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TECHNICAL NOTE

On seismic landslide hazard assessment

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KEYWORDS: earthquakes; landslides; slopes

INTRODUCTION

Seismic landslides are one of the most devastating effects of earthquakes, as evidenced by many historical records (Keefer, 1984; Turner & Schuster, 1996). To date, various models have been developed for spatially distributed landslide hazard assessment, and almost all of them are based on infinite slope idealisation combined with the Newmark sliding-block and pseudo-static analysis (e.g. Jibson *et al.*, 1998; Miles & Keefer, 2001). Although no slope perfectly satisfies the assumptions of the infinite slope model, many, if not most, natural landslides are predominantly translational, with relatively low thickness-to-length ratios. The infinite slope idealisation therefore provides a useful approximation that can help to identify the landslide hazard at the reconnaissance level.

In Newmark sliding-block theory, a potential landslide is modelled as a rigid friction-block resting on an inclined plane (Newmark, 1965). The seismic inertial force on the sliding mass is regarded as pseudo-static, and is usually represented using a horizontal seismic coefficient that is expressed as a percentage of the gravity. The Newmark analysis calculates the cumulative permanent displacement of the friction-block as it is subjected to the acceleration time-history of a given earthquake, by double-integrating those parts of the acceleration time-history that exceed the so-called yield acceleration. The seismic landslide hazard is often assessed using the calculated Newmark displacement. As an example, Table 1 gives the categories of landslide hazard that are proposed by the US Geological Survey using the normalised Newmark displacement for infinite slope models (Miles & Keefer, 2001). There are six levels of relative hazard, ranging from Low, with a normalised displacement of between 0 and 0.02, to Very High, with a normalised displacement of 0.5–1.0. Note that the Newmark displacement here should be considered as a relative index of slope performance rather than a real deformation.

The existing models have tended to focus on the horizontal seismic force and disregard the vertical acceleration, although a real earthquake will subject the sliding mass to both. This common practice is due partly to the consideration that most earthquakes produce a peak vertical acceleration that is small compared with the peak horizontal acceleration, and the effect of vertical acceleration is negligible (Day, 2002). A limited number of studies have discussed this effect on earth structures and appeared to offer different opinions. The elegant pseudo-static analysis of Sarma (1975) for stability of earth dams indicates that the effect is minor, whereas the analysis of Ling & Leshchinsky (1998) for dry, reinforced soil structures has suggested that the effect is significant.

Table 1. Categories of relative hazard for landslides (after Miles & Keefer, 2001)

Level of hazard	Normalised Newmark displacement*
Low (L)	0·00–0·02
Moderately low (ML)	0·02–0·05
Moderate (M)	0·05–0·10
Moderately high (MH)	0·10–0·20
High (H)	0·20–0·50
Very high (VH)	0·50–1·00

* Normalised by the value of 100 cm.

In dealing with this issue, some observations from recent earthquakes are worth noting. First, significantly large vertical accelerations have repeatedly been recorded in the near field of moderate and large earthquakes, which shows that the rule-of-thumb ratio, 1/2 or 2/3, between peak vertical and horizontal ground acceleration may not be a good descriptor (NCEER, 1997). For example, the peak vertical acceleration at the surface of a reclaimed site during the 1995 Kobe earthquake was found to be *twice* as high as the peak horizontal acceleration (Yang & Sato, 2000). Second, the ground motion amplification, particularly in the vertical direction, can be significantly affected by groundwater conditions (Yang & Sato, 2001; Yang *et al.*, 2002). These observations imply that various possible combinations of the groundwater and ground motion conditions may need to be examined to identify a particularly severe state of hazard for earth structures.

In view of the above observations, this study aims to explore the possible effect of vertical acceleration with regard to landslide hazard assessment, by taking account of a wide range of magnitudes of the vertical and horizontal accelerations and a varying water table. Effort is made to show how significant the effect could be on the factor of safety, yield acceleration and, particularly, permanent displacement that is directly associated with hazard estimation.

INFINITE SLOPE ANALYSIS

The simplified infinite slope model is shown in Fig. 1. This is a two-dimensional model describing a slope with a potential sliding plane that is taken parallel to the surface of the slope at a depth z, with the water table being at $z_w = mz$. Clearly, for dry slopes $m = 0$, and for saturated slopes $m = 1$.

Factor of safety

Consider a vertical slice of a unit width that has a weight W and is subjected to both vertical and horizontal ground accelerations (Fig. 1). The pseudo-static seismic forces act-

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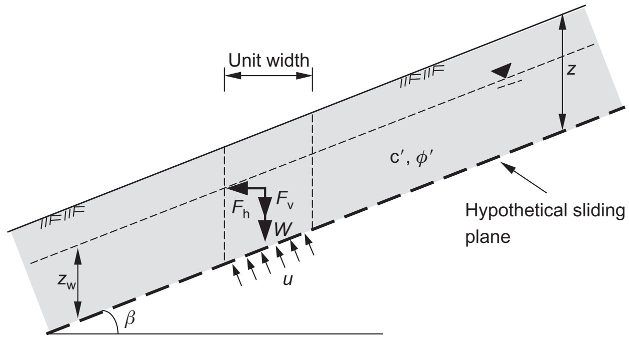

Fig. 1. Infinite slope model

ing on the slice are represented by $F_h = k_h W$ and $F_v = k_v W$, where k_h and k_v are known as the horizontal and vertical seismic coefficients respectively. In this paper, negative k_v corresponds to the upward inertial force.

