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Minimum flexural ductility design of high-strength concrete beams

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In the flexural design of reinforced concrete beams, apart from the provision of adequate strength, it is also necessary to provide a certain minimum level of ductility. Traditionally, this has been done by limiting the tension steel ratio or the neutral axis depth to no more than certain fixed maximum values. However, this would result in a variable level of curvature ductility depending on the concrete grade and the steel yield strength. Of greater concern is that this would lead to a lower level of curvature ductility than has been provided in the past to beams made of conventional materials when high-strength concrete and/or high-strength steel are used. It is proposed herein that instead of limiting the tension steel ratio and the neutral axis depth, it is better to set a fixed minimum to the curvature ductility factor. The maximum values of tension steel ratio and neutral axis depth corresponding to the proposed minimum curvature ductility factor for various concrete grades and steel yield strengths have been evaluated. Based on these maximum values, simplified guidelines for providing minimum flexural ductility have been developed.

Notation

A_{sc}	area of compression reinforcement
A_{st}	area of tension reinforcement
b	breadth of beam section
d	effective depth of beam section
d_1	depth of compression reinforcement
d_n	neutral axis depth
d_{nb}	neutral axis depth of the balanced section
E_s	Young's modulus of steel reinforcement
f_{co}	in situ uniaxial compressive strength of concrete
f_y	yield strength of steel reinforcement
f_{yc}	yield strength of compression reinforcement
f_{yt}	yield strength of tension reinforcement
h	total depth of beam section
M_p	peak resisting moment of beam section
ε_c	concrete strain
ε_{ce}	concrete strain at extreme compression fibre
ε_{co}	concrete strain at peak stress
ε_{sc}	steel strain in compression reinforcement
ε_{st}	steel strain in tension reinforcement
ϕ	curvature of beam section
ϕ_u	ultimate curvature of beam section
ϕ_y	yield curvature of beam section
μ	curvature ductility factor
ρ_b	balanced steel ratio of beam section
ρ_{bo}	balanced steel ratio of beam section without compression reinforcement
ρ_c	compression steel ratio ($\rho_c = A_{sc}/bd$)
ρ_t	tension steel ratio ($\rho_t = A_{st}/bd$)
σ_c	concrete stress
$(\ldots)_{min}$	minimum value of $(\ldots)$

Introduction

Traditional flexural design of reinforced concrete (RC) beams concentrates on the provision of adequate strength for resisting applied loads at ultimate limit state and sufficient stiffness for limiting the deflection at serviceability limit state. The post-peak behaviour is usually ignored and only nominal ductility is provided by imposing certain empirical rules on reinforcement detailing. This is understandable because while the flexural strength and stiffness can be evaluated using

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the ordinary beam bending theory, there exists no simple method for evaluating the flexural ductility of an RC beam. To evaluate the flexural ductility, it is necessary to conduct non-linear moment–curvature analysis, extended well into the post-peak range, of the beam section. The actual stress–strain curves of the constitutive materials have to be used in the analysis and the stress-path dependence of the tension reinforcement due to strain reversal, which may have significant effect on the post-peak behaviour, has to be taken into account.[1] Because of the difficulties involved, there have been few studies on the post-peak behaviour and flexural ductility of reinforced concrete members[2–4] and in all previous studies, the stress-path dependence of the tension reinforcement due to strain reversal has not been taken into account.

From the structural safety point of view, ductility is as important as strength. Possession of good flexural ductility would enable a structure to dissipate excessive energy through inelastic deformations within the potential plastic hinge regions while maintaining sufficient flexural strength to resist applied loads. A relatively high level of flexural ductility would provide the structure an increased chance of survival against accidental impact and seismic attack. Flexural ductility is particularly important when the capacity design philosophy for seismic resistant structures[5] is adopted. According to this philosophy, which is also called the 'strong column–weak beam' approach, the beams should yield before the columns yield and the beams should be required to have sufficient flexural ductility such that the potential plastic hinges in the beams are able to maintain their moment resisting capacities until the columns fail.

The flexural ductility of an RC beam is dependent mainly on the failure mode, which in turn, is governed by the reinforcement details. If the amount of tension reinforcement is relatively small such that the beam is under-reinforced, the tension reinforcement will yield before the concrete is crushed and the beam will fail in a ductile manner. If the amount of tension reinforcement is relatively large such that the beam is over-reinforced, the tension reinforcement will not yield even when the concrete is totally crushed and the beam will fail in a brittle manner. Thus, in order to ensure a ductile mode of failure, it has been universally imposed as a basic requirement that all beam sections are under-reinforced. For beams in seismic resistant structures, which are subjected to greater flexural ductility demands, more stringent requirements on the reinforcement detailing, such as the provision of confining reinforcement, are generally imposed.[6,7] The detailing practices for seismic resistance vary from one design code to another and are constantly being upgraded as researches on this topic progress and lessons are learnt after major earthquakes.

Nonetheless, even for beams in structures not expected to resist impact or seismic loads, it is generally considered that in the interests of safety, it is essential to provide a certain minimum level of flexural ductility and that for this purpose, just designing the beam sections to be under-reinforced is not sufficient. In most of the existing design codes,[6–10] reinforcement detailing rules, which impose limits on either the tension steel ratio or the neutral axis depth, have been incorporated to guarantee the provision of minimum flexural ductility, as highlighted below

(*a*) American code ACI 318:[6] Clause 10·3·3 of the code limits the tension steel ratio to no more than 0·75 of the balanced steel ratio.
(*b*) New Zealand code NZS 3101:[7] Clause 8·4·2 of the code restricts the neutral axis depth to no more than 0·75 d_{nb}, where d_{nb} is the neutral axis depth of the balanced section.
(*c*) British code BS 8110:[8] Clause 3·4·4·4 of the code specifies the neutral axis depth to be less than or equal to 0·5 d for all concrete with $f_{cu} \leqslant 100$ MPa, where d is the effective depth of the beam section and f_{cu} is the cube strength.
(*d*) European code EC 2:[9] Clause 2·5·3·4·2 of the code limits the neutral axis depth to no more than 0·45 d when $f_{cu} < 50$ MPa or 0·35 d when $f_{cu} \geqslant 50$ MPa.
(*e*) Chinese code GBJ 11:[10] Clause 6·3·2 of the code requires the neutral axis depth to be smaller than 0·35 d for all concrete grades.

