·38 for confined sections. Beyond the balanced axial load level, the moment decreases with the increase in axial load. It is very important to observe in Fig. 5 that the moment increases with the increase of the confining steel content, and the largest rate of increase occurs when the confining steel content is increased from 0 to 1%. It is also worth noting that for the interaction curves derived using the modified Mander *et al.*[3] stress-strain model with $\rho_s = 2\%$, the maximum axial capacity is lower than that with $\rho_s = 1\%$. This is because the effect of concrete strength enhancement in the concrete core due to confinement is offset by spalling of the concrete cover when the strain (ε_{cc}) reaches greater than the spalling strain, which is equal to 0·004. Table 1 lists the values of concrete strain corresponding to the concrete peak stress and the maximum axial load level for $f_{cu} = 50$ MPa.

ρ_s (%)	Model	f_{cc} (MPa)	ε_{cc}	$P/f_{cu}A_g$
0	BS 8110	33·5	0·0035	0·81
0	Modified Mander *et al.*[3]	37·5	0·0020	0·89
1	Modified Mander *et al.*[3]	43·2	0·0035	1·01
2	Modified Mander *et al.*[3]	48·5	0·0049	0·93
3	Modified Mander *et al.*[3]	53·4	0·0063	1·01

Table 1. Concrete strain corresponding to concrete peak stress and maximum $P/f_{cu}A_g$ for $f_{cu} = 50$ MPa

4. ANALYTICAL FLEXURAL STRENGTH ENHANCEMENT IN CONFINED REINFORCED CONCRETE COLUMNS

From the previous moment-curvature and axial load–moment analyses, it has been shown that the flexural strength increases with the increase of the confining steel content. In design practice, the flexural capacity of reinforced concrete columns, which always contain transverse steel, are always calculated based on the code's recommended concrete stress-strain relationship,[1, 2] in which the effect of confining steel is ignored.

In addition to strength enhancement, confinement also results in higher ductility. Ductility always gives beneficial effects to structural members. However, flexural strength enhancement must be treated with caution, as it will cause an increase in the shear demand, which might cause a brittle failure. Therefore, it is essential for practising structural engineers to estimate the flexural strength enhancement of confined reinforced concrete columns, so that premature shear failure will be prevented.

In addition to confinement, flexural strength enhancement depends on many more parameters, such as concrete compressive strength, amount and distribution of longitudinal reinforcement, as well as yield stress and shape of confining steel. These effects on the flexural strength enhancement will be investigated and quantified by means of the moment-curvature curves.

The flexural strength enhancement factor, ω, is defined in this study as the ratio of the actual moment capacity, M_c, of a confined reinforced concrete column to a reference moment capacity, M_{ref}. The former (M_c) is the maximum moment value obtained from the moment-curvature curve based on the modified Mander *et al.* stress-strain model,[3] and the latter (M_{ref}) is the ultimate moment capacity calculated using the equivalent rectangular concrete stress block recommended by BS 8110[2] without applying the partial safety factors for the concrete and longitudinal steel.

It is proposed in this study to relate the flexural strength enhancement factor with confinement effectiveness factor, which is expressed as $\rho_s f_{ys}/f_{cu}$, using a linear regression analysis. This confinement effectiveness factor is almost similar to the one proposed by Hoshikuma *et al.*,[5] except here the cylinder strength is replaced by cube strength. For a reinforced concrete column section, by varying the parameters ρ_s/f_{ys} (part of the confinement effectiveness factor) for a specific combination of f_{cu}, ρ (ratio of longitudinal steel area to gross section area) and $P/A_g f_{cu}$, a number of moment-curvature curves can be generated, each of which will give a value of maximum moment capacity (M_c). Each of these maximum moments is then compared to its corresponding unconfined moment capacity (M_{ref}), from which the flexural strength enhancement factor (ω) is obtained. Each coordinate point, (ω, $\rho_s f_{ys}/f_{cu}$), consisting of a pair of values of flexural strength enhancement factor and confinement effectiveness factor respectively, is plotted on the graph of ω against $\rho_s f_{ys}/f_{cu}$ (see Fig. 6). A straight line is later fitted in the graph by linear regression analysis. The best fitting line is expressed in the form of

$$\frac{M_c}{M_{ref}} = \alpha \frac{\rho_s f_{ys}}{f_{cu}} + \beta \tag{8}$$

where α and β are coefficients of the regression line.