The factor of safety of the infinite slope, FS, is determined as the ratio between the available shear strength and the shear stress developed on the failure plane, from

$$\mathrm{FS} = \frac{c' + (\sigma - u)\tan\phi'}{[(1+k_v)\sin\beta + k_h\cos\beta]\gamma z\cos\beta} \tag{1}$$

where c' is the effective cohesion, ϕ' is the effective friction angle, γ is the unit weight of the soil, σ is the total normal stress on the failure plane, and u is the pore water pressure.

Equation (1) can be further written as

$$\mathrm{FS} = \frac{\begin{gathered} c' + \{[(1+k_v)\cos\beta - k_h\sin\beta]\gamma \\ -(1+k_v)\gamma_w m\cos\beta\} z\cos\beta\tan\phi' \end{gathered}}{[(1+k_v)\sin\beta + k_h\cos\beta]\gamma z\cos\beta} \tag{2}$$

where γ_w is the unit weight of water. Because of the difficulty and uncertainty involved in determining the earthquake-induced excess pore water pressure, this effect is not included in the analysis, in order to retain the simplicity of the pseudo-static method.

The above general expression can be readily simplified for several special cases that have been discussed in the literature:

(*a*) *Special case 1*: Dry and static conditions, that is, $k_h = k_v = 0$, $m = 0$

$$\mathrm{FS} = \frac{c' + \gamma z\cos^2\beta\tan\phi'}{\gamma z\sin\beta\cos\beta} \tag{3}$$

(*b*) *Special case 2*: Saturated and static conditions, that is, $k_h = k_v = 0$, $m = 1$

$$\mathrm{FS} = \frac{c' + (\gamma - \gamma_w)z\cos^2\beta\tan\phi'}{\gamma z\sin\beta\cos\beta} \tag{4}$$

where γ should be taken as the saturated unit weight of the soil.

(*c*) *Special case 3*: Dry slopes subjected to horizontal acceleration only, that is, $k_v = 0$, $k_h \neq 0$, $m = 0$

$$\mathrm{FS} = \frac{c' + (\cos\beta - k_h\sin\beta)\gamma z\cos\beta\tan\phi'}{(\sin\beta + k_h\cos\beta)\gamma z\cos\beta} \tag{5}$$

(*d*) *Special case 4*: Saturated slopes subjected to horizontal acceleration only, i.e. $k_v = 0$, $k_h \neq 0$, $m = 1$

$$\mathrm{FS} = \frac{c' + [(\cos\beta - k_h\sin\beta)\gamma - \gamma_w\cos\beta]z\cos\beta\tan\phi'}{(\sin\beta + k_h\cos\beta)\gamma z\cos\beta} \tag{6}$$

Reduction of factor of safety due to vertical motion

To quantify the effect of vertical ground acceleration, it is useful to introduce a reduction factor for the factor of safety, defined as the ratio between the factor of safety under combined vertical and horizontal accelerations, $(\mathrm{FS})_{h+v}$, and the factor of safety under horizontal acceleration only, $(\mathrm{FS})_h$. In so doing, equation (2) is first rewritten in the form

$$(\mathrm{FS})_{h+v} = \frac{a_1 k_v + a_2 k_h + a_3}{a_4 k_v + a_5 k_h + a_6} \tag{7}$$

where

$$a_1 = (\gamma - \gamma_w m)z\cos^2\beta\tan\phi'$$
$$a_2 = -\gamma z\sin\beta\cos\beta\tan\phi'$$
$$a_3 = c' + (\gamma - m\gamma_w)z\cos^2\beta\tan\phi'$$
$$a_4 = \gamma z\sin\beta\cos\beta$$
$$a_5 = \gamma z\cos^2\beta$$
$$a_6 = \gamma z\sin\beta\cos\beta$$

From equation (7) the factor of safety for the slope subjected only to horizontal acceleration can be readily given as

$$(FS)_h = \frac{a_2 k_h + a_3}{a_5 k_h + a_6} \tag{8}$$

The reduction factor can then be established as

$$r_F = \frac{(\mathrm{FS})_{h+v}}{(\mathrm{FS})_h} = \frac{[(a_1 p + a_2)k_h + a_3](a_5 k_h + a_6)}{[(a_4 p + a_5)k_h + a_6](a_2 k_h + a_3)} \tag{9}$$

where $p = k_v/k_h$.