From the above, it is evident that it is not an easy task to provide simple guidelines for flexural ductility design. This paper looks at the problem of providing minimum flexural ductility, which is required even for beams in structures not expected to resist impact or seismic loads. The authors have recently developed a new method of analysing the complete moment–curvature behaviour of RC beams that takes into account the stress-path dependence of the stress–strain curve of the tension reinforcement[1] and using the new method of analysis conducted a series of parametric studies on the effects of various structural parameters on the flexural ductility of high-strength concrete (HSC) beams.[11] It has been found in these studies that the flexural ductility of an RC beam is dependent not only on the tension and compression steel ratios, but also on the concrete grade and the steel yield strength. This observed phenomenon hinted that, as will be shown in this paper, the current practices of providing minimum flexural ductility in the existing design codes would not really provide a consistent level of minimum flexural ductility. More importantly, when HSC and/or high-strength steel (HSS) are used, the flexural ductility so provided would be lower than what has been provided in the past to beams made of conventional materials.

With a view to providing a consistent level of minimum flexural ductility to all kinds of RC beams, including those made of HSC and/or HSS, it is proposed herein to specify a minimum value for the curvature ductility factor, which may be evaluated using the analytical method developed by the authors. The minimum curvature ductility factor to be specified may be established by making reference to the curvature ductility factors being provided in the various existing design codes. To save the trouble of evaluating the curvature ductility factor during beam design, the corresponding maximum values of tension steel ratio and neutral axis depth that would guarantee achievement of the proposed minimum curvature ductility factor have been determined for different combinations of concrete grade and steel yield strength. These maximum values may be used to replace those given in the existing design codes to provide a consistent level of minimum flexural ductility. Based on these maximum values, simplified guidelines for minimum flexural ductility design of normal and HSC beams have been developed.

Moment–curvature analysis

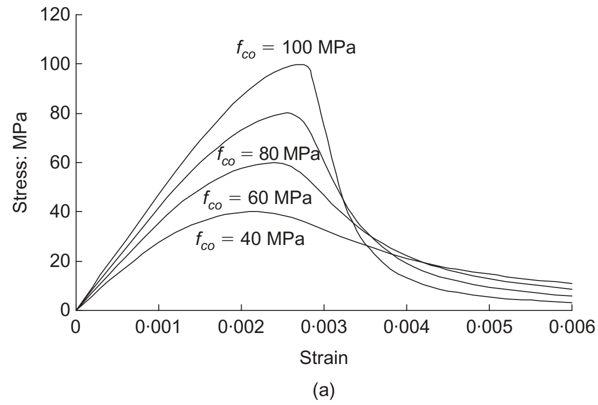
The concrete is assumed to be unconfined and the stress–strain curve model developed by Attard and Setunge,[12] which has been shown to be applicable to a broad range of concrete strength from 20 to 130 MPa, is adopted. The equation of the stress–strain curve is given by

$$\sigma_c/f_{co} = \frac{A(\varepsilon_c/\varepsilon_{co}) + B(\varepsilon_c/\varepsilon_{co})^2}{1 + (A-2)(\varepsilon_c/\varepsilon_{co}) + (B+1)(\varepsilon_c/\varepsilon_{co})^2} \quad (1)$$

where σ_c and ε_c are the compressive stress and strain at any point on the stress–strain curve, f_{co} and ε_{co} are the compressive stress and strain at the peak of the stress–strain curve, and A and B are coefficients dependent on the concrete grade. It should be noted that f_{co} is actually the in situ compressive strength, which may be estimated from the cylinder compressive strength or cube compressive strength using appropriate conversion factors. Fig. 1(a) shows some typical stress–strain curves so derived.

For the steel reinforcement, a linearly elastic–perfectly plastic stress–strain curve is adopted. Since there could be strain reversal in the steel reinforcement at the post-peak stage despite monotonic increase of curvature, the stress–strain curve of the steel is stress-path dependent. It is assumed that when strain reversal occurs, the unloading path of the stress–strain curve is linear and has the same slope as the initial elastic portion of the stress–strain curve. Fig. 1(b) shows the resulting stress–strain curve of the steel reinforcement.

Three basic assumptions have been made in the analysis, that: (*a*) the plane section remains plane after bending; (*b*) the tensile strength of concrete is negligible; and (*c*) there is no bond-slip between concrete and steel. These assumptions are widely accepted in the literature.[13] Fig. 2 shows a typical beam section analysed in this study. The moment–curvature behaviour of the beam section is analysed by applying prescribed curvatures to the beam section incrementally starting from zero. At a prescribed curvature, the strain profile is first evaluated based on the above assumptions. From the strain profile so obtained, the stresses developed in the concrete and the steel reinforcement are determined from their respective stress–strain curves. The stresses developed have to satisfy the axial equilibrium condition, from which the neutral axis depth is evaluated by iteration. Having determined the neutral axis depth, the



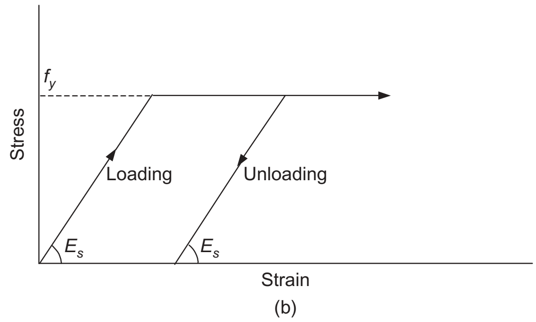


Fig. 1. Stress–strain curves of (a) concrete and (b) steel reinforcement

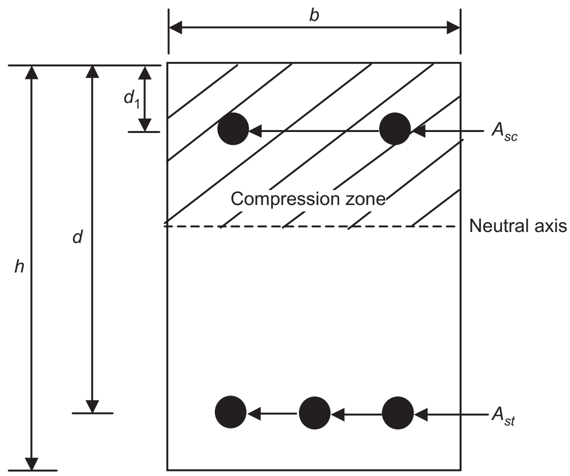


Fig. 2. Typical beam section analysed

resisting moment is calculated from the moment equilibrium condition. The above procedure is repeated until the curvature is large enough for the resisting moment to increase to the peak and then decrease to half of the peak moment.

