ROBUST GAIN BANDWIDTH OPTIMIZATION IN TWO-PUMP FIBER OPTICAL PARAMETRIC AMPLIFIERS WITH DISPERSION FLUCTUATIONS

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ABSTRACT

Gain bandwidth optimization in a two-pump fiber optical parametric amplifier (2P-OPA) with bounded zero-dispersion wavelength (ZDW) uncertainty is investigated. An analytical framework is devised for the design of maximum-bandwidth 2P-OPAs ensuring positive parametric gain, tunable gain spectrum quality, and robustness against ZDW fluctuations. By exploiting the polynomial nature of the phase mismatch, the design task is formulated as a non-convex optimization problem, which is then solved through convex programming techniques based on linear matrix inequality (LMI) relaxations. Compared to conventional nonlinear programming (NLP) algorithms such as genetic algorithm (GA), the proposed methodology exhibits superior computational efficiency, and guarantees convergence to globally optimal design parameters.

1. INTRODUCTION

Fiber optical parametric amplifiers (OPAs) constitute a class of practical amplifiers with high gain [1], large bandwidth [2] and polarization-independence [3], in both one-pump and two-pump configurations. The basic one-pump OPA (1P-OPA) is simpler to set up but its signal gain spectrum is nonuniform; this problem can be solved by using a two-pump OPA (2P-OPA) [3]. 2P-OPA provides an extra degree of freedom compared to 1P-OPA, such that a flattened gain spectrum can be achieved by trading with the gain bandwidth [4]. Besides, with complementary phase-dithering, one can obtain a narrow-linewidth idler spectrum, as well as effective suppression of stimulated Brillouin scattering (SBS) [5]. Two-orthogonal-pump OPA (2OP-OPA) has also been shown to achieve polarization-independent operation [3]. Mid-span spectral inversion (MSSI) using 2OP-OPA in a 320-km transmission link has already been demonstrated [6]. Fiber OPA relies on the phase matching conditions amongst pump(s), signal and idler, which in turn depends on the uniformity of zero-dispersion wavelength (ZDW), denoted by λ_0, along the fiber. However, the manufacturing process of any fiber inevitably introduces variation of the core diameter which causes ZDW fluctuations. Ref. [7] has investigated the effects of ZDW fluctuations on 2P-OPAs and showed that the amount as well as uniformity of gain reduce considerably because of the fluctuations. Degradation of performance is particularly significant when the fluctuations have long correlation lengths [8].

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Conventional analyses on the effects of ZDW fluctuations and the corresponding parameter optimization rely on stochastic models for ZDW variation [7–9].The outcomes are probabilistic indicators or general design guidelines regarding bandwidth and wavelength selections etc. No guarantee can be made about robust operation of a 2P-OPA [1] under all possible ZDW variations. Moreover, generic nonlinear programming (NLP) techniques such as genetic algorithm (GA) [9], simulated annealing etc., are marked by intensive computation, high dependence on initial condition for convergence, and often require deep physical insights for parameter tuning and function setups.

In this paper, we address the robust (namely, guaranteed positive parametric gain) design of maximum-bandwidth 2P-OPAs wherein the ZDW uncertainty is modeled as a bounded set. This is a more realistic assumption as experiments have shown the actual λ_0 fluctuations to be slow-varying (having long correlation lengths) and within a certain tolerance [10]. By exploiting the polynomial nature of the phase mismatch, the design task is formulated as a non-convex optimization problem, which is then solved through convex programming techniques based on linear matrix inequality (LMI) relaxations [11, 12]. The proposed methodology exhibits superior computational efficiency, has tunable gain spectrum quality, and guarantees convergence to globally optimal wavelength assignments.