Yield acceleration

The yield acceleration is conventionally determined to be the horizontal acceleration that results in a pseudo-static factor of safety equal to 1·0. From equation (7) the coefficient of yield acceleration, k_{hy}, can be given as

$$k_{hy} = \left(\frac{a_1 - a_4}{a_5 - a_2}\right)k_v + \left(\frac{a_3 - a_6}{a_5 - a_2}\right) \tag{10}$$

It is evident from equation (10) that the coefficient of yield acceleration without taking account of the effect of vertical acceleration has the form

$$k'_{hy} = \left(\frac{a_3 - a_6}{a_5 - a_2}\right) \tag{11}$$

Furthermore, a reduction factor for the yield acceleration r_y can be introduced to quantify the effect of vertical acceleration such that

$$r_y = \frac{k_{hy}}{k'_{hy}} = \frac{1}{1 - \chi p} \tag{12}$$

where $\chi = (a_1 - a_4)/(a_5 - a_2)$. Clearly, $\chi = \tan(\phi' - \beta)$ when $m = 0$.

Permanent displacement

A Newmark analysis calculates the cumulative displacement of the friction-block as it is subjected to the acceleration time-history of a given earthquake. Because of its simplicity, the method is widely used in seismic design of earth structures. A conventional Newmark analysis involves determination of the yield acceleration, selection of an appropriate earthquake record, and double integration of the parts in the acceleration time-history that exceed the yield acceleration. To facilitate applications of this method, several simplified models have been developed for estimating the Newmark displacements of slopes (e.g. Ambraseys & Menu, 1988; Jibson *et al.*, 1998). These models are usually established based on regression analyses for a large number of selected earthquake records.

For the first instance, the model of Ambraseys & Menu (1988) is used here to show how the permanent displacement is influenced by vertical acceleration. In this model, the permanent displacement d (in cm) is expressed as a function of the ratio a_y/a_{max}, where a_y is the yield acceleration and a_{max} is the peak horizontal acceleration.

$$\log d = 0{\cdot}90 + \log\left[\left(1 - \frac{a_y}{a_{max}}\right)^{2{\cdot}53}\left(\frac{a_y}{a_{max}}\right)^{-1{\cdot}09}\right] \tag{13}$$

Given a peak horizontal acceleration and the determined yield acceleration, equation (13) allows prediction of the permanent displacement.

As already indicated by equation (12), inclusion of vertical acceleration may affect the yield acceleration and, subsequently, the permanent displacement. Assuming that d and d' are the displacements calculated using a_y and a'_y respectively, where a_y is the yield acceleration that includes the effect of vertical acceleration and a'_y is the yield acceleration excluding this effect, an amplification factor A_d is introduced here to quantify the effect for the permanent displacement.

$$A_d = \frac{d}{d'} = \left(r_y^{-1{\cdot}09}\right)\left[\frac{1 - r_y\left(a'_y/a_{max}\right)}{1 - \left(a'_y/a_{max}\right)}\right]^{2{\cdot}53} \tag{14}$$

where r_y is the reduction factor for the yield acceleration, defined in equation (12).

NUMERICAL RESULTS AND DISCUSSION

The expressions established in the preceding section enable one to quantify the effect of vertical acceleration in a convenient way. Using equation (2), Fig. 2 shows the factor of safety for a slope of inclination angle 15° under different values of seismic coefficient. The slope is assumed to be in either a saturated or a dry state, with $c' = 0$ and $\phi' = 35°$. For a saturated slope the unit weight is taken as 20 kN/m^3, and for a dry slope it is assumed to be 17 kN/m^3. The depth of sliding plane is assumed to be a representative value, 3 m (Keefer, 1984).

Compared with the case where the effect of vertical acceleration is neglected, one may note that inclusion of positive k_v causes an increase in the factor of safety, whereas negative k_v reduces the factor of safety. This suggests that the upward inertial force gives a critical case. Therefore the following discussion will concentrate on the case of negative k_v. It is also noted that the effect of vertical acceleration is negligible when $k_h \leqslant 0{\cdot}2$, but tends to become significant when $k_h \geqslant 0{\cdot}4$. The reduction of the factor of safety can be better quantified using r_F, as shown in Fig. 3. Comparison of the two plots in Fig. 3 suggests that the degree of reduction is more significant for saturated than for dry slopes. For a saturated slope at $k_h = 0{\cdot}4$ and $k_v = (-0{\cdot}5)k_h$, the factor of safety is about 84% of that determined by ignoring the effect of vertical acceleration. At the same k_h but with $k_v = (-1{\cdot}0)k_h$, the factor of safety is reduced by about 35%. Note that it is not uncommon to record a peak vertical acceleration that is as great as the peak horizontal acceleration, as evidenced by recent ground motion data.

Figure 3 implies that the effect of vertical acceleration is related to groundwater conditions. To provide a better evaluation of this effect, the reduction of the factor of safety is calculated as a function of the groundwater table (Fig. 4). It can be seen that, under otherwise identical conditions, the factor of safety is reduced more for a higher groundwater table. Under the same groundwater conditions, the factor of safety decreases with increasing values of k_h and decreasing values of p (i.e. k_v/k_h).