Some selected moment–curvature curves of the beam sections analysed are plotted in Fig. 3. It can be seen that in the case of an under-reinforced section, the moment–curvature curve is almost linear before the peak moment is reached and there is a fairly long yield plateau at the post-peak stage before the resisting moment drops more rapidly till complete failure, while in the case of an over-reinforced section, the moment–curvature curve is more like a single smooth curve with a sharp peak. Comparing the moment–curvature curves, it is evident that an under-reinforced section is more ductile than an over-reinforced section.

To study the non-linear flexural behaviour, the variations of the neutral axis depth d_n, the concrete strain at extreme compression fibre ε_{ce}, the steel strains in the tension reinforcement ε_{st}, and the steel strain in the compression reinforcement ε_{sc} with the curvature ϕ in some typical sections are plotted in Fig. 4. It is seen that initially, the neutral axis depth remains almost constant. As the curvature increases and the concrete becomes inelastic, the neutral axis depth gradually decreases or increases depending on whether the section is under- or over-reinforced. However, regardless of whether the section is under- or over-reinforced, after entering into the post-peak stage, the neutral axis depth starts to increase rapidly such that the distance between the tension reinforcement and the neutral axis decreases quite quickly with the curvature and beyond a certain point on the moment–curvature curve, the strain in the tension reinforcement starts to decrease causing strain reversal. Such strain reversal of the tension reinforcement occurs in all beam sections. On the other hand, the strain in the compression reinforcement always increases monotonically.

The balanced steel ratios obtained in this study for singly reinforced sections are listed in the second columns of Tables 1–3. For doubly reinforced sections, it is found that at fixed concrete strength and steel yield strengths, the balanced steel ratio, ρ_b, increases linearly with the compression steel ratio, ρ_c. Correlating the balanced steel ratio to the compression steel ratio, the following equation has been derived

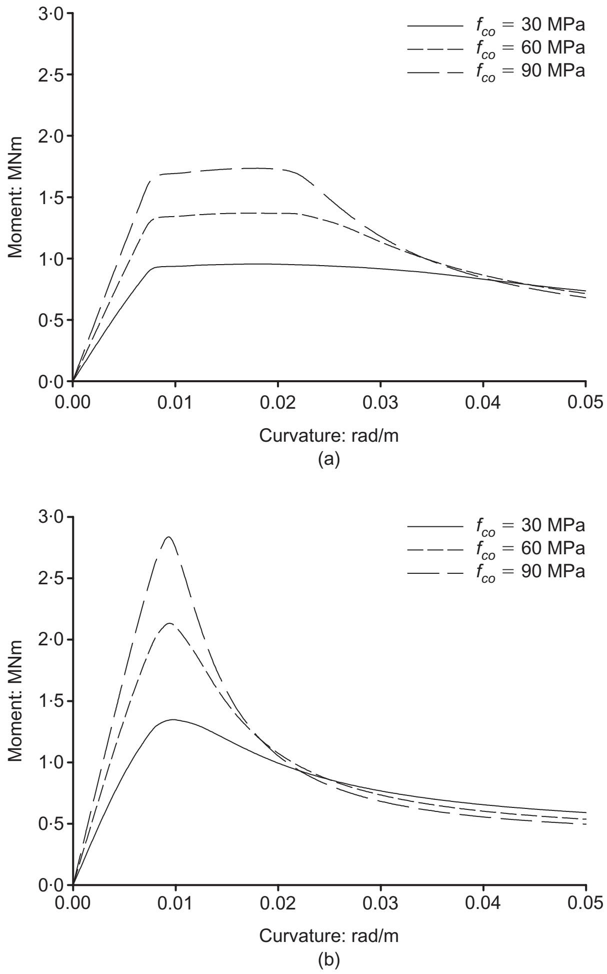


Fig. 3. Complete moment curvature curves of beam sections with $f_{yc} = f_{yt} = 460$ MPa: (a) under-reinforced section with $(\rho_t - \rho_c)/\rho_{bo} = 0{\cdot}5$ and $\rho_c = 1\%$; (b) over-reinforced section with $(p_t - p_c)/p_{bo} = 1{\cdot}2$ and $p_c = 1\%$

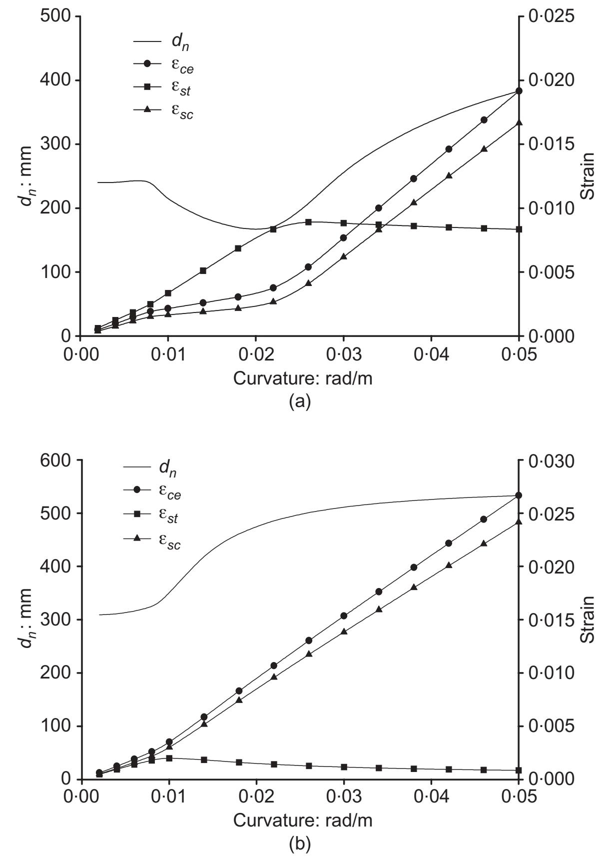


Fig. 4. Variation of neutral axis depth, concrete strain and steel strain with curvature for beam sections with $f_{co} = 60$ MPa and $f_{yc} = f_{yt} = 460$ MPa (a) under-reinforced section with $(\rho_t - \rho_c)/\rho_{bo} = 0{\cdot}5$ and $\rho_c = 1\%$; (b) over-reinforced section with $(p_t - p_c)/p_{bo} = 1{\cdot}2$ and $p_c = 1\%$