2. BACKGROUND AND MOTIVATION

In 2P-OPA analysis, assuming undepleted pumps, the signal (A_3) and idler (A_4) evolutions are described by [9, 13]

$$\frac{dA_3}{dz} = 2j\gamma(P_1+P_2)A_3 + 2j\gamma\sqrt{P_1P_2}e^{-j\theta z}A_4^* \quad (1a)$$

$$\frac{dA_4^*}{dz} = -2j\gamma(P_1+P_2)A_4^* - 2j\gamma\sqrt{P_1P_2}e^{j\theta z}A_3, \quad (1b)$$

with $\theta = \Delta\beta - 3\gamma(P_1+P_2)$. The notations are standard: P_1 and P_2 are respectively the pump powers at frequencies ω_1 and ω_2, γ is the nonlinear parameter, and $\Delta\beta$ is the linear propagation constant mismatch approximated by [7]

$$\Delta\beta = \left(\beta_3(\omega_c-\omega_0) + \frac{\beta_4}{2}(\omega_c-\omega_0)^2\right)\cdot \left((\omega_3-\omega_c)^2-\omega_d^2\right) + \frac{\beta_4}{12}\left((\omega_3-\omega_c)^4-\omega_d^4\right). \quad (2)$$

[1]By robustness we mean that the designed 2P-OPA should always exhibit positive parametric gain for the whole fiber section, for all possible ZDW variations within a bounded set.

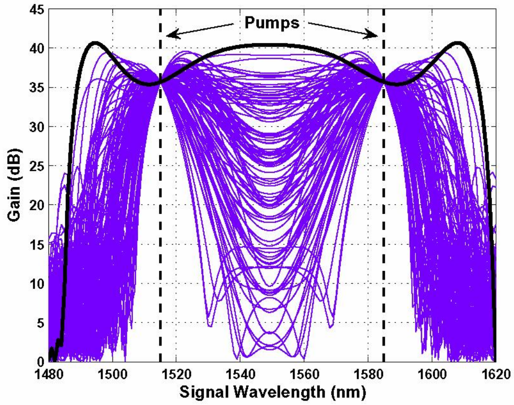

Fig. 1. Fiber-to-fiber variations in 2P-OPA spectra due to ZDW fluctuations (100 spectra in the plot). Here λ_0 is within 1550 ± 2nm, with correlation lengths randomly varying from 5m to 100m. The bold curve shows the spectrum with a constant $\lambda_0 = 1550$nm. (λ_1=1515nm, λ_2=1585nm, P_1=P_2=0.8W, γ=11.2W^{-1}/km, β_3=0.05ps^3/km, β_4=-2×10^{-4}ps^4/km, fiber length L=300m which is discretized into 300 sections for fluctuating ZDW simulations).

Here $\omega_c = (\omega_1 + \omega_2)/2$ is the center frequency of the two pumps and $\omega_d = (\omega_1 - \omega_2)/2$ is half their difference. Defining the total phase mismatch $\kappa = \gamma(P_1 + P_2) + \Delta\beta$, and the phase-shifted versions of the waves, $B_i = A_i e^{-2j\gamma(P_1+P_2)z}$, $i = 3, 4$, (1) can be rewritten as

$$\frac{dB_3}{dz} = 2j\gamma\sqrt{P_1P_2}e^{-j\kappa z}B_4^* \quad \text{(3a)}$$

$$\frac{dB_4^*}{dz} = -2j\gamma\sqrt{P_1P_2}e^{j\kappa z}B_3. \quad \text{(3b)}$$

Further letting $C_i = B_i e^{j(\kappa/2)z}$, $i = 3, 4$, it can be shown that

$$\frac{d}{dz}\begin{bmatrix} C_3 \\ C_4^* \end{bmatrix} = \begin{bmatrix} j\frac{\kappa}{2} & 2j\gamma\sqrt{P_1P_2} \\ -2j\gamma\sqrt{P_1P_2} & -j\frac{\kappa}{2} \end{bmatrix}\begin{bmatrix} C_3 \\ C_4^* \end{bmatrix}. \quad (4)$$