Figure 5 shows the variation of yield acceleration with the strength parameters and inclination angle for saturated slopes with the vertical motion effect excluded. It is clear that the yield acceleration increases with increasing shear strength, and decreases with increasing inclination angle. Depending

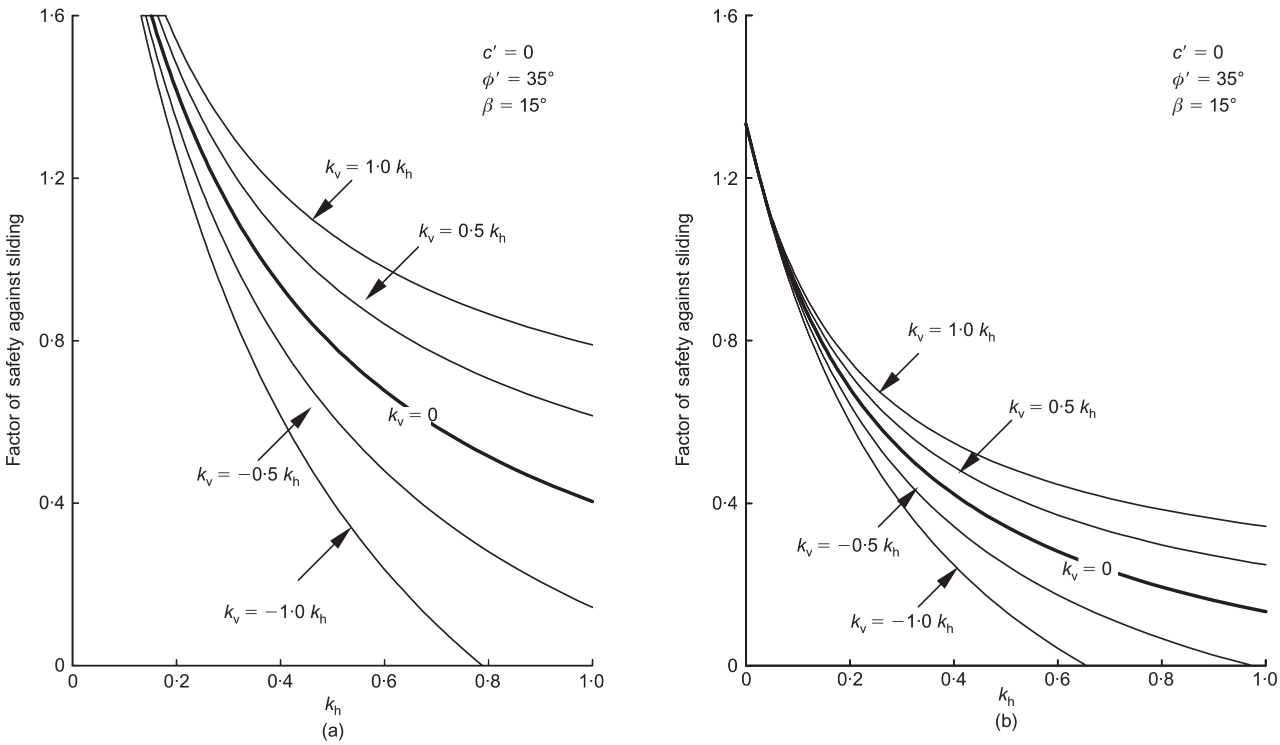

Fig. 2. Factor of safety of a saturated slope under various combinations of k_h and k_v: (a) dry conditions; (b) saturated conditions