The above procedure is repeated for other combinations of f_{cu} (from 25 to 60 MPa), ρ (from 1 to 6%) and $P/A_g f_{cu}$ (from 0·1 to 0·6). Fig. 6 shows some examples of the graphs. It can be observed from the figure that the flexural strength enhancement factor can reach as high as 1·4.

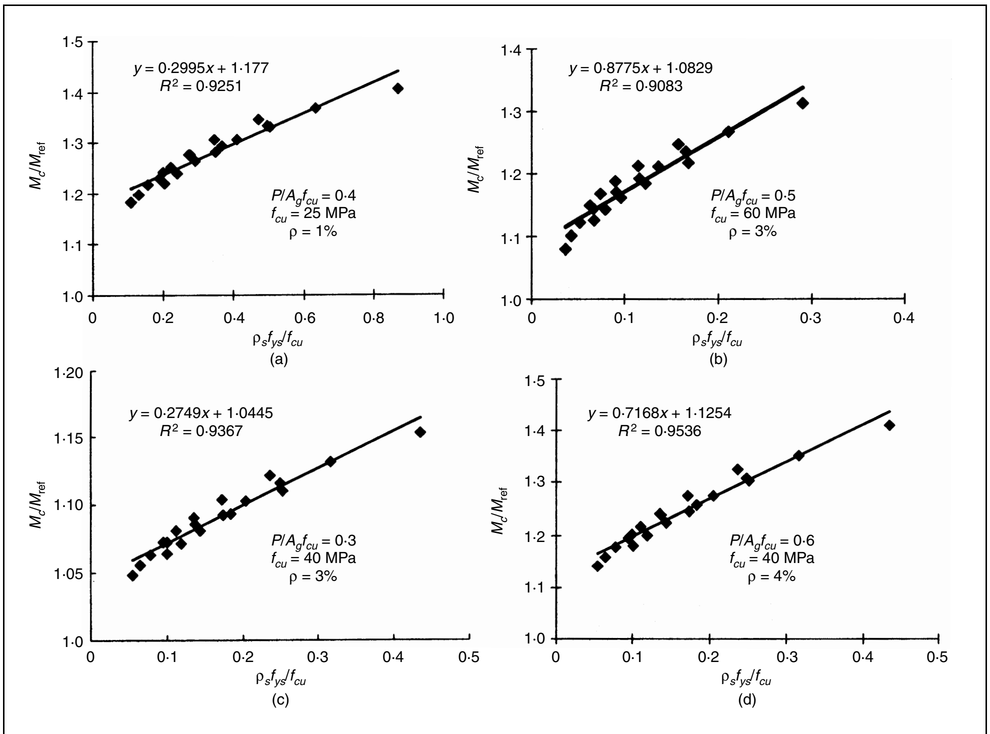

Fig. 6. Graphs of M_c/M_{ref} against $\rho_s f_{ys}/f_{cu}$

By performing regression analyses on all the obtained values of α and β, it is determined that they can be represented with good accuracy by

$$\alpha = 0{\cdot}0269\left(\frac{f_{cu}^{0{\cdot}75}}{\rho^{0{\cdot}5}}\right)\left(\frac{P}{A_g f_{cu}}\right)^2 \tag{9}$$

$$\beta = 0{\cdot}0678\,\frac{\left(\frac{P}{A_g f_{cu}}\right)^2}{f_{cu}^{0{\cdot}25}\rho^{0{\cdot}75}} + 1{\cdot}0109 \tag{10}$$

Figures 7 and 8 show the relationships of α and β as expressed in equations (9) and (10), respectively.

Shown also in Figs 6 to 8 are the coefficients of correlations (R^2), each of which is very close to 1.