This is simply a linear dynamical system (specifically, an autonomous system) with the 2-by-2 system matrix having eigenvalues $\pm g$, where

$$g = \sqrt{4\gamma^2 P_1 P_2 - \frac{\kappa^2}{4}}. \quad (5)$$

Note that $C_3^* C_3 = A_3^* A_3$. Assuming a constant λ_0 down the fiber, the signal amplification or power gain at $z = L$ is

$$\left|\frac{A_3(L)}{A_3(0)}\right|^2 = 1 + \left(1 + \frac{\kappa^2}{4g^2}\right)\sinh^2(gL).$$

A sound 2P-OPA design should have a positive parametric gain, namely, $g > 0$, in the useful signal bandwidth $\omega_3 \in [\omega_2, \omega_1]$. This is apparent by examining (4). In control-theoretic parlance, for large gain throughout the fiber section, we would like to maintain an unstable mode (corresponding to $g > 0$) so that the signal is amplified via absorption of power from pumps. A purely imaginary g corresponds to an oscillatory mode in which the signal is

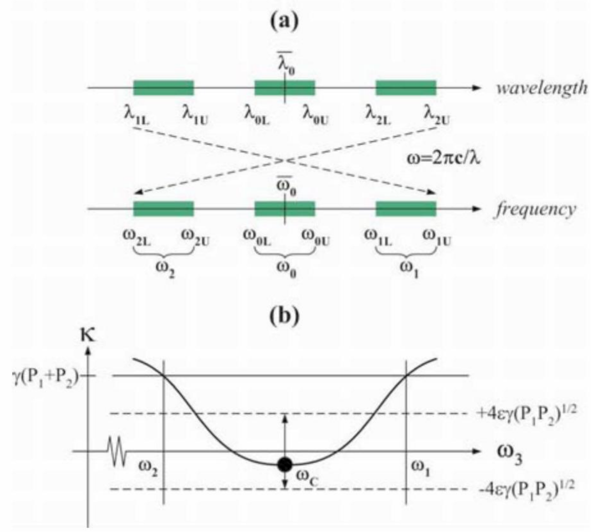

Fig. 2. (a) Notational conventions. The intervals denote the design/uncertainty ranges and the bar sign means the average value; (b) the κ curve evaluated along the ω_3-axis.

not amplified. Effects of a varying $\omega_0 (= 2\pi c/\lambda_0)$ can be studied by dividing the fiber into smaller sections and applying (4) iteratively. In this case, it is assumed that every finer section is small enough to have a relatively constant ω_0.

3. GAIN BANDWIDTH OPTIMIZATION

ZDW fluctuations result in a varying ω_0 which appears through $\Delta\beta$ (c.f. (2)) and therefore κ. It may happen that the amplitude of κ is large enough to render an imaginary g. When the fluctuations have long correlation lengths (in the order of 100m), this imaginary g may occur over a substantial portion in the z- and ω_3-dimension, causing detrimental effects on the gain profile [7,8,14]. In other words, a design that solely relies on the average ZDW ($\bar{\lambda}_0$) can be impractical. In fact, when the pump wavelengths are not carefully assigned, even slight fluctuations in the actual ZDW can lead to drastic changes in the gain spectra [7,8,14]. Fig. 1 depicts such a situation with non-optimized choices of λ_1 and λ_2.

In robust 2P-OPA design, an objective is to keep the parametric gain g positive, i.e.,

$$g^2 = 4\gamma^2 P_1 P_2 - \frac{\kappa^2}{4} > 0, \quad (6)$$

and as large as possible, in the signal bandwidth $\omega_3 \in [\omega_2, \omega_1]$. The following deals with the selection of ω_1 and ω_2 to maximize the useful bandwidth ($\omega_1 - \omega_2$) while ensuring positiveness of g against a bounded ω_0 uncertainty, $\omega_0 \in [\omega_{0L}, \omega_{0U}]$ (irrespective of any correlation length or pattern). For easy reference the notational conventions are illustrated in Fig. 2(a).