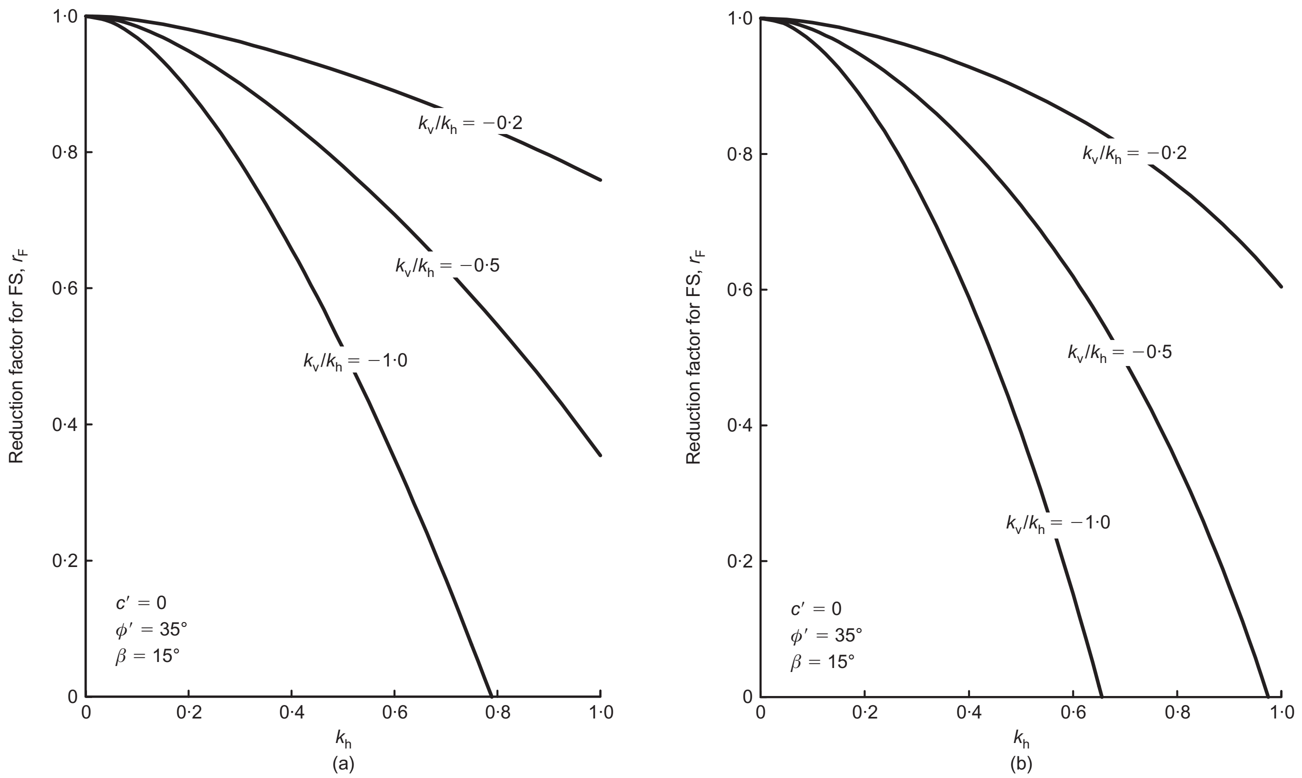

Fig. 3. Reduction factor for factor of safety under various combinations of *k*h and *k*v: (a) dry conditions; (b) saturated conditions

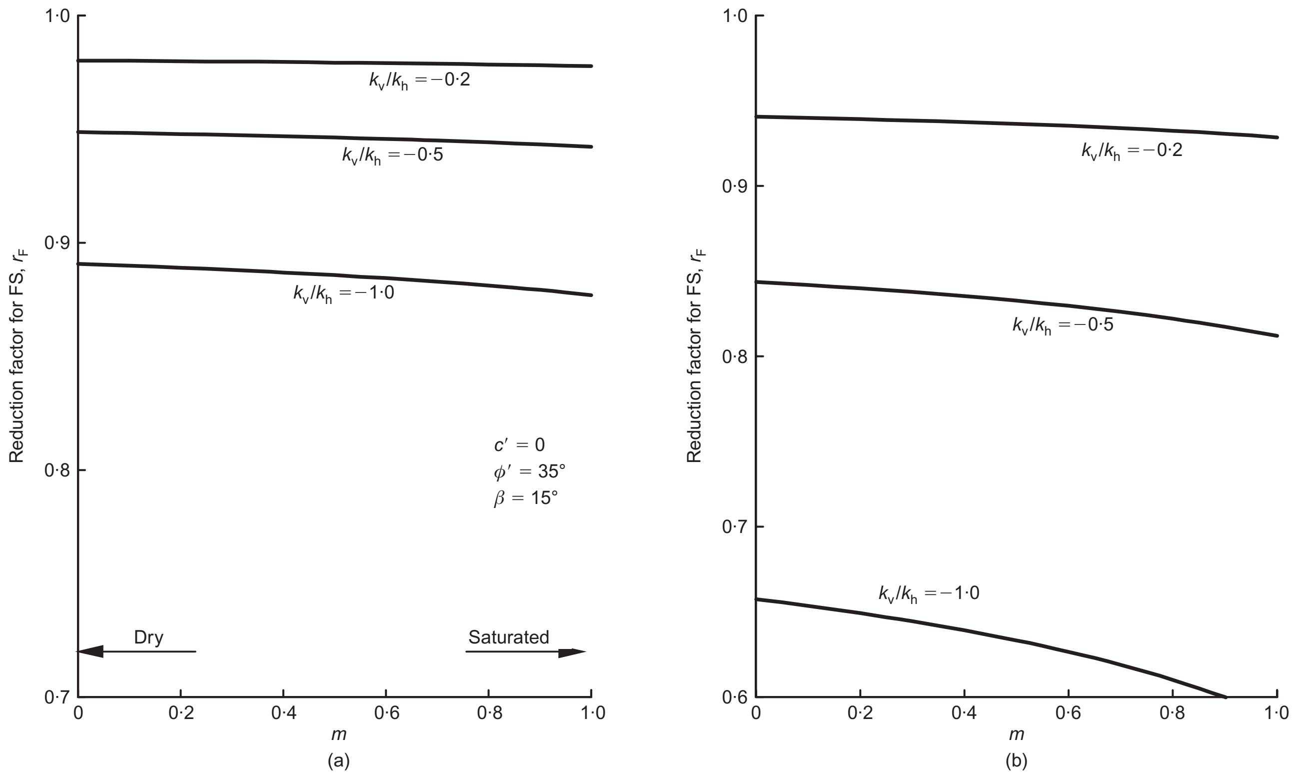

Fig. 4. Reduction factor for factor of safety as a function of water table: (a) $k_h = 0{\cdot}2$; (b) $k_h = 0{\cdot}4$

on the groundwater conditions and the value of p, the yield acceleration can be largely reduced by vertical acceleration, as shown in Fig. 6. Moreover, the reduction of yield acceleration appears to be more significant for gentle slopes, and slopes with a high friction angle.