Table 1. Balanced steel ratios and maximum values of $(\rho_t - \rho_c)$ for $\mu_{min} = 3{\cdot}32$ when $f_{yc} = f_{yt} = 250$ MPa

f_{co}: MPa	ρ_{bo}: %	Maximum value of $(\rho_t - \rho_c)/\rho_{bo}$	Maximum value of $\rho_t - \rho_c$: %
30	6·92	0·847	5·86
40	8·69	0·763	6·63
50	10·39	0·705	7·32
60	12·01	0·660	7·92
70	13·56	0·624	8·46
80	15·05	0·595	8·95
90	16·47	0·570	9·39

Table 2. Balanced steel ratios and maximum values of $(\rho_t - \rho_c)$ for $\mu_{min} = 3{\cdot}32$ when $f_{yc} = f_{yt} = 460$ MPa

f_{co}: MPa	ρ_{bo}: %	Maximum value of $(\rho_t - \rho_c)/\rho_{bo}$	Maximum value of $\rho_t - \rho_c$: %
30	3·19	0·750	2·39
40	3·95	0·676	2·67
50	4·69	0·624	2·93
60	5·39	0·584	3·15
70	6·06	0·552	3·35
80	6·70	0·527	3·53
90	7·30	0·505	3·69

Table 3. Balanced steel ratios and maximum values of $(\rho_t - \rho_c)$ for $\mu_{min} = 3{\cdot}32$ when $f_{yc} = f_{yt} = 600$ MPa

f_{co}: MPa	ρ_{bo}: %	Maximum value of $(\rho_t - \rho_c)/\rho_{bo}$	Maximum value of $\rho_t - \rho_c$: %
30	2·24	0·711	1·59
40	2·75	0·641	1·76
50	3·24	0·591	1·92
60	3·71	0·554	2·06
70	4·16	0·524	2·18
80	4·58	0·499	2·29
90	4·98	0·479	2·38

$$\rho_b = \rho_{bo} + (f_{yc}/f_{yt})\rho_c \qquad (2)$$

where ρ_{bo} is the balanced steel ratio of the beam section when no compression reinforcement is provided, and f_{yc} and f_{yt} are the yield strengths of the compression and tension reinforcement respectively. In general, the value of ρ_{bo} increases with the concrete strength f_{co} but not in direct proportion because the percentage increase in balanced steel ratio is generally smaller than the percentage increase in concrete strength. However, the value of ρ_{bo} decreases as the yield strength of the tension reinforcement f_{yt} increases.

Curvature ductility factor

The flexural ductility of the beam section may be evaluated in terms of a curvature ductility factor μ defined by

$$\mu = \phi_u/\phi_y \qquad (3)$$

where ϕ_u and ϕ_y are the ultimate curvature and yield curvature respectively. The ultimate curvature ϕ_u is taken as the curvature of the beam section when the resisting moment of the beam section has, after reaching the peak value of M_p, dropped to 0·8 M_p. On the other hand, the yield curvature ϕ_y is taken as the curvature at the hypothetical yield point of an equivalent linearly elastic–perfectly plastic system with an elastic stiffness equal to the secant stiffness of the beam section at 0·75 M_p and a yield moment equal to M_p.

From previous studies on the effects of concrete strength and steel yield strengths,[11] it has been found that the major factor determining the flexural ductility of a beam section is really the degree of the beam section being under- or over-reinforced, which, for a beam section with equal compression and tension steel yield strengths (i.e. $f_{yc} = f_{yt}$), may be measured in terms of $(\rho_t - \rho_c)/\rho_{bo}$. To illustrate the relation between the curvature ductility factor and the degree of the beam section being under/over-reinforced, μ is plotted against $(\rho_t - \rho_c)/\rho_{bo}$ in Figure 5. It is seen that in all cases, μ decreases as $(\rho_t - \rho_c)/\rho_{bo}$ increases until when $(\rho_t - \rho_c)/\rho_{bo} > 1$, μ becomes constant.

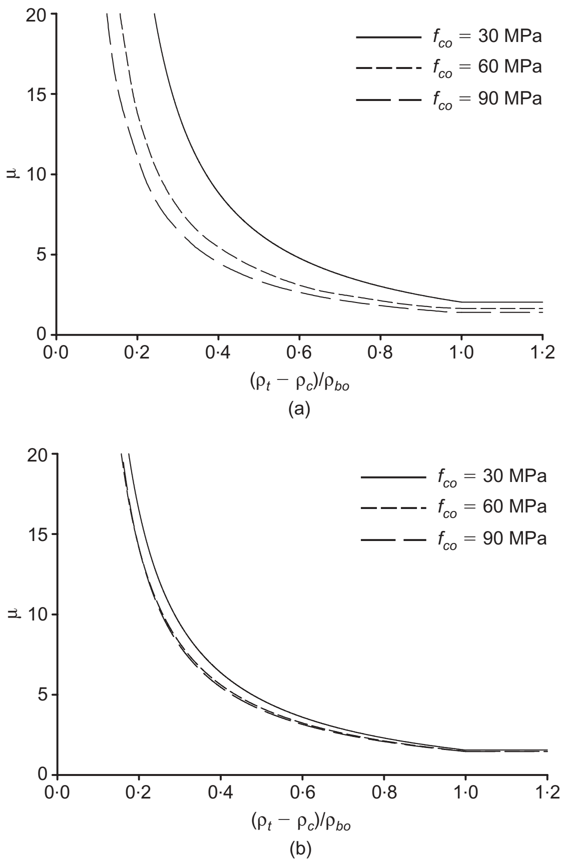


Fig. 5. μ versus $(\rho_t - \rho_c)/\rho_{bo}$ for different concrete grade and steel yield strength: (a) $f_{yc} = f_{yt} = 460$ MPa and $\rho_c = 1\%$; (b) see $f_{co} = 60$ MPa and $p_c = 1\%$

However, the relation between μ and $(\rho_t - \rho_c)/\rho_{bo}$ is dependent on the concrete strength and the steel yield strength. Basically, at a fixed degree of the beam section being under/over-reinforced, i.e. at a fixed value of $(\rho_t - \rho_c)/\rho_{bo}$, μ decreases as the concrete strength or the steel yield strength increases.