It is easy to see that the design objective translates into minimizing the amplitude of κ in $\omega_3 \in [\omega_2, \omega_1]$. Note that $\Delta\beta$ is a function of ω_0, ω_1, ω_2 and ω_3. On the signal wavelength or ω_3-axis, $\Delta\beta$ is symmetric about ω_c and identically zero at ω_1 and ω_2. This gives rise to the κ curve in Fig. 2(b). Differentiating κ

with respect to ω_3 and setting the derivative to zero, three extreme points are readily found, namely,

$$\omega_3 = \begin{cases} \omega_c \\ \omega_c \pm \sqrt{-\left(\frac{6\beta_3}{\beta_4} + 3(\omega_c - \omega_0)\right)(\omega_c - \omega_0)} \end{cases} .$$

The extreme point at $\omega_3 = \omega_c$ is always real. In physical settings, the other two extreme points are either imaginary conjugate pairs, or real pairs outside the optimized range $[\omega_2, \omega_1]$ (this can easily be incorporated as a post-optimization check, if necessary). A direct consequence is that the spectrum of κ in $\omega_3 \in [\omega_2, \omega_1]$ can be shaped by positioning the extreme point $\kappa|_{\omega_3=\omega_c}$. For (6) to hold, it dictates

$$4\epsilon\gamma\sqrt{P_1P_2} \geq \gamma(P_1 + P_2) + \Delta\beta|_{\omega_3=\omega_c} \geq -4\epsilon\gamma\sqrt{P_1P_2}, \quad (7)$$

where we have further introduced the *bound factor* $\epsilon \in [0, 1)$ for tuning the quality of the desired gain spectrum. Obviously, the smaller the ϵ, the higher the overall signal gain will be in $[\omega_2, \omega_1]$. Now we turn to the dependence of κ on the variation in ω_0. A key observation in (2) is that $\Delta\beta$ is only a quadratic function of ω_0. Similarly, by differentiating $\kappa|_{\omega_3=\omega_c}$ with respect to ω_0, the extreme point is found to be $\omega_0 = \omega_c + (\beta_3/\beta_4)$, which is also far outside the range $\omega_0 \in [\omega_{0L}, \omega_{0U}]$ for physical parameters. Therefore, depending on the signs of β_3 and β_4, the function $\kappa|_{\omega_3=\omega_c}$ is either monotonically increasing/decreasing on the ω_0-axis of interest. Subsequently, assuming a monotonically increasing $\kappa|_{\omega_3=\omega_c}$ in $\omega_0 \in [\omega_{0L}, \omega_{0U}]$ (modification for the opposite case is straightforward), the robust 2P-OPA design can be formulated as an optimization problem:

$$maximize \quad \omega_1 - \omega_2 \quad (8)$$

subject to

$$\begin{aligned} 4\epsilon\gamma\sqrt{P_1P_2} &\geq \gamma(P_1 + P_2) + \Delta\beta(\omega_1, \omega_2)|_{\omega_0=\omega_{0U},\omega_3=\omega_c}, \\ \gamma(P_1 + P_2) + \Delta\beta(\omega_1, \omega_2)|_{\omega_0=\omega_{0L},\omega_3=\omega_c} &\geq -4\epsilon\gamma\sqrt{P_1P_2}, \\ \omega_{1U} \geq \omega_1 &\geq \omega_{1L}, \\ \omega_{2U} \geq \omega_2 &\geq \omega_{2L}. \end{aligned}$$

This optimization problem is generally non-convex and normally requires NLP techniques. Fortunately, the multivariate polynomial nature of the objective and constraints in (8) allows it to be efficiently and optimally solved by convex programming techniques utilizing LMI relaxations. To this end, the solver *GloptiPoly* is employed [12]. Specifically, GloptiPoly solves the global optimization problem of minimizing (maximizing) a multivariate polynomial function subject to polynomial inequality, equality or integer constraints. Based on the theory of positive polynomials and moments, the solver builds and solves LMI relaxations of the optimization problem, thereby generating a series of lower (upper) bounds monotonically converging to the global optimum. The reader is referred to Ref.s [11, 12] for details. However, it should be stressed that the challenge falls more on the problem modeling and formulation. Once the problem is properly set up, as in (8) or (9) to follow, application of the solver is usually a routine job.