Of particular interest is the influence of vertical acceleration on slope displacement. Using equation (14), Fig. 7 shows the displacement amplification that is induced by the inclusion of vertical acceleration. At low values of the ratio a'_y/a_{max}, say 0·2, the amplification factor is between 1 and 2 for both saturated and dry slopes, where the lower bound is for $p = -0{\cdot}5$ and the upper bound is for $p = -1{\cdot}5$. At higher values of a'_y/a_{max}, say 0·8, the amplification factor varies from 5 to as much as 20 for dry slopes and from 2 to 6 for

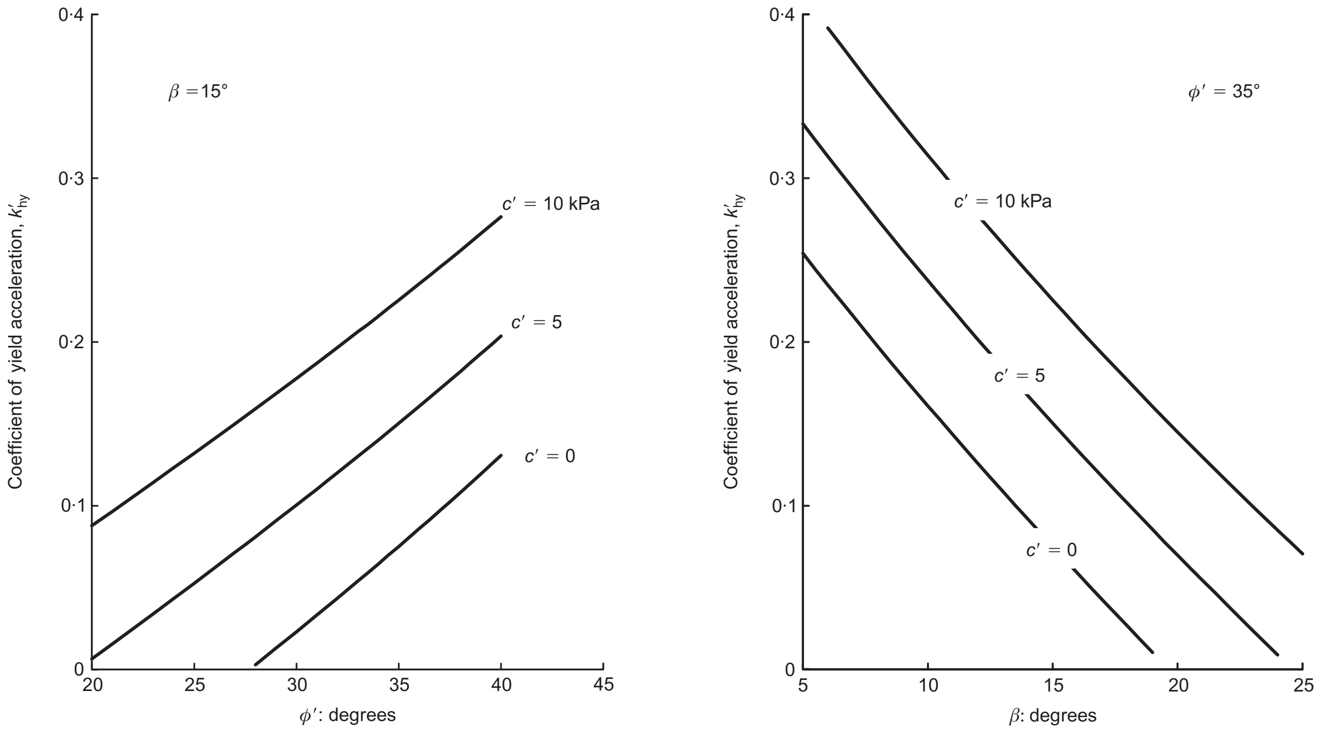

Fig. 5. Yield accelerations for saturated slopes, neglecting vertical motion effects