As an alternative, the degree of the beam section being under/over-reinforced may also be evaluated in terms of d_n/d_{nb}, in which d_n is the neutral axis depth of the beam section and d_{nb} is the neutral axis depth of the balanced section. Since the neutral axis depth actually varies with the loading stage, it is necessary to clarify when the neutral axis depths are measured. Although in many design codes, it has not been specified when the neutral axis depths are to be measured, the context implies that the neutral axis depths are the corresponding values at peak moment. To avoid ambiguity, it is clarified herein that all neutral axis depths referred to hereafter are the neutral axis depths at peak moment. To study the effect of d_n/d_{nb} on the flexural ductility, μ is plotted against d_n/d_{nb} in Fig. 6. It is seen that μ decreases as d_n/d_{nb} increases until when $d_n/d_{nb} > 1$, μ becomes constant. However, the relation between μ and d_n/d_{nb} is not the same as the relation between μ and $(\rho_t - \rho_c)/\rho_{bo}$. As before, the relation between μ and d_n/d_{nb} is dependent on the concrete strength and the steel yield strength.

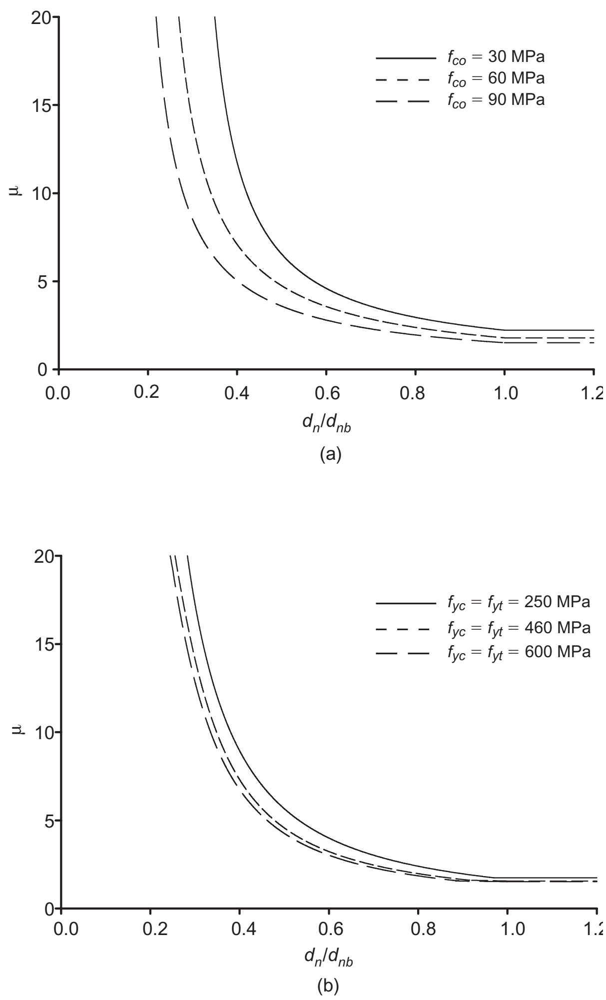


Fig. 6. μ versus d_n/d_{nb} for different concrete grade and steel yield strength: (a) $f_{yc} = f_{yt} = 460$ MPa and $\rho_c = 1\%$; (b) see $f_{co} = 60$ MPa and $p_c = 1\%$

From the above results, the minimum curvature ductility factors being provided by the various existing design codes may be worked out. It is seen that the minimum μ-values provided by the existing design codes actually vary with the concrete strength and the steel yield strength, being generally higher when lower strength materials are used and lower when higher strength materials are used. In other words, the minimum flexural ductility being provided is not consistent. More importantly, the minimum flexural ductility so provided to beams made of newer and higher strength materials would be significantly lower than what has been provided in the past to beams made of more conventional and lower strength materials. This is a dangerous situation, as high-strength materials are becoming more and more commonly used. The existing design codes need to be upgraded to cater for the flexural ductility design of beams made of high-strength materials.

The ranges of variation of the minimum μ-values being provided by the existing codes may be reflected by their respective μ-values at different material strength levels. Consider two possible cases: case 1 when $f_{co} = 30$ MPa and $f_{yc} = f_{yt} = 460$ MPa; and case 2 when $f_{co} = 60$ MPa and $f_{yc} = f_{yt} = 600$ MPa. The respective ranges of μ-values provided by the various existing codes are listed below

(*a*) ACI 318: μ varies from 3·32 in case 1 to 2·27 in case 2.
(*b*) NZS 3101: μ varies from 3·24 in case 1 to 2·23 in case 2.
(*c*) BS 8110: μ varies from 3·22 in case 1 to 1·80 in case 2.
(*d*) EC 2: μ varies from 3·69 in case 1 to 2·88 in case 2.
(*e*) GBJ 11: μ varies from 5·16 in case 1 to 2·88 in case 2.

In order to provide a consistent level of minimum flexural ductility, it is proposed to set a fixed minimum value for the curvature ductility factor. The minimum curvature ductility factor may be established by referring to the minimum curvature ductility factors being provided by the various existing codes. Herein, it is suggested to follow ACI 318, which, for a beam section made of conventional materials with $f_{co} = 30$ MPa and $f_{yc} = f_{yt} = 460$ MPa, yields a curvature ductility factor of 3·32. The proposed minimum curvature ductility factor μ_{min} is therefore set equal to 3·32.

Provision of minimum flexural ductility by limiting tension steel ratio

Since the flexural ductility of a beam section is influenced mainly by the degree of the beam section being under/over-reinforced, the provision of a minimum level of flexural ductility can be ensured by controlling the degree of the beam section being under/over-reinforced. As the degree of the beam section being under/over-reinforced may be measured in terms of either $(\rho_t - \rho_c)/\rho_{bo}$ or d_n/d_{nb}, there are at least two alternative ways of controlling the degree of the beam section being under/over-reinforced: setting maximum limits to the value of $(\rho_t - \rho_c)/\rho_{bo}$ or the value of d_n/d_{nb}. The method of limiting the value of $(\rho_t - \rho_c)/\rho_{bo}$ is considered in this section, while the method of limiting the value of d_n/d_{nb} is considered in the next section.