4. NUMERICAL EXAMPLES

Numerical consideration requires that optimization variables be defined or scaled to (approximately) the same order of magnitude for best accuracy. To achieve good numerical conditionings, let $\omega_a = (\omega_1 - \bar{\omega}_0)/2$ and $\omega_b = (\bar{\omega}_0 - \omega_2)/2$. Then, (8) is transformed into an equivalent problem

$$maximize \quad 2(\omega_a + \omega_b) \quad (9)$$

subject to

$$\begin{aligned} 4\epsilon\sqrt{P_1P_2} &\geq P_1 + P_2 + \tfrac{\Delta\beta(\omega_a,\omega_b)|_{\omega_0=\omega_{0U},\omega_3=\omega_c}}{\gamma}, \\ P_1 + P_2 + \tfrac{\Delta\beta(\omega_a,\omega_b)|_{\omega_0=\omega_{0L},\omega_3=\omega_c}}{\gamma} &\geq -4\epsilon\sqrt{P_1P_2}, \\ \omega_{1U} - \bar{\omega}_0 \geq 2\omega_a &\geq \omega_{1L} - \bar{\omega}_0, \\ \bar{\omega}_0 - \omega_{2L} \geq 2\omega_b &\geq \bar{\omega}_0 - \omega_{2U}. \end{aligned}$$

For completeness, expansion of $\Delta\beta(\omega_a, \omega_b)|_{\omega_0=\omega_{0\times},\omega_3=\omega_c}$, where $\omega_{0\times}$ is either ω_{0L} or ω_{0U}, verifies its polynomial nature

$$\begin{aligned} &\Delta\beta(\omega_a, \omega_b)|_{\omega_0=\omega_{0\times},\omega_3=\omega_c} = \\ &\quad -\tfrac{7\beta_4}{12}\omega_a^4 - \tfrac{\beta_4}{3}\omega_a^3\omega_b + \tfrac{\beta_4}{2}\omega_a^2\omega_b^2 - \tfrac{\beta_4}{3}\omega_a\omega_b^3 - \tfrac{7\beta_4}{12}\omega_b^4 \\ &\quad +(\beta_4\Delta\omega_0 - \beta_3)(\omega_a^3 + \omega_a^2\omega_b - \omega_a\omega_b^2 - \omega_b^3) \\ &\quad +(\beta_3 - \tfrac{\beta_4}{2}\Delta\omega_0)\Delta\omega_0(\omega_a^2 + 2\omega_a\omega_b + \omega_b^2), \quad (10) \end{aligned}$$

with $\Delta\omega_0 = \omega_{0\times} - \bar{\omega}_0$. Fig. 3 shows an optimized design with the same settings in Fig. 1 except that the pump wavelengths are now optimally chosen according to the solution of (9), with ϵ=0.5. It is immediately seen that the gain spectra are confined to a much smaller variation, with a minimum gain above 35dB. This agrees with the observations in [7] concerning better immunity for smaller bandwidths, but differs in that the optimized wavelengths are now systematically obtained and positive parametric gain is guaranteed for all $\lambda_0 \in [\lambda_{0L}, \lambda_{0U}]$, irrespective of its fluctuation pattern.

Table 1 shows the optimized pumps for different combinations of ZDW fluctuation amplitude α and bound factor ϵ. The trade-off is obvious: the more stringent the constraints are (namely, a smaller ϵ for a higher quality gain spectrum, or wider $[\lambda_{0L}, \lambda_{0U}]$ for better robustness), the smaller the optimized bandwidth is.