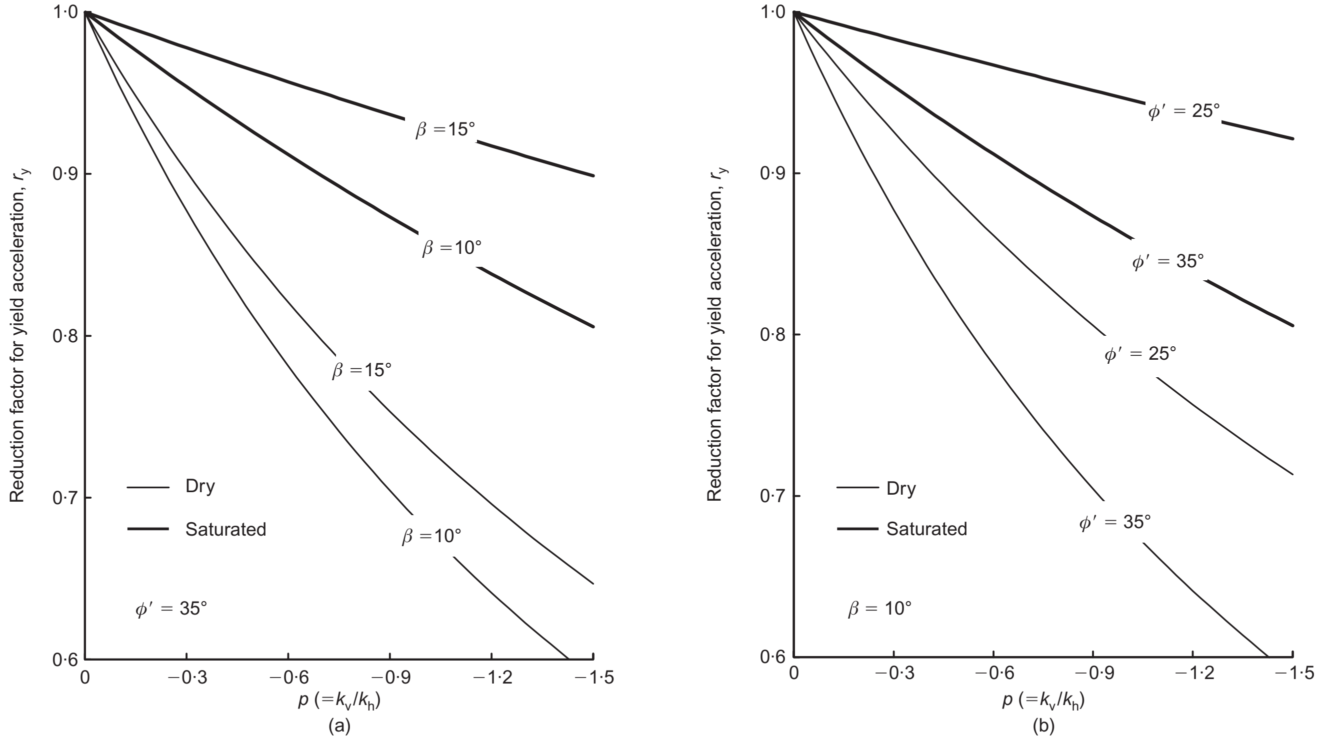

Fig. 6. Reduction factor for yield acceleration

saturated slopes. It is to be noted that the large amplification of displacements is in a relative sense, being associated with small displacements.

In order to identify the effect of vertical motion better, Table 2 gives comparisons of the Newmark displacements in three cases—$k_v/k_h = 0$, $-0{\cdot}5$ and $-1{\cdot}0$—for saturated slopes subjected to two scenario peak horizontal accelerations: $0{\cdot}3g$ and $0{\cdot}6g$. A range of friction angles and sloping angles are taken into account. Following the US Geological Survey classification system, the relative hazard for each case is also determined and included in the table. The comparisons suggest that the inclusion of vertical acceleration may cause a change in the Newmark displacement and, consequently, a redefinition of the hazard level in some cases.