In the design code ACI 318, minimum flexural ductility is provided by limiting the tension steel ratio ρ_t to no more than 0·75 ρ_b. This limit on ρ_t applies only when no compression reinforcement is provided. When compression reinforcement is provided, the portion of ρ_b equalised by compression reinforcement need not be reduced by the 0·75 factor. For a beam section with equal compression and tension steel yield strengths, this is equivalent to limiting the value of ρ_t to not more than (0·75 $\rho_{bo} + \rho_c$) or limiting the value of $(\rho_t - \rho_c)$ to not more than 0·75 ρ_{bo}. Hence, to some extent, the method proposed herein of limiting the value of $(\rho_t - \rho_c)/\rho_{bo}$ in order to provide minimum flexural ductility may be considered as an extension of the method being used by ACI 318.

From Fig. 5, it can be seen that for given material parameters and any specified value of minimum curvature ductility factor, there corresponds a maximum value of $(\rho_t - \rho_c)/\rho_{bo}$. Since the relation between the curvature ductility factor μ and the value of $(\rho_t - \rho_c)/\rho_{bo}$ is dependent on the concrete strength and the steel yield strength, the maximum value of $(\rho_t - \rho_c)/\rho_{bo}$ varies with the concrete strength and the steel yield strength. In the present study, the maximum values of $(\rho_t - \rho_c)/\rho_{bo}$ that would yield a minimum curvature ductility factor of 3·32 for beam sections with different concrete strength and steel yield strength have been evaluated by a trial-and-error process using the method of moment-curvature analysis presented herein. The maximum values of $(\rho_t - \rho_c)/\rho_{bo}$ so obtained for beam sections with equal compression and tension steel yield strengths are listed in the third columns of Tables 1–3. It is worth noting from these results that the maximum value of $(\rho_t - \rho_c)/\rho_{bo}$ decreases substantially as the concrete strength f_{co} increases from 30 to 90 MPa. Moreover, the maximum value of $(\rho_t - \rho_c)/\rho_{bo}$ decreases slightly as the steel yield strength increases from 250 to 600 MPa. In general, it may be said that a lower maximum limit should be set to the value of $(\rho_t - \rho_c)/\rho_{bo}$ when higher strength materials are used. It is therefore inappropriate to set a fixed maximum limit to the value of $(\rho_t - \rho_c)/\rho_{bo}$.

Multiplying the maximum values of $(\rho_t - \rho_c)/\rho_{bo}$ by the respective values of ρ_{bo} at the same material strength level, the corresponding maximum values of $(\rho_t - \rho_c)$ may be obtained, as listed in the fourth columns of Tables 1–3. It can be seen from these results that although the maximum value of $(\rho_t - \rho_c)/\rho_{bo}$ decreases as the concrete strength increases, the maximum value of $(\rho_t - \rho_c)$ still increases significantly with the concrete strength because the value of ρ_{bo} increases with the concrete strength. Therefore, the use of a higher strength concrete would allow a higher value of $(\rho_t - \rho_c)$ to be used, which in turn would allow a higher tension steel ratio to be employed to increase the flexural strength while maintaining the same minimum level of flexural ductility. On the other hand, it is also evident that the maximum value of $(\rho_t - \rho_c)$ decreases significantly as the steel yield strength increases because both the maximum value of $(\rho_t - \rho_c)/\rho_{bo}$ and the value of ρ_{bo} decrease as the steel yield strength increases. In other words, when a higher strength steel is used, a lower maximum limit has to be set to the value of $(\rho_t - \rho_c)$. It is therefore questionable whether the use of a higher strength steel for the reinforcement would really allow a higher flexural strength to be achieved while maintaining the same minimum level of flexural ductility.

In order to study the maximum flexural strength that could be achieved at various concrete strength and steel yield strength levels while maintaining the proposed minimum level of flexural ductility, the maximum tension steel ratio and the maximum flexural strength expressed in terms of $M_p/(bd^2)$ have been evaluated for each combination of concrete strength and steel yield strength, and the results are presented in Tables 4–6. It is revealed from these results that the maximum tension steel ratio increases as the concrete strength increases and it decreases as the steel yield strength increases. More importantly, while the maximum value of $M_p/(bd^2)$ increases significantly as the concrete strength increases, it decreases slightly as the steel yield strength increases. Therefore, the use of a higher strength steel for the reinforcement would not allow a higher flexural strength to be achieved while maintaining the same minimum level of flexural ductility. Nevertheless, the addition of compression reinforcement, which is generally quite costly, would allow a higher tension steel ratio to be employed to increase the flexural strength while maintaining the same minimum level of flexural ductility.

The advantages and disadvantages of using higher strength materials are now clear. The use of a higher strength concrete would allow a higher flexural strength to be achieved while maintaining the same minimum level of flexural ductility, albeit a higher strength concrete by itself is generally less ductile. On the other hand, the use of a higher strength steel

Table 4. Maximum tension steel ratios and maximum flexural strength for $\mu_{min} = 3{\cdot}32$ when $f_{yc} = f_{yt} = 250$ MPa

f_{co}: MPa	Maximum value of ρ_t: %			Maximum value of M_p/bd^2: MPa		
	$\rho_c = 0\%$	$\rho_c = 0{\cdot}5\%$	$\rho_c = 1{\cdot}0\%$	$\rho_c = 0\%$	$\rho_c = 0{\cdot}5\%$	$\rho_c = 1{\cdot}0\%$
30	5·86	6·36	6·86	10·83	11·97	13·10
40	6·63	7·13	7·63	12·86	13·99	15·13
50	7·32	7·82	8·32	14·63	15·77	16·90
60	7·92	8·42	8·92	16·18	17·31	18·45
70	8·46	8·96	9·46	17·57	18·70	19·84
80	8·95	9·45	9·95	18·83	19·97	21·10
90	9·39	9·89	10·39	19·97	21·11	22·24