By combining equations (8) to (10), the following equation can be derived to obtain the analytical flexural strength enhancement factor for reinforced concrete columns

$$\omega = 0{\cdot}0269\rho_s f_{ys}\,\frac{\left(\frac{P}{A_g f_{cu}}\right)^2}{f_{cu}^{0{\cdot}25}\rho^{0{\cdot}5}} + \left[0{\cdot}0678\,\frac{\left(\frac{P}{A_g f_{cu}}\right)^2}{f_{cu}^{0{\cdot}25}\rho^{0{\cdot}75}} + 1{\cdot}0109\right] \tag{11}$$

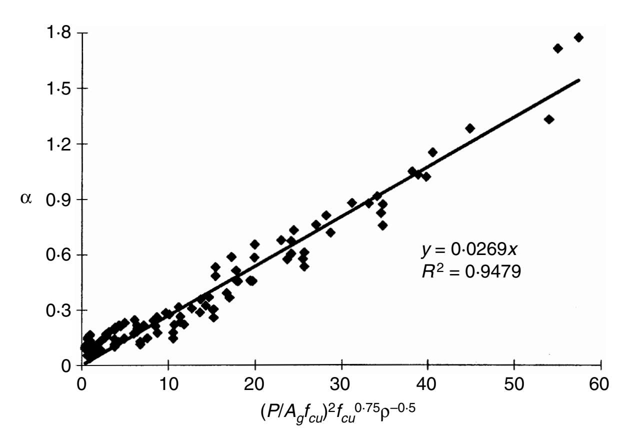

Fig. 7. Graph of α against $(f_{cu}^{0.75}/\rho^{0.5})(P/A_g f_{cu})$

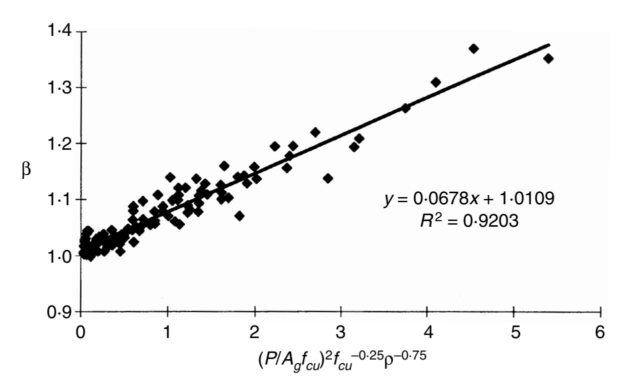

Fig. 8. Graph of β against $(P/A_g f_{cu})^2/f_{cu}^{0.25}\rho^{0.75}$

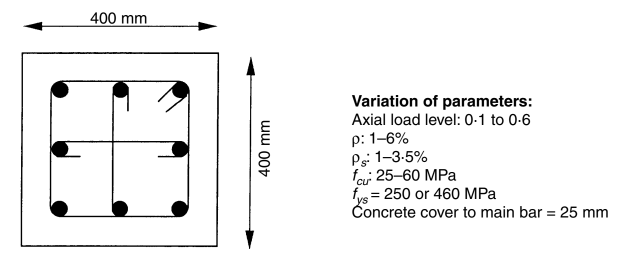

Fig. 9. Properties of the hypothetical column section

It has to be noted that all of the above analyses and calculation were performed using a hypothetical reinforced concrete column section described in Fig. 9.

5. FLEXURAL STRENGTH ENHANCEMENT FACTOR USED IN DESIGN PRACTICE

The above proposed flexural strength enhancement model is based on the comparison of the moment capacity calculated by the modified concrete stress-strain model of Mander *et al.*[3] and that calculated according to BS 8110[2] without taking into account the partial safety factors for the concrete and longitudinal steel. However, in design practice, these partial safety factors must always be considered when calculating the design moment capacity, M_{BS}, according to BS 8110. As the consequence, the actual moment capacity obtained by multiplying M_{BS} with the flexural strength enhancement factor (ω) obtained from equation (11) will be smaller than the actual moment capacity calculated by the modified concrete stress-strain model of Mander *et al.*[3] To compensate for the shortfall, the analytical flexural strength enhancement factor, ω, is adjusted to become design flexural strength enhancement factor, ω_d, which is expressed as

$$\omega_d = \omega\,\frac{M_{\text{ref}}}{M_{BS}} \tag{12}$$

where the value of ω_d will always be greater than 1.