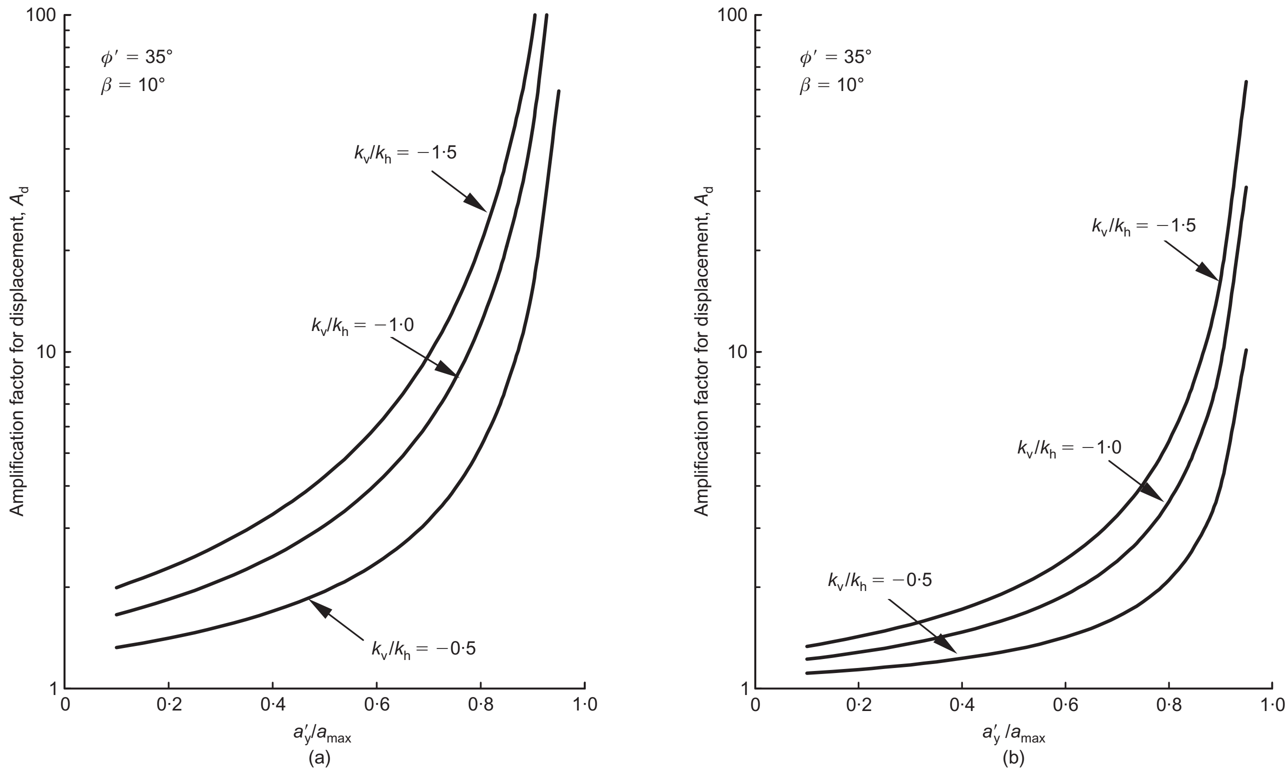

Fig. 7. Amplification factor for permanent displacement: (a) dry conditions; (b) saturated conditions

Table 2. Newmark displacements predicted for various conditions

ϕ': degrees	$a_{max} = 0{\cdot}3g$						$a_{max} = 0{\cdot}6g$					
	$\beta = 10°$			$\beta = 15°$			$\beta = 10°$			$\beta = 15°$		
	D_0: cm	D_1: cm	D_2: cm	D_0: cm	D_1: cm	D_2: cm	D_0: cm	D_1: cm	D_2: cm	D_0: cm	D_1: cm	D_2: cm
25	28·64 (H)	30·01 (H)	31·39 (H)	–	–	–	80·63 (VH)	83·74 (VH)	86·86 (VH)	–	–	–
30	7·97 (M)	9·05 (M)	10·16 (MH)	107·00 (>VH)	108·60 (>VH)	110·20 (>VH)	31·55 (H)	34·34 (H)	37·16 (H)	252·41 (>VH)	255·86 (>VH)	259·31 (>VH)
35	2·24 (ML)	3·00 (ML)	3·84 (ML)	17·35 (MH)	18·62 (MH)	19·90 (MH)	15·13 (MH)	17·62 (MH)	20·18 (H)	54·58 (VH)	57·56 (VH)	60·57 (VH)
40	0·41 (L)	0·83 (L)	1·37 (L)	4·63 (ML)	5·57 (M)	6·57 (M)	7·54 (M)	9·71 (M)	12·00 (MH)	22·49 (H)	25·15 (H)	27·86 (H)

Notes:
β = inclined angle of slope; ϕ' = friction angle of soil; $c' = 0$; $m = 1$ (saturated conditions); $\gamma = 20$ kN/m^3.
D_0 = displacement predicted by neglecting effect of vertical ground motion ($k_v/k_h = 0{\cdot}0$).
D_1 = displacement predicted by taking account of effect of vertical ground motion ($k_v/k_h = -0{\cdot}5$).
D_2 = displacement predicted by taking account of effect of vertical ground motion ($k_v/k_h = -1{\cdot}0$).
Letter in parentheses indicate relative hazard level.

CLOSING REMARKS

This study seeks to explore the possible effect of vertical ground motion, which has long been ignored in the practice of seismic landslide hazard analysis. For the infinite slope model, mathematical expressions have been established that allow a quick quantification of the effect on the factor of safety, yield acceleration and permanent displacement. The effect is shown to be related to several factors, including the shear strength of the soil (both c' and ϕ'), the groundwater condition, the slope angle, and the magnitudes of the vertical and horizontal accelerations. The study indicates that, under some combinations of these factors (e.g. large values of k_h and small values of p), simply disregarding the effect would not be appropriate. Using the US Geological Survey classification system, two sets of analyses have been conducted, which show that the inclusion of vertical acceleration may bring about a redefinition of the hazard level in some cases.

Finally, it is to be noted that current practice is based mainly on the pseudo-static approach and Newmark sliding-block theory. Although this provides a simple way of screening for various stability problems, it does not adequately account for some situations in real earthquakes, such as time-varying amplitudes and frequencies of vertical and horizontal ground accelerations, split in time of the peak values for vertical and horizontal ground accelerations, and deformations associated with significant build-up of excess pore pressures. When there is a need to go beyond pseudo-static analysis, comprehensive procedures such as effective-stress-based, non-linear finite element analyses in the time domain can provide a useful way of gaining insight into the performance of slopes and other earth structures.

ACKNOWLEDGEMENTS

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NOTATION

a_{max}	peak horizontal acceleration
a_y, a'_y	yield accelerations including and excluding effect of vertical motion respectively
A_d	amplification factor for Newmark displacement
c'	cohesion of soil
d, d'	Newmark displacements calculated using a_y and a'_y respectively
k_h, k_v	seismic horizontal and vertical coefficients respectively
k_{hy}, k'_{hy}	coefficients of yield acceleration including and excluding effect of vertical motion respectively
m	parameter characterising groundwater tables
p	ratio between seismic vertical and horizontal coefficients (k_v/k_h)
r_F, r_y	reduction factors for factor of safety and yield acceleration respectively
β	inclination angle of slope
γ, γ_w	unit weight of soil and unit weight of water respectively
ϕ'	effective friction angle of soil

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