Table 5. Maximum tension steel ratios and maximum flexural strength for $\mu_{min} = 3{\cdot}32$ when $f_{yc} = f_{yt} = 460$ MPa

f_{co}: MPa	Maximum value of ρ_t: %			Maximum value of M_p/bd^2: MPa		
	$\rho_c = 0\%$	$\rho_c = 0{\cdot}5\%$	$\rho_c = 1{\cdot}0\%$	$\rho_c = 0\%$	$\rho_c = 0{\cdot}5\%$	$\rho_c = 1{\cdot}0\%$
30	2·39	2·89	3·39	8·84	10·92	13·01
40	2·67	3·17	3·67	10·24	12·31	14·40
50	2·93	3·43	3·93	11·49	13·55	15·64
60	3·15	3·65	4·15	12·55	14·61	16·68
70	3·35	3·85	4·35	13·51	15·57	17·63
80	3·53	4·03	4·53	14·37	16·43	18·49
90	3·69	4·19	4·69	15·14	17·20	19·26

Table 6. Maximum tension steel ratios and maximum flexural strength for $\mu_{min} = 3{\cdot}32$ when $f_{yc} = f_{yt} = 600$ MPa

f_{co}: MPa	Maximum value of ρ_t: %			Maximum value of M_p/bd^2: MPa		
	$\rho_c = 0\%$	$\rho_c = 0{\cdot}5\%$	$\rho_c = 1{\cdot}0\%$	$\rho_c = 0\%$	$\rho_c = 0{\cdot}5\%$	$\rho_c = 1{\cdot}0\%$
30	1·59	2·09	2·59	7·92	10·48	13·21
40	1·76	2·26	2·76	9·05	11·61	14·25
50	1·92	2·42	2·92	10·07	12·64	15·23
60	2·05	2·55	3·05	10·90	13·49	16·09
70	2·18	2·68	3·18	11·71	14·31	16·92
80	2·29	2·79	3·29	12·40	15·02	17·63
90	2·38	2·88	3·38	12·98	15·61	18·23

would not allow a higher flexural strength to be achieved while maintaining the same minimum level of flexural ductility; it only allows the use of a smaller steel area for a given flexural strength requirement to save the amount of steel needed and to avoid steel congestion.

Tables 1–6 can be used directly as design aids for the flexural strength and ductility design of reinforced concrete beams. For given concrete strength and steel yield strength, the maximum values of $(\rho_t - \rho_c)/\rho_{bo}$ and $(\rho_t - \rho_c)$ may be obtained from the third and fourth columns of Tables 1–3. When given the flexural strength requirement expressed in terms of $M_p/(bd^2)$, the necessity to add compression reinforcement may be determined from the maximum values of $M_p/(bd^2)$ listed in Tables 4–6. If it is not decided yet whether to use HSC and/or HSS, the relative merits of using these higher strength materials may be evaluated using these tables.

However, it may not be practical to incorporate the above tables into a design code. For incorporation into a design code, simplified guidelines may be preferred. Referring to the maximum values of $(\rho_t - \rho_c)/\rho_{bo}$ listed in Tables 1–3, it can be seen that the effect of the steel yield strength on the maximum value of $(\rho_t - \rho_c)/\rho_{bo}$ is relatively small. Neglecting the effect of the steel yield strength, the following guidelines for limiting the value of $(\rho_t - \rho_c)$ in order to ensure provision of minimum flexural ductility are developed.

In the case of $f_{yc} = f_{yt} \leqslant 600$ MPa, the value of $(\rho_t - \rho_c)$ should not exceed 0·70 of ρ_{bo} when $f_{co} \leqslant 30$ MPa, should not exceed 0·60 of ρ_{bo} when 30 MPa $< f_{co} \leqslant 50$ MPa, and should not exceed 0·50 of ρ_{bo} when 50 MPa $< f_{co} \leqslant 80$ MPa.

Provision of minimum flexural ductility by limiting neutral axis depth

The method of limiting the neutral axis depth in order to ensure provision of minimum flexural ductility has been adopted by a number of design codes. However, different design codes set maximum limits to the neutral axis depth in different ways. For instance, the design code NZS 3101 limits the neutral axis depth to not more than a certain fraction of the neutral axis depth of the balanced section, while the design codes BS 8110, EC 2 and GBJ 11 limit the neutral axis depth to not more than a certain fraction of the effective depth. The maximum limits set to the neutral axis depth in these codes are applicable to both the case of singly reinforced sections with no compression reinforcement added and the case of doubly reinforced sections with compression reinforcement added.

As before, it can be seen from Fig. 6 that for given material parameters and any specified value of minimum curvature ductility factor, there corresponds a maximum value of d_n/d_{nb}. Since the relation between the curvature ductility factor μ and the value of d_n/d_{nb} is dependent on the concrete strength and the steel yield strength, the maximum value of d_n/d_{nb} varies with the concrete strength and the steel yield strength. In this study, the maximum values of d_n/d_{nb} that would yield a minimum curvature ductility factor of 3·32 for beam sections with different concrete strength and steel yield strength have been evaluated by a trial and error process and the results so obtained are presented, together with the corresponding values of d_{nb}/d, in Tables 7–9. It is noted that the maximum value of d_n/d_{nb} decreases significantly as the concrete strength f_{co} increases from 30 to 90 MPa. Moreover, the maximum value of d_n/d_{nb} decreases slightly as the steel yield strength increases from 250 to 600 MPa. Thus, in general, a lower maximum limit should be set to the value of d_n/d_{nb} when higher strength materials are used.

Multiplying the maximum values of d_n/d_{nb} by the respective values of d_{nb}/d at the same material strength level, the corresponding maximum values of d_n/d may be obtained, as listed in the fourth columns of Tables 7–9. It can be seen from these results that the maximum value of d_n/d decreases substantially when either the concrete strength or the steel yield strength increases because both the maximum value of d_n/d_{nb} and the value of d_{nb}/d decrease as the material strengths increase. The range of variation of the maximum value of d_n/d is generally larger than the corresponding range of variation of the maximum value of d_n/d_{nb}. Moreover, the variation of the maximum value of d_n/d with the steel yield strength is significantly larger than the variation of the maximum value of d_n/d_{nb} with the steel yield strength. For instance, at a concrete strength of $f_{co} = 60$ MPa, when the steel yield strength vary from $f_{yc} = f_{yt} =$ 250 MPa to $f_{yc} = f_{yt} = 600$ MPa, the maximum value of d_n/d decreases by 36% from 0·489 to 0·314 whereas the maximum value of d_n/d_{nb} decreases only by 16% from 0·661 to 0·555. Thus, if a maximum limit is to be imposed on the value of d_n/d in order to achieve a consistent minimum level of flexural ductility, the maximum limit has to be a variable limit depending on both the concrete strength and the steel yield strength.