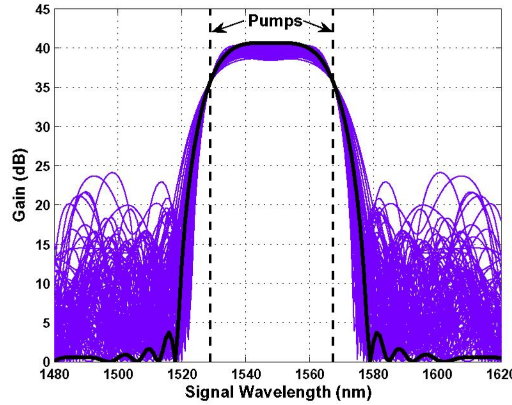

Fig. 3. Repeating the experiment in Fig. 1 with the optimized pump wavelengths of λ_1=1528.87nm and λ_2=1567.42nm, the bandwidth being 38.55nm (ϵ set at 0.5). Again, the bold spectrum corresponds to a constant λ_0 of 1550nm along the fiber.

Table 1. Optimized 2P-OPA Design Examples.

	ϵ=0.9	ϵ=0.7	ϵ=0.5	ϵ=0.3
	1498.99	1504.81	1511.51	1519.65
α=0.5	1602.51	1596.08	1588.63	1579.37
	103.52	91.27	77.12	59.72
	1513.38	1517.43	1522.03	1527.39
α=1.0	1586.53	1581.94	1576.54	1569.62
	73.16	64.51	54.52	42.23
	1519.64	1522.86	1526.43	1530.38
α=1.5	1579.37	1575.53	1570.95	1564.87
	59.72	52.67	44.51	34.49
	1523.28	1525.96	1528.87	1531.83
α=2.0	1575.00	1571.57	1567.42	1561.70
	51.72	45.61	38.55	29.87

**In each cell:
Optimized λ_1 (top), λ_2 (middle), bandwidth (bottom)
$[\lambda_{0L}, \lambda_{0U}]$=$[\bar{\lambda}_0 - \alpha, \bar{\lambda}_0 + \alpha]$, $\bar{\lambda}_0$=1550
$[\lambda_{1L}, \lambda_{1U}]$=[1490, 1545], $[\lambda_{2L}, \lambda_{2U}]$=[1555, 1610]
All wavelengths are in nm.
Other physical parameters as in Fig. 1.

4.1. Remarks

1. On a 3GHz PC with 1G RAM, Gloptipoly always finds the globally optimal solution to (9) within 10 seconds. This is in contrast to generic NLP algorithms like GA (e.g., [9]), simulated annealing etc., which typically take hours or days for a solution to converge (if at all), and the solution obtained is not necessarily the global optimum. Moreover, convergence of the latter approach is highly dependent on the quality of the initial guess, which often requires deep physical insights. Convex optimization and its variants, however, converge without the need of an initial condition.
2. Stochastic analyses generate metrics in a probabilistic sense [7–9]. To the contrary, the bounded uncertainty modeling here guarantees a positive parametric gain against all variations of λ_0 in $[\lambda_{0L}, \lambda_{OU}]$, including the notorious case of long correlation lengths in ZDW fluctuations [8].
3. Additional design constraints in the form of polynomial inequalities or equalities can readily be incorporated into the multivariate polynomial optimization framework. This provides an easy means for further fine-tuning and/or higher fidelity phase mismatch modeling.

5. CONCLUSION

This paper has presented a novel way of designing positive parametric gain 2P-OPAs with tunable gain spectrum quality and robustness against bounded ZDW uncertainty. The analysis takes advantage of the polynomial nature of phase mismatch, and formulates the bandwidth maximization problem as a non-convex, multivariate polynomial optimization task. Convex programming techniques based on LMI relaxations have been applied to arrive at globally optimal wavelength assignments. Compared to general nonlinear optimization, the proposed design framework is superior in terms of computational cost, systematic formulation, ease of deployment and optimality of solution.

6. REFERENCES

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