6. CALCULATION EXAMPLE

The following calculation example demonstrates the use of the proposed flexural strength enhancement model. The cross-section details and material properties are shown in Fig. 10. It will be clear from the example that the designer has to calculate two different moment capacities based on BS 8110.[2] The first moment capacity (M_{BS}) takes into account the partial safety factors for the material strengths, whereas the second moment capacity (M_{ref}) is without partial safety factors for the material strengths.

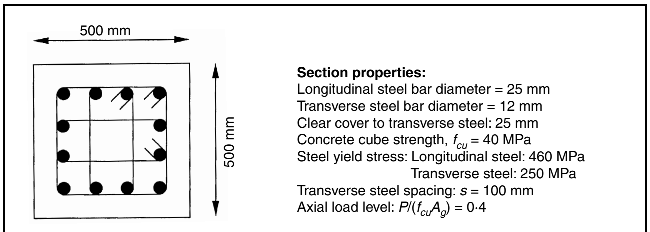

Fig. 10. Cross-section details and material properties for calculation example

The calculation procedure is as follows

(*a*) Longitudinal steel ratio

$$\rho = \frac{0{\cdot}25\pi 25^2 \times 12}{500^2} = 0{\cdot}0236$$

(*b*) Volumetric ratio of confining steel

$$\rho_s = \frac{0{\cdot}25\pi 12^2 \times (450 - 12) \times 8}{450^2 \times 100} = 0{\cdot}0196$$

(*c*) Analytical flexural strength enhancement factor

$$\omega = 0{\cdot}0269 \times 0{\cdot}0196 \times 250\left(\frac{0{\cdot}4^2}{40^{0{\cdot}25} \times 0{\cdot}0236^{0{\cdot}5}}\right) + \left[0{\cdot}0678\,\frac{0{\cdot}4^2}{40^{0{\cdot}25} \times 0{\cdot}0236^{0{\cdot}75}} + 1{\cdot}0109\right] = 1{\cdot}137$$

(*d*) Factored and unfactored moment capacity calculated according to BS 8110[2]

$$M_{BS} = 655{\cdot}4 \text{ kNm}$$
$$M_{\text{ref}} = 794{\cdot}3 \text{ kNm}$$

(*e*) Design flexural strength enhancement factor

$$\omega_d = 1{\cdot}137 \times \frac{794{\cdot}3}{655{\cdot}4} = 1{\cdot}378$$

(*f*) Actual moment capacity

$$M_c = \omega_d M_{BS} = 1{\cdot}378 \times 655{\cdot}4 = 903{\cdot}1 \text{ kNm}$$

It should be noted that if the actual moment capacity is calculated from the first principle using the theoretical moment-curvature curve derived from the modified concrete stress-strain model of Mander *et al.*,[3] the result is 922·9 kNm. Hence, the difference is about 2·1%, which is considered good for engineering design purposes.

7. SUMMARY AND CONCLUSIONS

The confined concrete stress-strain relationship proposed by Mander *et al.*[3] was modified and then used to generate a large number of moment-curvature curves of reinforced concrete columns with varying parameters (f_{cu}, ρ, ρ_s, f_y and $P/f_{cu}A_g$). The parameters in the analysis cover a wide range of values that are commonly found in the design practice. Each of the moment-curvature curves gives a maximum moment, which is considered as the actual moment capacity of the confined reinforced concrete column. This actual moment capacity was compared with that obtained using the code's method[2] by the analytical flexural strength enhancement factor.

The analytical flexural strength enhancement factor was developed using regression analysis in which the coefficients of correlation were all greater than 0·9. This analytical flexural strength enhancement factor was subsequently modified to become design flexural strength enhancement factor, so that it is ready for use by structural design engineers to obtain the actual moment capacity of confined reinforced concrete columns. It is shown that the flexural strength enhancement factor can reach as high as 1·4.

An example of how to apply the proposed method is given. It is shown in the example that the method gives very good results.

The proposed method of design aid in this research will enable practising structural design engineers to determine the actual moment capacity of confined reinforced concrete columns.

8. ACKNOWLEDGEMENTS

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