Table 7. d_{nb}/d and maximum values of d_n/d_{nb} and d_n/d for $\mu_{min} = 3·32$ when $f_{yc} = f_{yt} = 250$ MPa

f_{co}: MPa	d_{nb}/d	Maximum value of d_n/d_{nb}	Maximum value of d_n/d
30	0·766	0·838	0·642
40	0·753	0·757	0·570
50	0·746	0·701	0·523
60	0·740	0·661	0·489
70	0·736	0·628	0·462
80	0·733	0·602	0·441
90	0·730	0·579	0·423

Table 8. d_{nb}/d and maximum values of d_n/d_{nb} and d_n/d for $\mu_{min} = 3·32$ when $f_{yc} = f_{yt} = 460$ MPa

f_{co}: MPa	d_{nb}/d	Maximum value of d_n/d_{nb}	Maximum value of d_n/d
30	0·664	0·732	0·486
40	0·643	0·666	0·428
50	0·631	0·620	0·391
60	0·622	0·585	0·364
70	0·615	0·558	0·343
80	0·609	0·537	0·327
90	0·604	0·518	0·313

Table 9. d_{nb}/d and maximum values of d_n/d_{nb} and d_n/d for $\mu_{min} = 3·32$ when $f_{yc} = f_{yt} = 600$ MPa

f_{co}: MPa	d_{nb}/d	Maximum value of d_n/d_{nb}	Maximum value of d_n/d
30	0·617	0·687	0·424
40	0·592	0·627	0·371
50	0·577	0·586	0·338
60	0·566	0·555	0·314
70	0·558	0·529	0·295
80	0·550	0·511	0·281
90	0·544	0·493	0·268

The maximum values of d_n/d_{nb} and d_n/d listed in Tables 7–9 can be used to replace the existing values given in the various design codes to provide a consistent level of minimum flexural ductility. However, for incorporation into a design code, simplified guidelines, as developed in the following, may be preferred. Referring to the maximum values of d_n/d_{nb} listed in Tables 7–9, it can be seen that the effect of the steel yield strength on the maximum value of d_n/d_{nb} is relatively small. Neglecting the effect of the steel yield strength and expressing the maximum value of d_n as a fraction of d_{nb}, the following guidelines for the maximum value of d_n are developed.

In the case of $f_{yc} = f_{yt} \leqslant 600$ MPa, the value of d_n

should not exceed 0·70 of d_{nb} when $f_{co} \leqslant 30$ MPa, should not exceed 0·60 of d_{nb} when 30 MPa $< f_{co} \leqslant$ 50 MPa, and should not exceed 0·50 of d_{nb} when 50 MPa $< f_{co} \leqslant 80$ MPa.

The maximum value of d_n may also be expressed as a fraction of d instead of d_{nb}, as being given in some design codes. However, when the maximum value of d_n is expressed as a fraction of d, the maximum value has to vary with the steel yield strength because the effect of the steel yield strength on the maximum value of d_n/d is quite significant. Taking into account the effect of the steel yield strength and expressing the maximum value of d_n as a fraction of d, the following guidelines are developed.

In the case of $f_{yc} = f_{yt} \leqslant 460$ MPa, the value of d_n should not exceed 0·50 of d when $f_{co} \leqslant 30$ MPa, should not exceed 0·40 of d when 30 MPa $< f_{co} \leqslant 50$ MPa, and should not exceed 0·33 of d when 50 MPa $< f_{co} \leqslant$ 80 MPa.

In the case of 460 MPa $< f_{yc} = f_{yt} \leqslant 600$ MPa, the value of d_n should not exceed 0·45 of d when $f_{co} \leqslant 30$ MPa, should not exceed 0·35 of d when 30 MPa $< f_{co} \leqslant 50$ MPa, and should not exceed 0·28 of d when 50 MPa $< f_{co} \leqslant 80$ MPa.

Conclusion

The non-linear flexural behaviour and curvature ductility of reinforced concrete beams made of materials of widely varying strengths have been studied using a rigorous analysis method. Within the limitations of this investigation, it was found that the major factor determining the curvature ductility factor of a beam section is the degree of the beam section being under- or over-reinforced, which may be measured in terms of $(\rho_t - \rho_c)/\rho_{bo}$ or d_n/d_{nb}. However, the relation between the curvature ductility factor and $(\rho_t - \rho_c)/\rho_{bo}$ and the relation between the curvature ductility factor and d_n/d_{nb} are both dependent on the material strengths. Due to such dependence, the current practices in the various design codes of providing a minimum level of flexural ductility by limiting the tension steel ratio or the neutral axis depth would result in a variable level of curvature ductility depending on the concrete grade and the steel yield strength. Of greater concern is that this would lead to a lower level of curvature ductility than has been provided in the past to beams made of conventional materials when HSC and/or HSS are used.

In order to provide a consistent level of minimum curvature ductility, it is proposed to set a fixed minimum value for the curvature ductility factor, which, by referring to the curvature ductility factors being provided in the various existing design codes, is recommended to be 3·32. Using a trial-and-error process, the maximum values of $(\rho_t - \rho_c)/\rho_{bo}$ and d_n/d_{nb} corresponding to the proposed minimum curvature ductility factor for various concrete grades and steel yield strengths have been evaluated. From the numerical results, it was evident that both these two maximum values decrease significantly as the material strengths increase. Hence, it is inappropriate to set any fixed maximum limit to the value of $(\rho_t - \rho_c)/\rho_{bo}$ or the value of d_n/d_{nb}. Based on the maximum values of $(\rho_t - \rho_c)/\rho_{bo}$ and d_n/d_{nb} so obtained, simplified guidelines for limiting the value of $(\rho_t - \rho_c)$ or the value of d_n in order to ensure provision of the proposed minimum curvature ductility factor have been developed. These guidelines are applicable to both singly and doubly reinforced sections. It is proposed to modify the existing design codes by incorporating these guidelines, which would provide a much more consistent level of minimum flexural ductility regardless of the variations in material strengths.

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